TOTH

VOLUME 25 DECEMBER, 1937 NUMBER 12

PROCEEDINGS
of
The Institute of Radio Engineers

Application Blank for Associate Membership on Page XXI

Institute of Radio Engineers
Forthcoming Meetings

CHICAGO SECTION
December 17, 1937
January 7, 1938
January 21, 1938

CLEVELAND SECTION
December 23, 1937

DETROIT SECTION
December 17, 1937

EMPORIUM SECTION
December 13, 1937

LOS ANGELES SECTION
December 21, 1937

MONTREAL SECTION
December 8, 1937
January 12, 1938
January 26, 1938

NEW YORK MEETING
December 1, 1937
January 5, 1938

PHILADELPHIA SECTION
December 2, 1937
January 6, 1938

WASHINGTON SECTION
December 15, 1937

TOTH

PROCEEDINGS OF

The Institute of Radio Engineers

Volume 25 December, 1937 Number 12

Board of Editors

Alfred N. Goldsmith, *Chairman*

R. R. Batcher
P. S. Carter
F. W. Grover
J. W. Horton
G. W. Pickard
B. E. Shackelford
K. S. Van Dyke
H. P. Westman, *ex officio*
L. P. Wheeler
L. E. Whittemore
William Wilson

CONTENTS

1508

Copyright, 1937, by The Institute of Radio Engineers

The Institute of Radio Engineers

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

PROCEEDINGS. The PROCEEDINGS is the official publication of the Institute and in it are published all of the papers, discussions, and communications received from the membership which are accepted for publication by the Board of Editors. Copies are sent without additional charge to all members of the Institute. The subscription price to nonmembers is $10.00 per year, with an additional charge for postage where such is necessary.

RESPONSIBILITY. It is understood that the statements and opinions given in the PROCEEDINGS are views of the individual members to whom they are credited, and are not binding on the membership of the Institute as a whole. Papers submitted to the Institute for publication shall be regarded as no longer confidential.

REPRINTING PROCEEDINGS MATERIAL. The right to reprint portions or abstracts of the papers, discussions, or editorial notes in the PROCEEDINGS is granted on the express condition that specific reference shall be made to the source of such material. Diagrams and photographs published in the PROCEEDINGS may not be reproduced without making specific arrangements with the Institute through the Secretary.

MANUSCRIPTS. All manuscripts should be addressed to the Institute of Radio Engineers, 330 West 42nd Street, New York City. They will be examined by the Papers Committee and the Board of Editors to determine their suitability for publication in the PROCEEDINGS. Authors are advised as promptly as possible of the action taken, usually within two or three months. Manuscripts and illustrations will be destroyed immediately after publication of the paper unless the author requests their return. Information on the mechanical form in which manuscripts should be prepared may be obtained by addressing the secretary.

MAILING. Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage is provided for in the act of February 28, 1925, embodied in Paragraph 4, Section 412, P. L. and R., and authorization was granted on October 26, 1927.

Published monthly by

THE INSTITUTE OF RADIO ENGINEERS, INC.

Publication office, 450–454 Ahnaip St., Menasha, Wis.

BUSINESS, EDITORIAL, AND ADVERTISING OFFICES

Harold P. Westman, *Secretary*

330 West 42nd Street, New York, N. Y.

INSTITUTE SECTIONS

ATLANTA—Chairman, N. B. Fowler; Secretary, G. S. Turner, Federal Communications Commission, 411 Federal Annex, Atlanta, Ga.

BOSTON—Chairman, H. W. Lamson; Secretary, E. B. Dallin, 64 Oakland Ave., Arlington, Mass.

BUFFALO-NIAGARA—Chairman, George C. Crom; Secretary, E. C. Waud, 235 Huntington Ave., Buffalo, N. Y.

CHICAGO—Chairman, J. K. Johnson; Secretary, V. J. Andrew, 7221 S. Francisco Ave., Chicago, Ill.

CINCINNATI—Chairman, G. F. Platts; Secretary, M. M. Wells, Crosley Radio Corporation, 1329 Arlington St., Cincinnati, Ohio.

CLEVELAND—Chairman, R. A. Fox; Secretary, R. L. Kline, Winteradio, Inc., 3014 Lorain Ave., Cleveland, Ohio.

CONNECTICUT VALLEY—Chairman, F. H. Scheer; Secretary, C. E. Keeler, 59 Fairfield St., Springfield, Mass.

DETROIT—Chairman, R. L. Davis; Secretary, H. S. Gould, 214 Tuxedo Ave., Detroit, Mich.

EMPORIUM—Chairman, M. I. Kahl; Secretary, M. C. Hoffman, Hygrade Sylvania Corporation, Emporium, Pa.

INDIANAPOLIS—Chairman, V. C. MacNabb; Secretary, I. M. Slater, 6433 E. Shelley Ave., Indianapolis, Ind.

LOS ANGELES—Chairman, Douglas Kennedy; Secretary, F. G. Albin, United Artists Studios, 1041 N. Formosa Ave., Los Angeles, Calif.

MONTREAL—Chairman, A. M. Patience; Secretary, S. Sillitoe, 1261 Shearer St., Montreal, Que., Canada.

NEW ORLEANS—Chairman, L. J. N. Du Treil; Secretary, G. H. Peirce, Electrical Research Products, Inc., 1631 Canal Bank Bldg., New Orleans, La.

PHILADELPHIA—Chairman, A. F. Murray; Secretary, R. L. Snyder, 103 Franklin Rd., Glassboro, N. J.

PITTSBURGH—Chairman, R. T. Gabler; Secretary, A. F. Shreve, 459 Willow Dr., Mt. Lebanon, Pittsburgh, Pa.

ROCHESTER—Chairman, Lee A. Du Bridge; Secretary, H. A. Brown, 89 East Ave., Rochester, N. Y.

SAN FRANCISCO—Chairman, V. J. Freiermuth; Secretary, C. J. Penther, 1000 Aileen St., Oakland, Calif.

SEATTLE—Chairman, J. W. Wallace; Secretary, R. O. Bach, Pacific Telephone and Telegraph Company, Rm. 602, Northern Life Tower, Seattle, Wash.

TORONTO—Chairman, W. H. Kohl; Secretary, N. Potter, Canadian National Carbon Co., Ltd., Davenport Rd., Toronto, Ont., Canada.

WASHINGTON—Chairman, W. B. Burgess; Secretary, E. H. Rietzke, 3308 14th Ave., N. W., Washington, D. C.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

GEOGRAPHICAL LOCATION OF MEMBERS ELECTED NOVEMBER 3, 1937

Transferred to the Fellow Grade

Massachusetts	Cambridge, 30 State St.	Burke, C. T.
New York	New York, Bell Telephone Labs., Inc.	Lack, F. R.
Pennsylvania	Emporium, 220 E. Allegany Ave.	Wise, R. M.

Transferred to the Member Grade

Indiana	Indianapolis, 6433 E. Shelley Ave.	Slater, I. M.
New Jersey	Deal, Bell Telephone Labs., Inc.	Goodall, W. M.
	Oaklyn, 16 E. Oakland Ave.	Short, D. W.
New York	New York, 30 Rockefeller Plaza	Fitch, W. A.
	New York, Columbia Broadcasting System	Lodge, W. B.
	Schenectady, 2089 Plum St.	Bartelink, E. H. B.
Pennsylvania	St. Marys, Hygrade Sylvania Corp.	Dehlinger, W. H.
Canada	Toronto, 326 Elm Rd.	Poulter, R. C.
England	London, Northern Polytechnic	Hurren, S. A.

Elected to the Member Grade

Connecticut	New Haven, Yale University	Adams, N. I., Jr.
North Carolina	Charlotte, 912 Henley Pl.	Beloungy, J. J.

Elected to the Associate Grade

California	El Monte, 1621 Merced Ave.	White, C. E.
	Hollywood, Radio Station KNX	Bowman, L. H.
	Los Angeles, 65 Arcade Annex	Tomoda, F. J.
Florida	Miami Beach, 1241-13th St.	Coogan, T. P.
	Tallahassee, Radio Station WTAL	Bailey, J. H.
Georgia	Blakely, Thompson House	Thompson, L., Jr.
Illinois	Downers Grove, 4819 Wallbank	Kay, R. A.
Iowa	Des Moines, Radio Station WHO	Loyet, P. A.
Louisiana	New Orleans, P.O. Box 1076	Dollt, A.
	Shreveport, P.O. Box 17	Maddox, C. H.
Michigan	Negaunee, 500 Jackson St.	Blee, G. C.
New Jersey	Harrison, RCA Manufacturing Co., Inc., RCA Radiotron Div.	Hickok, W. H.
	Newark, 443 Jelliff Ave.	Sherman, B.
	New Brunswick, 152 Livingston Ave.	Evers, J. T.
New York	Brooklyn, 221 Westminster Rd.	Stodola, E. K.
	New York, 305 E. 183rd St.	Ponsolle, W. J.
	New York, c/o RCA Institutes, Inc., 75 Varick St.	Tam, G.
Ohio	Columbus, c/o Battelle Memorial Institute, 505 W. King Ave.	Sherwood, E. M.
	Dayton, 517 McLain St.	Senf, H. R.
Pennsylvania	Emporium, P.O. Box 226	Bachhuber, R. A.
	Emporium, 210 E. 4th St.	Herman, L. E.
	Emporium, 128 E. 5th St.	Schleimann, A.
	Norristown, 727 Buttonwood St.	Rehrer, W. E.
	Philadelphia, 1927 W. Airdrie St.	Sichel, A. T.
Texas	Stamford, c/o Geophysical Research Corp.	Day, C. E.
Washington	Seattle, 1802 Warren Ave.	Ruddell, L.
Wisconsin	Sheboygan, 914 Pennsylvania Ave.	Beck, H. M., Jr.
Australia	Laverton, Victoria, Royal Australian Air Force	Pither, A. G.
British West Indies	Bridgetown, Barbados, Manning and Co., Ltd.	Chandler, C. E.
England	Aylesbury, Bucks., 8 Chilton Rd., Wendover	Danielson, G. L.
	Chelmsford, Essex, Marconi College	Goolry, K. S.
Malay States	Muar P. O., Johore, Telegraph Eng. Dept., Posts and Telegraphs	Muthalaly, K. N. A.
South Africa	Johannesburg, 31 Lower Bldg., Plein St.	Krause, V. R.
	Pretoria, 480 V.D. Walt St.	Jones, O. B.

Elected to the Junior Grade

Illinois	Berwyn, 2506 S. Oak Park Ave.	Vinicky, J. A., Jr.
	Chicago, 4818 S. Parkway Ave.	Ford, E. V.

Elected to the Student Grade

California	Berkeley, 2634 Haste St.	Elliott, S. J.
	San Francisco, 2602 Sacramento St.	Faithorn, N. R.
Massachusetts	Waban, 46 Pine Ridge Rd.	Lamb, F. L.
New Jersey	Long Branch, R.F.D. 1	Maida, F. X.
North Dakota	Grand Forks, University Station	Kees, H.
South Carolina	Greenville, c/o Radio Station WFBC	Perry, C. S., Jr.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

APPLICATIONS FOR MEMBERSHIP

Applications for transfer or election to the various grades of membership have been received from the persons listed below and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before December 31, 1937. These applications will be considered by the Board of Directors at its meeting on January 5, 1938.

For Transfer to the Fellow Grade

New Jersey	Haddonfield, Haddonfield Manor	Murray, A. F.
Ohio	Columbus, Dept. of Electrical Engineering, Ohio State Univ.	Everitt, W. L.

For Transfer to the Member Grade

Connecticut	Stamford, c/o Cinaudagraph Corp.	Friend, H. H.
District of Columbia	Washington, 3403 Munitions Bldg.	Perkins, O. D.
England	Willesden Green, London N.W.10, 60 Donnington Rd.	Montague, D.

For Election to the Member Grade

District of Columbia	Washington, 1901 Wyoming Ave., N.W.	Cruse, A. W.
New Jersey	Harrison, RCA Manufacturing Co., Inc.	Bacher, E. J.
France	Paris VII e, 94 rue de Varenne	Besson, P.
India	Allahabad, Dept. of Physics, Univ. of Allahabad	Toshniwal, G. R.

For Election to the Associate Grade

California	Alhambra, 1612 W. Shorb St.	Foster, A. M.
	Los Angeles, 919 White Knoll Dr.	Hanson, C. T.
Connecticut	Stamford, c/o Cinaudagraph Corp.	Bozak, R. T.
District of Columbia	Washington, 3342-18th St., N.W.	Albanese, R.
	Washington, 3342-18th St., N.W.	Bell, W. S.
	Washington, 1400-18th Pl., S.E.	Dwight, S. J.
	Washington, 1375 Penn. Ave., S.E.	Larkin, G. G.
	Washington, 3020 Channing St., N.E.	McCue, J. J.
	Washington, 5231 Wisconsin Ave.	Treubig, K.
	Washington, 123-104th St., N.E.	Wheeler, J. P.
	Washington, Box 43, Bliss Electrical School	Wyatt, N. R.
Illinois	Chicago, 4509 N. Knox	Erickson, C. V.
	Chicago, 1638 N. Fairfield Ave.	Palik, F.
Indiana	Gary, 33 E. 8th Ave.	Lytle, C. W.
Kentucky	Anchorage, Route 1	Heick, B. K.
Massachusetts	South Braintree, 57 Holbrook Ave.	Saxe, R. K.
Michigan	Detroit, 2449 E. Grand Blvd.	Miller, O. V.
	East Lansing, 229 Division St.	Blakeslee, J. T.
New Jersey	Bloomfield, 15 Evergreen Ave.	Kofoid, M. J.
	Bound Brook, 97 Hazelwood Ave.	Housten, H. J
	Cranford, 27½ Hillcrest Ave.	Hall, F. T., Jr.
	Orange, 208 S. Center St.	Hazelhurst, E.
New York	Brooklyn, 109 Carroll St.	DiFusco, F. J.
	Brooklyn, 55 Hanson Pl.	Fong, L. B. C.
	New York, 52 Vanderbilt Ave., Rm. 1110	Freeman, W. H.
	New York, 420 Lexington Ave., Rm. 2354	Hellmann, R. K.
	New York, Shelton Hotel	Moffat, W. H.
	New York, 463 West St.	Shower, E. G.
	New York, 320 E. 158th St.	Soderquist, W. A.
	New York, 463 West St.	White, S. D.
	New York, 562 W. 148th St.	Zvorist, H.
	Schenectady, 69 Elder St.	Earls, H. G.
	Schenectady, General Electric Co.	Ramo, S.
Pennsylvania	Drexel Hill, 1032 Belfield Ave.	Schurch, J. H.
Tennessee	Johnson City, 109 W. Holston Ave.	Hine, R. T.
Texas	San Antonio, 721 Peck Ave.	Moseley, T. B.
West Virginia	Charleston, General Delivery	Collins, F.
Canada	Lethbridge, Alta., Radio Beacon Station	McDougal, D. A.
Chile	Santiago, Casilla 1060	Pinto, G.
	Santiago, Casilla 104 D	Ried, A.

England	Ashton-on-Mersey, Nr. Manchester, 67 Firs Rd	Dummer, G. W.
	Chelmsford, Essex, Marconi School	Wang, S. W.
	Finchley, London N.3, 41 Church Crescent	Pegrum, A. M.
	Gosforth, Newcastle-on-Tyne, "Langemarck," Bridge Park	Grove, S. H.
	Hampstead, London N.W.3, 54 Parliament Hill	Myers, L. M.
	Highgate, London N. 6, 29 Wood Lane	Smith, I. F.
	Iver, Bucks, Thatch End, The Ridings	Brecknell, C. J.
	Leigh-on-Sea, Essex, 20 Adalia Crescent	Inskip, F. A.
	London N.W.2, 34 Dartmouth Rd.	Traub, E. H.
	Lostock Hall, Nr. Preston, Lancs., Tardy Gate Hotel	Baxter, D.
	Richmond, Surrey, 1 Manor Park	Tolliday, K. L.
	South Kensington, London S.W.7, c/o Lloyds Bank Ltd., Onslow Sq.	Anand, L. S.
	Windsor, Berks., "Fernlea," Montagu Rd., Datchet	Brooker, F. C.
Germany	Berlin-Wilmersdorf, Holsteinische Str. 27	Hagen, G. B.
Holland	Eindhoven, Philips Gloeilampenfabrieken, 2 te Apparatifabriek	Schroter, E.
Italy	Torino, Via Assietta 33	Bertolotti, S.
New Zealand	Auckland, c/o New Zealand Radio College, Winstone Bldgs., Queen St.	Egan, R. A.
Philippine Islands	Manila, 719 Dakota St.	Spier, F. H.
South Africa	Uitenhage, Box 125	Reid, O. W.

For Election to the Junior Grade

Connecticut	New London, 42 Summit Ave.	Latham, W. S.
District of Columbia	Washington, 3342-18th St., N.W.	Armand, E. J. A.
	Washington, 1803 Monroe St., N.W.	Cuninghame, J. E.
	Washington, 3227-13th St.	Franck, F. O.
	Washington, 11-5th St., N.E.	Manning, D. F.
Pennsylvania	Philadelphia, 622 E. Cornwall St.	Kershaw, W. L.

For Election to the Student Grade

California	Berkeley, 2414 Telegraph Ave.	Fenn, W. H.
	San Jose, 1151 University Ave.	Abbott, W.
	Stanford University, Box 1117	Palmer, W.
Georgia	Rome, Radio Station WRGA	Swan, A. G.
Illinois	Champaign, 602 E. Springfield Ave.	Higgins, F. V.
Indiana	West Lafayette, 300 Sylvia St.	Dalrymple, H. C.
	West Lafayette, Electrical Engineering Dept., Purdue Univ.	Hylkema, C. G
	West Lafayette, 419 W. Stadium Ave.	Thomson, R. E.
Maryland	Baltimore, Box 510, Johns Hopkins Univ.	Larrick, C. V.
Massachusetts	Cambridge, M.I.T. Dormitories	Cohen, M. M.
	Cambridge, M.I.T. Dormitories	Popkin, J. R.
Michigan	Detroit, 808 W. Bethune	Beck, F. E., Jr.
New York	New York, 1230 Sheridan Ave.	Marshall, S. L.
Ohio	Cincinnati, 449 Riddle Rd.	McKenney, H.
	Cincinnati, 2355 Ohio Ave.	Strother, C., Jr.
Washington	Seattle, 6235-4th Ave., N.W.	Newsom, E. G.

OFFICERS AND BOARD OF DIRECTORS

(Terms expire January 1, 1938, except as otherwise noted)

President

H. H. Beverage

Vice President

P. P. Eckersley

Treasurer

Melville Eastham

Secretary

Harold P. Westman

Editor

Alfred N. Goldsmith

Directors

Stuart Ballantine, *Past President*
T. A. M. Craven
L. C. F. Horle
C. M. Jansky, Jr.
C. B. Jolliffe
A. F. Murray
E. L. Nelson
Haraden Pratt
B. J. Thompson
L. E. Whittemore

Serving until January 1, 1939

E. H. Armstrong
Virgil M. Graham
Alan Hazeltine, *Past President*

Serving until January 1, 1940

Ralph Bown
H. M. Turner

INSTITUTE NEWS AND RADIO NOTES

November Meeting of the Board of Directors

The regular monthly meeting of the Board of Directors was held in the Institute office on November 3. Those present were H. H. Beverage, president; E. H. Armstrong, Ralph Bown, Alfred N. Goldsmith, Virgil M. Graham, C. M. Jansky, Jr., A. F. Murray, E. L. Nelson, H. M. Turner, L. E. Whittemore, and H. P. Westman, secretary.

C. T. Burke, F. R. Lack, and R. M. Wise were transferred to the Fellow grade. E. H. B. Bartelink, W. H. Dehlinger, W. A. Fitch, W. M. Goodall, S. A. Hurren, W. B. Lodge, R. C. Poulter, D. W. Short, and I. M. Slater were transferred to the Member grade. N. I. Adams, Jr., and J. J. Beloungy were admitted to the grade of Member.

Thirty-four applications for Associate grade, two for Junior, and six for Student grade were approved.

In accordance with the Institute By-Laws, the Membership Committee proposed and the Admissions Committee approved a list of ten names of Members recommended for transfer to the Fellow grade and seventy Associates recommended for transfer to Member grade.

A letter from the National Bureau of Standards stated that the Radio Advisory Committee of the Bureau was being discontinued. The Institute was represented on this committee since its formation in 1926.

The Tellers Committee submitted its report which was accepted. Haraden Pratt was declared elected President for 1938, E. T. Fisk was named Vice President for 1938, and F. W. Cunningham, O. B. Hanson, and C. M. Jansky, Jr., were declared elected Directors to serve for the term 1938–1940.

An invitation to be represented in the American Documentation Institute was accepted.

Incorrect Addresses

At the end of the technical portion of this issue of the Proceedings there will be found a list of the names of Institute members whose correct addresses are not known to the Institute. Persons having more recent addresses for these members will confer a favor on them and the Institute by forwarding this information to the Institute.

Index for 1937

An index of all papers published during 1937 is included in this issue. This index includes a list of the papers in the chronological order of their publication, a cross index of authors' names, and a cross index by subjects. A complete index for all issues of the PROCEEDINGS published through 1936 is available at one dollar per copy.

Binders

Those who are interested in collecting together the twelve issues of the PROCEEDINGS published during 1937 may find our standard binders of convenience. These binders are of substantial construction and may be used either as temporary transfers or as permanent binders. They are available at $1.50 each and the member's name or PROCEEDINGS volume number will be stamped on it for fifty cents additional.

Committee Work

ADMISSIONS

The Admissions Committee met on November 3 in the Institute office and those present were C. M. Jansky, Jr., chairman; F. W. Cunningham, R. A. Heising, E. R. Shute, A. F. Van Dyck, and H. P. Westman, secretary. Three applications for Fellow grade were considered. Two were approved and one was tabled. Of four applications for transfer to Member, three were approved and one tabled. Four applications for admission to Member were approved.

The committee also considered the recommendations of the Membership Committee in regard to transferring of Institute members to higher grades and approved proposals for ten transfers to Fellow and seventy transfers to Member grade.

MEMBERSHIP

The Membership Committee met in the Institute office on October 28. Those present were F. W. Cunningham, chairman; L. E. Barton (representing Leslie Woods), T. H. Clark, J. M. Clayton, I. S. Coggeshall, E. D. Cook, C. R. Rowe, C. E. Scholz, and H. P. Westman, secretary. The committee devoted its time to consideration of a substantial number of names which were recommended by individual members of the committee as being suitable candidates for transfer to higher grades.

Standards Committee

The Standards Committee met in the Institute office on October 20. Those present were L. C. F. Horle, chairman, L. F. Curtis (representing H. A. Wheeler), Virgil M. Graham, H. F. Olson, J. C. Schelleng, L. P. Wheeler, J. D. Crawford, assistant secretary, and H. P. Westman, secretary. The committee approved of some modifications in the definitions report of the Technical Committee on Electroacoustics. It approved with minor changes reports on the testing of transmitters and the testing of antennas prepared by the Technical Committee on Transmitters and Antennas. The report of the Technical Committee on Radio Receivers on test procedures was approved. Some proposed changes in a report by this committee on definitions are being referred back to the technical committee.

Tellers Committee

The Tellers Committee, composed of E. H. Armstrong, chairman; Arthur Batcheller, J. D. Crawford, and H. P. Westman, secretary, met in the Institute office on October 29 and counted the ballots which were cast by the membership in the election of officers for next year.

Technical Committee on Radio Receivers

Sectional Committee on Radio

The Technical Committee on Radio Receivers, operating under the Sectional Committee on Radio of the American Standards Association, met on October 27 in the Institute office. Those present were G. L. Beers, chairman; D. E. Foster, C. J. Franks, J. W. Fulmer, F. A. Polkinghorn, J. D. Crawford (guest), and H. P. Westman, secretary.

The committee reviewed a substantial quantity of material which had been submitted for consideration at a meeting of the International Electrotechnical Commission to be held in Italy in the middle of November. The sectional Committee on Radio is the advisor on that subject to the United States National Committee which is a member of the International Electrotechnical Commission.

Institute Meetings

Atlanta Section

On October 21 a meeting of the Atlanta Section was held at the Georgia School of Technology. N. B. Fowler, chairman, presided and there were sixteen present.

"Signal Communications in the Army" was the title of a paper presented by I. H. Gerks of the electrical engineering department of the Georgia School of Technology. Having just completed a tour of duty at the Signal Corps School at Fort Monmouth, N. J., the author based his talk on this experience. He explained how reserve officers were chosen for this assignment and presented a general description of Fort Monmouth and its activities. The Signal Corps development and research laboratory were described and particular attention given to the ultra-high-frequency developments for field use which include directional microphones designed to exclude noise from gunfire. This was followed by a description of field telephone equipment, buzzer transmitters, telegraph systems, and a demonstration of the SCR 131 portable field set. Following the demonstration, Major Van Nostrand told of some of his experiences during the war at the post which is now Fort Monmouth.

Chicago Section

The Chicago Section met on September 24 in the auditorium of the Western Society of Engineers. J. K. Johnson, chairman, presided and there were sixty-seven present.

V. E. James, engineer for the Automatic Electric Company presented a paper on "Relays and Rotary Switches in the Radio Industry." He presented a description of various radio applications of relays and rotary switches. The variations possible in such characteristics as delay in operation or release, impedance of the operating coil, and sensitivity were described. The various spring and contact combinations generally offered were outlined.

A second paper on "Loud-Speaker Equipment Applied to Telephone Lines" was presented by R. H. Herrick a radio engineer for Associated Electrical Laboratories. He presented a general description of recent developments for providing microphone—loud-speaker services over regular telephone lines. A brief description was given of loud-speaker intercommunicating systems now commercially available.

Chairman Johnson presided at the October 15 meeting which was held at Fred Harvey's Union Station Restaurant and attended by 135.

"Manufacture and Application of Phenolic Products" was the subject of a paper by H. C. Steadman, sales engineer for the Spaulding Fibre Company. The major portion of this paper was composed of the projection of motion picture films showing manufacturing procedures and illustrating various applications of vulcanized fiber and phenolic products. A general discussion which followed was devoted factor and other characteristics of these products of their radio application.

...dretto, a communications engineer of the United Air ...port Corporation, presented a paper on "Some Principles ...tical Ground Radio Station Design." He outlined first the ...ce problems existing around airports and detailed some of ...s necessary to combat them. Crystal filters are used in re- ... to obtain high selectivity, remote antennas to reduce power ...ment interference, and specially shielded and filtered power supplies are used. Even then considerable interference between adjacent communication channels remains and is caused by transmitter harmonics. Modifications in transmitters to avoid this were discussed and consisted of special peak limiting circuits in the audio-frequency equipment and the installation of low-pass filters just ahead of the modulator to limit the modulation band to a maximum of 3500 cycles. This band permits satisfactory articulation for the particular surface. The paper was closed with a discussion of the frequency requirements of air lines and an outline of the characteristics desirable in transmitting equipment for this service. The paper was discussed by Messrs. Glover, Franke, Herrick, Johnson, Weber and others.

Connecticut Valley Section

On September 30, seventy-five members and guests of the Connecticut Valley Section met in the Hartford Electric Light Company auditorium. D. E. Noble, vice chairman, presided.

A paper on "Recent Achievements in Color Photography" was the subject of a paper by H. Johnson of the Eastman Kodak Company. He outlined briefly the history of photography and commented on the coordination of that subject with radio in the field of television. He then discussed the use of emulsions sensitive to ultraviolet and infrared radiations. In describing color photography, he discussed first the additive method in which pictures are taken through red, blue, and green filters and projected in the same way. He then described the subtractive method used in the Kodachrome process in which the three emulsions are yellow, magenta, and blue-green. The yellow, passing red and green will be sensitive to blue; the magenta, passing the red and blue will be sensitive to green; and the blue-green, passing those two colors will be sensitive to yellow. In processing, the whole film is first stained with a red dye and the top two layers bleached to remove the red. The top two layers are then stained with green dye and the top layer bleached to remove it. The top layer is dyed blue and the metallic silver is completely dissolved. The Technicolor process using three negatives and two light gates was described and sample pictures by this process were shown.

Detroit Section

On October 22 a meeting of the Detroit Section which was attended by forty-five was held in the Detroit News Conference Room with R. L. Davis, chairman, presiding.

A. A. Armer, field engineer of The Magnavox Company presented a paper on "Progress in Loud-Speaker Development." There was presented first an analysis of the basic operation of loud-speakers which was followed by a detailed description of the electrodynamic type. A brief history of this type was given and it was pointed out that Lodge used an almost identical piece of equipment in his resonant telegraph of 1898. The general response curve of a dynamic speaker was shown to have a gradual rise between thirty and two hundred cycles which depends on the specific design and baffle employed. At two hundred cycles a resonance peak occurs and from there to about a thousand cycles the curve is flat. At about a thousand cycles the cone ceases to vibrate as a piston and the curve beyond this point becomes highly irregular. A cone sufficiently rigid to vibrate only as a piston is unable to reproduce the higher frequencies and will usually show a very sharp cutoff in the vicinity of a thousand cycles. The efficiencies of dynamic speakers vary from four to fifteen per cent although it is possible to obtain efficiencies of the order of fifty per cent by properly matching the impedance of the speaker diaphragm to the air column. The application of speakers to automobile use was discussed and a new model demonstrated.

Emporium Section

M. I. Kahl, chairman, presided at the October 1 meeting of the Emporium Section held at the American Legion Rooms and attended by fifty-five. L. E. Barton of the engineering department of Philco Radio and Television Corporation presented a paper on "Some Applications for the Reverse Feed-Back Amplifier."

Several useful circuits were described. A typical circuit consisted of a 6Q7G as a second detector and first audio-frequency amplifier followed by a 6C6G as a final amplifier. An analysis of such a circuit was presented to show the effects of feedback on the volume control, the diode section, frequency characteristics, and frequency compensation. Push-pull output was also briefly analyzed. The greatest difficulty with these amplifiers is to obtain feedback which does not result in oscillation. The paper was discussed by Messrs. Acheson and McClintock and it was pointed out that a 6Y7G output tube with 400 volts on the plate was capable of delivering twelve watts at five per cent distortion when the circuit was adjusted for proper feedback.

Montreal Section

The October 14 meeting of the Montreal Section was held jointly with the Engineering Institute of Canada in the auditorium of that organization. C. J. DesBaillets of the Engineering Institute presided and there were 350 present.

L. W. Chubb, director of research of the Westinghouse Electric and Manufacturing Company, presented a paper on "Industrial Research." He divided his paper into three sections dealing with a new product, a suggested product, and a speculative product. Under the first, he described at some length the precipitron, a new type of electrostatic air cleaner. It operates by charging dust and smoke particles at about twelve kilovolts and then precipitating them onto grounded plates. It is satisfactory for dust, pollen, or germs and the power required to clean the air supply for a six-room house is about ten watts. A working model was demonstrated. As a suggested product, polarized light for the elimination of glare from automobile headlights and for small novelty signs was demonstrated. For the latter application various colors were polarized in different directions and separated by polarized screens. The atom smasher was the speculative product which is now at work using potentials as high as five million electron-volts. Many common substances can be rendered radioactive by it. A number of those present participated in the discussion of the paper.

Philadelphia Section

A. F. Murray, chairman, presided at the October 7 meeting of the Philadelphia Section which was attended by 142 and held at the Engineers Club.

"Some Notes On Television Progress in Great Britain and on the Continent" was the subject of a paper by E. W. Engstrom, director of general research for the RCA Manufacturing Company, RCA Victor Division. He described the experimental broadcast system that has been in operation for some time in Berlin and employs mechanical scanning for 180-line pictures projected twenty-five frames per second. As many as a thousand people see the programs each day at public viewing rooms. Laboratory systems were in operation for pictures of 441 lines, twenty-five frames per second, and interlaced scanning. Apparatus of this high definition type was under construction for radio exhibitions and the performance was considered very good.

In addition to work with older 180-line systems using mechanical scanning, the French were preparing for high definition television. Comparison of a number of studio systems was being made and a transmitter was being built for the Eiffel Tower.

Scheduled transmissions have been employed in England for some time and are of 405-line pictures, twenty-five frames, interlaced. Receivers are sold by several manufacturers and present prices range from $200 to about $700. Received images were approximately eight inches by ten inches in black and white. Transmission is from the Alexandra Palace in London. Mobile program pickup equipment has been used for a number of events occurring remote from the transmitters. The technical performance of the system was excellent. The paper was discussed by Messrs. Applegarth, Bean, Fraenckel, and Bingley.

The second paper was on "Quality and Uniformity Factors in Radio Receiving Tube Production" and was presented by R. M. Wise, chief radio engineer of the Hygrade Sylvania Corporation of Emporium, Pa.

The engineering problems in designing, for manufacture, new electronic tubes to meet circuit requirements were described. Under production conditions no two tubes are ever exactly identical in their electrical characteristics. With the trend toward increased values of transconductance in new tube designs, clearances between elements become so small as to increase greatly the problem of maintaining uniformity. Comparisons were given between the characteristics and element spacing of earlier tubes such as type 27 with the design of the 6J7. The effect of variations in element size and spacings in electrical characteristics were shown. Production limitations as applied to tube design are first ascertained in a small production section operated by the engineering department. New types are manufactured in substantial quantities over a sufficient length of time to determine the practicability of the design for production. This section produces an average of 30,000 tubes per month as compared with the factory production capacity of 65,000 tubes per day. Rejections for all causes on a commercial manufacturing basis run between two and five per cent with the older types of tubes and up to fifteen per cent or more for the newer or more difficult types.

Design difficulties caused by the temperature range encountered in automobile radio service were mentioned. The heater voltage may vary from 5 to 8.5 volts under service conditions and the effect of this on the cathode tab and the contact with the mica spacer on the distribution of temperature along the cathode were mentioned. The choice of materials for the component parts was discussed in detail. The paper was discussed by Messrs. Engstrom, Kellogg, McIlwain, and Woods.

Pittsburgh Section

R. T. Gabler, chairman, presided at the October 19 meeting of the Pittsburgh Section which was held at Carnegie Institute of Technology and attended by twenty-two.

R. O. Hurst of the engineering department of the Westinghouse Electric and Manufacturing Company presented a paper on "Large Modulation Transformers of WLW." He presented a detailed outline of the construction of the large modulation transformer used in the 500-kilowatt transmitter. It was pointed out that very close tolerances had to be observed for distortion occurring in the range between one thousand and five thousand cycles. At lower and higher frequencies, the amount of distortion was not so important as it is not readily noted by the human ear. The paper was closed with the description of some of the smaller transformers used in the transmitter and the discussion of the paper was participated in by Messrs. Berg, Gabler, Krause, Pickles, and Stark.

Toronto Section

W. H. Kohl, vice chairman, presided at the October 18 meeting of the Toronto Section which was attended by forty-two and held at the University of Toronto.

"Some Facts About Phenolic Laminated Materials" was the subject of a paper by E. O. Hausmann, control engineer of the Continental Diamond Fibre Company. The requirements for raw materials was first discussed and included the properties demanded of the cellulose and resins employed. Methods of manufacture were then presented and included those for flat stock and tubular forms. The important electrical properties such as power factor and dielectric losses were presented together with equipment and methods for their measurement. The humidity effects on various grades of material were discussed and their effect on the Q of the material outlined. The paper was discussed by Messrs. Buchan, Hepburn, and Thompson.

Washington Section

There were ninety-five present at the meeting of the Washington Section which was held in the Potomac Electric Power Company building on October 11. W. B. Burgess, chairman, presided.

"The New Volume Limiting Amplifier" was the subject of a paper by O. M. Hovgaard, engineer for the Bell Telephone Laboratories. A description was presented of the design, construction, and performance of the system. It was pointed out that while considerably more compression is possible, the instrument is designed primarily to provide compression up to three decibels, and permits a broadcast station to maintain the highest practical modulation level without exceeding one hundred per cent modulation on the peaks. Thus the use of this amplifier will allow improvement in the received signal level without the accompanying distortion of overmodulation which is so frequently experienced when no automatic method of peak limiting is employed.

Personal Mention

P. J. Ambrose, formerly an instructor in Indiana Technical College, is now an electrical and radio engineer for the Consumer Power Company of Jackson, Mich.

G. W. Barnes has joined the engineering staff of the Pacific Telephone and Telegraph Company of Sacramento, California, having previously been with Bell Telephone Laboratories.

G. H. Brown has left the research division of RCA Manufacturing Company to form the firm of Godley and Brown, consultants, at Montclair, N. J.

O. R. Buchanan has joined the radio inspection staff of the Federal Communications Commission at Grand Island, Nebraska, having previously been with WHA.

Formerly with F. W. Sickles Company, E. T. Cahalan has joined the staff of Automatic Winding Company of East Newark, N. J.

J. B. Campbell is now on the engineering staff of the Federal Telegraph Company at Newark, N. J., having previously been with the Tung-Sol Lamp Works.

L. F. Curtis has joined the research staff of the Bayside Laboratory of the Hazeltine Service Corporation having formerly been with the United American Bosch Corporation.

E. F. Dillaby has left the Raytheon Production Corporation to become a vacuum tube engineer for the Hytron Corporation at Salem, Mass.

Bernard Drake, previously with A. C. Cossor, Ltd., is now in the transmission laboratory of the Telephone Manufacturing Company, London, England.

A. J. Fischer is now with the Continental-Diamond Fibre Company of Newark, Del, having formerly been with the Haveg Corporation.

R. L. Freeman, previously with Farnsworth Television, has joined the development staff of the Hazeltine Service Corporation, Bayside Laboratory.

L. F. Guaragna has left the Lumiton Picture Company to join the Engineering Department of RCA Victor Argentina at Buenos Aires.

R. M. Heintz has left Heintz and Kaufman, Ltd., to join the staff of Bendix Aviation Corporation at East Orange, N. J.

F. A. Hinners is now chief engineer of the Fada Radio and Electric Company of Long Island City, N. Y., having previously been connected with the Hazeltine Service Corporation.

Formerly with Crosley Radio Corporation, C. A. Hultberg is now a member of the engineering department of General Household Utilities Company, Chicago, Ill.

D. G. Jachness previously with Pye Radio, Ltd., is now with Scophony, Ltd., of London, England.

J. K. Johnson formerly with Wells-Gardner and Company is now engineer in charge of the Chicago Division of the Hazeltine Service Corporation Laboratory.

H. O. Klinke has become a development engineer for the Western Electric Company at Baltimore, Md., having previously been with the RCA Manufacturing Company.

Graham Madgwick formerly with the Public Works Department of the Hong Kong Government has been made manager of Cable and Wireless, Limited, Nairobi, Kenya, East Africa.

Formerly with Astatic Microphone Laboratory, G. E. Makinson is now with the Ohio Bell Telephone Company in Cleveland.

A. W. Marriner, Major, U.S.A., has been transferred from Washington to Maxwell Field, Montgomery, Ala.

Previously with First National Television, Inc., K. H. Martin is now with Midland Television, Inc., of Kansas City, Mo.

F. H. McIntosh formerly with Bell Telephone Laboratories is now with the Graybar Electric Company, San Francisco, Calif.

G. G. Mendenhall previously with the General Electric Supply Corporation is now chief engineer of Communication Engineers, Inc., Seattle, Wash.

P. J. Neimo, Lieutenant, U.S.N., has been transferred from the Philippines to the U.S.S. *Seattle* basing at Brooklyn, N. Y.

Previously with Wright and Weaire, Ltd., R. S. Roberts has joined the teaching staff of Northern Polytechnic Institute in London, England.

H. M. Smith, field engineer for the Canadian Radio Broadcasting Commission, has been transferred from Ottawa to Montreal.

Previously with E. K. Cole, Limited, F. G. Story has become an engineer for Weston Electrical Instrument Company, Limited, of London, England.

W. O. Swinyard of the Hazeltine Service Corporation has been transferred from New York City to Chicago.

B. F. Tyson has joined the research staff of the Hazeltine Service Corporation at Bayside, N. Y.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

TECHNICAL PAPERS

MINIMUM NOISE LEVELS OBTAINED ON SHORT-WAVE RADIO RECEIVING SYSTEMS*

By
KARL G. JANSKY
(Bell Telephone Laboratories, Inc., New York City)

Summary—*The theoretical minimum noise level of receivers in the absence of any interference, the source of which is external to the receiver, is discussed and compared with the limit actually measured on various antennas over a limited frequency range in the short-wave spectrum. It is pointed out that, on the shorter wave lengths and in the absence of man-made interference, the usable signal strength is generally limited by noise of interstellar origin. The powers obtained from this noise with the various antennas and for different times of the day are given.*

Recently, man-made interference, of which that caused by diathermy machines constitutes the greatest part, has become so extensive that it is now the limiting noise during most of the daylight hours. Data are given on the intensity and extent of this form of interference.

INTRODUCTION

EXPERIENCE in the past has taught us that it is usually atmospheric noise from thunderstorms, either near or distant, which limits the usefulness of radio receiving systems on wave lengths of thirty meters or longer.[1] This is also occasionally the case on shorter wave lengths. It is the purpose of this paper to present data on the limiting noise of receivers taken at times and on wave lengths such that the atmospheric noise was not the limiting noise.

The apparatus used consisted of a double detection measuring set of conventional design having a band width of 1586 cycles, an automatic energy operated recorder for recording the output of the set,[2] and various types of antennas as described later.

Calibration of the system was achieved by connecting the measured output of a thoroughly shielded high-frequency signal generator to the input circuits of the receiver through a transmission line of eighty ohms characteristic impedance. Attenuation pads made up of small carbon resistors were inserted in this line until the output of the receiver was within the range of the automatic recorder. A photograph

* Decimal classification: R270. Original manuscript received by the Institute, May 24, 1937. Presented before joint I.R.E.–U.R.S.I. meeting, Washington, D. C., April 30, 1937.

[1] E. Bruce, "Developments in short-wave directive antennas," PROC. I.R.E., vol. 19, pp. 1406–1433; August, (1931).

[2] W. W. Mutch, "A note on an automatic field strength and static recorder," PROC. I.R.E., vol. 20, pp. 1914–1919; December, (1932).

of the signal generator with the cover opened is shown in Fig. 1. Two of the attenuation pads are seen mounted in the rack in the cover and a short section of the transmission line is seen plugged into the output jack of the box at the left. The meter in the lower left-hand corner is connected to the thermocouple used for measuring the output of the oscillator.

A calibration of this type permits the evaluation of the power in micromicrowatts at the terminals of the antenna. The relationship between this power and the field strength of the received signal for

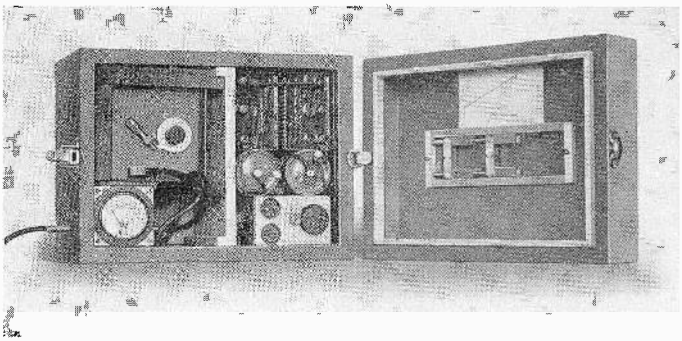


Fig. 1—Signal generator used for calibrating the measuring equipment.

various antennas over different types of ground has been discussed by others.[3]

It has been shown that a noise voltage due to the thermal agitation of electric charge appears between the grid and ground of the first tube of a radio receiver which is proportional to the square root of the resistive component of the complex impedance across those points and to the absolute temperature.[4] Since the noise power due to this voltage is measured at the output of the receiver after passing through an amplifier which in most cases has a much narrower band width than the effective band width of the first circuit, the power as measured is proportional to the effective band width of the receiver.

Johnson has shown[4] that the effective noise voltage produced in a

[3] C. B. Feldman, "The optical behavior of the ground for short radio waves," Proc. I.R.E., vol. 21, pp. 764–801; June, (1933).
H. T. Friis, C. B. Feldman, and W. M. Sharpless, "The determination of the direction of arrival of short radio waves," Proc. I.R.E., vol. 22, pp. 47–78; January, (1934).

[4] H. Nyquist, "Thermal agitation of electric charge in conductors," *Phys. Rev.* II, vol. 32, p. 110; July, (1928).
J. B. Johnson, "Thermal agitation of electricity in conductors," *Phys. Rev.* II, vol. 32, p. 97; July, (1928).
F. B. Llewellyn, "A study of noise in vacuum tubes and attached circuits," Proc. I.R.E., vol. 18, pp. 243–265; February, (1930).

circuit by thermal agitation is given by the equation

$$E_n{}^2 = 4kTR(f_2 - f_1),$$

where,

E_n = effective noise voltage
k = Boltzmann's constant = 1.37×10^{-23}
T = absolute temperature = 300 degrees Kelvin
$f_2 - f_1$ = effective band width
R = resistance of circuit with antenna connected.

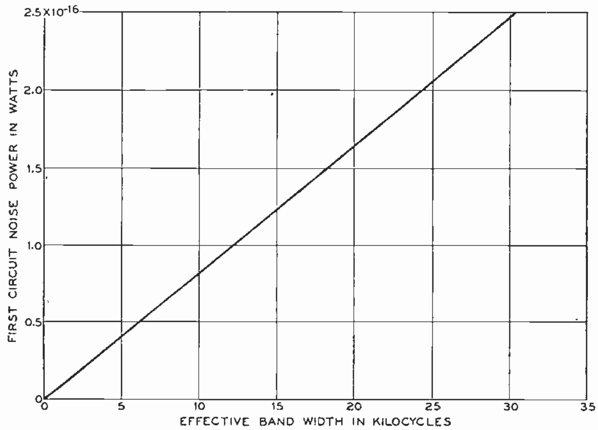


Fig. 2—Noise power due to thermal agitation of electric charge in first circuit of receiver.

Now in our case $R = R'/2$ where R' is the resistance of the first circuit measured with the antenna disconnected. Therefore

$$E_n{}^2 = 4kT\frac{R'}{2}(f_2 - f_1),$$

but the power P dissipated in R' is equal to $E_n{}^2/R'$ or

$$P = 2kT(f_2 - f_1).$$

In Fig. 2 the average power P is plotted as a function of the effective band width. We see that for the receiver used having a band width of 1586 cycles $P = 0.13 \times 10^{-4}$ micromicrowatts which is 48.9 decibels below one micromicrowatt.

It is possible by the proper design and construction of a receiver to reduce the noise from other sources within the receiver to the point where that generated in the grid circuit of the first tube predominates. In the measuring set used, the noise from other sources was reduced to the point where the actual noise level was forty-seven decibels below a micromicrowatt or within two decibels of the theoretical limit.

It is apparent that, if the signal power dissipated in the first circuit is less than this noise power, the signal will be useless for the purpose of conveying intelligence.[5]

If this "resistance noise" were the limiting factor as regards radio reception, that is, if there were no external source of noise, then, when an antenna is connected to a receiver through a transformer which matches the antenna impedance to that of the first circuit, a decrease in the noise level should be obtained. However, experience in the operation of radio receivers has shown that this reduction in noise level is very seldom, if ever, realized, but that an increase in noise is obtained instead. This increase in noise level is experienced even at times when there is no atmospheric noise or man-made interference present. It has been pointed out previously that the source of some, if not all, of this noise is external to the solar system,[6] and data will be presented below on its intensity and the variation of that intensity with time and with the frequency on which it is received.

The taking of these data has been greatly hampered during the daylight hours by a form of interference on the short waves that is relatively new. In fact when the experiments described in the papers before mentioned[6] were performed there was no evidence of this interference whatsoever. Since it first appeared in the radio spectrum, however, it has steadily increased until now it presents a very serious problem in the operation of short-wave radio circuits. The interference referred to is that caused by diathermy or similar machines, and the second part of this paper deals with the intensity and extent of this interference.

Either these machines are not operated during the nighttime or else the condition of the ionized layers is such that the radiation from

[5] When the ratio of carrier-to-noise power is fifteen decibels for this type of noise, experiments have shown that the speech will be intelligible providing the modulation approximates 100 per cent and providing a good quality speech band of about 5000 cycles is used. The carrier level was measured when there was no modulation present and the noise level was measured in the absence of the carrier. With a 5000-cycle band the theoretical noise level becomes 43.8 decibels below one micromicrowatt so that if we allow a 15-decibel signal-to-noise ratio, the signal must furnish power at the terminals of the antenna equal to 28.8 decibels below one micromicrowatt or greater to be intelligible. A 25-meter signal having a vertical angle of arrival of fifteen degrees and an *incident* field strength of 27.8 decibels below one microvolt per meter would furnish this power at the terminals of a $\lambda/2$ horizontal doublet antenna located one wave length above the ground at Holmdel, New Jersey. See Fig. 5 of the second paper given under reference 3.

[6] Karl G. Jansky, "Directional studies of atmospherics at high frequencies," Proc. I.R.E., vol. 20, pp. 1920–1932; December, (1932).

Karl G. Jansky, "Electrical disturbances apparently of extraterrestrial origin," Proc. I.R.E., vol. 21, pp. 1387–1398; October, (1933).

Karl G. Jansky, "A note on the source of interstellar interference," Proc. I.R.E., vol. 23, pp. 1158–1163; October, (1935).

only a very few is refracted to the receiver. For this reason and because the wave lengths upon which this study was made are seldom used during the nighttime, excellent records, entirely free of man-made interference have been obtained during the dark hours.

PART I

The Noise Level in the Absence of Atmospherics and Man-Made Interference.

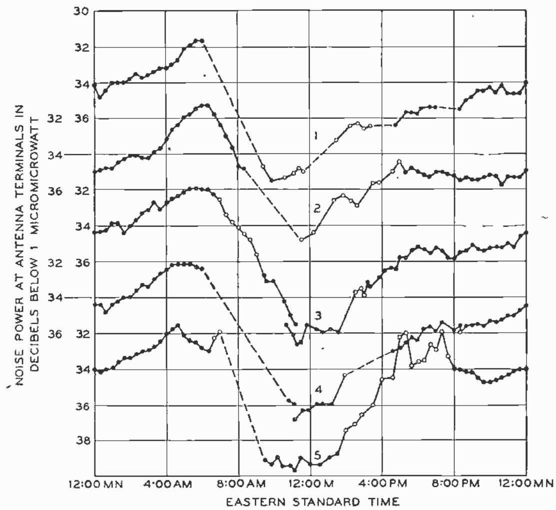


Fig. 3—Minimum noise levels measured on 16.7 meters with a rhombic antenna directed N.50°8′E.

1. December 29–30, 1936
2. January 4–5, 1937
3. January 11–12, 1937
4. January 18–19, 1937
5. January 25–26, 1937

Fig. 3 shows data obtained during observations made on several different days on sixteen meters with a rhombic antenna oriented so as to receive from a direction fifty degrees and eight minutes east of north.[7] Each curve in the figure represents the data obtained on one day. The values of noise power at the antenna terminals are given as ordinates, there being one scale for each of the curves, and eastern standard time is given as abscissas. Those points marked by open circles on the curves were obtained during times when either atmospheric noise or some other form of interference was present and are probably not very accurate.

Examination of the figure shows that on the days when the data

[7] E. Bruce, A. C. Beck, and L. R. Lowry, "Horizontal rhombic antennas," PROC. I.R.E., vol. 23, pp. 24–46; January, (1935).

were taken, the noise power rose to a maximum in the early morning hours and dropped to a very low minimum around noon, there being one maximum and one minimum value for each day. If we assume that the source of this noise lies in the Milky Way and that the radiations therefrom are unaffected by the ionized layers of the earth, then it is to be expected that the maximum values would be obtained when the antenna points in the direction of the Milky Way, and, as has been demonstrated before,[6] the times at which the maximum and minimum values are reached should occur four minutes earlier for each passing day.

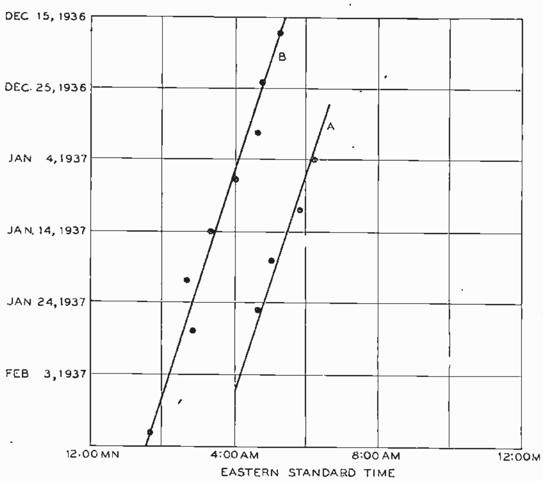


Fig. 4—Time of occurrence of maximum noise power obtained on 16.7 meters with rhombic antennas directed *A*, N.50°8′*E*, B, S.50°8′W.

On curve *A* of Fig. 4 the times at which the maximum value occurs, as shown by the curves of Fig. 3, are plotted against the day of the year. It will be noticed that, except for slight variations, this time of maximum value does change by four minutes a day.

Inspection of Fig. 3, however, shows that the minimum value is reached at approximately the same time each day indicating that there are unaccounted factors involved. Furthermore, the antenna used points towards the Milky Way twice each day and we should expect, therefore, two maxima to appear on the curve instead of only one as observed. One of the maxima should be slightly greater than the other, for at one time the antenna points towards a section of the Milky Way which is very near the center of the Galaxy and at the other time it points out towards the edge. On January 11, 1937, (curve 3 of Fig. 3), these maxima should have occurred at 5:10 A.M. and 1:40 P.M. The one at 5:10 A.M. should have been the larger. Actually the early morning maximum was reached at a time close enough to 5:10 A.M. to be considered as conforming to the theory. On the other hand the curve shows a deep minimum at 1:40 P.M. instead of the expected maximum. But at 12:00 noon the ionization due to the ultraviolet light from the sun reaches its highest value. It is quite plausible, therefore, that conditions in the ionized layers are such at 1:40 P.M. that

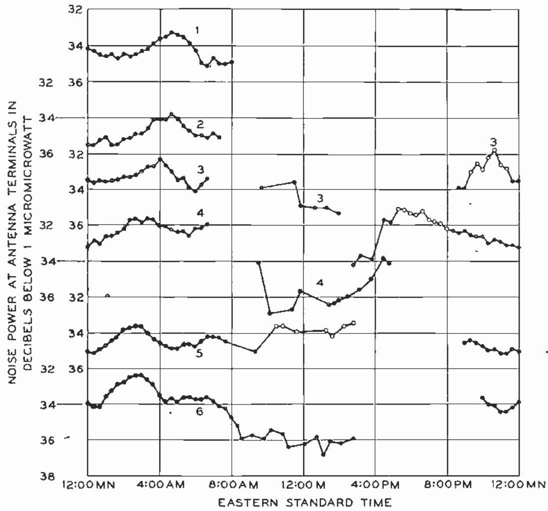


Fig. 5—Minimum noise levels measured on 16.7 meters with a rhombic antenna directed S.50°8′W.

1. December 24, 1936
2. December 31, 1936
3. January 7–8, 1937
4. January 14–15, 1937
5. January 21–22, 1937
6. January 28–29, 1937

the radiations are either refracted away from the earth or are absorbed before they reach the receiver. This could account very nicely for the deep minimum that is obtained every day at about 12:00 noon.

The lowest noise level measured on this particular antenna was 40.8 decibels below one micromicrowatt which is 8.1 decibels above the theoretical noise level of the receiver. The maximum value obtained during periods entirely free of interference and atmospheric noise was 31.3 decibels below one micromicrowatt.

Fig. 5 shows similar curves which were taken with the same type of antenna, but one which was pointed in just the opposite direction. This antenna also points towards the Milky Way twice during each twenty-four hours, but to different sections than the other so that the noise levels obtained would not be expected to be the same. As with the other antenna, one of the periods when the level should reach a maximum occurs during the daytime and a minimum is recorded instead, presumably because of the increased absorption at that time. The other maximum occurs during the nighttime and makes its appearance on the curves. The times at which this maximum is reached are given on curve *B* of Fig. 4. Again the data check the theory of an interstellar origin for this noise. There are two points on this curve which were obtained from data which are not given in Fig. 5.

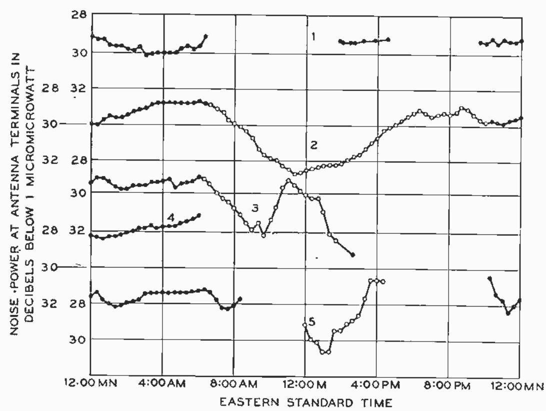


Fig. 6—Minimum noise levels measured on 16.7 meters with a $\lambda/2$ horizontal doublet with axis directed W.50°8′N to E.50°8′S.

1. December 28–29, 1936
2. January 31, 1937
3. February 1, 1937
4. February 9, 1937
5. February 15–16, 1937

With this southwest rhombic antenna, as in fact was the case with all antennas used on sixteen meters except the northeast rhombic, the interference caused by diathermy machines during the daytime was so severe that good records were obtained very rarely. Nevertheless, as before, very good data were obtained during the nighttime.

The minimum value measured with this antenna was 36.9 decibels below one micromicrowatt which is twelve decibels above the theoretical minimum noise level of the receiver. The maximum value obtained when there was no interference was 31.6 decibels below one micromicrowatt.

Although rhombic antennas of the type used have relatively high gains, they also have sharp directional characteristics in both the horizontal and vertical planes so that with them noise energy is received from only a very limited area of the Milky Way at any given instant. The noise level obtained may, therefore, be lower than that obtained with a low gain antenna which has a broad directional characteristic such as a horizontal doublet. This is especially true if the horizontal doublet is oriented so that its axis is perpendicular to the plane of the Milky Way at some time during the day, for at that time it will pick up energy from nearly one half of the Milky Way and the noise level should be correspondingly high. The data given in Fig. 6 were obtained with a horizontal doublet which was oriented in a very favorable though not quite the best direction for receiving this noise. It is to be noted that both the maximum and minimum noise levels measured

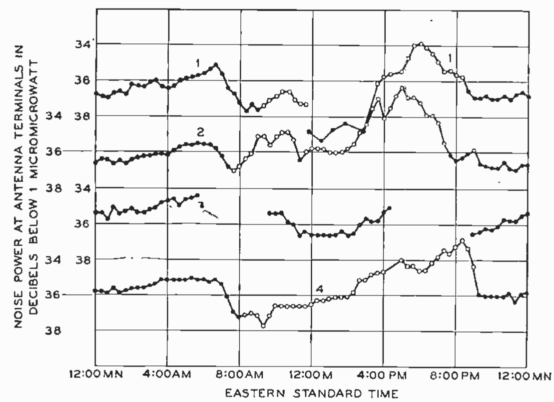


Fig. 7—Minimum noise levels measured on 14 meters with a $\lambda/2$ vertical antenna.

1. January 27–28, 1937
2. February 3–4, 1937
3. February 10–11, 1937
4. February 17–18, 1937

exceed those obtained with either of the rhombic antennas. Except in a very few cases diathermy interference again prevented reliable measurements from being made during the daytime. The lowest noise level measured was 33.2 decibels below one micromicrowatt which is 14.5 decibels above the theoretical minimum noise level of the receiver. The highest level measured at a time of no interference was 26.4 decibels below one micromicrowatt.

Figs. 7, 8, and 9 show exactly similar data taken with $\lambda/2$ vertical antennas on wave lengths of 14, 16.7, and 32.2 meters, respectively. These data were taken with the idea in mind of determining, if possible, the variation in the intensity of the interstellar noise with frequency. On 32.2 meters, however, ordinary atmospheric noise was always the limiting noise except for a very short time during the middle of the day. At this time, as said before, the influence of the sun on the ionized layers of the atmosphere is greatest, the effect of which may vary from day to day so that data taken on different days are not comparable. The similarity between these curves and those given by Potter for atmospheric noise on ten megacycles should be noted.[8]

A comparison of the noise levels measured at midnight on 16.7 and 14 meters shows that, on the average, the power obtained on 16.7

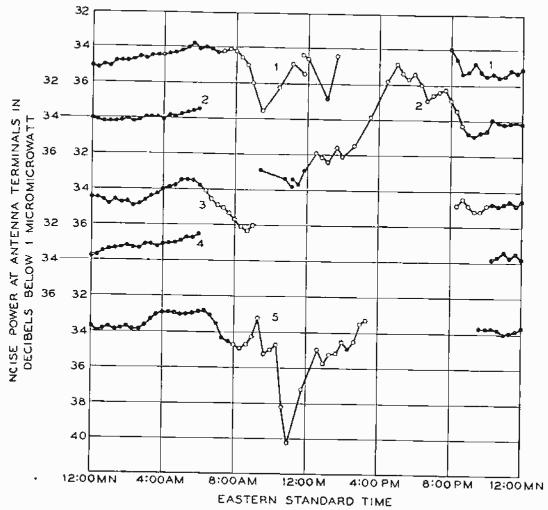


Fig. 8—Minimum noise levels measured on 16.7 meters with a $\lambda/2$ vertical antenna.

1. January 19–20, 1937
2. January 26–27, 1937
3. February 2–3, 1937
4. February 9–10, 1937
5. February 16–17, 1937

meters was two decibels greater than that obtained on 14 meters. This is slightly more than the 1.5-decibel difference which must be attributed to the difference in the effective heights of the antennas, but the data are not numerous enough to be conclusive.

Curve 5 of Fig. 8 for February 16 and 17, 1937, is especially interesting for on February 16 a short period high-frequency radio "fade-out" was reported to have occurred between 10:58 A.M. and 11:16 A.M. E.S.T.[9] At this time the interference caused by diathermy

[8] R. K. Potter, "High-frequency atmospheric noise," Proc. I.R.E., vol. 19, pp. 1731–1765; October, (1931).

[9] J. H. Dellinger, "A new radio transmission phenomenon," *Phys. Rev.* vol. 48, p. 705; October 15, (1935).

J. H. Dellinger, "Direct effects of particular solar eruptions on terrestrial phenomena," *Phys. Rev.*, vol. 5, p. 1189; December 15, (1936).

A. G. McNish, "Magnetic effects associated with bright solar eruptions and radio 'fade-outs'," *Nature.*, vol. 139, p. 244; February 6, (1937).

machines and atmospherics dropped to a very low value. There was still some interference present, however, and the noise levels measured are probably somewhat higher than the level of the interstellar noise alone. The value of 40.2 decibels below one micromicrowatt was over two decibels lower than any other value obtained at that time of day indicating that the "fade-out" affected the interstellar noise as well as terrestrial signals.

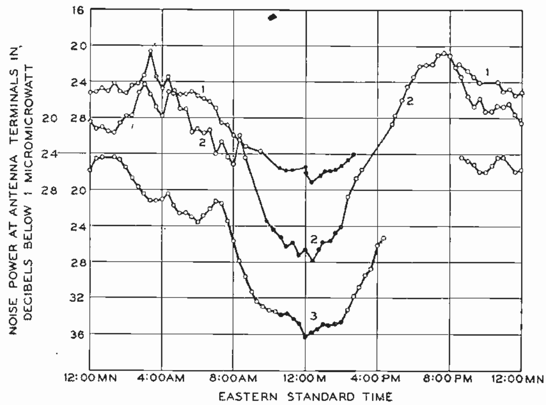


Fig. 9—Minimum noise levels measured on 32.2 meters with a $\lambda/2$ vertical antenna.
1. January 13–14, 1937 2. January 20–21, 1937
3. January 18–19, 1937

In concluding this section, it should be emphasized that at no time was the noise level of the receiver ever reached, but that the interstellar noise always set the lower noise limit, the value of which depended upon the directional characteristic and orientation of the antenna, the time of day, the day of the year, and the condition of the ionosphere. The lowest value measured was 40.8 decibels below one micromicrowatt. This is 8.1 decibels above the theoretical and over six decibels above the actual noise level of the receiver.

Part II

Man-Made Noise

Data were taken on the level of the noise caused by diathermy machines as follows. Beginning at one end of a 1.3-megacycle band centered on 16.7 meters and gradually tuning through the band, the value of the power obtained from every diathermy machine heard was jotted down. These measurements were made in the morning and afternoon and on all the antennas described above which were used on 16.7 meters. The data obtained are given in Figs. 10, 11, 12, and 13, on which the powers measured at the antenna terminals are plotted as abscissas and the percentages of the total number of readings giving powers equal to or greater than the corresponding abscissa are plotted as ordinates.

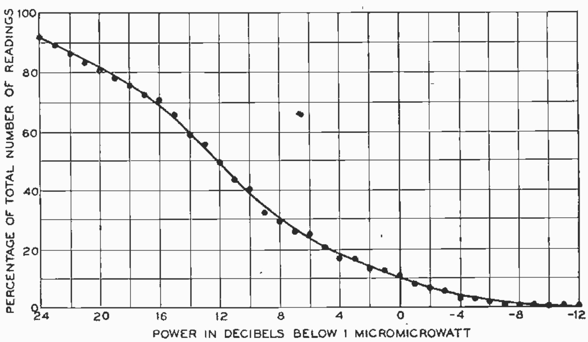


Fig. 10—Diathermy noise power measured on 16.7 meters with a $\lambda/2$ vertical antenna. Number of readings = 137.

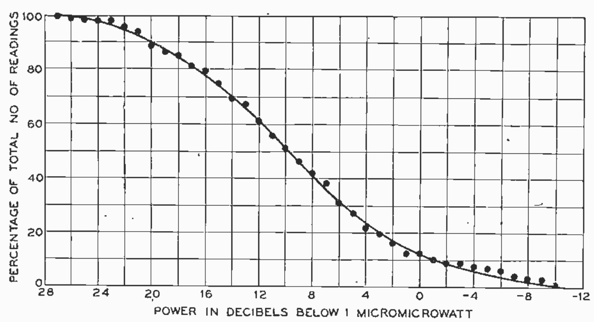


Fig. 11—Diathermy noise power measured on 16.7 meters with a $\lambda/2$ horizontal doublet. Axis directed W.50°8′N. to E.50°8′S., number of readings = 109.

Thus, from Fig. 10, we see that with the $\lambda/2$ vertical antenna the power level was 12.2 decibels below one micromicrowatt or higher for fifty per cent of the readings made. The highest value noted was twelve decibels above one micromicrowatt which is 60.9 decibels above the theoretical noise level of the receiver.

For the horizontal doublet, (Fig. 11), the values are very similar. Fifty per cent of the measurements gave a value of 9.7 decibels below one micromicrowatt or higher and the maximum value obtained was ten decibels above one micromicrowatt.

With the northeast rhombic antenna, (Fig. 12), fifty per cent of the readings gave values of 25.4 decibels below one micromicrowatt or higher, and the maximum value was only ten decibels below one micromicrowatt. These values are definitely lower than those obtained with the other two antennas indicating that the majority of the machines are located in a direction other than northeast of the receiver. This is

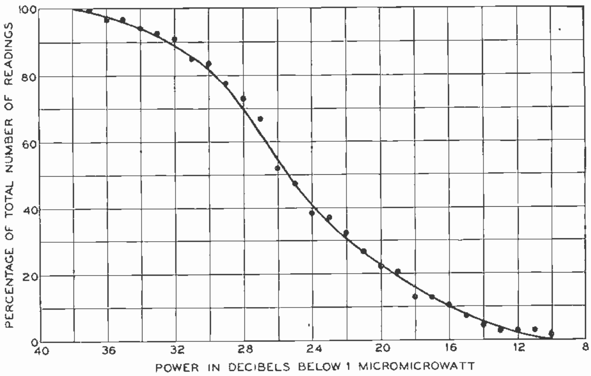


Fig. 12—Diathermy noise power measured on 16.7 meters with a rhombic antenna directed N.50°8′E. Number of readings = 67.

further substantiated by the fact that the values obtained with the southwest rhombic antenna are again much higher (Fig. 13), being comparable to the values measured with the $\lambda/2$ vertical antenna.

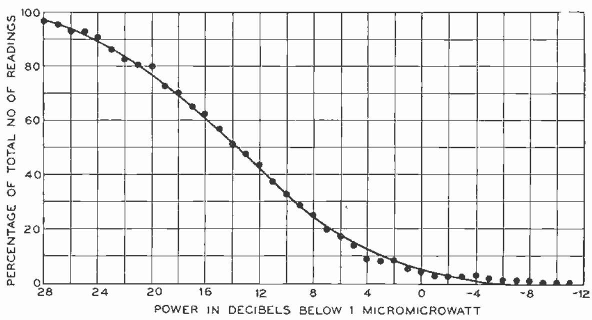


Fig. 13—Diathermy noise power measured on 16.7 meters with a rhombic antenna directed S.50°8′W. Number of Readings = 130.

These intensity values do not tell all that it might be desired to know about this interference, but it is not intended to go into the matter any further in this paper other than to give the following few observations made during the progress of the experiments described above.

This interference was found to be very much worse on 16.7 meters than on 32.2 meters or 14 meters. In fact, not even enough measurements were obtained on the latter wave lengths to permit the drawing of curves similar to those drawn with the data taken on 16 meters.

The frequency of some of the machines would vary erratically by at least as much as 200 kilocycles.

The average duration of a single diathermy treatment appeared to be about ten minutes.

Often two or more machines could be heard at the same time.

Although, some interference was encountered when using the northeast rhombic, nevertheless, it was always easy to avoid it and to find clear channels upon which measurements could be made of the interstellar or atmospheric noise levels. With the other 16.7 meter antennas, the interference was so severe that at times hours would have to be spent going over the band before a channel could be found that would remain clear long enough for a measurement to be made.

Conclusions

These experiments have shown that, on the shorter wave lengths, in the absence of atmospherics or man-made interference, the noise level measured is several decibels above the noise due to the thermal agitation of electric charge in the first circuit of the receiver. The intensity depends upon the directional characteristic and orientation of the antenna, upon the time of day and day of year, and upon the condition of the ionosphere.

This noise level, which is apparently due to radiations of interstellar origin, would be the limiting noise on these wave lengths for a large percentage of the time were it not for man-made interference. Such interference was found to be very severe on all the 16.7-meter antennas used except the northeast rhombic. The maximum power obtained from this interference was twelve decibels above one micromicrowatt which was 60.9 decibels above the theoretical noise level of the receiver.

The experiments described above were performed and the equipment used was developed at the Holmdel Radio Laboratories of Bell Telephone Laboratories, Inc.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

MEASURING THE REFLECTING REGIONS IN THE TROPOSPHERE*

By
A. W. Friend and R. C. Colwell
(West Virginia University, Morgantown, West Virginia)

Summary—*During the routine investigation of the E and F regions of the ionosphere, it was found that the supposed ground wave was undergoing severe fluctuations. This phenomenon pointed to reflections from regions at low atmospheric heights. In order to separate these reflections from the ground wave, it was necessary to design a device for producing very short pulses and to make a rapid sweep for the oscilloscope. A positively synchronized thyratron modulator instrument gave a regular succession of pulses each lasting for four microseconds. The receiving antenna was in the form of a loop rotatable about both the horizontal and vertical axes. This permitted a weakening of the ground wave, thus producing better resolution of the received pulses. The short pulse receiver consisted of three stages of radio-frequency amplification and a detector operated from a 110-volt alternating-current circuit. A voltage limiting arrangement on the oscilloscope gave a maximum velocity synchronized sweep of 9000 inches per second. With this equipment, several stratified layers from one to 12 kilometers high were discovered, with occasional echoes from 15 to 65 kilometers. Frequencies of 1614, 2398, and 3492.5 kilocycles were used.*

A FEW years ago it was observed that the fading of broadcast stations in the evening was directly related to the rise and fall of the barometer.[1] It was evident that these fluctuations, beginning at the sunset period, could be due only to ionospheric phenomena. At first the changes were ascribed entirely to the E region, but later observations showed a close correlation between signal variations and changes in the troposphere. It is well known to radio operators that there is a definite connection of some kind between weather cycles and the propagation of radio waves. Since little of the change could be happening in the E and F regions, it was thought that the variations of the D region[2] should be investigated. Observations were made upon the virtual heights of the F, E, and D regions using a base line 2.5 kilometers long. A rapid fluctuation of the supposed ground wave was noticed. Careful investigation showed that the transmitter was emitting a steady stream of identical, nonfluctuating pulses. Even when different types of pulse generators were used, the received signal continued to fluctuate.

In order to examine this new phenomenon, two new pulse genera-

* Decimal classification: R113.6. Original manuscript received by the Institute, June 29, 1937.

[1] R. C. Colwell, "Cyclones, anticyclones, and the Kennelly-Heaviside layer," Proc. I.R.E., vol. 21, pp. 721–763; May, (1933).

[2] S. K. Mitra, and P. Syam, *Nature*, vol. 135, p. 953; June, (1935).

tors were invented.[3] One of these consisted of a rotating fluid stream commutator, while the other more satisfactory type was a positively synchronized thyratron modulator device.

With these devices in use in the simplified circuit (Fig. 1) the only limits to the shortness of the pulses which could be generated were the damping constant of the resonant output circuit of the transmitter and the time required for one cycle of the desired radio frequency to be completed.

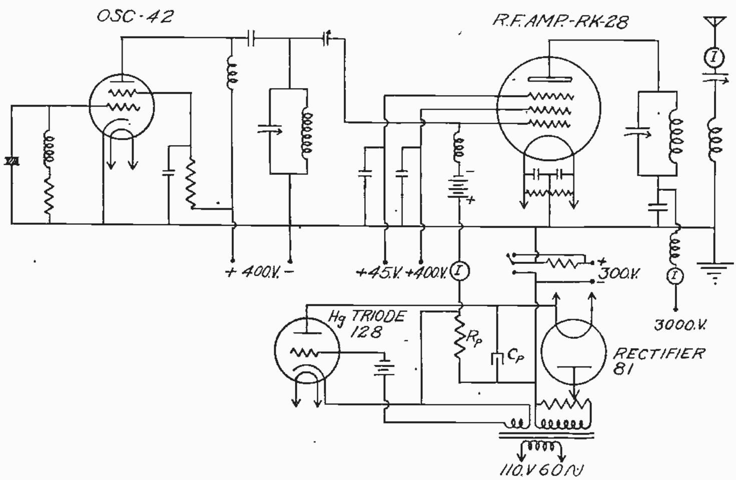


Fig. 1—The transmitter and pulse generator circuits.

The pulses produced by the transmitter were tested in every conceivable way to see if each pulse was separate and distinct. Both the direct-current modulating pulse and the radio-frequency output pulse were checked by the use of a rapidly sweeping cathode-ray oscillograph and found to be perfect and single. Various types of detectors and other receivers were used for picking up the radiated pulses directly from the transmitter and checking them for duality. No such double pulses were found in the output from the transmitter.

The Receiving Equipment

During this development period another difficulty arose from the fact that the linear sweep circuit of the standard oscillograph which was synchronized with the sixty-cycle pulse frequency by means of the power lines would not give sufficient resolution between the received ground and ionosphere pulses for the observation of any reflections or

[3] R. C. Colwell, A. W. Friend, and N. I. Hall, *Rev. Sci. Inst.*, vol. 7, p. 420; November, (1936).

other phenomena having time intervals equivalent to regions lying below 40 kilometers. Reliable measurements could be made only above a 65-kilometer equivalent height.

High harmonic operation of the linear sweep circuit was tried by the application of greatly distorted synchronous wave forms to the oscilloscope synchronizing terminals. This procedure allowed fairly accurate observations of received phenomena arriving with a time separation interval of ten microseconds. This arrangement was not satisfactory for continued operation, however, since when it was operated on the twentieth harmonic of the pulse frequency, there were nineteen extra sweep lines playing across the screen. It was also found difficult to maintain operation always on the same harmonic frequency.

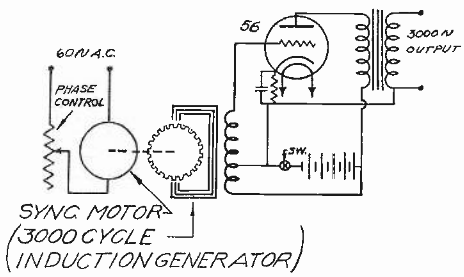


Fig. 2—3000-cycle induction alternator for calibration of time scale.

Because of these difficulties another experiment was performed from which a better system was developed. An ordinary radio receiver type of power supply transformer having a complete root-mean-square secondary voltage of 750 was connected with its high voltage winding across the horizontal deflecting plates of the cathode-ray tube so as to give a maximum spot velocity of 5000 inches per second across the screen. A variable voltage radio B battery was connected in series with this circuit so as to allow for a slight centering adjustment and a 100,000-ohm protective resistance was also included in series in order to prevent any possible destructive currents from flowing. A continuously variable applied primary voltage made possible minute adjustments of sweep velocity.

A small induction alternator was designed so as to give a 3000-cycle timing wave when driven by an 1800-revolution-per-minute synchronous motor (Fig. 2). When this timing device was used the sweep was found to be so perfectly linear that no deviation from linearity could be detected. The one and only fault of this sweep is the visible return sweep which has been displaced slightly from the forward sweep by the use of a small sixty-cycle voltage on the vertical deflection plates. The return has not been objectionable enough to warrant any additional effort for its elimination. It could be eliminated easily by the use of a pulsating magnetic field applied so as to deflect the return sweep from the screen of the tube. Several electrical arrangements would also be satisfactory for this purpose.

The receiving device for the very short pulses presented the greatest difficulties of the entire system. A very excellent make of communications type superheterodyne receiver proved quite satisfactory for measuring reflections from any regions of height greater than ten kilometers, but the multitude of tuned circuits and the lower intermediate frequency (465 kilocycles) prevented measurements of time intervals less than 50 microseconds.

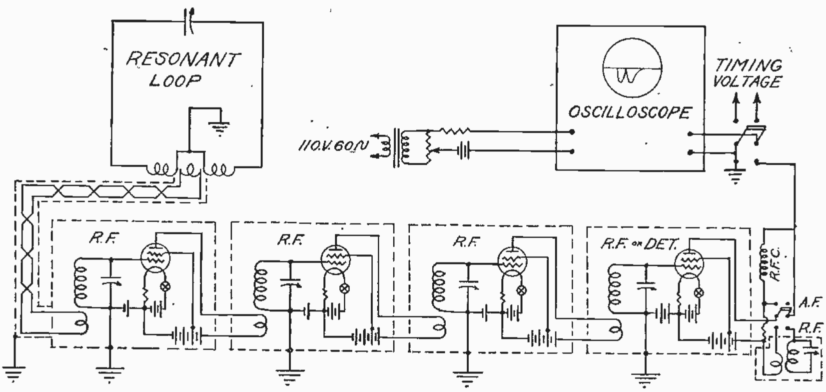


Fig. 3—The battery receiver and oscillograph sweep circuits.

In order to eliminate all time constant effects in the first experimental receiver, a tuned radio-frequency set was designed with separately shielded tuned radio-frequency stages, with each stage shield can containing its own power supply batteries (Fig. 3). No resistors or by-pass condensers were used (except for the filament rheostats). This excellent shielding provided almost complete freedom from oscillation troubles so that observation of the radio-frequency output on an oscilloscope screen was possible.

When this receiver was used either with or without an output rectifier the resolution on the oscilloscope screen was sufficient to permit the measurement of time intervals as small as two microseconds provided *the timing impulses were of approximately equal amplitude.* In ionospheric or radio wave velocity measurements with the transmitter very near the receiver, the chief difficulty in measuring such small time intervals lies in the fact that the resonant receiver circuits have currents of such high amplitude set up in them that they remain in damped oscillation until after the second timing impulse has been received. Two arrangements were devised for correcting this trouble. The first was by the use of fairly low Q tuned circuits, and the second and much more necessary device was a special loop antenna. The loop designed for this purpose consisted of a balanced single turn tuned coil mounted so as to be rotatable about *both the vertical and the horizontal axes.* By a suitable adjustment of the loop the timing impulse received directly from the transmitter may be reduced in amplitude to a value equal to or less than that of the reflected impulse which is to be used as the second timing impulse. Fig. 4 is a photograph of this loop system. For

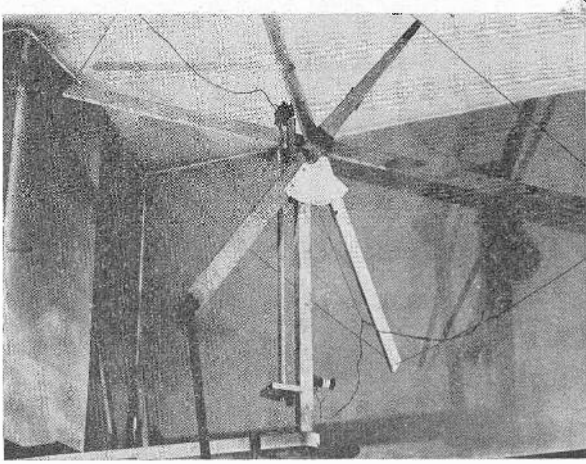


Fig. 4—Double rotation loop antenna.

very short waves a rotatable doublet antenna may be used. By the use of this equipment, which could resolve radio-frequency timing impulses having a time separation of more than four microseconds, the reflections from the so-called C reflecting region of the troposphere were discovered.[4] The observed direct and reflected impulses have been thoroughly checked every day for a period of over one and one half years, and there is *no possibility* that the observed reflection is merely a product of the equipment.

It was found that the fluctuations in the originally observed ground wave were not due to any fault of the equipment, but to the presence of a strong reflection from a region lying between one and twelve

[4] R. C. Colwell and A. W. Friend, *Nature*, vol. 137, p. 782; May, (1936).
R. C. Colwell, A. W. Friend, N. I. Hall, and L. R. Hill, *Nature*, vol. 138, p. 245; August, (1936).
R. C. Colwell and A. W. Friend, *Phys. Rev.*, vol. 50, p. 632; October, (1936).

kilometers above the earth in the troposphere. *The first definite decision concerning the external origin of this apparent reflection was made when*

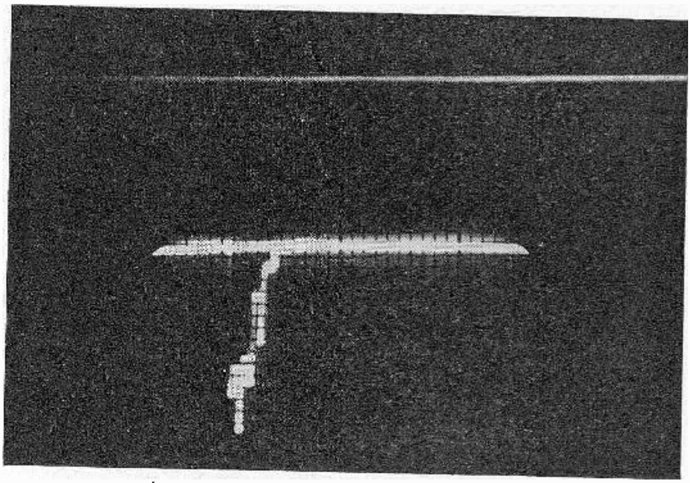


Fig. 5—Circuit parasitics.

it was found that by the use of the loop antenna (rotatable about two axes) the first received impulse (the ground wave) could be entirely eliminated or the second received impulse (the C region reflection) could likewise be en-

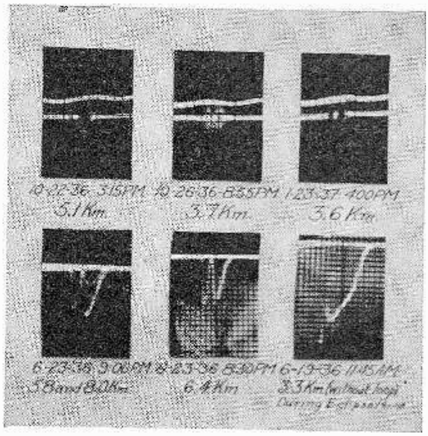


Fig. 6—The ground-wave and C-region reflection.

tirely eliminated, merely by rotating the loop antenna. By using an intermediate position of the loop antenna the two impulses could be made of equal amplitude for the purpose of accurate measurement of the elapsed time interval between the two. Equal amplitude adjustment eliminates any difference in the transit time of the wave fronts through the receiving equipment.

Before these results could be achieved it was necessary, as previously mentioned, to eliminate from the receiving circuits all appreciable traces of resonant circuit interaction by extremely careful shielding, resonant circuit damping, and carefully balanced grounding arrangements. Any circuit interaction will be likely to cause an apparent series of multiple reflections which is due to the wave form of several damped circuit oscillations beating with one another (Fig. 5). Fig. 6 shows several unretouched photographs of the ground wave and reflected signals received while using this receiver.

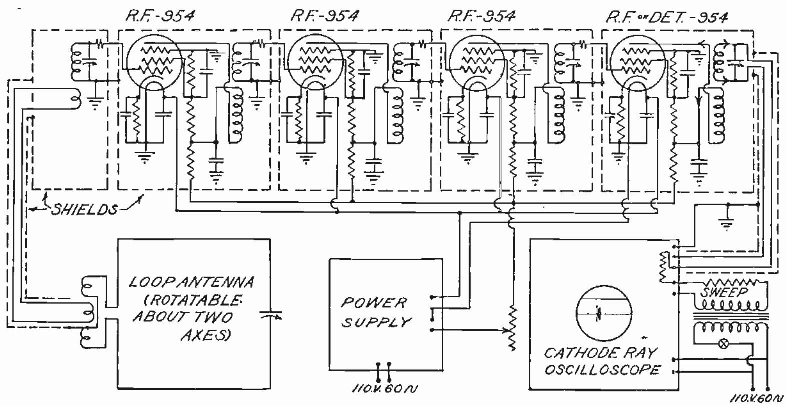


Fig. 7—The improved receiver circuit.

A second short pulse receiver similar to that of Fig. 3 was later designed in order to eliminate the troubles incident to maintaining the battery system of the original set and to increase simultaneously the sensitivity and stability. Acorn pentode tubes (type 954) were used for better high-frequency performance. Fig. 7 shows the circuit diagram of this receiver. It will be noticed that each grid lead contains a 3500-ohm resistor. These resistors were used for the purpose of eliminating the troublesome effects which cause the pulse carrier oscillations to set the cascaded radio-frequency amplifier circuit into a momentary damped oscillation of excessive duration. The resistors produced the damping required for making observations of complete pulses which, after passing through the receiver, had a duration of less than five microseconds. By using the pulse peaks, measurement of time intervals of as little as 1.5 microseconds was possible.

This receiver could be used either as a four-stage radio-frequency amplifier or as a three-stage radio-frequency amplifier and detector, the change being effected by substituting for the plug-in output coil a low-pass filter network. After all feed-back troubles were eliminated by complete shielding and the filtering of all power leads, this receiver proved to be quite excellent for the purpose. Fig. 8 shows the completed receiver.

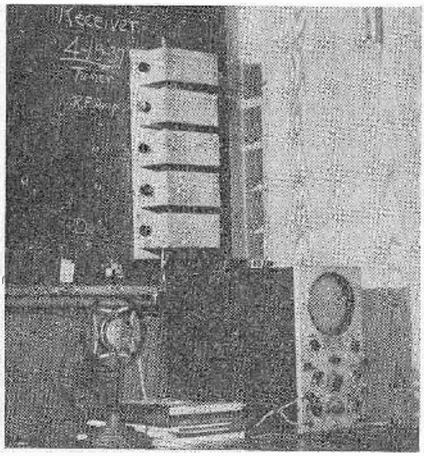


Fig. 8—The new receiver—no batteries.

As the first year of continuous observations progressed it became obvious that due to the increasing sunspot activity the mean C region height was becoming much lower. In order to maintain a good degree of resolution on the oscilloscope screen a much faster sweep was de-

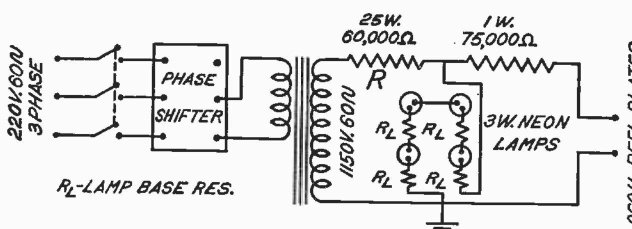


Fig. 9—The improved rapid sweep circuit.

signed. Since the sixty-cycle sine wave sweep voltage was already above the rated maximum for the cathode-ray tube, if a higher voltage was to be used for producing the more rapid sweep, the peak voltage applied to the deflecting plates had to be reduced. For this purpose a very simple voltage limiting device was designed so as to allow a sweep velocity of 9000 inches per second without exceeding the maximum rated peak deflecting plate voltage. Fig. 9 shows the diagram of the sweep circuit. The voltage limiting was provided by employing gaseous discharge lamps to produce a high voltage drop in R when a maximum desired voltage had been reached. This voltage was equal to the ionizing potential of all of the neon lamps used in series plus the voltage drop across their series resistors installed in the lamp bases. By the use of a phase shifter the linear phase of this sweep circuit may be shifted to any desired position. This linear sweep device has proved to be quite flexible and very satisfactory.

Conclusions

During an attempt to measure accurately the known ionospheric regions an unaccounted for fluctuation of the supposed ground-wave signal was noticed. Observations to determine the cause of this phenomenon revealed the fact that this supposed ground wave consisted of a true ground wave, a tropospheric reflection (from heights of one to 12 kilometers) at all times, and other echo signals occasionally received from heights of 15 to 65 kilometers. The lowest region from which signals are returned has been called the C region and it has been assumed that the higher region is the same as that called the D region by Mitra, Syam and others.[2,5] Sudden changes in the D-region signals have been connected with some types of rapid broadcast fading and general correlations have been made between average signal strength variations and C-region changes. When the barometer rises the C region falls and vice versa. Fluctuations in height have also been connected with magnetic and solar disturbances.

New equipment for the accurate measurement of very short time intervals has been developed and applied to this work and also to the measurement of the velocity of radio waves over both short and medium distances. Velocities ranging from 50 to 85 per cent of the velocity of electromagnetic waves in vacuum have been measured over distances as short as fifteen miles. It is probable that this reduced velocity is definitely connected with the reflecting (or refracting) regions of the lower atmosphere.

Addendum

Since this article was submitted to the Proceedings, a paper has appeared in England[6] giving the results of a similar research. The layers observed there appeared at heights of 8.39, 9.33, 10.26, and 10.76 kilometers, respectively. The length of the pulses used (20 microseconds duration for the ground ray sent out and 60 to 80 microseconds in the

[5] P. Syam, *Indian Jour. Phys.*, vol. 10, part 1, p. 13; January, (1936)
[6] R. A. Watson Watt, A. F. Wilkens, and E. G. Bowen, *Proc. Roy. Soc.*, ser. A., vol. 161, p. 181; July, (1937).

receiver) prevented the detection of the lower lying regions found in the United States. On the whole there is fair agreement between these two researches, conducted in two separate countries under very different conditions and with different types of transmitters and receivers.

A somewhat unusual photograph (Fig. 10) of the observed reflections, as seen on the cathode-ray oscilloscope, was taken on the night of August 1 and 2, 1937. At this time the rapid sweep circuit of Fig. 9 was in use, and the excellent resolution may be observed. Each scale

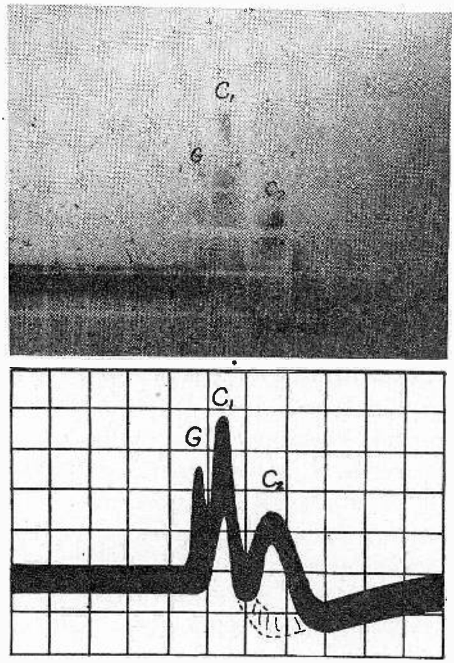


Fig. 10—Cathode-ray oscilloscope pattern with short pulse and 9000-inches per-second horizontal sweep. Two reflections from the C region; taken at 10:00 P.M., E.S.T., August 1, 1937, just before an auroral display. *G*, ground pulse, C_1, first reflection, C_2, second reflection.

division represents a virtual height of 1.515 kilometers. The very narrow pulse at the left represents the direct signal from the transmitter, when reduced by means of the special receiving antenna. The center pulse represents a reflected signal from a height of 0.78 kilometer; and the pulse to the right represents a virtual height of approximately 2.56 kilometers. It may be noted that while the direct signal was a very short pulse, the first and second pulse reflections were lengthened considerably. This is construed to mean that these reflections are from somewhat diffuse regions.

Two very strong reflections such as these occur only occasionally. The usual observation shows only one very strong reflection from this lowest region, although a more sensitive receiver may be used at any time desired to observe the reflections from heights of about 8 to 20 kilometers. The receiver of Fig. 7 was designed with such a high degree of damping (or broad resonance curve) that its sensitivity is not normally sufficient for reception of the reflections from above 10 kilometers. These higher reflections, which have also been observed in England,[6] must, therefore, be of much lower intensity than those from the region between 0.5 and 10 kilometers.

All of this work has been done on frequencies of 1614, 2398, and 3492.5 kilocycles with a few tests on 4797.5 kilocycles. It is planned that the observations shall be extended to ultra-high frequencies in the near future. At the present time all daily measurements are taken on 2398 kilocycles. The observations made in England have been made entirely on higher frequencies.

The photograph of Fig. 10 is believed to be most unusual since it was taken during a period which was afterward found to coincide with a brilliant auroral display.[7] Due to constant shifting of the reflection pattern and also to the very rapid oscilloscope sweep, it was difficult to obtain a clear photograph of the phenomena. In order to make the results more obvious, a tracing of the oscilloscope pattern has been appended below the photograph. Even in the short exposure time of one-half second it may be observed that the C_2 reflection has shifted once.

[7] Ernest Cherrington, Jr., *Science*, vol. 86, no. 2229, p. 265; September 17, (1937).

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

EXPERIMENTS WITH UNDERGROUND ULTRA-HIGH-FREQUENCY ANTENNA FOR AIRPLANE LANDING BEAM*

By

H. Diamond and F. W. Dunmore
(National Bureau of Standards, Washington, D.C.)

Summary—*Experiments are described on the electrical properties of an ultra-high-frequency transmitting antenna operating very near to and below the ground surface. The work was done with the purpose of locating the landing beam in the center of an airport in order to secure a steeper approach path and to provide for landing service for different wind directions. The effect of the proximity of the ground to the transmitting antenna upon the low angle distribution of energy in the radiated field and upon the polarization of the field is described. An approximate mathematical analysis is given of the mechanism of setting up a landing path when the transmitting antenna is below the ground surface.*

I. Introduction

In the course of experiments with the radio landing beam[1] an investigation was made during 1933–1934 on the operation of an ultra-high-frequency transmitting antenna very near to and below the ground surface. The work was done with the purpose of locating the landing beam antenna in the center of a landing field. In this location the approach path becomes steeper for a given point of contact of the landing airplane with the airport surface and thereby permits full utilization of long runways without requiring a very flat approach to the airport. A further advantage of this location is that landing beam service may be provided for all directions of approach to the airport to meet varying wind conditions. The investigation led to what appears to be a practicable solution of locating the landing beam antenna in the center of an airport. In addition, a number of interesting phenomena on the effects of the proximity of the ground upon the transmitted wave were observed and are reported in this paper.

* Decimal classification: R526.3×R525. Original manuscript received by the Institute, May 7, 1937; revised manuscript received by the Institute, August 12, 1937. Presented before joint I.R.E.-U.R.S.I. meeting, Washington, D. C., April 30, 1937. Published in *Nat. Bur. Stand. Jour. Res.*, vol. 19, pp. 1–20; July, (1937). Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] H. Diamond and F. W. Dunmore, "A radio beacon and receiving system for blind landing of aircraft," *Bur. Stand. Jour. Res.*, vol. 5, pp. 897–931; October, (1930) (RP238); Proc. I.R.E., vol. 19, pp. 585–626; April, (1931). H. Diamond, "Performance tests of radio systems of landing aids," *Bur. Stand. Jour. Res.*, vol. 11, pp. 463–490; October, (1933) (RP602); abstract, Proc. I.R.E., vol. 22, pp. 120–121; January, (1934).

II. Increased Steepness of Approach

Consideration of the principle of operation of the radio landing beam will indicate the factors which control the steepness of the landing path and hence the rate of descent of a landing airplane following the path. The low angle distribution of energy in the vertical plane about an ultra-high-frequency transmitting antenna is such that any line of constant field intensity represents a possible landing path. The landing airplane may be guided along a given line of constant field intensity by maintaining fixed output from a receiver having a predetermined fixed sensitivity. The lines of constant intensity are tangential to the airport surface at the transmitting antenna and curve upward with increasing horizontal distance from the transmitter. The particular line chosen is that one which most nearly approximates the normal approach path of the modern airplane during the final stages of a landing. For a given transmitter output, it is necessary only to adjust the sensitivity of the airplane receiver to obtain a path of proper slope.

However, a second condition must be satisfied which does not always permit use of a landing path having the desired slope. Assuming that the transmitter is located at one end of the landing runway, it is necessary that, in following the landing path, the airplane contact the airport surface at sufficient distance from the transmitter to permit of stopping gradually without sudden applications of the brakes. This requires that the height of the landing path at the desired point of contact be equal to the height of the airplane receiving antenna above the bottom of the airplane landing wheels. This automatically fixes the shape of the landing path without regard to the desired steepness of approach. In practice, it was found that for points of contact up to about 1800 feet from the transmitter, satisfactory steepness of approach was attained. At large airports, however, where the desired point of contact is often 3000 feet or more from the transmitter location, the landing path became much too flat. The center-of-field location of the transmitting antenna overcomes this operational defect by shortening the distance between the transmitter location and the point of contact.

The steeper approach path obtainable by locating the transmitting antenna at the center of the field may be seen from Fig. 1. The landing paths of Fig. 1 are computed on the basis of (1) of Section V. While this equation applies only when the transmitting antenna is at least one-half wave length above ground, and hence holds only for the edge-of-the-field location, it is sufficiently close for the center-of-field antenna condition to serve as a satisfactory basis of comparison.

Curve A is the landing path with the landing beam transmitter located at the far edge of the field and the sensitivity of the airplane receiver adjusted so that the point of contact of the landing airplane with the airport surface is at a point 1000 feet on the approach side of the center of the field. Curve B shows the landing path corresponding to the same point of contact but with the transmitter at the center of the field. The length of airport runway in this illustration is assumed 5000 feet and the height of the airplane receiving antenna, ten feet. It is of interest to note that the obstacles in the approach would prevent the possibility of following path A, the airplane clearing the edge of the airport by only ten feet (the difference between the height of

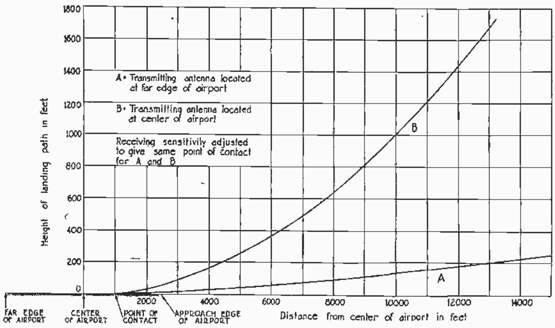


Fig. 1—Approach paths corresponding to edge-of-field and center-of-field landing beam locations.

the landing path at that point, twenty feet, and the height of the receiving antenna above the bottom of the landing gear). In the case of path B, the steepness of approach is more normal and the airplane clears the edge of the field by fifty-three feet.

Not only does the center-of-field location provide a steeper approach path for a given point of contact but also it affords considerable flexibility for varying the steepness of approach without appreciable change in the point of contact. Thus the clearance at the approach end of the field may be doubled by moving the point of contact by only 250 feet toward the center of the field. With the transmitter located at the far edge of the field, this clearance would require moving the point of contact by 2000 feet, that is, to within 1500 feet of the far edge of the field, thereby seriously reducing the length of runway available for coming to a stop.

III. Tests with Transmitting Antenna at Small Distances Above the Ground Surface

Because of its location in the center of an airport, the choice of the transmitting antenna to be employed was necessarily restricted to the simplest possible type. Accordingly, a horizontal half-wave transmitting antenna was adopted and was used throughout the tests.

The use of a simple half-wave antenna in place of the directive antenna array previously employed for setting up a landing path led to a consideration of whether the transmitter power and receiver sensitivity employed were sufficient. Also, while optical theory showed that the slope of the landing path would be the same for both types of antennas, it was desired to determine the effect on the path of placing the transmitting antenna close to the ground surface. Tests were accordingly made to investigate these features.

The transmitter utilized two 500-watt tubes in push-pull, operating at a frequency of 90,800 kilocycles (3.3 meters), and the half-wave antenna was fed from it by a short two-wire parallel conductor transmission line. The transmitter was completely shielded and the transmission line properly terminated so that the radiation was confined to the antenna proper. The receiving equipment consisted of a detector and a two-stage audio-frequency amplifier fed by a parallel conductor transmission line from a half-wave receiving antenna. The output of the receiver was rectified and applied to the landing beam indicator (a direct-current microammeter). The receiver sensitivity corresponding to half-scale ("on-course") deflection of this indicator was approximately 5000 microvolts. The antenna was mounted on a portable support so that its center was ten feet above ground and it could be rotated laterally and vertically about its center in order to investigate the polarization of the received wave. The height of ten feet above ground corresponded to the average height of the landing beam receiving antenna on an aircraft, so that the distance from the transmitter at which the "on-course" indication was obtained on the indicator represented the distance of the point of contact for an average airplane.

TABLE I

Height of transmitting antenna above ground	Distance of point of contact from transmitting antenna*
28 centimeters	1300 feet
51 centimeters	1550 feet
89 centimeters	1800 feet
165 centimeters (one-half wave length)	2000 feet

* For evaluating this value from the transmitter power and receiver sensitivity, see the equation on p. 476 of RP602 of reference 1.

From the point of view of using a landing beam, the location of the point of contact represents an over-all figure of performance. Table I shows the experimentally determined variation of the distance of point of contact from the transmitter as a function of the height above ground of the transmitting antenna. Remembering that in Table I the distances from the various points of contact to the transmitter correspond to distances from the center of the field, it is evident that a receiver sensitivity of the order of 5000 microvolts is satisfactory for use at even the largest field for any of the transmitting antenna heights considered.

Flight tests were next made using a transmitting antenna height of thirty centimeters and adjusting the sensitivity of the receiver for contacts at 1300 feet and 800 feet, respectively. The landing paths obtained in these flight tests are shown in Fig. 2. The flexibility afforded for varying the steepness of the approach path without materially changing the length of runway available for coming to a stop is clearly

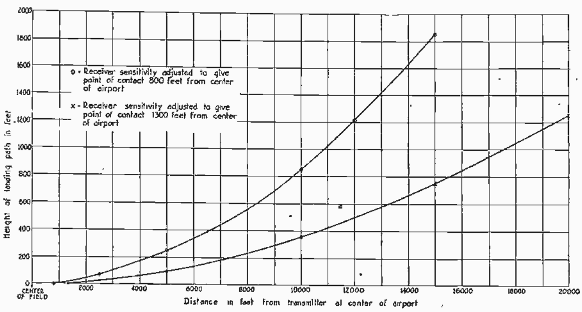


Fig. 2—Typical approach paths possible with landing beam located at center of airport.

apparent. An interesting point in connection with Fig. 2 is that the landing paths actually obtained are considerably flatter than expected from theory. This point will be discussed in a later section in the paper.

IV. Tests with Transmitting Antenna in Pit

Having demonstrated the desirability of locating the transmitting antenna at the center of the field, it is necessary to consider some of the practical features incident to such location. The simplest arrangement is to place the transmitter in a pit and the antenna some twelve inches above the ground surface, feeding the antenna from the transmitter by a simple transmission line. Reasonable attention to preventing the accumulation of water or snow in the vicinity of the antenna would be sufficient to safeguard its electrical operation. However, the presence of such an antenna in the center of the airport presents some hazard. It is likely that this hazard might be reduced to a negligible degree by making the half-wave antenna and support collapsible and controlled from the control tower so as to be in operation only while an airplane is landing. Some reduction in hazard may also be obtained by making the antenna and support quite fragile, so that an airplane, in accidentally running over it, would not be damaged. There are obviously some objections to either arrangement or to their combination.

From the point of view of hazard, a more effective solution is to place both the transmitter and antenna in the pit. Tests were accordingly made to investigate all possible effects upon the electrical opera-

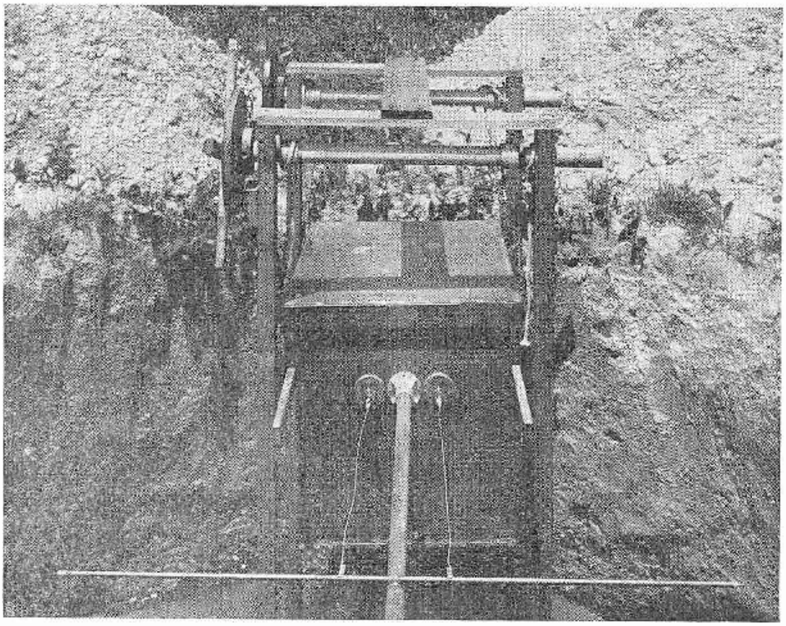


Fig. 3—Experimental transmitting equipment used in the pit tests.

tion caused by placing the antenna in a pit. The portable transmitter and antenna arrangement, and the means used in the pit experiments for varying the position of the unit as a whole below and above the pit surface, are shown in Fig. 3. Two different depths of pit were used, approximately one-fourth and one-half wave length, respectively. The effects of change in water level inside and outside the pit were also studied.

The first tests made were with a pit 80 centimeters in depth and 245 centimeters in diameter. The purpose of these tests was to see if, with the antenna below the pit surface, a true landing path was obtained, and to determine the amount of reduction in the distance of the point of contact due to the expected decreased radiation. A few simple experiments on the ground showed that a true landing path was obtained. These consisted in noting the variation of intensity of received signal with height for fixed distances from the transmitter and the variation of intensity with distance for a fixed height of receiving antenna. The reduction in the distance of the point of contact as the transmitting antenna was brought down to and below the pit surface is shown in Table II. A receiver sensitivity of 5000 microvolts is again seen to be of the correct order for use at even the largest field. (See Figs. 1 and 2.)

TABLE II

Depth of pit (centimeters)	Location of transmitting antenna relative to surface of pit (centimeters)	Distance of point of contact from transmitter (feet)
82.5 (one-fourth wave length)	82.5 above surface	2000
82.5 (one-fourth wave length)	60 above surface	1750
82.5 (one-fourth wave length)	30 above surface	1200
82.5 (one-fourth wave length)	15 above surface	1100
82.5 (one-fourth wave length)	At surface	950
82.5 (one-fourth wave length)	10 below surface	900
82.5 (one-fourth wave length)	20 below surface	900

Tests were next made to determine the shape of the landing path produced when using a pit 165 centimeters deep and 245 centimeters in diameter, with the transmitting antenna 45 centimeters below the

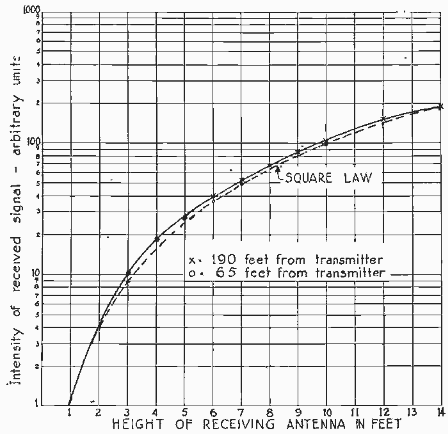


Fig. 4—Vertical distribution of intensity of received signal in field radiated from transmitting antenna in pit.

surface. Ground measurements were made in lieu of airplane flights because of the practical difficulties of experimenting with a pit at an airport. A special receiving set was used with self-contained batteries and receiving antenna to permit hoisting up and down a pole. Fig. 4 shows the variation of the intensity of received signal with height for two distances from the transmitter, 65 and 190 feet, respectively. The two graphs are plotted to adjusted ordinate scales so that they overlap, forming a single smooth curve. Fig. 5 shows the variation of the intensity of received signal with distance from the transmitter for the

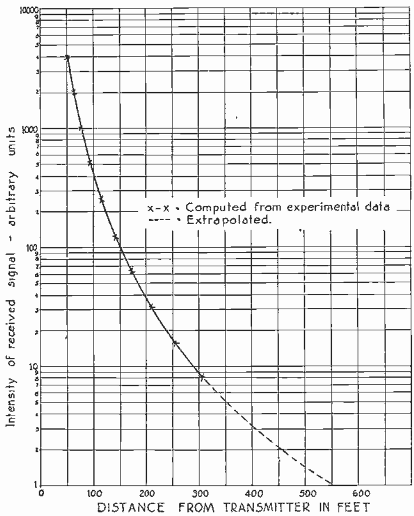


Fig. 5—Variation of the intensity of received signal with distance from the pit transmitting antenna, corresponding to a constant height of the receiving antenna.

receiving antenna at a fixed height. This latter graph was derived from data taken as follows: The receiver was adjusted for an arbitrary

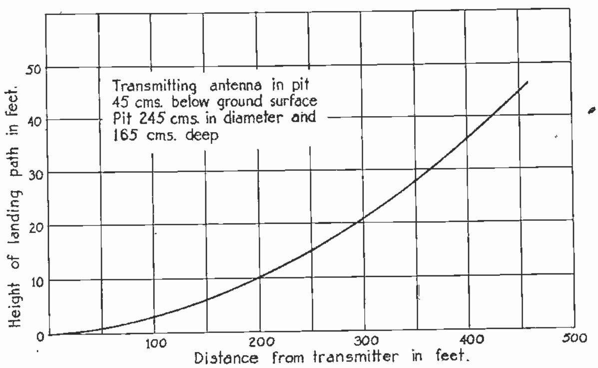


Fig. 6—Shape of a landing path computed from the data of Figs. 4 and 5.

volume output at fifty feet from the transmitter and carried away from the transmitter until the signal intensity was halved. The volume output was then adjusted to its original value and the procedure repeated again and again, the distance from the transmitter being recorded corresponding to each adjustment of volume. A value of 8 was arbitrarily chosen for the signal intensity at the farthest point measured, 310 feet.

From Figs. 4 and 5 it is possible to compute the landing path corresponding to a given signal intensity. Thus assume that at fifty feet from the transmitter, the output volume indicator deflects to one-half scale at a height of one foot. At 100 feet from the transmitter, the received signal at one foot height is one-tenth of that at fifty feet distance (see Fig. 5). To return to the original received signal, it is necessary to raise the receiving antenna to 3.2 feet (see Fig. 4). This is a second point on the landing path. The complete landing path, derived in this way, is shown in Fig. 6. There is, of course, a family of such paths, the steepness depending entirely on the intensity of the line considered. Corresponding to a height of ten feet at distances of 800 and 1300 feet from the transmitter, the landing paths would be somewhat flatter than those shown in Fig. 2 (for the transmitting antenna thirty centimeters above ground).

V. Study of Shape of Landing Paths for Transmitting Antenna at Various Heights Above Ground

It is of interest to note the effect of the proximity of the ground to the transmitting antenna upon the shape of the lines of constant received signal forming the landing paths. As will be shown later in this section, the theoretical equation of the landing path may be stated as follows (See Fig. 7):

$$y = y_0 \left(\frac{r}{r_0}\right)^2 \tag{1}$$

where,

y_0 = the height of the receiving antenna above the bottom of the landing gear,

r_0 = the distance of the point of contact from the transmitter,

r = distance from the transmitter,

y = corresponding height of the landing path.

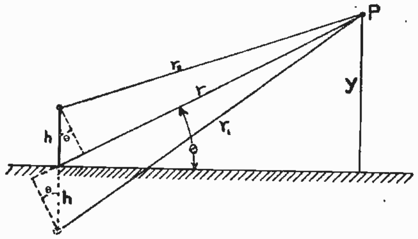


Fig. 7—Mechanism of setting up a landing path with the transmitting antenna above ground.

Experimentally this equation was checked closely when using directive transmitting antenna arrays with their centers located from three-fourths to one wave length above ground.[1] However, with a half-wave transmitting antenna close to and below the ground surface, the landing paths were found to be considerably flatter. Empirical equations of the form of (1) but with different exponents were determined to fit these landing paths. For the antenna thirty centimeters above ground (see Fig. 2), the exponent was found to be 1.85 instead of 2, while for the antenna in the pit forty-five centimeters below the ground surface (see Fig. 6), the exponent was found to be 1.75. There would appear to be a steady transition in the shape of the path as the antenna is brought down to and below the ground surface. As a check on the trend of this transition, a further test was made with the transmitting antenna fifteen centimeters above the ground surface. For this case, the shape of the path was found to be very nearly the same as for the antenna in the pit. The difference in the shape of the paths under the different test conditions is possibly due to the fact that when the antenna is near the ground, the wave incident on the ground is no longer plane, so that (1) which is based on the assumption of plane wave ground reflection does not hold.

The derivation of (1) showing that other assumptions involved do not contribute to the apparent departure from the plane wave theory follows (see Fig. 7): Given an antenna h units above the ground and its image at similar distance below the ground. For unit antenna current, the electric field set up at a point P at a height y above ground, at the distance r from the point on the ground surface just below the antenna and making an angle θ with the ground is

$$E_p = C\left\{\frac{1}{r_2}\cos\omega\left(t-\frac{r_2}{V}\right)+\frac{A_h}{r_1}\cos\omega\left(t-\frac{r_1}{V}\right)\right\} \tag{2}$$

where A_h is the complex reflection coefficient for horizontally polarized waves and r_2 and r_1 are the distances between the point P and the antenna and its image, respectively. The amplitude of E_p may be written as (3), remembering that $\omega/V = 2\pi/\lambda$,

$$E_p = \frac{C}{r_1 r_2}\sqrt{r_1^2 + A_h^2 r_2^2 + 2A_h r_1 r_2 \cos\frac{2\pi}{\lambda}(r_1 - r_2)}. \tag{3}$$

In (3), $r_1 - r_2 = 2h\sin\theta$ within an extremely close approximation.

Now, for any landing path, E_p = constant. Also, at the point of contact, $r = r_0$, $\theta = \theta_0$ and $A_h = A_{h0}$. Hence, the general equation for the landing path becomes

$$\frac{C}{r_1 r_2}\sqrt{r_1^2 + A_h^2 r_2^2 + 2A_h r_1 r_2 \cos\left(\frac{4\pi h}{\lambda}\sin\theta\right)}$$
$$= \frac{C}{r_{1_0} r_{2_0}}\sqrt{r_{1_0}^2 + A_{h_0}^2 r_{2_0}^2 + 2A_{h_0} r_{1_0} r_{2_0} \cos\left(\frac{4\pi h}{\lambda}\sin\theta_0\right)}. \tag{4}$$

Equation (1) may be derived from (4) on the basis of the assumptions that $r_1 = r_2$ and $A_h = -1 + j0$, noting that $\sin \theta = y/r$. It is evident therefore that for the shape of the landing path to vary with the height of the transmitting antenna above the ground, the variation would have to be produced by the difference in the values of A_h corresponding to various values of y, assuming the same point of contact but different transmitting antenna heights. A large number of computations, corresponding to possible practical values of h, y_0, r_0 and the electrical ground constants, showed that this is not the case. It would therefore appear that the variation must be due to the lack of planeness of the radio wave.

VI. Theory Underlying Operation of Landing Beam Antenna in Pit

The fact that a landing path is set up with the transmitting antenna located inside the pit presents a study in the optical behavior of the ultra-high-frequency radiations. In an attempt to arrive at a theoretical analysis of the phenomena involved, two approaches present themselves. The first is that the rays penetrate the sides of the pit and emerge at the earth's surface. The vertical distribution of energy obtained may then be assigned to the fact that rays at the lower angles have the longer optical paths and hence the greater attenuation. There is quite strong evidence, however, that this is not the actual case. First, an experiment was made in which the walls of the pit were lined with a copper shield. This resulted in negligible change in attenuation of the transmitted wave. At a distance of 190 feet from the transmitter, the intensity of the received signal and its variation with height were very nearly the same with and without the shield. Furthermore, an examination of the angles involved shows that even were the rays transmitted through the walls of the pit, they would reach the ground surface at such angles as to require total internal reflection; there could thus be no rays emerging at the ground surface.

The next likely explanation of the phenomena involved is that the rays are diffracted around the rim of the pit, the intensity of radiation dropping off as a function of the angular deviation below the marginal rays just clearing the rim. The experimental evidence pointed to this theory as a plausible one. A marked change in the water content in the surrounding ground and also shielding the walls of the pit, introduced no appreciable change in either the intensity of the received signal or its vertical distribution. However, the fact that the shape of a line of constant received signal was so very nearly the same as that for the case of the transmitting antenna above ground required that ground reflection of the diffracted rays enter into the picture. We are indebted to Chester Snow of the National Bureau of Standards for assistance in working out an approximate mathematical analysis of this problem which serves to give a clear idea of the phenomena involved.

Referring to Fig. 8, the transmitting antenna is taken perpendicular to the plane of the paper. The marginal rays 1 and 2 of a right section of the wave emerging through the surface of the pit make angles with the ground surface equal respectively to ϕ_0 and $\pi-\phi_0$. Consider the receiving point P at a distance r from the transmitting antenna and at an angle θ above the horizontal. By Huyghens' prin-

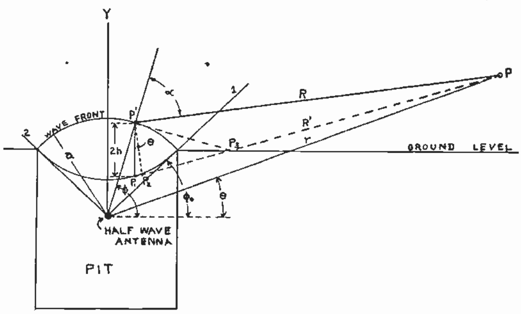


Fig. 8—Mechanism of setting up a landing path with the transmitting antenna in a pit.

$$E_p = \frac{4\pi A a}{\lambda r}\{2\cos\phi_0 - (\pi - 2\phi_0)\sin\phi_0\}\sin\theta\cos\omega\left(t - \frac{a+r}{v}\right).$$

ciple, each element of the wave front, P' (at an angle ϕ with the horizontal), becomes a new source and radiates energy in all directions. Part of this energy reaches the point P directly along the path R while part reaches it by reflection from the ground along the path $P'P_3P$. The latter appears to come from the virtual image of P' located at P_1, directly below P' by a distance

$$2h = 2a(\sin\phi - \sin\phi_0). \tag{5}$$

It is possible to assume an image under the general point P' even though the surface of the reflecting plane is cut away from below it, since, for the angles of θ involved, the actual reflecting point P_3 always falls beyond the rim of the pit. The intensity at P due to the two rays from P' may then be set up in terms of the various distances and angles indicated on Fig. 8 and it may be summed for all the elementary points on the wave front between the limits of the marginal rays as indicated in the following expression:

$$E_p = \frac{A}{r}\int_{\phi_0}^{\pi-\phi_0}\{1+\cos\phi\}\left\{\sin\omega\left[t - \frac{a+R}{V}\right] - \sin\omega\left[t - \frac{a+R'}{V}\right]\right\}d\phi \tag{6}$$

where $(1+\cos\phi)$ is the Stokes' obliquity fact or taking into account that the new wavelet at p' tends to be propagated with maximum effect in the direction of propagation of the original wave front at this point. Equation (6) involves the assumption that $\alpha=\phi$, since θ, being always less than three degrees, is small compared to ϕ. In (6) $(a+R)/V$ is the phase retardation with respect to the phase at the antenna of the wave reaching P along the path R, and $(a+R)/V$ is the phase retardation of the wave reaching P via $P'P_3P=R'$. The negative sign is taken before this term to indicate a negative image. Placing

$$R = r - \frac{R' - R}{2}$$

and

$$R' = r + \frac{R' - R}{2}$$

we may write from (5) and from the fact that $R'=R+P_1P_2=R+2h\sin\theta$ approximately

$$\left.\begin{aligned} R &= r - a\sin\theta(\sin\phi - \sin\phi_0) \\ R' &= r + a\sin\theta(\sin\phi - \sin\phi_0) \end{aligned}\right\}. \tag{7}$$

Substituting (7) in (6), we have

$$E_p=\frac{A}{r}\int_{\phi_0}^{\pi-\phi_0}\{1+\cos\phi\}\left\{\sin\omega\left(t-\frac{a+r}{V}+\frac{a\sin\theta(\sin\phi-\sin\phi_0)}{V}\right)\right.$$
$$\left.-\sin\omega\left(t-\frac{a+r}{V}-\frac{a\sin\theta(\sin\phi-\sin\phi_0)}{V}\right)\right\}d\phi. \tag{8}$$

Simplifying

$$E_p=\frac{2A}{r}\cdot\cos\omega\left(t-\frac{a+r}{V}\right)\int_{\phi_0}^{\pi-\phi_0}\{1+\cos\phi\}$$
$$\cdot\left\{\sin\left[\frac{\omega a}{V}\sin\theta(\sin\phi-\sin\phi_0)\right]\right\}d\phi. \tag{9}$$

But since θ is small,

$$\sin\left[\frac{\omega a}{V}\sin\theta(\sin\phi-\sin\phi_0)\right]=\frac{\omega a}{V}\sin\theta(\sin\phi-\sin\phi_0).$$

Also,

$$\frac{\omega a}{V}=\frac{2\pi a}{\lambda}.$$

Therefore,

$$E_p = \frac{4\pi A a}{\lambda r} \cdot \cos \omega \left(t - \frac{a + r}{V} \right)$$

$$\sin \theta \int_{\phi_0}^{\pi - \phi_0} (1 + \cos \phi)(\sin \phi - \sin \phi_0) d\phi. \tag{10}$$

Integrating,

$$E_p = \frac{4\pi A a}{\lambda r} \cdot \{ 2 \cos \phi_0 - (\pi - 2\phi_0) \sin \phi_0 \} \sin \theta \cos \omega \left(t - \frac{a+r}{V} \right). \tag{11}$$

Equation (11) gives the field intensity at the point P in terms of the angle of elevation θ, the dimensions of the pit, and the wave length in air. For a pit of given dimensions and with the antenna in a given position, (11) resolves into

$$E_p = C \sin \theta \cos 2\pi f \left(t - \frac{a + r}{V} \right). \tag{12}$$

The latter equation indicates that the intensity at the point P is a sine function of the angle of elevation of the point P. Since for small angles, $\sin \theta = \theta$, the vertical distribution of intensity is seen to be a linear function of the height. In our experiments, we obtained a square-law function. (See the dotted curve in Fig. 4.) However, the receiver used was of the triple detection type of which the law of relation between output and input was probably close to a square law, so that there is fair agreement between the theory and the experimental data.

Referring back to (11), the first group of factors indicates that for a given antenna location in the pit, the intensity P increases with the opening of the pit (i.e., with its diameter) and decreases with an increase in the wave length used. The portion of (11) in brackets shows that for a given pit diameter, the intensity at P is a function of the angle of the marginal ray; i.e., of the depth of the antenna in the pit. A study of this term discloses that the intensity is large for small angles of θ_0 and decreases as θ_0 is increased becoming zero for $\phi_0 = \pi/2$. The factor $\sin \theta$ in (11) gives the relation of the intensity with the angle of elevation of the receiving point while the remaining cosine function indicates the phase of the resultant field at the receiving point.

From the foregoing analysis, it becomes apparent why the path of a line of constant field intensity is of very nearly the same shape with the transmitting antenna in the pit as for the antenna a short distance above the ground surface. The wave front emerging from the pit is practically equivalent to a physical antenna above the ground surface, so that the phenomena of interference between a direct and reflected wave may occur. From the trend of change of path shape with proximity of the antenna to the ground (discussed in Section V), the equivalent height of the experimental combination used to set up the landing path of Fig. 6 was approximately fifteen centimeters.

VII. Polarization of the Received Wave

A study of the polarization of the electric field corresponding to different positions of the transmitting antenna above the ground surface and above and below the pit surface revealed further evidence of

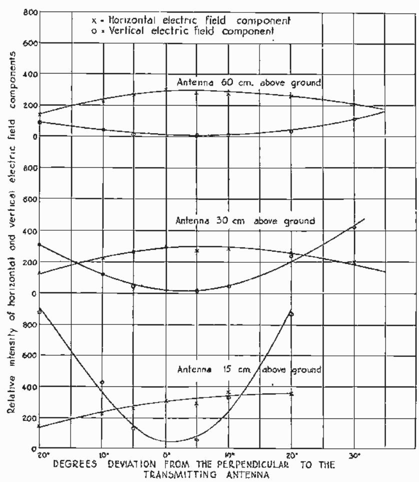


Fig. 9—Relative magnitudes of horizontally and vertically polarized components of the received wave as a function of the height of the transmitting antenna above the ground surface.

the effect of the ground proximity. It was found that the ratio of a vertically to horizontally polarized electric field component radiated from the horizontal antenna at various azimuth angles on either side of the normal to the length of the antenna was much greater than expected as the antenna was brought close to the ground surface. For the receiving antenna at a distance of 100 meters and at a height of three meters the ordinary theory requires that the ratio be very small because of the very small angles of elevation involved.

Fig. 9 shows the data obtained for the transmitting antenna at various heights above the ground surface. Each set of curves on this figure corresponds to a fixed position of the transmitting antenna and shows the relative amplitudes of the horizontally and vertically polarized electric field components as a function of angle (in the horizontal plane) on either side of the normal to the transmitting antenna. The magnitude of the vertically polarized electric field and the rapid increase in the ratio of vertical to horizontal field component as the transmitting antenna is brought closer to the ground are far beyond what would be expected from the plane wave theory. Fig. 10 shows similar data for the transmitting antenna at various positions with respect to the surface of a pit approximately one-fourth wave length in depth (80 centimeters). A study of Fig. 10 in comparison with Fig. 9 brings out the following points:

(1) For equivalent heights above the pit surface and above actual

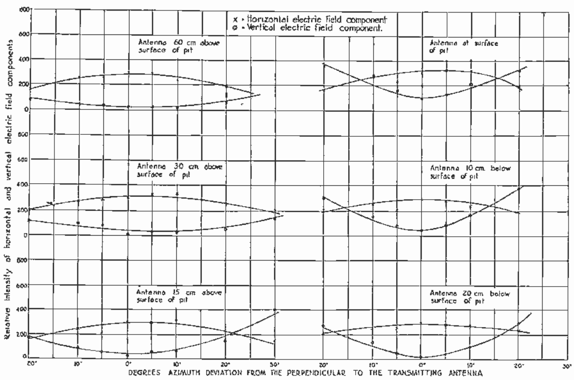


Fig. 10—Relative magnitudes of horizontally and vertically polarized components of the received wave as a function of the position of the transmitting antenna with respect to the pit surface.

ground surface, the relative amount of vertically polarized component is considerably lower for the former.

(2) The relative amount of vertically polarized component for departures from the normal of ± 15 degrees is negligible until the antenna approaches within 60 centimeters of either the pit or the ground surface.

(3) In the case of the experiments with the pit having a depth of one-fourth wave length, the relative amount of vertical component increases as the antenna approaches from 60 centimeters above the surface to the surface and then would appear to decrease gradually as the antenna is brought to 20 centimeters below the surface.

There was a twofold reason for our interest in the presence of an appreciable vertical component in the radiated field. First, such a component results in a tilt of the plane of polarization of the total electric field, so that tilting of the airplane receiving antenna on either side of its normal horizontal position would result in different readings of the landing path "course" indicator. Second, since the vertical component is not useful it represents an actual waste of energy. The first reason does not become important until the ratio of vertical to horizontal component becomes greater than unity for angular departures of less than ± 10 degrees from the normal direction to the transmitting antenna. A little study will show that this is the case particularly since, in normal use of the landing beam, the transmitting antenna would be oriented at all times perpendicularly to the existing wind direction.

The next test was made to see if the depth of the water level inside or outside the pit would affect the ratio of vertical and horizontal

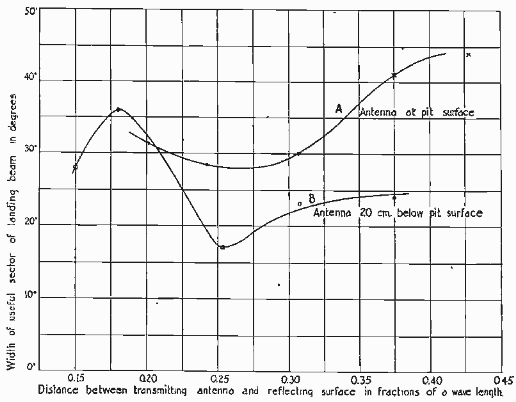


Fig. 11—Effect of distance between transmitting antenna and reflecting surface at bottom of pit upon relative magnitudes of horizontal and vertical field components. Ordinates denote the width of sector, normal to the transmitting antenna, wherein the horizontal component is greater than the vertical.

electric field components. Any variation in this ratio would indicate a transfer of a portion of the total available energy from one component to the other and would therefore result in a change in the landing path, as followed with a receiving set of fixed sensitivity fed from an antenna responsive only to the horizontal component. A series of measurements was made for various conditions of water level inside the pit and in the ground surrounding the pit. To extend the data secured, two depths of pit were used, approximately one-fourth and one-half wave length. The rise and fall of the water level in the surrounding ground was observed to have a negligible effect upon the relative amount of vertical and horizontal electric field components. On the other hand, changing the water level and hence the reflecting surface inside the pit was found to have a marked effect upon the relative values of these components.

The results obtained from this series of measurements are correlated in Fig. 11. The abscissas represent distance in fractions of a wave length between the transmitting antenna and the surface of the water in the pit (or the bottom of the pit, if dry). The ordinates represent the total width in degrees of the sector, substantially at right angles to the length of the transmitting antenna, in which the vertical component is less than the horizontal. This may be termed the effective useful sector of the landing beam. Graph A is for the transmitting antenna at the surface of the pit, while graph B is for the antenna twenty centimeters below the pit surface. All of the data obtained for the antenna in these two positions corresponding to both depths of pit used are plotted in Fig. 11. The graphs show a definite minimum width of useful sector for a height of antenna above the reflecting surface equal to one-fourth wave length and a maximum width for a height of one-half wave length. This is in agreement with what is normally expected for the case of a transmitting antenna above the ground.

Unlike the case for a transmitting antenna above ground, the relative amount of vertical component in the radiated field from the antenna in the pit is very much greater. This is probably due to the effect of the proximity of the ground to the high voltage ends of the transmitting antenna which may result in the production of a vertical current. There is some experimental basis for believing this to be the case. In some of the tests made, it was observed that varying the position of the antenna below the pit surface and at the same time keeping the effective reflecting surface at a constant distance below the antenna, did not result in the same relative amount of vertical component. The difference in the net proximity effect of the sides of the pit for the various antenna positions may be offered as an explanation of this phenomenon. This effect is indicated by the graphs of Fig. 11, graph A for the antenna at the pit surface showing, on the average, a greater width of useful sector than graph B which corresponds to the antenna at twenty centimeters below the pit surface.

From the practical point of view, insofar as use of the landing beam transmitting antenna in a pit is concerned, it is possible to provide for an unchanging ratio of the two components by waterproofing the pit. This will result in an unvarying amount of energy in the horizontal electric field (for a given transmitter power) and hence in a fixed landing path.

VIII. Conclusions

As a result of the various experiments outlined, the following conclusions may be drawn:

A very material increase in the flexibility of use of the landing beam, particularly in providing a suitably steep approach path to large airports, may be obtained through employing a half-wave transmitting antenna located at the center of the airport. The antenna may be a small fraction of a wave length above the ground surface or in a pit below the ground surface. In either case the proximity of the ground to the horizontal transmitting antenna introduces interesting effects upon the electric field radiated.

As the transmitting antenna is brought closer to the ground surface, the shape of a line of constant field intensity in the radiated field (for angles of elevation less than three degrees) departs from a parabola, becoming somewhat flatter. The effect appears to be the same for the antenna in a pit as when it is just above the ground surface. This arises from the phenomena involved in the radiation of an electric field from the transmitting antenna located in the pit. The wave front emerging from the pit operates as a large number of new sources, which produce direct radiation to the receiving point and also indirect radiation by way of reflection from the ground surface. The two sets of radiation produce an interference pattern very similar to that produced by a transmitting antenna a short distance above ground.

The proximity of the ground to the transmitting antenna also increases the relative amount of vertically polarized electric field in the emitted wave. This effect may be limited through use of the pit. The depth of the pit should be of the order of one-half wave length corresponding to which the width of useful sector of the landing beam is a maximum. The pit should be waterproofed so that water cannot enter it from the surrounding ground and thereby change its effective depth. The proximity effect may be further limited by keeping the walls of the pit away from the ends of the transmitting antenna; the minimum cross-sectional dimension should be at least three fourths of a wave length.

The walls and bottom of the pit may be lined with shielding material in order to render constant the radiation losses to the surrounding ground. The roof of the pit, required for the protection of landing airplanes, must be of a nonconducting material of low dielectric constant to permit of free emergence of the radiated wave.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

ON THE OPTIMUM LENGTH FOR TRANSMISSION LINES USED AS CIRCUIT ELEMENTS*

By
Bernard Salzberg
(RCA Manufacturing Company, Inc., RCA Radiotron Division, Harrison, New Jersey)

Summary—*The existence of an optimum length for transmission lines which are tuned by low-loss capacitor to give maximum sending-end impedance is discussed. This optimum length is found to be 0.185λ for a shorted line and 0.472λ for an open-circuited line, resulting in impedances 14 and 3 per cent higher, respectively, than can be obtained from lines without tuning condensers.*

As THE application of the higher radio frequencies for communication purposes grows, the transmission line finds increasing use as a circuit element. Many properties of such circuit elements which are useful to the design engineer have been described.[1,2,3,4] For example, when the line consists of two coaxial conductors, it has been pointed out that the highest sending-end impedance is obtained when the ratio of the inside diameter of the outer conductor to the outside diameter of the inner conductor is equal to 9.18. The existence of an optimum *length* for transmission lines which gives maximum sending-end impedance, however, does not appear to have been treated in the literature, nor does it seem to be known generally to engineers working with such devices. Several months ago the author had occasion to analyze the transmission line used as a circuit element. Among other things of interest was the following problem: Suppose a transmission line, either short-circuited or open-circuited at one end and tuned by a variable capacitor of negligible losses at the other end, is used as a resonant circuit element. Is there an optimum ratio of line length to operating wave length, l/λ, which results in maximum absolute impedance across the capacitor? If there is, what is this ratio?

The result of a simple analysis, which will be reproduced below, was that there exists an optimum value of the ratio l/λ for the line, which gives maximum impedance at the capacitor end of the line, this value being $l/\lambda \cong 0.185$ for the short-circuited line and $l/\lambda \cong 0.472$ for the open-circuited line. The capacitance in both cases must be adjusted

* Decimal classification: R320. Original manuscript received by the Institute, August 20, 1937.

[1] C. S. Franklin, British Patent No. 284,005 and corresponding U. S. Patent No. 1,937,559.

[2] E. J. Sterba and C. B. Feldman, "Transmission lines for short-wave radio systems," Proc. I.R.E., vol. 20, pp. 1163–1202; July, (1932).

[3] F. E. Terman, "Resonant lines in radio circuits," *Elec. Eng.*, vol. 53, pp. 1046–1061; July, (1934).

[4] C. W. Hansell and P. S. Carter, "Frequency control by low power factor line circuits," Proc. I.R.E., vol. 24, pp. 597–619; April, (1936).

to resonance with the sending-end inductance of the line. The resistance of the capacitor is also assumed to be small in comparison with the sending-end resistance of the line: this can be realized by suitable construction. This result ignores radiation, and hence is strictly true only for the completely enclosed coaxial line.

The maximum absolute impedance of a circuit consisting of a variable capacitance, C, shunted by an inductance, L, in series with a resistance, R, is obtained when the capacitance is adjusted to equal the ratio of the inductance to the square of the absolute impedance of the series combination of inductance and resistance. The phase angle of the absolute impedance of the shunt circuit is then zero, and its magnitude is $r=R+(\omega^2L^2/R)$. This corresponds to the condition of antiresonance.

The impedance of a transmission line at its sending-end terminals may be represented by a series combination of resistance and reactance, $Z=R+jX$. For a low-loss radio-frequency line, short-circuited at its distant end, this is

$$Z \cong \frac{Z_0 \cdot k\theta}{\cos^2\theta + k^2\cdot\theta^2\cdot\sin^2\theta} + j\cdot\frac{Z_0\cdot\sin\theta\cdot\cos\theta}{\cos^2\theta + k^2\cdot\theta^2}, \tag{1}$$

and for a low-loss radio-frequency line, open-circuited at its distant end, this is

$$Z \cong \frac{Z_0 \cdot k\theta}{\sin^2\theta + k^2\cdot\theta^2\cdot\cos^2\theta} - j\cdot\frac{Z_0\cdot\sin\theta\cdot\cos\theta}{\sin^2\theta + k^2\cdot\theta^2}. \tag{2}$$

In (1) and (2), Z_0 = surge impedance; $\theta=\beta l=2\pi l/\lambda$; $k=2Z_0/\pi\cdot 1/r_0$, for the short-circuited line, and $k=Z_0/\pi\cdot 1/r_0$, for the open-circuited line; $r_0=8Z_0^2/R_0\lambda$ = quarter-wave resonant impedance for the short-circuited line, and $r_0=4Z_0^2/R_0\lambda$ = half-wave resonant impedance for the open-circuited line; R_0 = distributed series resistance of the line; and λ = operating wave length.

The antiresonant impedance is thus given by

$$\frac{r}{Z_0} \cong \frac{k\theta}{\cos^2\theta + k^2\cdot\theta^2\cdot\sin^2\theta} + \frac{\sin^2\theta}{k\theta}\cdot\frac{1}{1+\dfrac{k^2\cdot\theta^2}{\cos^2\theta}} \tag{3}$$

for the short-circuited line, and by

$$\frac{r}{Z_0} \cong \frac{k\theta}{\sin^2\theta + k^2\cdot\theta^2\cdot\cos^2\theta} + \frac{\sin^2\theta}{k\theta}\cdot\frac{1}{\tan^2\theta+\dfrac{k^2\cdot\theta^2}{\cos^2\theta}} \tag{4}$$

for the open-circuited line.

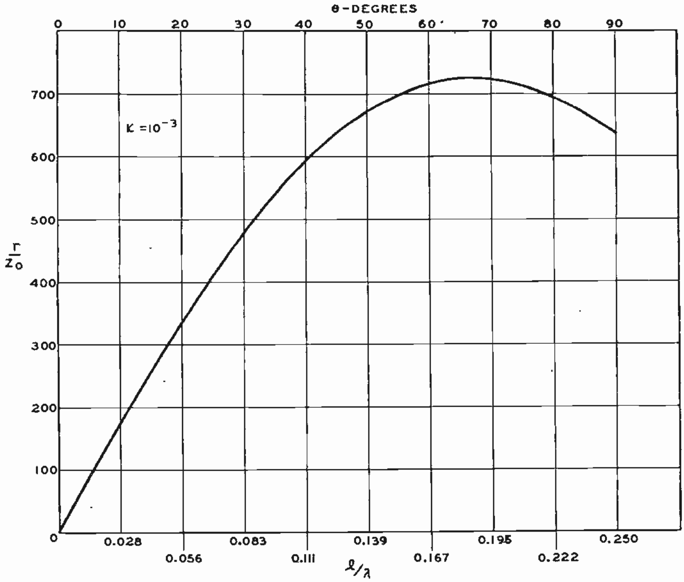


Fig. 1—The variation of the resonant impedance with line length, for a typical short-circuited line.

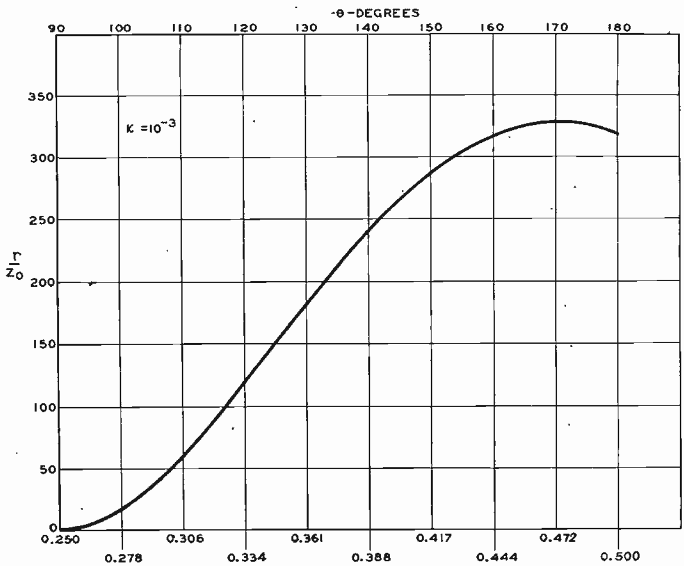


Fig. 2—The variation of the resonant impedance with line length, for a typical open-circuited line.

The variation of the resonant impedance with line length is shown in Figs. 1 and 2. These are plots of (3) and (4) for the case of a reasonably low-loss line; e.g., $k=10^{-3}$, corresponding to a surge impedance of approximately 133 ohms, and a quarter-wave resonant impedance of approximately 85,000 ohms or a half-wave resonant impedance of approximately 42,500 ohms.

Examination of (3) reveals that for the usual low-loss line the first term is negligible in comparison with the second term, except very near $\theta=\pi/2$. Also, except very near $\theta=\pi/2$, the second term may be approximated by $\sin^2\theta/k\theta$, or

$$\frac{r}{r_0}\cdot\frac{2}{\pi}\cong\frac{\sin^2\theta}{\theta}. \tag{3a}$$

Maximization with respect to θ indicates that an optimum occurs at $\theta\cong 66.5$ degrees, corresponding to $l/\lambda\cong 0.185$. For this value the resonant impedance is approximately fourteen per cent higher than that of a quarter-wave line.

Similar examination of (4) reveals that for an efficient line the first term is negligible in comparison with the second term, except very near $\theta=\pi/2$ and $\theta=\pi$. Also, except very near $\theta=\pi$, the second term may be approximated by $\cos^2\theta/k\theta$, or

$$\frac{r}{r_0}\cdot\frac{1}{\pi}\cong\frac{\cos^2\theta}{\theta}. \tag{4a}$$

Maximization with respect to θ indicates that an optimum occurs at $\theta\cong 170$ degrees, corresponding to $l/\lambda\cong 0.472$. For this value the resonant impedance is approximately three per cent higher than that of a half-wave line.

It should be pointed out that not only is the improvement due to operating at the optimum length considerably less in the case of the half-wave line than that for the quarter-wave line, but that the short-circuited line has twice the resonant impedance for $l/\lambda=0.25$ than the open-circuited line for $l/\lambda=0.50$. Consequently, it would be best to use the short-circuited, short line, even for such uses as require the prevention of direct-current flow through the line: in this case, the line is short-circuited at the far end by a blocking condenser.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

NOTE ON LARGE SIGNAL DIODE DETECTION*

BY

S. BENNON

(Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pa.)

Summary—*Several articles have appeared in the last few years on large signal detection, which placed emphasis upon the avoidance or minimizing of distortion. The most serious form of distortion in ordinary diode circuits is due to "nontracking," or the failure of the tube to become conducting for each peak of the impressed voltage. It is shown that the previously derived criterion for just avoiding nontracking is closely connected with the condition for maximum detected signal. Experimental results which revealed the connection are given together with possible explanations for the existing deviations.*

THE material presented here is a part of an experimental study of large signal diode detection made in the graduate laboratory of the Moore School of Electrical Engineering. It deals with an investigation of the factors affecting the amplitude of the detected signal. The work led to results indicating an interesting connection between the condition for the start of nontracking and the condition for maximum detected signal. It is to be understood that the maximum fundamental component of the detected signal is meant when the maximum detected signal is mentioned.

Terman and Morgan[1] gave the first derivation of a criterion for nontracking distortion. In their analysis the diode was supposed to have zero resistance during the conducting intervals. The analysis was made for a diode circuit with a load consisting of a resistance R in parallel with a capacitance C. During conduction the capacitance is brought to the voltage of the applied wave, which for simplicity is assumed to be a carrier modulated by a single frequency. The conducting period ends when the instantaneous applied voltage falls below the condenser voltage, so that the condenser is left with a voltage equal to the amplitude of the modulation envelope at that instant. During the succeeding nonconducting interval the condenser discharges into the load resistance. The condition for tracking, that is for the tube to become conducting for each peak of the applied voltage, requires that the decrease in the condenser voltage during a carrier cycle be greater than or equal to the decrease of the modulation envelope for the same interval for all points on the modulation envelope.

* Decimal classification: R134. Original manuscript received by the Institute, June 15, 1937.

[1] F. E. Terman and N. R. Morgan, "Some properties of grid leak detection," PROC. I.R.E., vol. 18, pp. 2160–2175; December, (1930).

By using a first approximation to these voltage decreases, that is by evaluating the derivatives at the start of the nonconducting interval and multiplying by the period corresponding to a carrier cycle, the following criterion was obtained for a carrier modulated by a single frequency:

$$\omega_s RC \leqq \sqrt{\left(\frac{1}{k}\right)^2 - 1},$$

where ω_s is the periodicity of the modulating signal and k is the modulation factor.

However, a consideration of the form of the voltage variation across the diode load, as illustrated in Fig. 1, would lead one to expect a close relationship between the start of nontracking and the condition for maximum detected signal. Fig. 1(b) shows the variation of the voltage for $\omega_s RC = \sqrt{(1/k)^2 - 1}$. Here the voltage across the condenser just manages to fall off quickly enough to track the applied electromotive force at the most critical part of the modulation envelope. If now $\omega_s RC$ is made considerably smaller than $\sqrt{(1/k)^2 - 1}$, we get the condition shown in Fig. 1(a). Here the deep saw-tooth variations of the voltage are due to the sharp exponential voltage decreases during the nonconducting intervals. The peaks of the saw-tooth variations follow the upper side of the modulation envelope. It will be noticed that the saw-tooth variations on the high parts of the modulation envelope are much larger than the variations for the low parts. This is due to the fact that exponential voltage decreases during equal intervals are proportional to the initial voltage values. As a result there will be a decrease in the detected signal. If now $\omega_s RC$ is made considerably greater than $\sqrt{(1/k^2) - 1}$ we will have a condition illustrated by Fig. 1(c). Here at the start of nontracking the voltage variation becomes an exponential discharge curve, and the diode will not become conducting again until the exponential portion intersects the modulation envelope. This again results in a decrease in the detected signal. Therefore we would expect a close connection between the condition for maximum signal and that for the start of nontracking.

The point on the modulation envelope at which nontracking begins varies approximately from p to q as shown in Fig. 1(c) as the modulation factor varies from zero to unity. This shifting of the point at which nontracking starts to the lower parts of the troughs of the modulation envelope is due to the previously mentioned fact that the exponential decrease in condenser voltage is proportional to the initial value. Since the troughs of the modulation envelope become lower as the modulation factor is increased, the critical point tends to shift toward the bottom of the troughs.

This shift of the point at which nontracking starts might explain why it is still possible to obtain good detection at unity modulation, even though the criterion given states that nontracking is inevitable.

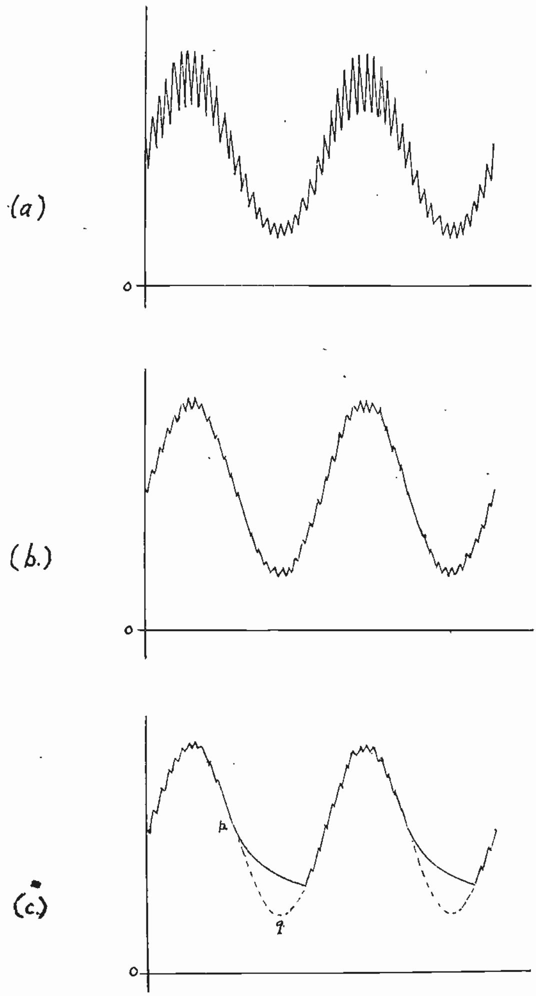


Fig. 1—Variation of voltage across diode load.

Near unity modulation the start of nontracking occurs practically at the bottom of the trough of the modulation envelope, so that the portion of the envelope variation cut out by the smooth exponential discharge curve during nontracking is relatively small.

The criterion developed by Terman and Morgan is too restricted to be applied to actual detectors since it does not take into account the effect of the coupling arrangement to the next circuit. This necessary extension was made by Roberts and Williams[2] for the case of resistance-capacitance coupling. The general criterion they derived reduces, for the arrangement used in the experimental study, to,

$$\omega_s\left(\frac{RR_0}{R+R_0}\right)C \leqq \sqrt{\left[\frac{R_0}{(R+R_0)k}\right]^2 - 1},$$

where R_0 is the resistance in the grid circuit of the succeeding tube. This criterion reduces, as it should, to the one given by Terman and Morgan when R_0 is made infinite.

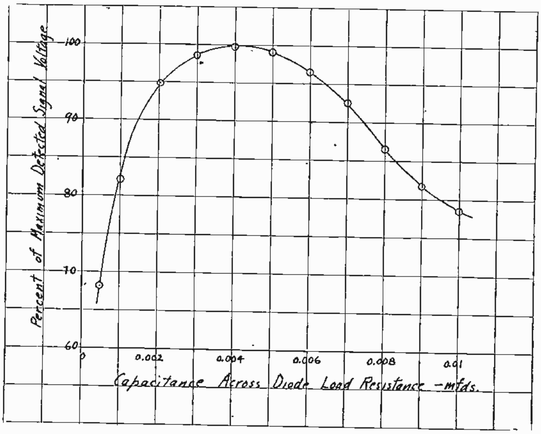


Fig. 2—Variation of detected signal with capacitance.

Carrier frequency = 2080 cycles
Signal frequency = 98 cycles
Modulation factor = 0.49
Diode load resistance = 0.45 megohm
Peak input = 20 volts
R_0 = 3 megohms

Experimental Results and Discussion

The experimental work was carried out at audio frequencies to allow accurate measurements on the Moore-Curtis harmonic analyzer[3] in the Moore School graduate laboratory. One pair of elements of a

[2] F. Roberts and F. C. Williams. *Jour. I.E.E.*, pp. 379–388, September, (1934).

[3] C. R. Moore and A. S. Curtis, *Bell Sys. Tech. Jour.*, pp. 217–229, April, (1927). C. Paul Young, "An audio-frequency wave analyzer," Master's Thesis, Moore School of Electrical Engineering, University of Pennsylvania, June, (1932).

type 6H6G, double diode, was used as the detector. The value of capacitance for maximum detected signal, or the optimum capacitance, for any set of conditions was determined by setting the analyzer to the modulating signal frequency, with the amplified detected signal impressed on the analyzer, and varying the capacitance (in the form of a decade capacitance box) across the diode load resistance until a maximum reading was obtained on the output meter of the harmonic analyzer. In difficult cases the average of the two values of capacitance producing a definite decrease from the maximum meter reading was taken.

Fig. 2 illustrates the general variation of the amplitude of the detected signal with capacitance across the diode load resistance. In

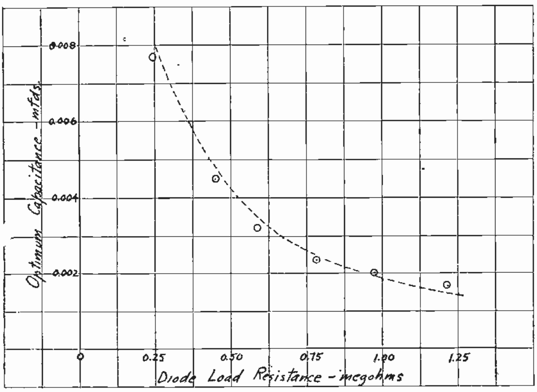


Fig. 3—Variation of optimum capacitance with load resistance.

Carrier frequency = 2080 cycles
Signal frequency = 102 cycles
Modulation factor = 0.56

Peak input = 20 volts
R_0 = 3 megohms

Dotted curve plots $\omega_s\left(\dfrac{RR_0}{R+R_0}\right)C = \sqrt{\left[\dfrac{R_0}{(R+R_0)k}\right]^2 - 1}$

Circles represent experimental points

all succeeding curves each point corresponds to the value of capacitance giving a maximum point on a curve like this, for example this particular curve would yield a value of optimum capacitance of approximately 0.0041 microfarad. Fig. 3 shows the variation of the optimum capacitance with diode load resistance. The dotted curve is a plot of the criterion for the start of nontracking. This figure together with Fig. 4, which shows the variation with modulating frequency, show a general agreement between the theoretical and experimental data. Fig. 5 shows the variation with percentage modulation. This shows a close agreement for moderate percentages of modulation but deviations occur at both extremes. To explain the deviations two possibilities suggested themselves. The first was the possibility that the connection deduced between the condition for maximum detected signal and the start of nontracking requires some modification, and the second was the possibility that the deviations were due to the first approximations used in deriving the nontracking criterion. The latter possibility will be treated first.

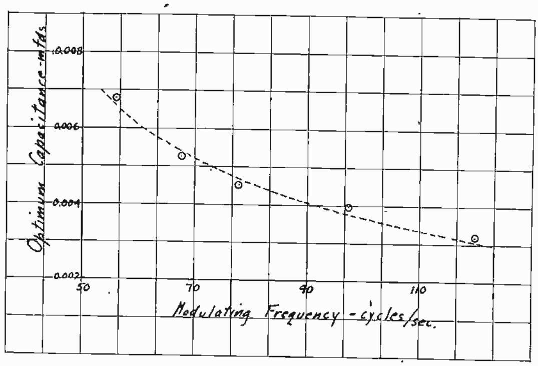


Fig. 4—Variation of optimum capacitance with modulating frequency.

Carrier frequency = 2080 cycles
Modulation factor = 0.65
Diode load resistance = 0.45 megohm
Peak input = 20 volts
$R_0 = 3$ megohms

Dotted curve plots $\omega_s \left(\dfrac{RR_0}{R+R_0}\right) C = \sqrt{\left[\dfrac{R_0}{(R+R_0)k}\right]^2 - 1}$

Circles represent experimental points

If a voltage $e = E_m(1 + k \cos \omega_s t) \cos \omega_p t$ is impressed on the diode circuit, the accurate formulation of the criterion set up by Terman and Morgan would be the following:

$$\text{(A)} \qquad \frac{e^{-\alpha/f_pRC}}{\omega_s RC} \geqq \frac{k \sin\left(\omega_s t_0 + \dfrac{\omega_s \beta}{f_p}\right)}{1 + k \cos \omega_s t_0}$$

where ω_s is the periodicity of the signal frequency, $\omega_p = 2\pi f_p$ is the periodicity of the carrier frequency, and α and β by the theorem of mean value[4] lie between zero and unity. Terman and Morgan took α and β as zero. In this particular case α and β must have values between

[4] E. S. Crawley and P. A. Caris, "A First Course in Calculus," (textbook), pp. 134–135.

zero and one half due to the fact that the exponential curve and the portion of the cosine curve under consideration both have continuously decreasing first derivatives. The introduction of the factor α tends to decrease the critical value of capacitance, while the factor β tends to increase it. In order to get the extreme limits of variation possible (but not necessarily the closest) we can use the two curves obtained for $\alpha=0$, $\beta=\frac{1}{2}$ and for $\alpha=\frac{1}{2}$, $\beta=0$. To determine t_0 we must maximize the right-hand member of relation (A),

$$\frac{d}{dt}\left[\frac{k \sin\left(\omega_s t + \dfrac{\omega_s \beta}{f_p}\right)}{1 + k \cos \omega_s t}\right] = 0$$

$$[1 + k \cos \omega_s t_0] \cos\left(\omega_s t_0 + \frac{\omega_s \beta}{f_p}\right) + k \sin\left(\omega_s t_0 + \frac{\omega_s \beta}{f_p}\right) \sin \omega_s t_0 = 0$$

$$\cos\left(\omega_s t_0 + \frac{\omega_s \beta}{f_p}\right) + k \cos\left(\omega_s t_0 + \frac{\omega_s \beta}{f_p} - \omega_s t_0\right) = 0$$

$$\cos\left(\omega_s t_0 + \frac{\omega_s \beta}{f_p}\right) = -k \cos\left(\frac{\omega_s \beta}{f_p}\right).$$

In making calculations for points on the two curves k was replaced by $(R+R_0)k/R_0$ and R by $RR_0/(R+R_0)$ to take account of the coupling circuit. The two curves calculated in this manner are drawn in fine dotted lines on Fig. 5. These limiting curves are misleading in the sense that they appear to allow considerable variation from the criterion derived by Terman and Morgan. Actually, however, the possible variation is much smaller for two reasons, the first is that α and β are considerably less than one half, especially for moderate and low percentages of modulation, thus bringing the limiting curves closer together, and the second is that the two opposing tendencies introduced by α and β exist simultaneously and partly cancel each other. It should also be realized that the possible variation from the nontracking criterion decreases as the ratio of the carrier frequency to the modulating signal frequency is increased. In the experimental study this ratio was approximately twenty, which is small compared with that encountered in most applications. For example, if we consider a station in the middle of the broadcast band with a carrier of 900 kilocycles and a signal frequency of 2000 cycles per second, the ratio is 450, and the two limiting curves become indistinguishable from the regular criterion. Even for the case of an intermediate frequency of say 175 kilocycles, the ratio is still over eighty and the limiting curves are close to the values given by the criterion. However, to return to the case under consideration, an examination of the limiting curves does reveal the fact that the deviation of the experimental points, at least for the lower percentages of modulation, cannot be due to the approximations just analyzed.

The other possible explanation for the deviation of the experimental points for extreme values of per cent modulation may be given in a qualitative manner. In the discussion previously given the existence of a condition for maximum detected signal, as the capacitance was varied, was based upon the increase of the saw-tooth variations as the

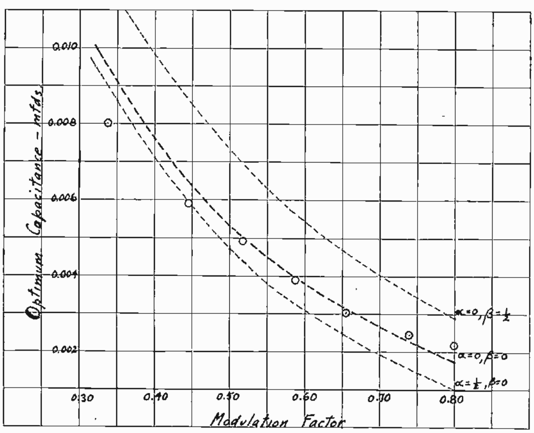


Fig. 5—Variation of optimum capacitance with modulation factor.
Carrier frequency = 2080 cycles
Signal frequency = 100 cycles
Diode load resistance = 0.45 megohm
Peak input = 20 volts
R_0 = 3 megohms

Heavy dotted curve plots $\omega_s \left(\frac{RR_0}{R + R_0}\right) C = \sqrt{\left[\frac{R_0}{(R + R_0)k}\right]^2 - 1}$

Circles represent experimental points

capacitance was decreased below the critical value and the start of nontracking when it was increased above it. However, for the value of capacitance given by the nontracking criterion, as illustrated in Fig. 1(b), small saw-tooth variations still exist and continue to decrease in magnitude as the capacitance is increased past the critical value (i.e., the value given by the nontracking criterion). This will in general tend to increase the optimum value of capacitance. At the same time the resistance of the diode, neglected up to this time, introduces an opposite tendency. This resistance produces a drop of voltage across the diode which increases with the increase of capacitance across the diode load resistance. This would tend to lower the optimum value of capacitance. Now although these opposing tendencies exist simultaneously, the effect of the voltage drop across the diode would be expected to predominate at the lower percentages of modulation, while the effect of the saw-tooth variations would be expected to become important for the higher percentages. This is true because the variation of the detected signal produced by the saw-tooth variations is directly dependent upon the difference between the high and the low parts of the modulation envelope. This difference is small for low percentages and large for high percentages of modulation. On the other hand, the drop across the diode will be larger at low percentages and smaller at high percentages of modulation. This is true because the value of the tube drop depends upon the value of capacitance across the diode load resistance, and an examination of the values given by the nontracking criterion shows that the capacitance is small for high percentages of modulation and increases rather rapidly for lower percentages of modulation. As a result, these considerations might account for the general character of the deviations.

The chief difficulty in the study was determining the exact value of the optimum capacitance for those cases where the change in the amplitude of the detected signal, as the capacitance was varied about the optimum value, was slight. This corresponds to a flattening of the curve in Fig. 2, which would necessitate the use of the averaging method previously referred to. A small amount of difficulty was experienced due to a slight frequency drift in the signal generators during measurements. The effect of this was minimized by allowing the apparatus to run for a length of time before making any measurements, and by averaging the frequency values before and after a series of measurements.

ACKNOWLEDGMENT

Thanks are due to Dr. Carl C. Chambers of the Moore School faculty for his helpful suggestions during the course of the study.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

THEORY OF LOOP ANTENNA WITH LEAKAGE BETWEEN TURNS*

BY

PAUL B. TAYLOR

(War Department, Aircraft Radio Laboratory, Wright Field, Dayton, Ohio)

Summary—*The theory of the receiving loop antenna of two turns with distributed leakage, conductive and reactive, is developed. Equations are derived for currents and voltages along the loop and at the terminals. For terminal currents and voltages the effect of leakage may be expressed by means of a single function H, and the loop may be represented by an equivalent two-mesh circuit of lumped constants. The effect of leakage upon the Q of a loop is discussed, and methods are described for the precise measurement of Q in the presence of leakage. Experimental applications are described.*

I. THE CONVERSION FACTORS OF A FIELD STRENGTH METER

THIS research was undertaken as part of an examination into the possibilities and limitations of the loop antenna in the quantitative measurement of radio field strength, and presents refinement of existing theory of the loop by including consideration of effects of distributed electromotive force and of distributed leakage between turns.

At the outset it is to be remarked that the mathematical treatment here presented is in strictness limited to the loop of only two turns. The solution of a problem even so simplified as this is, of course, of value in itself; but it is believed that the mathematical theory developed will be found to have considerably wider scope than just in the solution of this restricted problem. In the first place, the results obtained appear practically to apply with considerable precision to the loop of many turns. Second, the theory appears to be quite fundamental to the consideration of any circuit in which distributed electromotive force occurs. For instance, it is applicable to the transmission line in which either pickup or radiation occurs. Finally, the mathematical analysis forms a basis for extension to a rigorous theory of the loop of many turns, and to a general theory of leakage losses in inductance coils. It is proposed so to extend the theory in a subsequent article.

In any conventional field strength meter, three quantitative relations must be established.

* Decimal classification: R125.3. Original manuscript received by the Institute, March 22, 1937; revised manuscript received by the Institute, August 16, 1937.

(1) The conversion of field strength into induced electromotive force in the loop.
(2) The conversion of induced electromotive force into output voltage of the loop.
(3) The conversion of the output voltage into the reading of the indicating meter.

Concerning the last relation nothing need be said here. The first relation will be touched on briefly, but it is the relation between the electromotive force induced in the loop and the loop output to which main consideration will be given.

In practice a field strength meter is set up on level ground in an open space with the plane of the loop vertical and rotated about a vertical axis to the position of maximum signal reception. The turns of the loop (here restricted to two) are like polygons side by side in parallel planes, the plane of the loop. This plane is then perpendicular to the wave front and contains the direction of wave propagation and the electric vector. The short leads connecting the turns are perpendicular to the electric vector, so that in them no electromotive force is induced.

The field strength ϵ is constant in amplitude and direction, but varies in phase along the axis of wave propagation. The electromotive force induced in a differential length of wire dx will be

$$\epsilon \sin \theta \, dx$$

where θ is the complement of the angle between the wire and the electric vector. Writing the induced electromotive force per unit length as e we have

$$e = \epsilon \sin \theta .$$

The total electromotive force induced in one turn E is given as

$$E = \int_{\circ} e \, dx = \int_{\circ} \epsilon \sin \theta \, dx,$$

where the integration extends around the turn.[1]

In a loop of n turns it will be

$$nE .$$

It is convenient to resolve the field at each point into two components ϵ' and ϵ'' respectively in phase with the field at the center of the loop and 90 degrees out of phase. Resolving the induced voltage likewise into components E' and E'' we have

[1] The notation with certain noted exceptions is that of electric vectors.

$$nE' = n \int_{\circ} \epsilon' \sin \theta \, dx$$

$$nE'' = n \int_{\circ} \epsilon'' \sin \theta \, dx.$$

The first of these integrals may be shown to vanish. In what follows then we shall take e and E to refer to the out-of-phase components solely, as the only components which contribute to the induced electromotive force.

In general the integral is readily computed. In most cases, in which the loop dimensions are small compared to the wave length, it becomes

$$nE = j2\pi n \frac{A}{\lambda} \epsilon,$$

ϵ being the field at the center of the loop. As the formula shows, E is at 90 degrees to ϵ in phase.

The expression

$$j2\pi n \frac{A}{\lambda},$$

or some equivalent, is the required conversion factor between the field strength and the electromotive force induced in the loop.

The output voltage of the loop is related to the induced electromotive force by the scalar relation

$$nV = nQE,$$

where V is the output voltage per turn and Q is a function of the circuit constants. It is customary to tune the loop with a paralleling condenser across the output, so that V is larger than E and Q may be thought of as the amplification factor or gain of the circuit. The elementary derivation of Q considers a simple series circuit with lumped impedances and electromotive force and expresses Q as the ratio of loop reactance to effective circuit resistance. However, if the turns of the loop are sufficiently close together or the frequency sufficiently high, the procedure is not accurate, and the distributed nature of the induced electromotive force and the leakage between turns must be taken into account. This we proceed to do.

II. Representation of the Loop Circuit

To fix the attention, consider a loop antenna of two turns in a radio field, as shown schematically in Fig. 1. Fig. 2 shows the same loop in still more idealized form. The electromotive force induced by the radio field drives a current through the loop in a positive direction as shown. The output terminals of the loop, 2 and 3, are shown joined through a terminating impedance $2Z_b$. This impedance commonly consists of a tuning capacitance and includes also the (relatively high) impedance of the voltmeter which measures the output of the loop. The

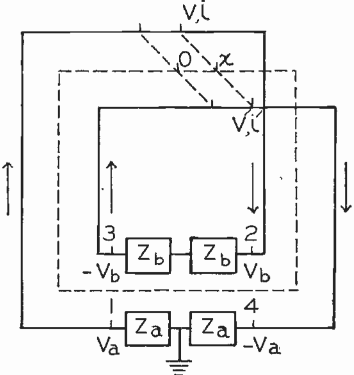


Fig. 1—Loop antenna with terminations.

loop circuit is shown opened at its center, 1, 4, and an impedance $2Z_a$, is shown inserted. This is for the purpose of increasing the generality of treatment and because a method of loop calibration to be described makes use of an impedance inserted at this point. In normal operation of a loop antenna $2Z_a$ is zero. The mid-point of $2Z_a$ is taken as ground.

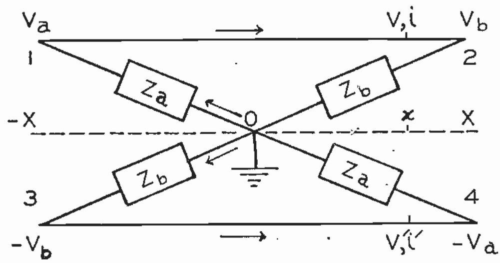


Fig. 2—Loop antenna idealized to show symmetries.

Points on the loop are designated by the ordinate system shown in Fig. 2. An origin is taken on each turn midway between terminals, and ordinate x measured positive and negative therefrom. Thus points directly opposite on the two turns will have the same ordinate. Electrical quantities appertaining to a point on the loop will bear the ordinate of that point as subscript, those belonging to one of the turns will be distinguished by primes from those belonging to the other turn.

The conductor impedance per unit length of wire z is composed of a large reactance term due to the self-inductance of the loop plus a small resistance term, which includes radiation resistance. The leakage admittance between turns per unit length y is due to capacitance between turns and to conductance across the insulation. This conductance term is small compared to $1/z$, but is not necessarily small compared to the reactive part of y. z and y are to be taken as constants. In most loops this condition is met sufficiently, although not always rigorously. At any rate this is the correct first approximation and to attempt any closer approximation would complicate the discussion to a degree out of all proportion to the value of the results obtained.

The induced electromotive force per unit length is in general not uniform, but varies along the wires. However, it is the same in both wires at the same ordinate. Further, since loops are usually constructed symmetrically about their vertical axis, we may take e as the same at equal distances on both sides of the origin. Analytically this restricts e to be an even function of the ordinate.

It is adequate to take the leakage current as between points on the two turns opposite to each other, that is, between points having the same ordinate.

From the prescribed symmetries it is seen that the mid-point of $2Z_b$ is at the same potential as the mid-point of $2Z_a$, that is, at ground.

The derivation of the circuit equations now proceeds as follows:

The eletromotive force e is subject to the symmetrical relations

$$e_x = e_x' = e_{-x} = e_{-x}'. \tag{1}$$

Likewise the current i and the potential v are subject to the relations

$$i_x = i_{-x}' \tag{2}$$

$$v_x = -v_{-x}'. \tag{3}$$

The potential in one turn at the ordinate x is given by

$$v_x = \int_0^x (e - iz)dx + v_0. \tag{4}$$

At the same ordinate in the other turn it is given by

$$v_x' = \int_0^x (e - i'z)dx - v_0. \tag{5}$$

The leakage current between points of the same ordinate is

$$-di = y(v_x - v_x')dx. \tag{6}$$

Substituting for v_x and v_x',

$$\frac{di}{dx} = yz\int_0^x (i - i')dx - 2yv_0 \tag{7}$$

$$\begin{aligned}\frac{d^2i}{dx} &= yz(i_x - i_x') \\ &= yz(i_x - i_{-x}).\end{aligned} \tag{8}$$

The derivation of the solution of this differential equation will be found in the Appendix. The solution is

$$i_x = i_0 + B \sinh cx, \tag{9}$$

where,

$$c = \sqrt{2yz}, \tag{10}$$

and B is a constant of integration to be evaluated.

Since the hyperbolic sine is an odd function we have

$$i_{-x} = i_0 - B \sinh cx. \tag{11}$$

The potential may now be derived by substituting in (4) the expression just obtained for the current.

$$\begin{aligned} v_x &= \int_0^x edx - z\int_0^x (i_0 + B \sinh cx)dx + v_0 \\ &= \int_0^x edx - i_0 zx + \frac{zB}{c}(1 - \cosh cx) + v_0 \end{aligned} \tag{12}$$

$$\begin{aligned} -v_x' &= v_{-x} \\ &= -\int_0^x edx + i_0 zx - \frac{zB}{c}(1 - \cosh cx) + v_0. \end{aligned} \tag{13}$$

The form of the integral is justified because e is an even function of x. The potential difference between turns at ordinate x is given by

$$v_x - v_x' = \frac{2zB}{c}(1 - \cosh cx) + 2v_0. \tag{14}$$

The undetermined constant B is now to be evaluated by equating this expression for the potential difference between turns to the current-impedance drop across the leakage path. Over a differential length of line the leakage current is $-di$ and the leakage admittance ydx, so that the potential difference between turns is also given by

$$v_x - v_x' = \frac{-\,di}{ydx} = -\frac{cB}{y}\cosh cx, \tag{15}$$

the right-hand member being obtained by differentiation of (9).

From (14) and (15)

$$-\frac{2zB}{c}\cosh cx + \frac{2zB}{c} + 2v_0 = -\frac{cB}{y}\cosh cx. \tag{16}$$

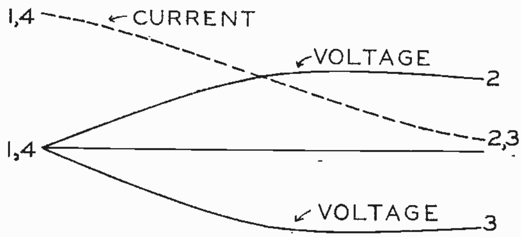


Fig. 3—Distribution of currents and voltages about loop (Z_a zero, c and z reactive).

Applying the method of undetermined coefficients we obtain for the constant term

$$\frac{2zB}{c} + 2v_0 = 0,$$

whence,[2]

$$B = -\frac{c}{z}v_0. \tag{17}$$

This value for B in (9) and (11) yields

$$i_x = i_0 - \frac{c}{z}v_0 \sinh cx \tag{18}$$

$$i_{-x} = i_0 + \frac{c}{z}v_0 \sinh cx, \tag{19}$$

and in (12) and (13)

$$v_x = \int_0^x edx - i_0 zx + v_0 \cosh cx \tag{20}$$

$$v_{-x} = -\int_0^x edx + i_0 zx + v_0 \cosh cx. \tag{21}$$

These four equations are a solution of the problem and by their

[2] The coefficients of the hyperbolic cosine terms are reconciled by reference to (10).

form show plainly the distribution of current and voltage along the loop.

By addition and subtraction we may obtain also

$$i_x + i_{-x} = 2i_0 \tag{22}$$

$$i_x - i_{-x} = -\frac{2c}{z} v_0 \sinh cx \tag{23}$$

$$v_x + v_{-x} = 2v_0 \cosh cx \tag{24}$$

$$v_x - v_{-x} = 2\left(\int_0^x e dx - i_0 zx\right). \tag{25}$$

III. Terminal Currents and Voltages

The currents and voltages at the loop terminals are of particular interest and importance for succeeding developments. For them a special notation is desirable. Let X denote the length of one-half turn (the ordinate of terminals 2 and 4), and let the terminal currents and voltages be designated by subscripts a or b according as they pertain to a or b terminating impedances. Thus we write

$$\begin{aligned} i_X &= i_b = i_{-X}' \\ i_{-X} &= i_a = -i_X' \\ v_X &= v_b = -v_{-X}' \\ v_{-X} &= v_a = -v_X'. \end{aligned} \tag{26}$$

Let us also denote by Z the series impedance of one-half turn

$$Z = zX, \tag{27}$$

and by E the total electromotive force induced in one whole turn

$$E = 2\int_0^X e dx. \tag{28}$$

For the terminating currents and voltages (22) to (25) now appear as

$$i_a + i_b = 2i_0 \tag{29}$$

$$i_b - i_a = -\frac{2c}{z} v_0 \sinh cX \tag{30}$$

$$v_a + v_b = -v_0 \cosh cX \tag{31}$$

$$v_b - v_a = E - 2i_0 Z. \tag{32}$$

Eliminating v_o and i_o from these four equations, we obtain

$$v_b + v_a = (i_a - i_b)\frac{z}{c}\coth cX \tag{33}$$

$$v_b - v_a = E - (i_a + i_b)Z. \tag{34}$$

In addition we have for the circuits terminating the loop

$$v_a = -\, i_a Z_a \tag{35}$$

$$v_b = i_b Z_b. \tag{36}$$

These four equations (33) to (36) completely define the terminal currents and voltages, and their explicit solution may be written in terms of the circuit constants and the induced electromotive force.

At this point it is desirable to introduce the notation

$$H = \frac{z}{c}\coth cX. \tag{37}$$

The equations (33) to (36) then yield explicit solutions for the terminal currents in terms of the circuit constants:

$$i_a = \frac{E(Z_b + H)}{(Z + Z_a)(Z_b + H) + (Z + Z_b)(Z_a + H)} \tag{38}$$

$$i_b = \frac{E(Z_a + H)}{(Z + Z_b)(Z_a + H) + (Z + Z_a)(Z_b + H)}. \tag{39}$$

IV. Significance of the Leakage Function H

The significance of the term H is more than just a convenience of notation. In this single term is concentrated the entire effect of the distributed leakage. When leakage disappears H becomes infinite, thereby reducing all the circuit equations to those of a single series circuit.

It may serve to fix in mind the nature of the quantity H by observing how it depends upon c and in turn upon the leakage admittance y. When cX is sufficiently small,

$$\coth cX \doteq \frac{1}{cX},$$

and

$$H \doteq \frac{z}{c^2X} = \frac{1}{2yX}; \tag{40}$$

so the H might be thought of roughly as the lumped impedance between turns of the whole of the leakage path.

If predominantly z is inductive and y capacitive, as commonly they would be considered, and cX is no longer restricted to small values,

$$cX \doteq j|c|X$$

$$\coth cX \doteq \coth j|c|X = -j\cot|c|X$$

$$H \doteq -j\left|\sqrt{\frac{z}{y}}\right|\cot|c|X, \tag{41}$$

and is a capacitive impedance so long as cX is less than $\pi/2$.

If y is predominantly conductive, z being still inductive, H will be complex.

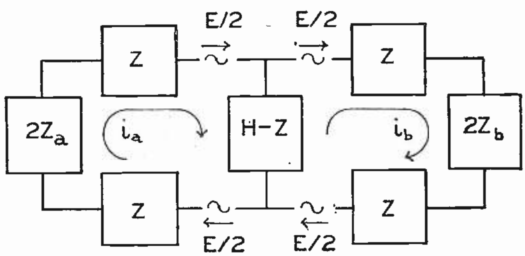


Fig. 4—Equivalent loop circuit with lumped constants.

The rôle played by the distributed leakage in the loop is further elucidated by showing that there exists a doubly branched circuit of lumped constants equivalent to the loop and invariant to the terminating impedances. The circuit is represented in Fig. 4. The equivalence may be shown by demonstrating that (33) to (36) are satisfied by such a circuit. Here the leakage is shown lumped into a single circuit element, the value of which, however, is not H but $H-Z$. Since H is a function of the loop constants only and not of the terminations, the invariance of the equivalent circuit to termination is established.

It is possible to discuss the loop, insofar as its effects on external circuits are concerned, by reference solely to this equivalent network and most of the results here presented were first obtained in that manner. However, in this paper we shall continue to consider the loop circuit itself.

V. Three-Resistor Method for Measuring Loop Gain

Having now in hand a complete general theory of the two-turn loop antenna, we are ready to examine its output, particularly in its bearing on the use of the loop for the quantitative measurements of radio field strengths. We shall derive usable expressions for the gain of the loop and for determining the effect of leakage upon the gain, and a method of self-calibration of the loop from the measurement of output voltages.

In order to discover the properties of a loop we propose to measure its output under four different terminations. This being done, we shall apply the condition of resonance. The results expressed in terms of electric vectors will then be recast in terms of tensor values and so reduced to terms of actual voltmeter readings.

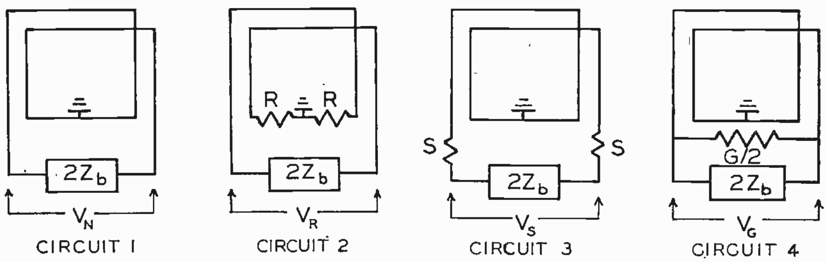


Fig. 5—Circuits for measurement of loop gain.

Denote the vector ratio of output voltage of the loop to input voltage by q, its reciprocal by p. We have

$$q = \frac{2v_b}{2E} = \frac{i_b Z_b}{E} \cdot \tag{42}$$

By (39)

$$\frac{E}{i_b} = (Z + Z_b) + (Z + Z_a)\frac{(Z_b + H)}{(Z_a + H)} \tag{43}$$

$$p = \frac{1}{q} = \frac{E}{i_b Z_b} = \frac{Z + Z_b}{Z_b} + \frac{(Z + Z_a)(Z_b + H)}{Z_b(Z_a + H)} \cdot \tag{44}$$

Thus far no restriction whatever has been placed upon the nature of Z_a. We now restrict it to be a small pure resistance so that terms in Z_a^2 may be neglected in comparison with the squares of reactive terms and in comparison with H^2. In general we shall neglect similarly all second-order resistive terms. Accordingly,

$$\begin{aligned} p &= \frac{Z + Z_b}{Z_b} + \frac{Z + Z_a}{Z_b}\left(1 + \frac{Z_b}{H} - \frac{Z_a}{H} - \frac{Z_a Z_b}{H^2}\right) \\ &= \frac{D}{Z_b H} + \frac{Z_a(Z_b + H)(H - Z)}{Z_b H^2}, \end{aligned} \tag{45}$$

where,

$$D = 2ZH + Z_b(Z + H). \tag{46}$$

We shall now write expressions for p for the four circuit variations which are shown in Fig. 5.

Circuit 1. The normal circuit. In normal operation the loop is grounded directly at its mid-point, that is,

$$Z_a = 0.$$

Write the values of p, q, and v_b for this case as p_N, q_N, v_N.

$$\begin{aligned} p_N &= \frac{1}{q_N} = \frac{E}{v_N} = \frac{D}{Z_b H} \\ &= \frac{2ZH + Z_b(Z + H)}{Z_b H.} \\ &= 1 + \frac{2Z}{Z_b} + \frac{Z}{H} \cdot \end{aligned} \tag{47}$$

Circuit 2. Let a small known pure resistance $2R$ be inserted in the loop at the mid-point.

$$Z_a = R.$$

Let p_R and v_R be the values of p and v_b in this circuit. From (45) and (47)

$$p_R = p_N + \frac{R(Z_b + H)(H - Z)}{Z_b H^2} \cdot \tag{48}$$

Form the function f_R, defined thus:

$$f_R = \frac{1}{R}\left(\frac{v_N}{v_R} - 1\right) = \frac{1}{R}\left(\frac{p_R}{p_N} - 1\right) = \frac{(Z_b + H)(H - Z)}{ZH} \cdot \tag{49}$$

Circuit 3. Let now the normal circuit (Z_a zero) be modified by insertion of a small known pure resistance $2S$ between the loop and the tuning capacitor (impedance $2Z_b$) so that the new terminating impedance is $2(Z_b+S)$, and the current becomes by (39)

$$i_b = \frac{EH}{2ZH + (Z_b + S)(Z + H)} \cdot$$

But let us continue to measure voltage across the capacitor $2Z_b$ (and not across the loop terminals, which would be across $(2Z_b+S)$). Denote this voltage by $2v_S$ and define p_S as follows:

$$\begin{aligned} p_S &= \frac{E}{v_S} = \frac{E}{i_b Z_b} = \frac{2ZH + (Z_b + S)(Z + H)}{Z_b H} \\ &= p_N + \frac{S(Z + H)}{Z_b H} \cdot \end{aligned} \tag{50}$$

Form the function f_S defined by

$$f_S = \frac{1}{S}\left(\frac{v_N}{v_S} - 1\right) = \frac{1}{S}\left(\frac{p_S}{p_N} - 1\right) = \frac{Z+H}{D}. \tag{51}$$

Circuit 4. Finally, let a third modification of the normal circuit be set up. As in the normal circuit Z_a is to be zero. Across the normal termination $2Z_b$ let be shunted a small known pure conductance $G/2$, so that the terminating admittance in this case becomes

$$\frac{1}{2}\left(\frac{1}{Z_b} + G\right).$$

Let the voltage across this termination, that is, across the loop, be in this case $2v_G$, and define p_G by

$$p_G = \frac{E}{v_G} = 1 + 2Z\left(\frac{1}{Z_b} + G\right) + \frac{Z}{H}$$

$$= p_N + 2ZG \tag{52}$$

by (47).

Define f_G thus:

$$f_G = \frac{1}{G}\left(\frac{v_N}{v_G} - 1\right) = \frac{1}{G}\left(\frac{p_G}{p_N} - 1\right) = \frac{2ZZ_bH}{D}. \tag{53}$$

Up to this point no restriction has been placed upon the impedance Z_b except that it maintain the same fixed value in all four of the p functions. We shall now consider that it is tuned to resonance with the loop. This is, of course, universal in practical operation and for the remainder of this paper we shall imply without specific notation that resonance obtains.

The condition for resonance in the normal circuit is given from (46) as

$$D = 2ZH + Z_b(Z+H) \doteq 0. \tag{54}$$

This is also the condition of resonance in the three other circuits. The solution is

$$Z_b = \frac{-2ZH}{Z+H}. \tag{55}$$

In a physical circuit, the above value of Z_b cannot be obtained identically since the right-hand member of (55) contains a small negative resistance and the tuning capacitor and voltmeter whose impedance constitutes Z_b, must contribute a positive resistance term.

However, except for special cases in which the resistive component of Z_b becomes important, this solution is permissible, the error in tensor value being of the second order. Thus we substitute for Z_b from (55) in (49) and (53), obtaining

$$q_N = \frac{-2ZH^2}{(Z+H)D} \tag{56}$$

$$f_G = \frac{-(2ZH)^2}{(Z+H)D}. \tag{57}$$

The substitution for Z_b in f_R, (49), may be shown to be permissible as long as the resistive term in Z_b is small compared to H. The result is

$$f_R = \frac{(H-Z)^2}{(Z+H)D}. \tag{58}$$

The next step is to connect the four functions q_N, f_R, f_S, f_G. By (57) and (58)

$$\sqrt{-f_G f_R} = \frac{2ZH(H-Z)}{(Z+H)D}. \tag{59}$$

By (53) and (57)

$$\sqrt{-f_G f_S} = \frac{2ZH(Z+H)}{(Z+H)D}. \tag{60}$$

From (56), (59), and (60) it follows that

$$\tfrac{1}{2}[\sqrt{-f_G f_R} + \sqrt{-f_G f_S}] = \frac{2ZH^2}{(Z+H)D} = -q_N. \tag{61}$$

Equation (61), it will be seen, relates q_N, the gain of the loop, to a set of output voltage measurements through the functions f_R, f_S, f_G. The four voltages v_N, v_R, v_S, v_G are vectors, but all very nearly parallel, so that to form the ratios v_N/v_R, v_N/v_S, v_N/v_G we may use their tensor values as read on an ordinary vacuum tube voltmeter. Write these meter readings as V_N, V_R, V_S, V_G, respectively. The functions f_R, f_S, f_G are substantially real and may be calculated from the observed voltmeter readings by means of (49), (51), and (53).

q_N is substantially a pure imaginary and its tensor value is the quantity commonly written Q, the ratio of output voltage of the loop to electromotive force induced in the loop. Hence the form of (61) for computation from observable data is

$$Q = \tfrac{1}{2}\sqrt{f_G}(\sqrt{f_R} + \sqrt{f_S})$$
$$= \frac{1}{2}\sqrt{\frac{1}{G}\left(\frac{V_N}{V_G} - 1\right)}\left[\sqrt{\frac{1}{R}\left(\frac{V_N}{V_R} - 1\right)} + \sqrt{\frac{1}{S}\left(\frac{V_N}{V_S} - 1\right)}\right]. \quad (62)$$

Equation (62) expresses the gain of the loop in terms of three calibrated resistors R, S, $1/G$, and four output voltages V_N, V_R, V_S, V_G. None of the circuit constants of the loop nor the field strength needs to be known. Even the output voltages need to be known only relatively. Because of freedom from auxiliary equipment and auxiliary standards (except the resistors) this method for calibrating a loop may be classed as a self-calibration. It has been described previously by the author, but without proof.[3]

Less exact methods for finding loop gain consist in inserting the series resistor at but one point, and using the approximation

$$Q \doteq \sqrt{f_G f_R}, \quad (63)$$

or,

$$Q \doteq \sqrt{f_G f_S}. \quad (63a)$$

In the ordinary loop (Z inductive, H capacitive) the former value is too large and the latter too small. The fractional error in the former is approximately

$$\left|1 - \frac{Z}{H}\right| - 1,$$

and in the latter

$$\left|1 + \frac{Z}{H}\right| - 1.$$

VII. Capacitor Method for Measuring Loop Gain

As an alternative to the use of the auxiliary conductance $G/2$ in finding the loop gain the value of the terminating impedance Z_b may be used. This requires that the tuning capacitor be calibrated and that the signal frequency be known. On the other hand it avoids the uncertainties as to the exact value of high resistance elements at radio frequencies, and is to be preferred for measurements of the highest precision.

In this method f_R and f_S are found as in the three-resistor method. We have by (51) and (58),

$$\tfrac{1}{2}(1 + \sqrt{f_R/f_S}) = \frac{H}{Z + H},$$

[3] "A compact radio field strength meter," Proc. I.R.E., vol. 22, pp. 191–200; February, (1934).

and by (51) and (55)

$$Z_b f_S = \frac{-2ZH}{D}.$$

Hence,

$$\tfrac{1}{2}Z_b(f_S + \sqrt{f_R f_S}) = \frac{-2ZH^2}{(Z+H)D}.$$

This by (56) is q_N. The equation as it stands is in shape for numerical computation. Thus we write

$$Q = \tfrac{1}{2}\left| Z_b \right| (f_S + \sqrt{f_R f_S}). \tag{64}$$

The method commonly employed for finding loop gain by means of a calibrated capacitor and a single series resistor in the loop is a simplification of this method, and uses the approximation

$$Q \doteq \left| Z_b \right| f_R, \tag{65}$$

or,

$$Q \doteq \left| Z_b \right| f_S. \tag{65a}$$

It may be shown that the fractional error in the former of these is approximately

$$\left| 1 - \frac{3Z}{H} \right| - 1,$$

and in the latter

$$\left| 1 + \frac{Z}{H} \right| - 1.$$

It is thus seen that less inaccuracy is incurred in the common method by inserting the series resistor at the loop output (S) than at the loop mid-point (R).

These approximate methods of measuring the loop gain all become more exact as the leakage becomes less. The expressions given are the errors in the measured value of the loop gain when these approximate methods are used on loops subject to leakage; that is, they indicate how much the leakage affects the measurements.

VIII. Effect of Leakage on Loop Gain

We propose now to examine how much the leakage affects the actual gain. To do so we return to the expression for reciprocal gain (47)

$$p_N = \frac{1}{q_N} = 1 + \frac{2Z}{Z_b} + \frac{Z}{H}.$$

It might appear that the extent of the effect of leakage was comprised in the term Z/H. However, the loop with leakage is not tuned to reasonance by the same value of Z_b as is the loop without. Accordingly, we write for $1/Z_b$ in the light of the discussion following (55)

$$\frac{1}{Z_b} = -\left|\frac{Z+H}{2ZH}\right|_i + \left|\frac{1}{Z_b}\right|_r. \tag{66}$$

The subscripts i and r denote respectively the imaginary and real parts of the vector. The real part of $1/Z_b$ is the shunt leakage of the insulation and voltmeter paralleling the capacitor and does not change with tuning.

$$p_N = 1 - 2Z\left|\frac{Z+H}{2ZH}\right|_i + 2Z\left|\frac{1}{Z_b}\right|_r + \frac{Z}{H}. \tag{67}$$

Using a prime to denote the reciprocal gain of the loop free from leakage (H infinite),

$$p_N' = 1 - 2Z\left|\frac{1}{2Z}\right|_i + 2Z\left|\frac{1}{Z_b}\right|_r \tag{67a}$$

$$\begin{aligned} p_N - p_N' &= Z\left|\frac{1}{Z} - \frac{Z+H}{ZH}\right|_i + \frac{Z}{H} \\ &= -Z\left|\frac{1}{H}\right|_i + \frac{Z}{H} \\ &= Z\left|\frac{1}{H}\right|_r. \end{aligned} \tag{68}$$

The three terms of (68) are all substantially pure imaginaries. It is instructive to note the effect of the character of the leakage admittance on the loop gain. When the leakage is purely reactive H is substantially pure imaginary, and it follows from (68) that the reactive leakage does not affect the gain. It does affect, however, the measured value of the gain as given by the approximate methods which neglect leakage, (63) and (64). When the leakage is either partly conductive or entirely so, H is complex and the gain is reduced.

IX. Illustrative Example

An experimental application of these ideas to a concrete case may be of interest. The loop studied was 30 inches square, of two turns of No. 22 tinned copper wire (0.02535 inch in diameter) with 0.5 inch between turns. The turns were supported at the corners only on bakelite spacers. Measurements were made at 4050 and 6000 kilocycles. The turns were terminated in fuse holders at each end into which could be inserted the resistors S or R or circuit closing jumpers. A vacuum tube voltmeter was connected across one half of the loop (between the grounded mid-point and one high voltage terminal). The data are taken for one side of the circuit. The loop was placed in a fairly uniform field which was held constant during the measurements.

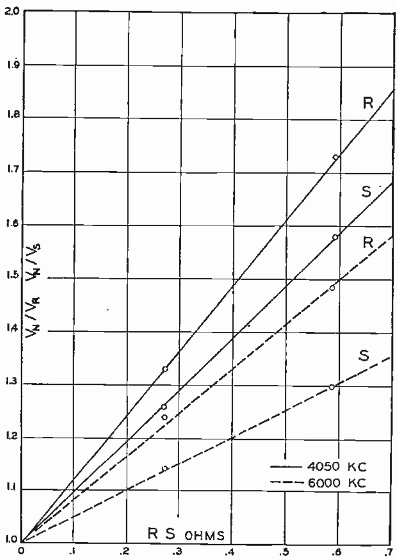


Fig. 6—Graph for determination of f_R and f_S using (69).

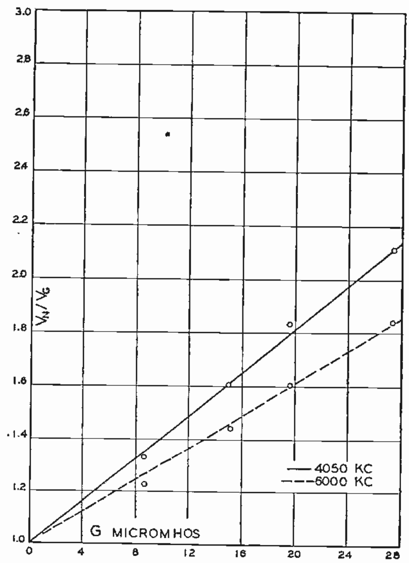


Fig. 7—Graph for determination of f_G.

The function f_R is best found by observing a series of values of V_R for a set of resistors R and plotting V_N/V_R against R. Equation (49) may be written

$$\frac{V_N}{V_R} = Rf_R + 1; \tag{69}$$

so that the points should lie on a straight line through the point 0,1. The slope of this line is the value of f_R desired. In similar fashion f_S and f_G are found. The graphs for finding f_R, f_S, f_G are shown in Figs. 6 and 7, and the values found, together with the values for Q computed therefrom by (62), are given in Table I.

TABLE I
COMPUTATION OF Q

Frequency	f_R	f_S	f_G	Q
4050 kc	1.230	0.985	0.0307×10^6	185
6000	0.843	0.514	0.0407	165

The difference between f_R and f_S is due to the presence of the distributed capacitance. If Q were computed by any method which ignores this distributed capacitance, appreciable error would result. Thus the use of (63) would lead to a value of Q six per cent too high at 4050 kilocycles and ten per cent high at 6000 kilocycles.

For the computation of Q by the capacitor method the tuning condenser used was calibrated on a capacitance bridge and (64) applied with results as shown in Table II.

TABLE II
COMPUTATION OF Q

Frequency	Tuning Capacitance	$2Z_b$	$\frac{1}{2}(f_S+\sqrt{f_R f_S})$	Q
4050 kc	106 $\mu\mu$f	370 ohms	1.042	193
6000	45	588	0.586	172

The values of Q obtained by the two methods agree to the precision with which the tuning capacitors and the G resistors were known. Apparently a little capacitance was introduced by the fuse blocks which was not included in the measurement of the tuning capacitance.

The simplified capacitor methods of Q measurement would be entirely unsuitable at these frequencies. The former of (65) yields a result 18 per cent in error at 4050 kilocycles and 44 per cent in error at 6000 kilocycles.

The relative magnitudes of the series impedance of the loop and the leakage impedance may be obtained from the relation

$$\frac{Z}{H} = \frac{\sqrt{f_R} - \sqrt{f_S}}{\sqrt{f_R} + \sqrt{f_S}}. \tag{70}$$

The relation follows from (51) and (58).

If the tensor values of the radicals are used in the right-hand member of (69) the real part of $-Z/H$ is obtained very approximately; that is, the ratio of the reactive part of the leakage impedance of the loop to one quarter of the series reactance. It is worth while to compare the values of $-Z/H$ obtained by means of (70) with those obtained from the computed distributed capacitance and self-inductance of the loop.

The geometrically computed self-inductance of the loop was 12.5 microhenrys, from which Z and z, the impedance per centimeter of wire, may be computed.

The geometrically computed capacitance between turns was 0.0757 micromicrofarads per centimeter.

Assuming that the principal source of leakage between turns is capacitive, y and c may be computed and hence H by means of (37).

This has been done and the value of $-H/Z$ found compared in Table III with that found by means of the observed data and (70).

TABLE III
VALUES OF $-Z/H$
(Units are reactive ohm and centimeter)

Frequency	z	y	c	X	Z	$-H$
4050 kc	0.522	1.926×10^{-6}	0.001419	152.4	79.6	1728
6000	0.777	2.853×10^{-6}	0.002106	152.4	118.0	1189
Frequency	$-Z/H$ (by geometry)		$-Z/H$ (by (70))		Ratio of Frequencies Squared	
4050 kc	0.0460		0.056			
6000	0.0992		0.123			
Ratio	0.464		0.455		0.456	

The agreement between the computed and observed values of $-Z/H$ is sufficient to verify the ideas here advanced. The disparity appears to be due not to error in observation but rather to additional capacitances in the fuse blocks at the terminals, which were not considered in the computed capacitance. The ratio $-Z/H$ should vary approximately as the square of the frequency in the range of impedances here considered and in this regard the computed and observed values agree much better.

APPENDIX

Solution of a Differential Equation

The solution of the differential equation

$$\frac{d^2 i}{dx^2} = yz(i_x - i_{-x}) \tag{8}$$

is obtained by observing that the right-hand member is an odd function of x. The left-hand member must then be odd also and may be expanded in a series of odd powers of x.

$$\frac{d^2 i}{dx^2} = 3\cdot 2A_3 x + 5\cdot 4A_5 x^3 + \cdots.$$

The successive integrals are

$$\begin{aligned}
\frac{di}{dx} &= A_1 + 3A_3x^2 + 5A_5x^4 + \cdots \\
i_x &= A + A_1x + A_3x^3 + A_5x^5 + \cdots \\
-\, i_{-x} &= -\, A + A_1x + A_3x^3 + A_5x^5 + \cdots \\
i_x - i_{-x} &= 2A_1x + 2A_3x^3 + 2A_5x^5 + \cdots \\
&= 2(i_x - A).
\end{aligned}$$

Substituting this for $(i_x - i_{-x})$ in (8) we obtain

$$\frac{d^2 i}{dx^2} = 2yz(i - A).$$

This is a linear differential equation of the second order with constant coefficients. The standard solution is

$$i = A + B \sinh cx + C \cosh cx,$$

where,

$$c = \sqrt{2yz},$$

and the coefficients A, B, C are constants to be determined. By reference to (8) it is seen that the cosh term cannot appear in the solution desired. Also since at the origin the sinh function vanishes we have

$$A = i_0,$$

and the solution may be written

$$i = i_0 + B \sinh cx. \tag{9}$$

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

THE CLARIFICATION OF AVERAGE NEGATIVE RESISTANCE WITH EXTENSIONS OF ITS USE*

By

CLEDO BRUNETTI

(Formerly, University of Minnesota, Minneapolis, Minnesota; Now, Department of Electrical Engineering, Lehigh University, Bethlehem, Pennsylvania)

Summary—*A treatment of the subject of negative resistance and the application to the problem of nonlinear oscillations shows the necessity of clarifying the meaning attached to the commonly used term "average" negative resistance. An average effective resistance* R_n, *based on a simple energy consideration, is introduced which when applied to the above problem affords a correct prediction of the amplitude of oscillation and supplies a meaning for the term "average" negative resistance. An earlier method known as the secant method of predicting the amplitude of oscillation is investigated and found to yield results not in accord with experimental behavior.*

The quantity R_n, *being a function of the oscillation voltage* V *suggests a new type of curve called the* $R_n - V$ *curve which serves to predict the possible amplitudes of oscillation when the negative resistance device is connected to a parallel R, L, and C circuit. Following this a criterion for amplitude stability is deduced which allows one to determine whether or not a possible amplitude is also stable by simply observing the slope of the* $R_n - V$ *curve at the point corresponding to the possible amplitude. A problem of so-called "oscillation hysteresis" earlier reported in the literature is described, and it is shown how the* $R_n - V$ *curve for this particular case offers a simple explanation of the phenomenon.*

Finally a set of curves known as constant R_n *curves are introduced which serve to predict the amplitude of oscillation when the values of the tuned circuit constants are held fixed and the operating point is changed. Incomplete curves have been previously published showing the variation of oscillation amplitude with the operating point. Because of the impossibility of obtaining constant oscillations in the regions of instability the complete curves were therefore not given. A comparison of the results deduced from the quantity* R_n *with the Appleton-van der Pol solution and analysis shows the power of the concept to yield useful results simply.*

INTRODUCTION

ON reviewing the literature on the subject of negative resistance with the intention of obtaining some practical ideas on the mechanism involved one soon finds himself either in a labyrinth of theoretical treatments or else following some interpretation of the former. The theoretical attack[1] is essential even though it may not always be easy to adapt its results in practice.

Many interpretations of the more theoretical works have appeared in the past fifteen years. These being of a secondary nature they are

* Decimal classification: R133. Original manuscript received by the Institute, June 22, 1937.

[1] B. van der Pol, "The nonlinear theory of electric oscillations," PROC. I.R.E., vol. 22, pp. 1051–1086; September, (1934).

often characterized by the presence here and there of a certain vagueness in the principles emphasized and the actual meanings attached to the terms employed. For example, we find the words "negative resistance" used unsparingly, yet little attention is ever given to the fact that one not initiated to the theories of nonlinear oscillation is likely to think of the term as the opposite of positive resistance. Thus one might possibly expect that if he were given a box with two terminals brought out labeled "negative resistance" and if he were to connect in series with these terminals a positive resistance of the same value, then any electrical oscillation once set up in the closed circuit would persist at a constant value indefinitely or until some other disturbance came along to change it. Furthermore, if in series with this combination one were to connect a small electromotive force a current would flow whose magnitude would increase without limit. Such phenomena, however, do not take place with devices which possess what is commonly known as negative resistance.

In order to understand what is known as the negative resistance of a device it is necessary to consider its current-voltage characteristic (Fig. 2). This characteristic displays a negative slope over some portion of its range and according to the conventional definition[2] of plate or variational resistance applied to vacuum tubes possesses a negative variational resistance over the region of negative slope. When operating on this region a slight change in either the current or voltage is accompanied by a corresponding negative change in the other. In general this negative property is displayed only for changes in either the current or voltage and the instantaneous values of both of these quantities is at all times positive. An exception is the case of the dynatron which, by virtue of the fact that a part of its characteristic extends below the voltage axis, possesses a negative direct-current as well as a negative variational resistance. The important point to note is that the negative variational resistance property is restricted to a limited part of the characteristic. It is this factor which prevents the current, in the instance cited previously, from increasing without limit.

Negative resistance devices are classified into two types[3] known as current and voltage controlled. The carbon arc is an example of the first type while the dynatron serves to illustrate the second type. It has been pointed out[4] that the phenomena associated with one may be described in terms of analogous phenomena of the other.

[2] E. L. Chaffee, "Theory of Thermionic Vacuum Tubes," McGraw-Hill Book Co., (1933).

[3] K. Steimel, "Stability and self-oscillation of electric circuits," *Zeit. für Hochfrequenz.*, vol. 36, pp. 161–170; November, (1930).

[4] E. W. Herold, "Negative resistance and devices for obtaining it," Proc. I.R.E., vol. 23, pp. 1201–1223; October, (1935).

The Negative Transconductance Device

Another voltage-controlled device and the type adopted for this study is the negative transconductance arrangement shown in Fig. 1. The results obtained using this type are general and applicable to the other types as well. A type 58 tube is connected as shown. The voltage E_{32} is chosen so as to make grid No. 3 negative with respect to the

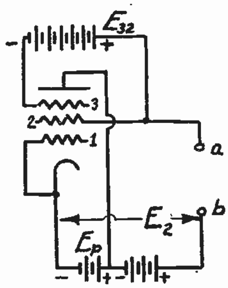


Fig. 1—Negative transconductance device.

cathode. Electrons attracted by the high positive potential of grid No. 2 (anode) are repelled by the negative potential of grid No. 3. A slight negative increment in voltage across *ab* is transmitted simultaneously to both the anode and grid No. 3 causing the latter to repel more electrons and the net current to the anode to increase. The transconductance between grid No. 3 and the anode is therefore negative. The cur-

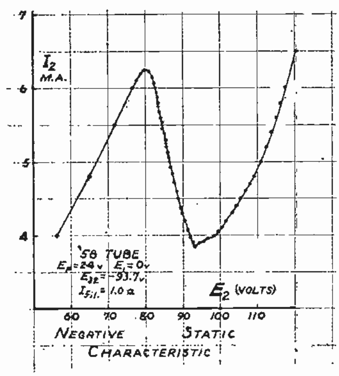


Fig. 2

rent flowing to the plate is not utilized. By applying a small negative bias to grid No. 1 the magnitude of the above effect may be controlled and thus the negative slope of the current-voltage characteristic may be varied. A more practical circuit for alternating-current phenomena may be had by replacing the bias between grids Nos. 2 and 3 with a large condenser. The bias for grid No. 3 is then applied directly from the cathode through a high resistance.

A static characteristic for the circuit of Fig. 1 is shown in Fig. 2. The negative variational resistance at the center of the characteristic is about 3800 ohms. It has been possible to obtain characteristics with a negative variational resistance as low as 1300 ohms. These low values are especially desirable.[5]

Behavior of the Negative Resistance Device

If a small external alternating voltage is impressed across the terminals *ab* of Fig. 1, the battery voltage on grid 2 being suitably chosen,[6] an alternating current will flow in a direction opposite to the impressed electromotive force. The average alternating-current power supplied by the external source will be negative and equal to the voltage squared divided by twice the variational resistance (considered constant). The device therefore delivers power to the external system. The source of this power is the battery, and the tube regulates the delivery of the power in conformance with the demand made on it by the external controlling voltage. From the standpoint of power obtainable it is advisable at any particular value of voltage to have the variational resistance of the tube as small as possible.

If a condenser C in parallel with an inductance L and its associated resistance R is connected to a negative resistance device oscillations in the parallel tank circuit begin when L/RC is just equal to the negative variational resistance at the operating point. If L/RC is increased the amplitude of oscillations increases. An attempt at explanation is made by saying[7,8,9,10,11] that the amplitude increases until L/RC is equal to some "average" negative resistance which is a function of the amplitude. The reader is left to guess at what this "average" negative resistance may be and in what manner it varies with the oscillation amplitude. Evidently a clarification of the exact meaning of the term "average" negative resistance is necessary.

[5] H. Iinuma, "A method of measuring the radio-frequency resistance of an oscillatory circuit," Proc. I.R.E., vol. 18; pp. 537–543; March, (1930), and "Resonant impedance and effective series resistance of high-frequency parallel resonant circuits," Proc. I.R.E., vol. 19, pp. 467–478; March, (1931).

[6] The battery or direct potential on this grid determines what shall be called the "operating point." Thus in Fig. 2 the operating point is the point on the curve corresponding to the direct potential.

[7] F. E. Terman, "Measurements in Radio Engineering," McGraw-Hill Book Co., (1935).

[8] E. V. Appleton, "Thermionic Vacuum Tubes," p. 93.

[9] Morecroft, "Principles of Radio Communication," Second Edition, (1927), p. 573.

[10] T. Hayasi, "Experimental and analytical studies of negative resistance oscillators," *Report of Radio Research Board of Japan*, vol. 4, March, (1934).

[11] M. G. Scroggie, "Applications of the dynatron," *Wireless Engineer and Exp. Wireless*, p. 527, October, (1933).

AVERAGE EFFECTIVE RESISTANCE OF A NEGATIVE RESISTANCE DEVICE

If a source of pure sine wave voltage is connected to a negative resistance device such as that of Fig. 1, the device will feed power back into the source. The average amount of power depends on the voltage maintained by the external source and can be regarded as due to that voltage impressed on some average effective resistance which is negative. We have to deal with a nonlinear quantity whose magnitude varies with the magnitude of the impressed voltage. It is convenient to define this average effective resistance R_n by the equation

$$\frac{V^2}{2R_n} = \begin{matrix}\text{average power supplied per cycle} \\ \text{to the negative resistance device}\end{matrix} \tag{1}$$

or,

$$R_n = \frac{V^2}{2(\text{power supplied})} \tag{2}$$

where R_n will be called the *average effective resistance*, and where V is the maximum value of the impressed sinusoidal electromotive force. By (2) it is clear that when the power supplied to the device is negative, R_n will be negative. For a negative resistance device R_n will be in general negative but under certain conditions may also take on positive values. This definition is seen to depend on the two fundamental factors involved in the operation of the negative resistance device, namely the power transfer and the controlling voltage.[12]

The current-voltage characteristic of a negative resistance device can in general be represented by the series

$$i = \alpha v + \beta v^2 + \gamma v^3 + \delta v^4 + \epsilon v^5 + \cdots \tag{3}$$

(where v is the impressed electromotive force and i is the resulting current). The origin of the system of co-ordinates is taken at the operating point on the characteristic. The number of terms required for a satisfactory approximation depends on the range of oscillation over the characteristic.

Let us now consider the problem of a voltage $v = V \sin \omega t$ impressed on a device for which (3) holds. The resulting current may be reduced trigonometrically to

[12] For a complex steady-state wave of voltage impressed on the device the definition may be extended as follows:

$$\bar{R}_n = \frac{V_{\mathrm{I}}^2 + V_{\mathrm{II}}^2 + V_{\mathrm{III}}^2 + V_{\mathrm{IV}}^2 + \cdots}{2(\text{total power supplied})}$$

where the subscripts denote the maximum values of the harmonic components of the total voltage.

$$i = I_0 + I_{\mathrm{I}} \sin \omega t + I_{\mathrm{II}} \cos 2\omega t + I_{\mathrm{III}} \sin 3\omega t + I_{\mathrm{IV}} \cos 4\omega t + \cdots$$

where,

$$I_0 = \beta V^2/2 + \tfrac{3}{8}\delta V^4 + \cdots$$

$$I_{\mathrm{I}} = \alpha V + \tfrac{3}{4}\gamma V^3 + \tfrac{5}{8}\epsilon V^5 + \cdots$$

$$I_{\mathrm{II}} = -\beta V^2/2 - \delta V^4/2 - \cdots \text{ etc.}$$

The average power delivered by v per cycle is

$$P = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} ivdt = \frac{I_{\mathrm{I}}V}{2}.$$

Consequently by definition the average effective resistance is

$$R_n = \frac{V^2}{2P} = \frac{V}{I_{\mathrm{I}}} = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \cdots}. \tag{4}$$

The definition of R_n makes R_n inversely proportional to the power delivered by the device, but since the device is essentially a source of power it would seem more proper to describe its behavior in terms of an average effective conductance defined as the reciprocal of R_n or

$$G_n = \alpha + \tfrac{3}{4}\gamma V^2 + \tfrac{5}{8}\epsilon V^4 + \cdots = \frac{2(\text{power supplied})}{V^2}. \tag{5}$$

Such a term would be more convenient; however, because of the trend of the literature on the subject it is deemed best to continue the discussion on the basis of an average effective resistance. All the results deduced for one of course can be immediately converted to the other by considering the reciprocal of the first.

Upon inspection of (4) we find R_n to be equal to the ratio of the impressed voltage to the resultant fundamental harmonic component of current. This is the result of the special case where the voltage impressed is a sinusoid. As will be shown later for most practical purposes it is sufficient to consider only a pure sine wave of voltage for the external driving force. Equation (4) also shows that R_n depends only on the voltage and on α, γ, ϵ, etc., (the coefficients of the odd powers of voltages in (3)). In most cases it is sufficient to consider only the first two or three of these coefficients in order to explain the phenomena observed as, for example, when a tuned circuit is connected to the device. For very small values of the voltage, $R_n \cong 1/\alpha$ which is the reciprocal of the tangent to the characteristic at the point of operation.

The Negative Resistance Oscillator

The problem of the oscillations occurring when a tuned parallel circuit is connected to a negative resistance device has been carefully studied by van der Pol,[1] Appleton,[13] Usui[14] and others, and solutions both mathematical and graphical have been developed. While such studies have been useful they do not always lend themselves to simple interpretations. It is for this reason that the simple but incomplete methods such as the one mentioned previously are resorted to for explaining the establishing of equilibrium of oscillations.

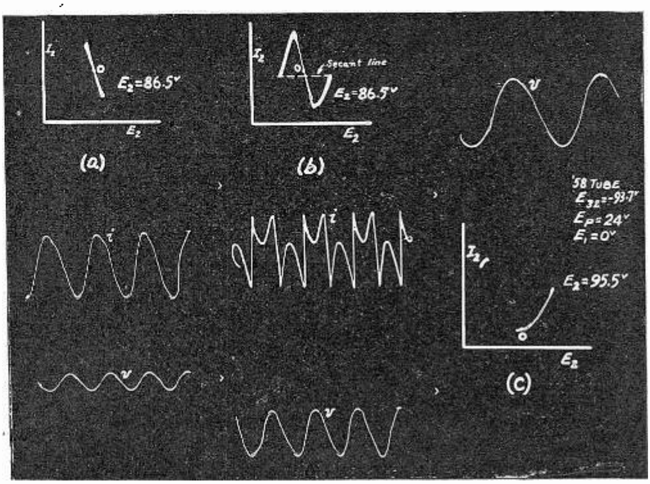


Fig. 3—Wave forms of a negative resistance oscillator.
(a) Linear case
(b) Horizontal secant case
(c) Operating point (*o*) on lower bend

One important feature of voltage-controlled negative resistance oscillators is the exceptional wave form of voltage which occurs for most normal conditions arising in practice. It has been consistently reported that even with coils of fairly low Q and operating over large portions of the bends in the tube characteristic the harmonic voltages have not been more than two or three per cent.[11,15,16] In Fig. 3 are shown

[13] E. V. Appleton and Balth. van der Pol, "Oscillation hysteresis in simple triode generators," *Phil. Mag.*, vol. 43, p. 177; January, (1922).

[14] R. Usui, "Nonlinear theory of electric oscillators," *Report of Radio Research Board of Japan*, vol. 5, no. 2, p. 40, (1935).

[15] F. M. Gager and J. B. Russell, "A quantitative study of the dynatron," Proc. I.R.E., vol. 23, pp. 1536–1566; December, (1935).

[16] E. B. Moullin, "Effect of curvature of the characteristic on frequency of dynatron generator," *Jour. I.E.E.* (London), vol. 73, no. 440, p. 186, (1933).

the results of a tuned parallel circuit connected to the negative resistance device of Fig. 1. The cathode-ray oscillogram of the current-voltage characteristic was obtained with the horizontal plates of the oscillograph recording the alternating voltage across the tube and the vertical plates recording the tube alternating current.

Fig. 3(a) was obtained using a low value of L/RC. Both the tube current and voltage, shown independent of one another, are very nearly sinusoidal. In Fig. 3(b), the value of the capacitance C in the tuned circuit was decreased until the secant line joining the ends of the excursion over the characteristic was horizontal. Although the current wave is badly distorted the voltage is still very nearly sinusoidal. The results of Fig. 3(c) were obtained by shifting the operating point away from the center of the characteristic towards the lower bend. These results show that even with the circuit oscillating under the abnormal conditions of Figs. 3(b) and 3(c) the wave form of the voltage across the tuned circuit is still very nearly sinusoidal. It is obvious that the power consumed by the harmonics in such a case is negligible in comparison with the power consumed by the fundamental. One can, therefore, without great error, consider the oscillation voltage across the parallel circuit to be $V \sin \omega t$. If the value of R is small the frequency of oscillation will be such as to make the equivalent impedance of the parallel circuit a pure resistance of value L/RC. The sum of the total power absorbed by the tuned circuit and the negative resistance device must be zero; hence we have

$$\frac{V^2}{2R_n} + \frac{V^2}{2L/RC} = 0$$

which yields

$$R_n = -L/RC. \tag{6}$$

Thus the definition of R_n given leads to the expression so commonly used to express the condition of equilibrium, namely, that "the amplitude of the oscillations will increase until the "average" negative resistance of the tube is equal to L/RC." The quantity R_n thus supplies a meaning for the term "average" negative resistance as used in the above statement. Equating the value of R_n from (4) to that in (6) we obtain

$$-\frac{L}{RC} = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \cdots}. \tag{7}$$

One immediately sees that the amplitude of oscillation V is determined by (7) which may be solved for V in terms of the constants L/RC,

α, γ, ϵ, etc. It is interesting to note that this solution coincides identically with that obtained formally by Appleton and van der Pol[13] employing the same assumptions. Their solution is expressed in the form of an infinite series. The simple method of approach used in this treatment yields the same results as their extensive mathematical treatise. Complicated integrals have been replaced by a simple energy balance which presents a clearer picture of the actual physical phenomenon involved. This illustrates the utility and convenience of the term R_n.

It is interesting to compare the above method of solution with the so-called "Secant Method."[10,17,18] A secant line of slope $-RC/L$ is drawn through the operating point on the tube characteristic. The points of intersection of this line with the characteristic are taken as the limit of the amplitude excursion from which the amplitude is determined. That this does not correctly represent the operation has been surmised by some writers who in the course of their experiments with negative resistance oscillators have observed oscillations to take place which did not at all correspond with the above theory.[11] Oscillations can be obtained in which the slope of the secant line is actually positive. In Fig. 3(b) is shown the true operation of a tuned parallel circuit connected to a negative resistance device. It is seen that the slope of the secant is zero. This would correspond in the secant method to a value of L/RC equal to infinity. Actually L/RC was 42,000 ohms. Increasing L/RC to 44,000 ohms resulted in a secant line or chord having a positive slope. Often the line joining the ends of the excursion over the characteristic does not pass through the operating point at all. Usually no basic theory is offered by those who proceed in this manner to justify equating $-RC/L$ to the slope of the secant line.[19]

Experimental Study of the Conditions Existing during Oscillation

An experimental method for obtaining R_n is suggested by (4). It is necessary only to measure the ratio between the fundamental of tube voltage and tube current under normal operating conditions. For comparison purposes the value of R_{sec} may be determined at the same time by measuring the amplitude of the oscillation voltage, then referring to a static plot of the current-voltage characteristic, proceed to find the slope of the secant. The reciprocal of this slope will be R_{sec}.

[17] Van der Bijl, "Thermionic Vacuum Tubes," p. 268, (1920).

[18] E. N. Dingley, "Development of a circuit for measuring the negative resistance of pliodynatrons," Proc. I.R.E., vol. 19, pp. 1948–1950; November, (1931).

[19] In what follows the reciprocal of the slope of the secant line shall be referred to as R_{sec}.

The comparison will yield sufficient information to show the inadequacy of the secant method and the error in assuming that the reciprocal slope of the secant line is equal to L/RC.

A circuit consisting of an inductance L and a resistance R in parallel with a condenser C was connected to the negative resistance device of Fig. 1. The oscillation voltage V corresponding to a given value of L/RC was measured. Then a suitable negative resistance bridge was substituted for the tuned circuit. The external voltage supplying the bridge was adjusted until the same voltage V appeared

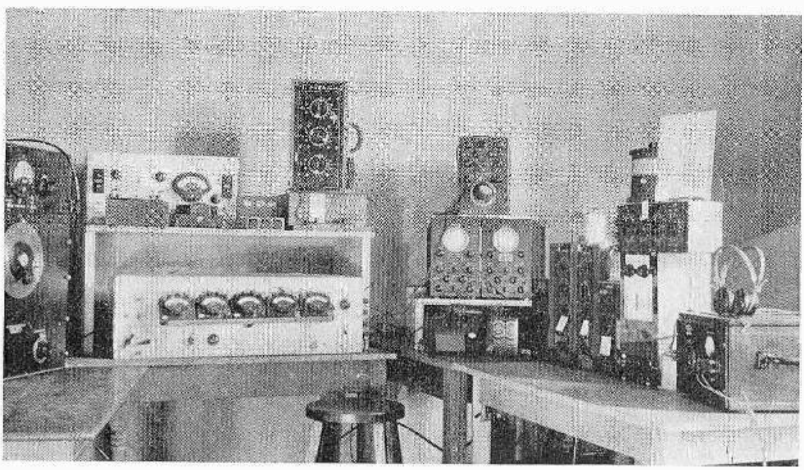


Fig. 4—Photograph of complete experimental arrangement.

across the tube, as before. Since the operating point and the plate and grid voltages on the tube were not changed the internal tube conditions were then the same as when the tube was oscillating in conjunction with the tuned circuit. It was verified that tube conditions were unchanged by observing the tube characteristic traced out on a cathode-ray oscilloscope. It was found to be identical for both cases. The bridge was then balanced for the fundamental and the value of R_n determined from the bridge readings. R_{sec} was obtained by the method described above. The quantity L/RC was varied by varying the capacitance C. The complete circuit diagram, description of the bridge and other experimental details are included in the Appendix. A photograph of the experimental arrangement is shown in Fig. 4.

The results of the comparison between L/RC and both R_n and R_{sec} are shown in Fig. 5. The straight line R_n represents the relation $R_n = -L/RC$. The points shown on this line are the experimental values of R_n. The agreement between R_n and L/RC is perfect. R_{sec}, however, is seen to differ considerably from the value of L/RC and at a value of the latter equal to 42,000 ohms passes through infinity from negative to positive values. The point at which it passes through infinity is further illustrated by Fig. 3(b).

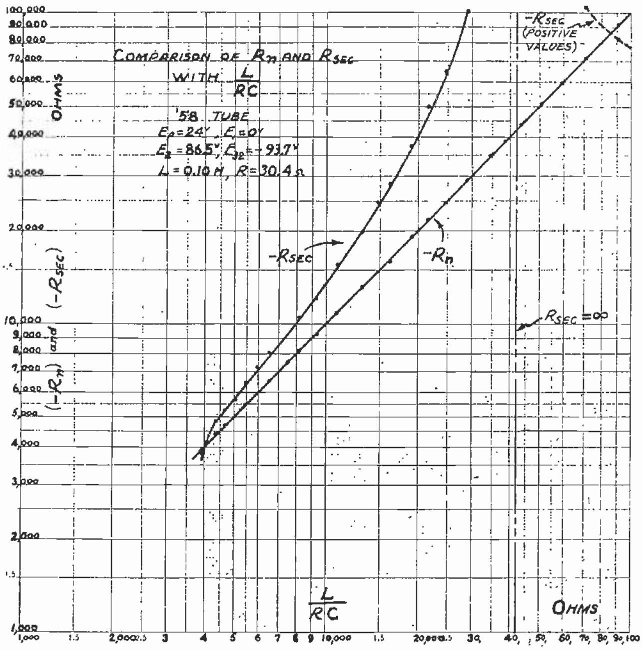


Fig. 5—Comparison of L/RC with R_n and R_{sec}.

$R_n - V$ CURVES

A convenient method for predicting the amplitude of oscillations is suggested by (4). This equation may be written in the form

$$R_n = \frac{1}{\alpha + \frac{3}{4}\gamma V^2 + \frac{5}{8}\epsilon V^4 + \cdots}. \tag{4a}$$

Fig. 6 shows the relation of R_n determined from (4a) to the voltage V. This is a special case where α is negative, γ positive, and the other odd power coefficients ϵ, etc., are assumed to be zero. If we plot $-R_n$ instead of R_n then the amplitude of oscillation V' at any given value of L/RC may be obtained by finding the value of V corresponding to the ordinate L/RC as shown. The justification for this is given by (6). Oscillations cannot start unless L/RC is at least equal to $-1/\alpha$. As

L/RC is increased further the oscillations increase accordingly. We shall refer to the curve of Fig. 6 as an $R_n - V$ curve.

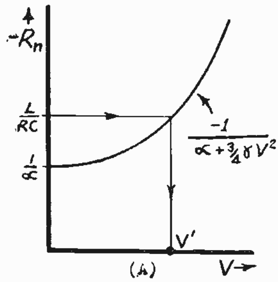


Fig. 6—$R_n - V$ curve for a negative resistance device.

A point to be emphasized in connection with the $R_n - V$ curves is the simplicity with which they may be obtained experimentally. Thus all that is necessary is to impress an external sine wave of voltage across

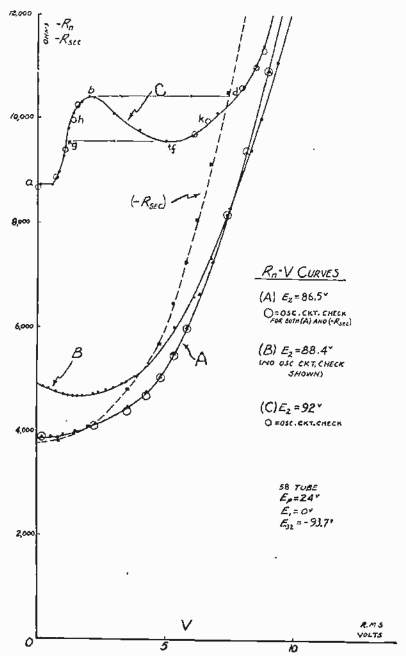


Fig. 7—$R_n - V$ curves.

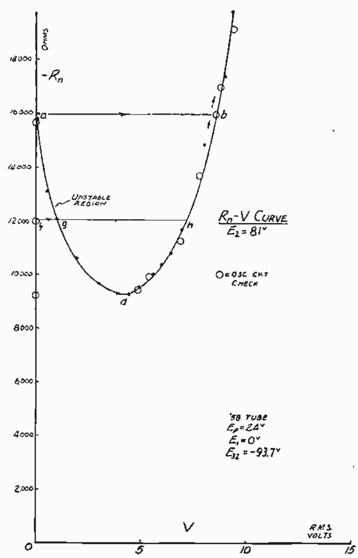


Fig. 8—$R_n - V$ curve. (Operating point near left bend of tube characteristic.)

the negative resistance device and measure the fundamental component of the current flowing. The ratio between these two gives R_n for that particular value of impressed voltage. By varying the voltage and obtaining the corresponding value of R_n the complete $R_n - V$ curve may be obtained. A second method and the one employed here was to use the negative resistance bridge described in the Appendix from which the value of R_n at any voltage impressed across the tube could be had directly. In addition to predicting the correct amplitudes of oscillation the $R_n - V$ curves serve the further useful purpose of presenting a simple method for studying the problem of amplitude stability. This problem which ordinarily is dealt with by very complicated methods is immediately solved by a direct inspection of the $R_n - V$ curve which because of its directness and simplicity brings out many

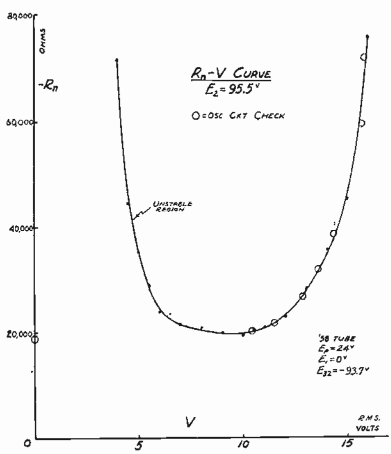


Fig. 9—$R_n - V$ curve. (Operating point beyond right bend of tube characteristic.)

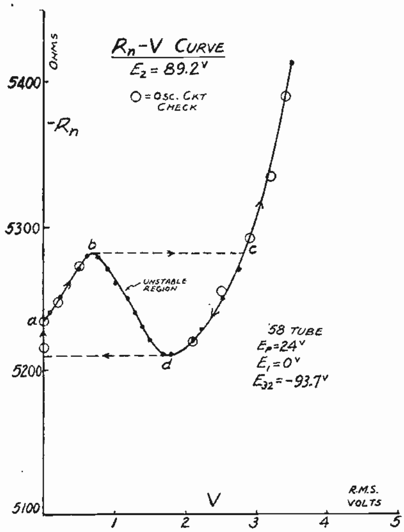


Fig. 10—$R_n - V$ curve displaying the Appleton-van der Pol "oscillation-hysteresis."

important points regarding the operation of the tube which otherwise might remain hidden.

A complete family of these curves was obtained for the negative resistance device of Fig. 1. They are illustrated in Figs. 7, 8, 9, and 10 for different operating points on the same tube characteristic (Fig. 2). The operating point is determined by the direct voltage E_2 on the anode. Fig. 7 shows three curves, A, B, and C, taken with the operating point on the negative slope portion of the tube characteristic. The shape of these curves varies considerably with the operating point. All of the experimentally determined $R_n - V$ curves possess the general shape predicted by (4a). Curve A was taken near the center of the characteristic and is similar to the case discussed in Fig. 6. Note the large values of R_n when the operating point is taken near the bends in the characteristic (Figs. 8 and 9). In all of these curves the possible amplitudes of oscillation may be determined by drawing a horizontal line at the ordinate $-R_n = L/RC$.

The points of intersection of this line with the curve give the predicted amplitudes. Thus in curve A there is only one amplitude possible for each value of L/RC. In curve B two amplitudes are predicted in the lower region. In curve C as many as three amplitudes are seen to be possible since it is possible for an ordinate to intersect the curve in three places.

Not all of these possible amplitudes are stable, however, and consequently will not all be available in a practical oscillator. The problem of stability will be treated in more detail later. It is sufficient to mention at this point that the $R_n - V$ curves will predict all of the possible amplitudes, whether stable or unstable.

To verify this prediction as well as to obtain a measure of its accuracy an R, L, C parallel circuit was connected to the device and the oscillation voltage at each value of L/RC was measured. As before, the quantity L/RC was varied by varying the capacitance C. The points determined by the measured values of L/RC and V are represented by the small circles on the $R_n - V$ curves of the four Figs. 7, 8, 9, and 10. The agreement with the predicted values is excellent. Let us consider one case in detail. In Fig. 8 the oscillations do not start until the value of L/RC has been increased to the value given by the point a. At this point the oscillations suddenly set in at a large magnitude equal to the abscissa $a\,b$. Further increase in L/RC results in a corresponding increase in amplitude. If L/RC is decreased below the point b the oscillations will now persist until the point d is reached beyond which the amplitude suddenly drops to zero. If one desires to start the oscillations again it is necessary to increase L/RC to the value given by a. When the oscillating circuit is in a state of rest no self-starting oscillations are possible until the value of L/RC is increased to the point where the $R_n - V$ curve cuts the vertical axis. It is possible, however, to start oscillations below the point a if a sufficiently large electrical disturbance is applied to the circuit by external means. This phenomenon and the fact that no stable oscillations are possible in the region $a\,d$ will be discussed in the section on stability. Since the curve of Fig. 9 does not intersect the vertical axis, self-starting oscillations are not possible at any value of L/RC. As in the case previously described at large values of L/RC an external impulse, if sufficiently large, can start the oscillations. The dotted line in Fig. 7 shows the amplitudes predicted by the secant method for the same operating point as for curve A. This operating point, near the center of the characteristic is the one most likely to give approximately correct results by the secant method. A comparison with the actual oscillation amplitudes for that case (the small circles on the A curve) shows a fairly good agreement at the low amplitudes but as the value of L/RC is raised the discrepancy between predicted amplitudes and experimental results is clearly apparent.

The $R_n - V$ curves also show how the amplitude of oscillation varies with the frequency. Thus as the value of L/RC is raised by decreasing C, the frequency increases accordingly. For large values of L/RC and therefore higher frequencies the $R_n - V$ curves approach a parallel to the vertical axis. Consequently the amplitude does not change much with frequency at these values. This is another of the advantageous characteristics of the negative resistance oscillator.

The Amplitude Stability of Oscillations

A stable point or a stable voltage amplitude is one at which any slight change in the amplitude results in a condition which acts to restore the voltage to the original value. At any unstable point if the amplitude is disturbed the voltage will tend to continue to change until a new and stable amplitude is arrived at.[20,21]

It is possible to derive a very simple criterion for amplitude stability based on the quantity R_n. The average power delivered by the tube is $-V^2/2R_n$: that dissipated by the tuned circuit is $V^2/2(L/RC)$. Let us write

$$\Delta P = \frac{V^2}{2}\left(\frac{1}{R_n} + \frac{RC}{L}\right)$$

which is the average power absorbed per cycle by the tuned circuit less the power delivered by the tube per cycle. If ΔP is negative the tube supplies more power than the tuned circuit dissipates. The additional power, or energy, goes to swell the stored average energy in the tuned circuit ($1/2\ CV^2$) and consequently the amplitude V increases. If ΔP is positive the tube supplies less power than is dissipated by the tuned circuit and the latter must therefore draw on its stored energy thus decreasing the amplitude. Let V represent a possible amplitude of oscillation. For V to be stable it is necessary that when V increases slightly ΔP becomes positive so that less power will be supplied than

[20] H. Nyquist, "Regeneration theory," *Bell Sys. Tech. Jour.*, vol. 11, p. 126; January, (1932).

[21] E. J. Routh, "Stability of Motion," London (1877) and (1905, Sixth Edition).

is dissipated and V is forced to decrease again. When V decreases slightly it is necessary that ΔP be negative in order to increase the stored energy and restore the voltage. Therefore for stable oscillations we must have

$$\frac{d(\Delta P)}{dV} > 0$$

or,

$$\frac{d(\Delta P)}{dV} = \frac{\partial(\Delta P)}{\partial R_n}\frac{dR_n}{dV} + \frac{\partial(\Delta P)}{\partial V}$$

$$= -\frac{V^2}{2R_n^2}\frac{dR_n}{dV} + V\left(\frac{1}{R_n} + \frac{1}{L/RC}\right) > 0. \qquad (8)$$

In order for the amplitude V to be possible (6) must be satisfied, hence (8) reduces to

$$\frac{d(-R_n)}{dV} > 0 \qquad (9)$$

which is our criterion for stability.

Equation (9) states that in order for an amplitude to be stable it must lie on a point on the $R_n - V$ curve which has a positive slope. We are thus led to a very simple method of determining not only the amplitude but the stability at the same time. Here too it is interesting to note that the above criterion coincides with a similar one deduced by Appleton and van der Pol[13] by the use of routine mathematical methods but expressed in the form of an infinite series difficult to interpret.

Application of the Criterion of Stability

Throughout the literature on the subject of negative resistance oscillators we find reported many instances of peculiar behavior of the amplitude of oscillations as some parameter or other is varied.[10,13,14,22] It is not necessary to treat these cases as difficult singularities. Such phenomena may be explained very simply with the aid of the $R_n - V$ curves. A representative set of $R_n - V$ curves which contains most of the conditions found in practice has already been displayed (Figs. 7, 8, 9, 10).

Returning to the discussion of Fig. 8 it is easy now to see why the region *a d* is unstable since it does not possess a positive slope, which is the requirement for stable operation. If the magnitude of L/RC is raised to the value corresponding to the ordinate f and the

[22] T. Hayasi, "Problems of the dynatron oscillator," *Jour. I.E.E.* (Japan), p. 35, April, (1934).

circuit is initially in a state of rest no oscillations will occur. However, if an external electrical impulse comes along large enough to raise the amplitude past the point g oscillations will set in. Point g, however, is not a stable point, for the slope of the $R_n - V$ curve is negative at that point and consequently the criterion $d(-R_n)/dV > 0$ is not satisfied. The oscillations will therefore continue to grow until the stable point h is reached. These predictions are easy to verify experimentally. The external impulse may be applied in many different ways, for example, by sweeping a magnet across the coil L, by initially charging the condenser C, and then discharging it into the oscillatory circuit, or by a sudden opening and then closing of the direct-current power supply to create a transient disturbance in the circuit. The particular shape of the $R_n - V$ curve of Fig. 8 is due to the constants α and γ being negative and ϵ positive in (4a). These three constants are all that are needed to approximate the curve satisfactorily.

No sudden jumping of the amplitude is observed with curve A of Fig. 7 since $-R_n$ increases steadily with voltage and consequently any L/RC line will intersect the curve at only one point. This case represents operation with the operating point at the center of the negative portion of the tube current-voltage characteristic (Fig. 2).

Curve (C) of Fig. 7 presents a very interesting behavior. It is obtained by taking the operating point determined by $E_2 = 92$ volts which places the point on the negative portion of the tube characteristic but near the right bend. The behavior is as follows: As the value of L/RC is slowly raised oscillations set in gradually at a which continue to increase with L/RC along the portion a b until the point b is reached at which the oscillations suddenly increase, the amplitude jumping from b to d. This is illustrated rather clearly by Fig. 11 which shows a cathode-ray oscillogram of the oscillation parameters over the tube characteristic for this case. Fig. 11(a) represents the oscillation just as point b was reached. It would not persist at this value very long, however, and some slight fluctuation within the circuit would cause it to go over the hump to point d which is illustrated by Fig. 11(b). Further increase of L/RC was followed by a gradual increase in the amplitude. Decreasing L/RC below the point d caused the oscillations to decrease accordingly until f was reached. Fig. 11(c) shows the oscillation over the characteristic at this point. In the same manner as explained before, the oscillations would now spontaneously drop from f to g. The small oscillation at g is shown in Fig. 11(d).

From the criterion of stability it is easy to see why it was possible to check only certain portions of the $R_n - V$ curves experimentally by self-excitation with a parallel R, L, C circuit. The portions possessing negative slopes represent unstable amplitudes, consequently no oscillation can persist at these amplitudes in a practical oscillator.

Appleton and van der Pol[13] have described an interesting type of phenomenon which they called "oscillation-hysteresis." A sketch illus-

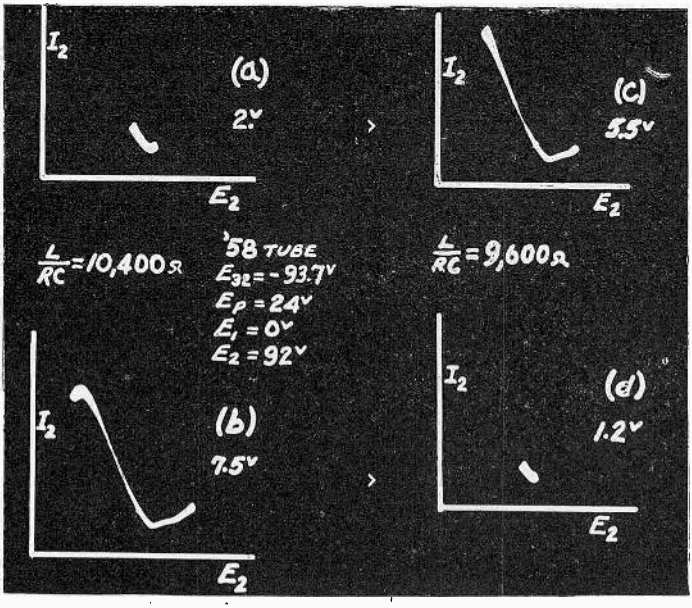


Fig. 11—Spontaneous jumping of amplitude of oscillation.

trating their observations is shown in Fig. 12. They found that as the resistance R of their tuned circuit was decreased oscillations would gradually set in at a, as shown, and gradually increase until point b

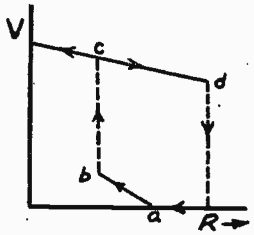


Fig. 12—Appleton-van der Pol "oscillation-hysteresis."

was reached at which the oscillations would suddenly jump to the magnitude given by c. Further decrease in R would result in a gradual increase in V. If R was increased from the value at point C the oscillations would gradually decrease until point d was reached when they would suddenly drop to zero. To understand this behavior we need only consider an $R_n - V$ curve such as shown in Fig. 10 which is of the same family as those of Figs. 7, 8, and 9. On the basis of the previous discussions it is easy to see that oscillations will start when L/RC reaches the magnitude corresponding to point a. The oscillations then gradually increase in amplitude with L/RC until b is reached at which they jump to c. If L/RC is now decreased the oscillations will also decrease gradually in amplitude until point d is reached at which they drop to zero. Here then is the type of "oscillation-hysteresis" described by Appleton and van der Pol. In fact if Fig. 10 is rotated counterclockwise about the origin through ninety degrees a figure similar to that of Fig. 12 is obtained. Appleton and van der Pol did not work out the complete solution of this problem but mentioned the fact that it would require at least seven terms in the equation for the tube characteristic (equation (3)) in order to arrive at a satisfactory explanation of the phenomenon. That their statement is correct will be seen when it is considered that it requires at least four terms in the equation for R_n in terms of V (equation (4a)) in order to obtain the type of $R_n - V$ curve of Fig. 10. This means that at least the first seven terms in the equation of the tube characteristic must be considered. The phenomenon here cited affords a striking illustration of the advantages obtained by predicting the behavior of the tube with its associated circuits from the derived $R_n - V$ curve rather than attempting to do so by a mathematical analysis involving the E, I tube characteristics directly.

Another application is suggested by the fact that it is possible to obtain families of $R_n - V$ curves for fixed direct anode voltages and different bias voltages on grid No. 1 (Fig. 1). Such curves should yield interesting information for problems such as automatic amplitude control,[23,24] and the measurement of resistance in an oscillatory circuit,[5] which involve the variation of this bias.

Constant R_n Curves

Another family of curves which is useful in studying the operation of a negative resistance oscillator is shown in Fig. 13. As the name implies each curve is taken for a constant value of R_n. They represent the variation in voltage V (impressed across the negative resistance device) required to maintain R_n constant as the operating point is changed by varying the direct voltage E_2 applied to the anode of the negative resistance tube. The curves are obtained experimentally with the apparatus

[23] L. B. Arguimbau, "An oscillator having a linear operating characteristic," Proc. I.R.E., vol. 21, pp. 14–28; January, (1933).

[24] J. Groszkowski, "Oscillators with automatic control of the threshold of regeneration," Proc. I.R.E., vol. 22, pp. 145–151; February, (1934).

described in the Appendix. The negative resistance bridge is adjusted for a constant value of R_n. Then for each value of E_2 the external oscillator voltage is varied until a balance is noted in the earphones, and the corresponding voltage V across the tube is recorded. By varying E_2 the complete curve may be obtained.

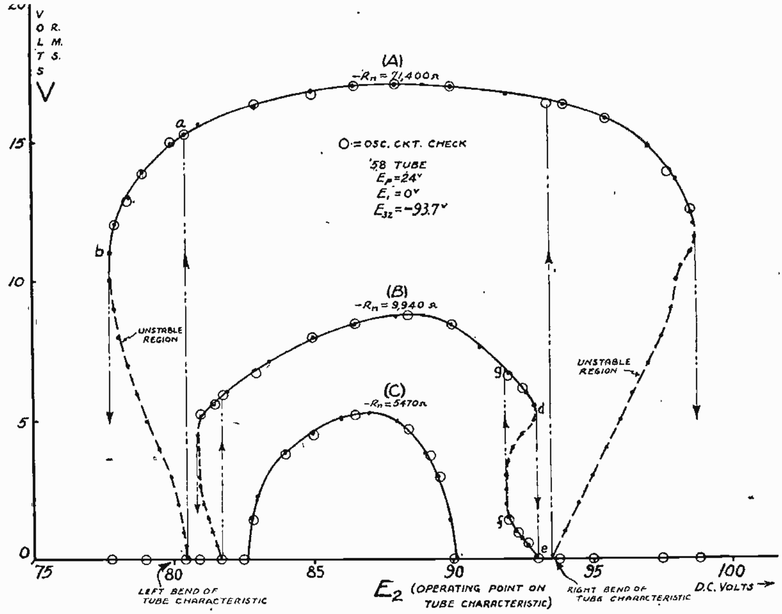


Fig. 13—Constant R_n curves.

These curves serve to predict the amplitude of oscillation when the values of the tuned circuit constants are held fixed and the operating point is changed. Thus each curve of constant R_n corresponds to a constant value of L/RC. After obtaining these curves a parallel R, L, C circuit was connected to the negative resistance device and their value in correctly predicting oscillation phenomena was experimentally verified. The small circles show the close agreement of the oscillation amplitude with the predicted values as the operating point was varied. Note that no experimental check was possible in the regions shown dotted. These are unstable regions and occur when the operating point is near either bend of the tube characteristic. In order for oscillations to start spontaneously at any given value of L/RC, it is necessary that the operating point be brought completely within the corresponding curve of constant R_n. Thus if $L/RC = 71{,}400$ ohms corresponding to curve (A) oscillations will not start until the increasing operating point reaches 80.5 volts at which the oscillations will suddenly set in at a very large amplitude jumping to point *a*. If now, the anode voltage E_2 is decreased again large oscillations will persist until the point *b* is reached at which they suddenly drop to zero. It should be observed that point *b* is considerably to the left of the maximum of the left bend in the E, I tube characteristic.

An interesting behavior is presented by curve (B). With the tube oscillating, as the operating point approaches the right bend in the curve, the amplitude gradually decreases. At *d* the amplitude drops to zero. If now the anode voltage is gradually decreased small oscillations set in until the point *f* is reached when the amplitude jumps suddenly to the large value at *g*.

Such phenomena are easy to understand with the aid of the constant R_n curves. The unstable portions of these curves coincide identically with the unstable parts of the $R_n - V$ curves. In like manner the stable parts of these two sets of curves correspond with each other. Thus the amplitudes predicted by points *f* and *g* at $E_2 = 92$ volts on the constant R_n curve B, namely 1.4 and 6.6 volts are the same as the amplitudes predicted by points *h* and *k* on the $R_n - V$ curve (C) of Fig. 7.

The interesting phenomenon[10,15] of a dynatron maintaining large oscillations with the plate voltage reduced to zero or even made negative can be explained with the use of a curve of constant R_n. In order for such oscillations to be maintained it is necessary to use large values of L/RC. This will correspond to a large value of R_n. In Fig. 13 we saw that for large values of R_n it was possible to reduce the anode voltage below the point corresponding to the left bend in the tube characteristic before the oscillation would suddenly drop to zero. In the dynatron, because of the shape of the characteristic, this effect is greatly enhanced. For large values of R_n (and therefore L/RC) the curve of constant R_n extends far into the negative anode voltage region and in fact slopes gently upward. Consequently an oscillation once started (with the anode voltage at the negative portion of the tube characteristic) will persist when the anode voltage is reduced to zero and even made negative.

Finally the curves of Fig. 13 show that for large values of L/RC (or R_n) it makes little difference where the operating point is chosen so far as the amplitude of oscillation is concerned as long as the operating point is near the center of the negative portion of the tube characteristic. Thus small changes in the direct voltage supplying the anode produce a negligible effect on the amplitude of oscillation.

Acknowledgment

The author wishes to express his appreciation to Dr. Henry E. Hartig of the Department of Electrical Engineering at the University of Minnesota for his thoughtful guidance and for the invaluable aid received through timely suggestions.

Appendix

Experimental Details

The experimental arrangement is shown in Figs. 4 and 14. The negative resistance bridge[7,18,25] is composed of the elements R_1, R_2, R_3 and R_0 and C_0. The switch S_1 serves to connect either the bridge or the tuned circuit (R, L, C) to the negative resistance terminals BD with a

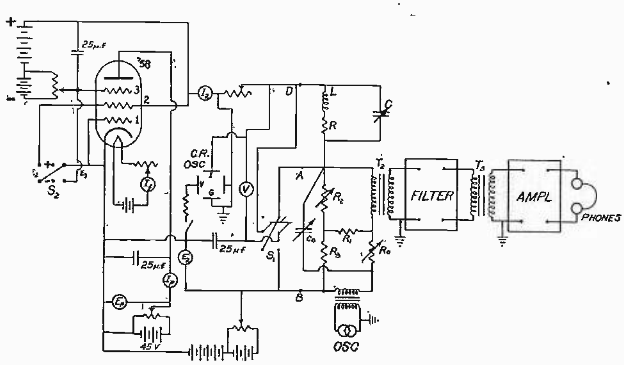


Fig. 14—Circuit for complete study of negative resistance device.

minimum of disturbance of the operating conditions.[7] The bridge detector circuit consisted of a low-pass filter and a resistance coupled amplifier connected to a pair of earphones. A frequency of 300 cycles was used in the bridge. It was found that R_n was practically independent of the frequency at which it was measured. An external beat-frequency oscillator supplied this signal voltage. The elements R_0 and C_0 served only to balance out the tube and extraneous capacitances (C_n). If $C_0 = C_n$, $R_0 = R_1 + R_3$, and if R_3 is small compared with R_n it is easy to show that at balance, $R_n = R_2\ (1 + R_3/R_1) + R_3$. An alternating-current vacuum tube voltmeter with an input impedance of five megohms was used to measure V which inspection on a cathode-ray oscillograph showed to be very nearly sinusoidal at all times. Switch S_2 leads to a direct-current vacuum tube voltmeter for measuring the direct anode voltage E_2 applied to grid No. 2.

[25] A. Pinciroli, "Risistenze negative de tubi elettronici e loro misura," *Alta Freq.*, vol. 3, pp. 5–19; February, (1934).

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

EFFECTS OF TUNED CIRCUITS UPON A FREQUENCY MODULATED SIGNAL*

By
HANS RODER
(General Electric Company, Schenectady, New York)

Summary—*Prior investigations indicated that the frequency modulated receiver would always respond to the signal having the largest amplitude. Thus, selective circuits would be required to pick out a desired signal existing simultaneously with a number of other signals.*

The first item considered in this paper is that the signal carrier is tuned to the steep side of the resonance curve. It is found that in this case conversion from frequency modulation into amplitude modulation can be effected. It is required that the amplitude and phase characteristic of the circuit be linear with respect to frequency over the whole frequency interval occupied by the modulated signal. In order to derive a faithful audio signal reproduction from the complete detection process, a phase modulated signal must be transformed into frequency modulation first.

Next, the case is considered in which the signal carrier is tuned to the peak of the resonance curve. The single tuned circuit is taken up first and two methods of solution are presented. It is found that with certain values of the modulation index, the tuned circuit may cause very serious nonlinear distortion of the output. For large values of the modulation index the resulting variation in amplitude and frequency can be determined statically, while for very small values of the modulation index the effect of the tuned circuit is exactly analogous to that encountered with amplitude modulation. To eliminate the nonlinear distortions, the circuit should provide a uniform amplitude and a linear phase characteristic over the operating frequency range. This statement is confirmed by investigating a circuit consisting of two coupled circuits which provide essentially a band-pass characteristic and yield very much less linear distortion than a corresponding single tuned circuit. Curvature of the phase characteristic will cause nonlinear audio distortions, while curvature of the amplitude characteristic may cause additional distortions if the amplitude happens to drop below the limiter operating voltage.

In the last section, the interference problem between two frequency modulated signals is studied. It is found that the relations are entirely different from the case of amplitude modulation. The helpful phenomenon of "demodulation" does not exist. If the two carriers are spaced at the frequency interval equal to their maximum frequency shift, and if the interfering carrier is greater than the desired carrier, then the interference is of the order of 100 times greater than in the corresponding amplitude modulation case. An increase in selectivity will not reduce the interference. It is required to have sufficient spacing between two signals, in order to eliminate mutual interference. This spacing should be at least equal to the total *signal band width occupied by each signal.*

I. INTRODUCTION

THE principle of frequency modulation, which was repeatedly proposed and discussed during the last fifteen years in radio publications, has recently again aroused great interest. The

* Decimal classification: R141.2×R148. Original manuscript received by the Institute, June 9, 1937.

fact that wide-band frequency modulation[1] possesses the unique property of practically eliminating interfering signals and interfering noises, provided the interference is not excessively strong, threw an entirely new light on this type of modulation and indicated that wide-band frequency modulation might possibly some day serve to provide a high fidelity broadcast transmitting system at ultra-high frequencies practically unaffected by interference from static or man-made noise and from other signals.

In a broadcast system based on wide-band frequency modulation it will undoubtedly be required to operate a certain number of stations simultaneously on frequencies assigned within a certain band, the same as is now the case for the broadcast band. In a recent paper,[2] the author has discussed in detail the phenomenon of noise suppression on a theoretical basis and also has arrived at some conclusions regarding the simultaneous operation of several transmitters broadcasting at "carrier" frequencies spaced at frequency intervals about equal to their side-band coverage. He found that, unless sufficient selectivity is provided, the receiver always would respond to the strongest signal only, suppressing all other signals as unwanted interference. In the case of three or more signals, existing simultaneously and all having about equal signal strengths, it was found that conditions could arise which would not permit receiving any one of the signals undisturbed. These results indicated that selective circuits would be required to pick out a desired signal existing simultaneously with a number of other signals.

The theory of sinusoidal frequency and phase modulation has been highly developed during recent years. The side-band coverage and the mutual relations of the two types of modulation are now commonly known. Little, however, is known about what effects a tuned circuit will have upon a frequency or phase modulated signal. In amplitude modulation we know that a signal passing through a selective circuit is affected in a twofold manner: first, the amplitude of the envelope is reduced (due to "side-band cutting") and second, the envelope of the output is delayed in phase with respect to the envelope of the input.

To study the effect which a selective circuit has upon a phase or frequency modulated signal passing through it we shall consider three different cases:

1. Signal carrier tuned to the steep side of the resonance curve (detector case).

[1] E. H. Armstrong, "Method of reducing disturbances in radio signaling by a system of frequency modulation," Proc. I.R.E., vol. 24, pp. 689–740; May, (1936).

[2] H. Roder, "Wide-band frequency modulation," *Electronics*, vol. 10, pp. 22–64; May, (1937).

2. Signal carrier tuned to the peak of the resonance curve (amplifier case).
3. Interference produced by an undesired modulated signal in an adjacent channel, assuming high adjacent channel attenuation by means of band-pass filters.

Before we start with the main subject of this paper we must pay brief attention to some peculiar features encountered both at the transmitter and at the receiver with this type of modulation which makes it differ fundamentally from amplitude modulation.

At the transmitter a frequency modulated signal is generated. The amount of the frequency shift is made proportional to the amplitude of the audio signal; the frequency of the frequency shift is equal to the audio frequency. (Example: Carrier frequency, 60,000 kilocycles. Maximum frequency shift corresponding to maximum audio signal amplitude is equal to 100,000 cycles, corresponding to 100 per cent modulation in amplitude modulation. A 1000-cycle signal, whose amplitude is 50 per cent of maximum audio signal, will then require a frequency shift of $\pm 50{,}000$ at a rate of 1000 times per second.)

The receiver comprises radio-frequency, converter, intermediate-frequency stages, second detector, and audio output stages similar in principle to those in a conventional superheterodyne. Between the last intermediate-frequency stage and the second detector, however, we have a limiter, which is followed by a frequency-amplitude converter. The limiter may be defined as a device which produces a radio-frequency or intermediate-frequency output whose amplitude is fixed at a constant predetermined value, regardless of the input amplitude. The instantaneous phase of the signal, however, must be unaffected by the limiting action. The extent to which a device of this kind can physically be realized is beyond the scope of this discussion; we shall simply postulate its existence. Necessarily, it will be required that the input amplitude shall not drop below a certain value (minimum limiter input). The frequency-amplitude converter can in principle be represented by a resonant circuit which is so tuned that the maximum slope of its resonance curve occurs at carrier frequency. The output of this device is an amplitude modulated intermediate frequency, which is supplied to the second detector in the conventional manner. The action of the frequency-amplitude converter will be studied in detail in the next section.

With the arrangement and operating principle of the receiver established, it becomes evident what procedure of attack must be chosen to solve the problems we set out to investigate. We first determine the instantaneous phase of the incoming signal by means of the fundamental relation

$$\phi = \int \omega dt. \tag{1}$$

The signal then passes through the tuned circuit or has an interfering signal added to it, resulting in changes in amplitude and changes in phase. After having determined the output phase, ϕ_0, we have for the output frequency

$$\omega_0 = \frac{d\phi_0}{dt}. \tag{2}$$

If the frequency-amplitude conversion is perfect and if a linear second detector is used, the audio-frequency output will be a true reproduction of the frequency ω_0. We must, furthermore, solve for the envelope of the radio frequency which enters the input of the limiter. The amplitude of this envelope must not fall below the minimum limiter input; if it does then nonlinear distortions will result on account of superimposed amplitude modulation. It becomes apparent, therefore, that we have to solve for amplitude, instantaneous phase, and instantaneous frequency for both input and output signal.

II. Tuned Circuit as Frequency-Amplitude Converter

Let us consider a network in which, for constant amplitude and phase of the input, the amplitude and phase of the output become linearly variable with frequency. A circuit of this type is most simply represented by a resonant circuit operated over a narrow frequency interval at the steepest slope of the resonance curve. Conditions are illustrated in Fig. 1. We have

$$s = \frac{\Delta E_2}{\Delta\omega} = \text{slope of the amplitude function}.$$

For the input we choose a phase or frequency modulated signal

$$\text{input} = e_1 = E_1 \sin(\omega t + m \sin \mu t) \tag{3}$$

in which expression ω and μ represent the radio and audio frequency, respectively. m is the "modulation index," with $m = \Delta\omega/\mu$. The instantaneous phase of this signal is

$$\omega t + m \sin \mu t,$$

while the instantaneous frequency is

$$\omega + \Delta\omega \cos \mu t = \omega + m\mu \cos \mu t.$$

By transferring the above expression into the standard side-band equation,[3] we get for the input signal

$$e_1 = E_1[J_0(m) \sin \omega t + J_1(m)(+ \sin (\omega + \mu)t - \sin (\omega - \mu)t) + J_2(m)(+ \sin (\omega + 2\mu)t + \sin (\omega - 2\mu)t) + \cdots], \tag{4}$$

where the $J_n\ (m)$ are Bessel functions of the first kind of order n for the argument m.

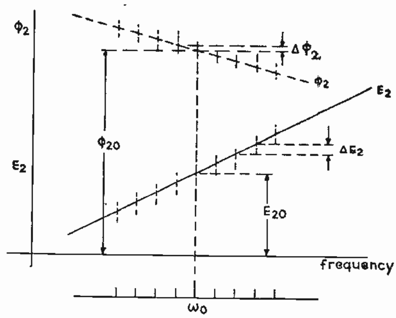


Fig. 1—Linear amplitude and phase characteristic.

We can now find the output signal if we take into account the amplitude and phase change inflicted upon each side-band component by the network. Referring to Fig. 1, the output becomes (choosing ϕ_{20} as new phase reference)

$$\begin{aligned} e_2 = E_{20}[&J_0(m) \sin \omega t \\ &+ J_1(m)(1 + s\mu) \sin (\omega t + (\mu t - \Delta\phi_2)) \\ &- J_1(m)(1 - s\mu) \sin (\omega t - (\mu t - \Delta\phi_2)) \\ &+ J_2(m)(1 + 2s\mu) \sin (\omega t + 2(\mu t - \Delta\phi_2)) \\ &+ J_2(m)(1 - 2s\mu) \sin (\omega t - 2(\mu t - \Delta\phi_2)) \\ &+ \cdots]. \end{aligned}$$

If we put $\mu t - \Delta\phi_2 = \mu t_1$, this expression can be written as follows:

$$\begin{aligned} e_2 = E_{20}[J_0 \sin \omega t + 2(&J_1 \cos \omega t \sin \mu t_1 \\ &+ s\mu J_1 \sin \omega t \cos \mu t_1 \\ &+ J_2 \sin \omega t \cos 2\mu t_1 \\ &+ 2s\mu J_2 \cos \omega t \sin 2\mu t_1 \\ &+ J_3 \cos \omega t \sin 3\mu t_1 \\ &+ 3s\mu J_3 \sin \omega t \cos 3\mu t_1 \\ &+ \cdots)]. \end{aligned} \tag{5}$$

[3] H. Roder, "Amplitude, phase, and frequency modulation," Proc. I.R.E., vol. 19, pp. 2145–2176; December, (1931).

To show that (5) has amplitude modulation in addition to its frequency modulation, we compare the bracketed term in (5) against the following function:

$$(1 + k \cos \mu t_1) \sin (\omega t + m \sin \mu t_1). \tag{6}$$

By applying the well-known expansions due to Jacobi[4] this function becomes after some manipulation (writing, for brevity, J_n in place of $J_n(m)$)

$$\begin{aligned} J_0 \sin \omega t &+ 2J_1 \cos \omega t \sin \mu t_1 \\ &+ k(J_0 + J_2) \sin \omega t \cos \mu t_1 \\ &+ 2J_2 \sin \omega t \cos 2\mu t_1 \\ &+ k(J_1 + J_3) \cos \omega t \sin 2\mu t_1 \\ &+ \cdots . \end{aligned}$$

From the addition theorem of Bessel's functions we have the relation

$$J_{(n-1)}(m) + J_{(n+1)}(m) = \frac{2n}{m} J_n(m).$$

Hence the above series becomes

$$\begin{aligned} J_0 \sin \omega t &+ 2J_1 \cos \omega t \sin \mu t_1 \\ &+ 2\frac{k}{m} \sin \omega t \cos \mu t_1 \\ &+ 2J_2 \sin \omega t \cos 2\mu t_1 \\ &+ 2\frac{2k}{m} \cos \omega t \sin 2\mu t_1 \\ &+ 2J_3 \cos \omega t \sin 3\mu t_1 \\ &+ 2\frac{3k}{m} \sin \omega t \cos 3\mu t_1 \\ &+ \cdots . \end{aligned} \tag{7}$$

If we make $k = s\mu m$, then (7) becomes identical with the bracketed term in (5). We thus obtain an output signal

$$\text{output} = e_2 = E_{20}(1 + s\mu m \cos \mu t_1) \sin (\omega t + m \sin \mu t_1) \tag{8}$$

which is an amplitude modulated signal whose radio-frequency envelope is

[4] *Loc. cit.*, equations (12) and (13).

$$1 + s\mu m \cos \mu t_1 .$$

The term $s\mu m$ is the factor of percentage modulation. The envelope is shifted by a phase delay of $\Delta\phi_2$ with respect to the input, which fact, however, has no particular practical consequence.

So far we have not yet decided whether the input shall be phase or frequency modulated. Assuming phase modulation we have

1. The phase shift is m radians.
2. The phase shift is proportional to the audio amplitude.
3. The modulation depth, $am\mu$, is therefore proportional to μ. This means that low frequencies are suppressed and high frequencies are exaggerated. In other words the output is linearly distorted.

With a frequency modulated input, conditions are as follows:

1. The audio signal amplitude is proportional to $\Delta\omega = m\mu$.
2. The phase shift is $m = \Delta\omega/\mu$; i.e., inversely proportional to μ.
3. The modulation depth is $am\mu = a\Delta\omega$; i.e., independent of μ and in direct proportion to $\Delta\omega$ and consequently to the audio signal amplitude.

It is thus seen that the above circuit arrangment can serve as a perfect frequency-amplitude converter for a frequency modulated input signal. A phase modulated input signal would produce a linearly distorted audio output, unless it is first converted into a frequency modulated signal. Arrangements are known for this type of conversion.[5]

It is evident from the foregoing discussion that a network which is to serve as a perfect frequency amplitude converter must provide an amplitude and phase characteristic which are both linear functions of frequency over the whole band width occupied by the frequency modulated signal.

III. The Effect of a Single Tuned Circuit and Two Coupled Tuned Circuits upon a Frequency Modulated Signal

While the foregoing example referred to the case of the carrier being tuned to the steepest slope of the resonant circuit (case of the frequency amplitude converter), we shall next investigate the case in which the carrier is tuned to the peak of the resonance curve. This case applies when a frequency modulated signal is amplified by a tuned radio-frequency or intermediate-frequency stage.

[5] H. Roder, Discussion on reference 3, Proc. I.R.E., vol. 20, p. 885, May, (1932).

The circuit is illustrated in Fig. 2. We put

$$\omega_0 L = \frac{1}{\omega_0 C} = X_0$$

$$\omega_0 = \text{resonant frequency} = \text{carrier frequency}$$

$$\frac{\omega}{\omega_0} = 1 + \delta$$

$$\frac{\omega - \omega_0}{\omega_0} = \frac{\mu}{\omega_0} = \delta$$

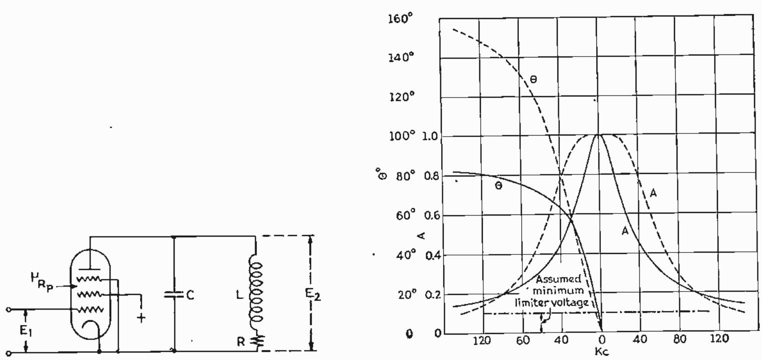
$$\mu = \text{audio (modulation) frequency}$$

$$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = 2\delta \qquad (9)$$

$$Q = \frac{X_0}{R} = \text{quality factor of coil}$$

$$\frac{X_0}{R_p} + \frac{R}{X_0} = \frac{X_0}{R_p} + \frac{1}{Q} = p = \text{power factor of circuit}$$

$$\mu_0 = \text{amplification factor of tube}$$
$$R_p = \text{plate resistance of tube}$$



(a) (b)

Fig. 2—Simple tuned amplifier.

For the transmission characteristic or gain of the stage we find

$$E_2 = \frac{\mu_0}{R_p} E_1 \frac{X_0}{p + j\,2\delta}\,.$$

If we write

$$\tan\theta = +\frac{2\delta}{p} = 2\frac{\mu}{\omega_0}\frac{1}{\dfrac{1}{Q} + \dfrac{X_0}{R_p}},$$

we get

$$E_2 = E_{20} \cos \theta \epsilon^{-j\theta}, \tag{10}$$

where,

$$E_{20} = \frac{\mu_0 E_1}{R_p} \frac{X_0}{p}.$$

The magnitudes E_2/E_{20} and θ, both plotted versus frequency, are shown in Fig. 2(a) (solid lines).

A frequency modulated input signal of the type indicated by (3) and (4) is now applied at the input terminals. The output can be determined by means of (10); each side band is reduced in amplitude by a factor $\cos \theta$ and shifted in phase by an angle $-\theta$. The output becomes (writing again J_n in place of $J_n(m)$)

$$\begin{aligned} e_2 = E_{20}[J_0 \sin \omega t & \\ &+ J_1 \cos \theta_1(\sin (\omega t + (\mu t - \theta_1)) - \sin (\omega t - (\mu t - \theta_1))) \\ &+ J_2 \cos \theta_2(\sin (\omega t + (2\mu t - \theta_2)) + \sin (\omega t - (2\mu t - \theta_2))) \\ &+ \cdots] \end{aligned}$$

whereby

$$\tan \theta_n = \frac{2}{p} \frac{n\mu}{\omega_0} = a\, n \tag{11}$$

$$a = \frac{2}{p} \frac{\mu}{\omega_0} \tag{12}$$

n = order of side band.

By rearranging

$$\begin{aligned} e_2 = E_{20}\Bigg[&+ \sin \omega t \Bigg(J_0 + \sum_{n=2,4,\cdots} (2J_n \cos \theta_n \cos (n\mu t - \theta_n)\Bigg) \\ &+ \cos \omega t \Bigg(\sum_{n=1,3,5,\cdots} (2J_n \cos \theta_n \sin (n\mu t - \theta_n)\Bigg)\Bigg]. \end{aligned} \tag{13}$$

Using the relations

$$\cos \theta_n = \frac{1}{\sqrt{1 + (a\,n)^2}},$$

$$\sin \theta_n = \frac{an}{\sqrt{1 + (a\,n)^2}},$$

we can write for the coefficient of the sin ωt term

$$\begin{aligned} R &= J_0 + \sum_{n=2,4,\cdots} (2J_n \cos\theta_n \cos(n\mu t - \theta_n)) \\ &= J_0 + \sum_{n=2,4,\cdots} (2J_n \cos^2\theta_n \cos n\mu t + 2J_n \sin\theta_n \cos\theta_n \sin n\mu t) \\ &= J_0 + \sum_{n=2,4,\cdots} \left(\frac{2J_n}{1+(na)^2}\cos n\mu t + \frac{2J_n na}{1+(na)^2}\sin n\mu t\right). \end{aligned} \tag{14}$$

Then

$$a\frac{dR}{d(\mu t)} = \sum_{n=2,4,\cdots} \left(\frac{-2J_n na}{1+(na)^2}\sin \mu nt + \frac{2J_n(na)^2}{1+(na)^2}\cos n\mu t\right).$$

Consequently, by using the Jacobi expansion formulas,

$$R + a\frac{dR}{d(\mu t)} = J_0 + \sum_{n=2,4,\cdots} (2J_n \cos n\mu t) = \cos(m \sin \mu t). \tag{15}$$

For the coefficient of the cos ωt term in (13) we find analogously

$$\begin{aligned} S &= \sum_{n=1,3,5,\cdots} (2J_n \cos\theta_n \sin(n\mu t - \theta_n)) \\ &= \sum_{n=1,3,5,\cdots} (2J_n \cos^2\theta_n \sin n\mu t - 2J_n \sin\theta_n \cos\theta_n \cos n\mu t) \\ &= \sum_{n=1,3,5,\cdots} \left(\frac{2J_n}{1+(na)^2}\sin n\mu t - \frac{2J_n na}{1+(na)^2}\cos n\mu t\right), \end{aligned} \tag{16}$$

and

$$S + a\frac{dS}{d(\mu t)} = \sum_{n=1,3,5} (2J_n \sin n\mu t) = \sin(m \sin \mu t). \tag{17}$$

We thus have for (13)

$$e_2 = E_{20}(R \sin \omega t + S \cos \omega t), \tag{18}$$

wherein R and S are complicated functions of μt. Equation (18) can be represented graphically by the diagram of Fig. 3. The angle ϕ_0 is to be determined, because $\omega_0 = d\phi_0/dt$, where ω_0 is the instantaneous value of the output frequency. Furthermore, the amplitude A must also be found because A must not fall below the minimum limiter input voltage. A and ϕ_0 are best determined graphically, after R and S are known. To find R and S two methods have been worked out.

The *first method* is a graphical one. We write for (14)

$$R = J_0 + \sum_{n=2,4,\cdots} (r_{n1} + r_{n2}) \tag{14a}$$

and for (16)

$$S = \sum_{n=1,3,5,\cdots} (s_{n1} - s_{n2}). \tag{16a}$$

The significance of the terms r_{n1}, r_{n2}, s_{n1}, and s_{n2} is evident when comparing (14) with (14a) and (16) with (16a). Referring now to Fig. 4, it will be seen that the magnitudes $(r_{n1}+r_{n2})$ and $(s_{n1}-s_{n2})$ can quite

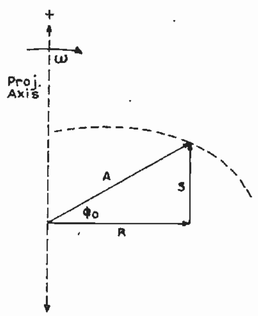


Fig. 3—Vector representation for modulated signal.

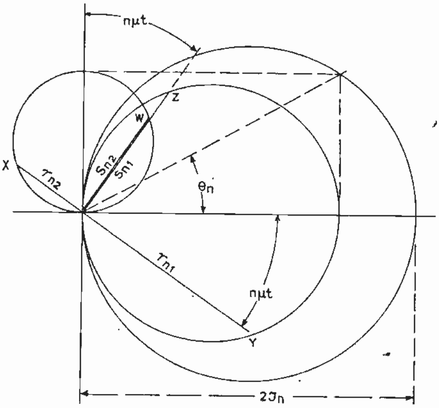


Fig. 4—Graphical representation of side-band amplitudes.

conveniently be determined by drawing the diagram of Fig. 4 for each side-band of order n and reading

$$\text{distance } XY = r_{n1} + rn_2,$$
$$\text{distance } WZ = s_{n1} - s_{n2}.$$

The proof will follow from the geometrical relations of the figure and need not be gone through here in detail. In this manner, by determining each component under the summation signs in (14) and (16) for a sufficiently large number of values of μt in the interval 0 to π, both R and S can be determined as functions of μt. This method can be used for relatively small values of m, say, up to $m=6$. Up to this value, not more than about nine terms or less in the summations need be considered, and μt time intervals of fifteen degrees are sufficient. For greater values of m, the series containing $J_n(m)$ do not converge fast enough; also, in order to get sufficient accuracy, one must compute R and S for time intervals smaller than fifteen degrees. The method becomes quite awkward and time-consuming for greater values of m.

The *second method* for finding R and S is based on the fact that these functions can be defined by differential equations of the first degree, (15) and (17). These differential equations are of a form which makes it relatively simple to obtain their solution by mechanical integration by means of a "differential analyzer."[6] Through the courtesy of the staff of the Moore School of Electrical Engineering, the use of the differential analyzer of that institute was made available and a number of solutions were obtained for various parameters by means of the machine. By the use of five integrators, of which four were used to generate the function sin (m sin X) or cos (m sin X), respectively, the operation of the machine was made entirely automatic. $m=24$ was the highest value of the parameter m for which a solution was drawn up by the machine; higher values of m could not well be handled due to mechanical limitations (high speed of integrators, available gears,

TABLE I

$\mu/2\pi$ cycles	$\omega/2\pi$ kc	$\Delta\omega/2\pi=m\mu/2\pi$ kc	% mod.	a	m
20,000	4,000	20	20	1	1.0
20,000	4,000	40	40	1	2.0
20,000	4,000	60	60	1	3.0
20,000	4,000	80	80	1	4.0
20,000	4,000	100	100	1	5.0
16,667	4,000	100	100	5/6	6.0
14,286	4,000	100	100	5/7	7.0
12,500	4,000	100	100	5/8	8.0
10,000	4,000	100	100	1/2	10.0
4,170	4,000	100	100	1/5	24.0

backlash etc.). However, it was found unnecessary to go to higher values of m than about 24, because of a more direct approach becoming available for high values of m than by means of (14) to (18).

The circuit of Fig. 2 was choosen because it represents the most simple amplifier circuit employing one tuned circuit. Of the two analytical methods just discussed, the second method is confined to this type of circuit; the first one, however, is applicable to more complicated transmission networks, such as, for instance to standard type intermediate-frequency transformers comprising two loosely coupled tuned circuits. The numerical data choosen for the circuit of Fig. 2 were

$$X_0 = 2500 \text{ ohms}$$

$$Q = 133$$

$$R_p = 1{,}000{,}000 \text{ ohms}$$

$$p = 1/100.$$

The carrier frequency was assumed equal to 4000 kilocycles, while a frequency shift of ± 100 kilocycles was taken to correspond to 100 per cent modulation. The cases shown in Table I were investigated.

[6] V. Bush, "Mechanical differential analyzer," *Jour. Frank. Inst.*, vol. 212, p. 447; October, (1931).

As seen from (15) and (17) the only independent parameters of the problem are m and a. Thus, for instance, the case $a=1/2$, $m=10$ in the above tabulation would also correspond to the following set of values:

$$\frac{\mu}{2\pi} = 1000 \text{ cycles}; \quad \frac{\omega}{2\pi} = 400 \text{ kilocycles}; \quad \frac{\Delta\omega}{2\pi} = 10 \text{ kilocycles}$$

or, to the following set,

$$p = 1/50; \frac{\mu}{2\pi} = 500 \text{ cycles}; \frac{\omega}{2\pi} = 100 \text{ kilocycles}; \frac{\Delta\omega}{2\pi} = 5 \text{ kilocycles}.$$

The data for the circuit and for the signal are choosen such that, if the carrier is tuned to the circuit resonant frequency, an attenuation of 1/0.196 will result ($\cos\theta=0.196$) for a frequency deviation of ± 100 kilocycles. We assume that the minimum voltage required for satisfactory operation of the limiter is say 10 per cent of the maximum voltage occurring (at $\omega=\omega_0$) during the audio cycle. The minimum voltage to which the signal amplitude will fall during one audio cycle (at low audio frequency) is twenty per cent of the maximum; consequently there is at all times sufficiently high voltage for the limiter to work on (Fig. 2 (a)).

The functions R and S in (18), are functions of μt. To obtain the instantaneous ("instantaneous" with reference to the modulation frequency) amplitude and phase of the signal, (equation (18)), R is plotted as a horizontal and S as a vertical vector. According to Fig. 3, a spiral diagram will result in this manner, as shown in Figs. 6 to 14. The desired amplitude of e_2 then becomes $\sqrt{R^2+S^3}$ and the desired phase angle $\phi_0=\arctan S/R$. Both magnitudes can be readily determined from the spiral diagram; they are also plotted in Figs. 6 to 14. We are mostly interested in the magnitude ϕ_0 because

$$\frac{d\phi_0}{dt} = \omega_0 = \text{output signal}.$$

ω_0, the output magnitude, has not been plotted in the following figures, because it simply is the derivative of ϕ_0. Considering distortions in the ϕ_0 wave, however, it must be remembered, that an n^{th} harmonic component of a per cent in ϕ_0 will cause a corresponding harmonic component of na per cent in ω_0. In other words, the harmonics in the ω_0 wave are always larger than those of the ϕ_0 wave. ϕ_0 and ω_0 behave analogous to the terminal voltage e across a capacitor and the current i flowing through it, with reference to their respective harmonics.

Before beginning with the discussion of the diagrams it is desirable to consider what diagram will be obtained if the input is plotted in a manner analogous to that used for plotting the output. We find from (3) that

$$e_1 = E_1(\sin \omega t \cos (m \sin \mu t) + \cos \omega t(\sin m \sin \mu t)).$$

Hence, for the input signal,

$$R = \cos (m \sin \mu t),$$

$$S = \sin (m \sin \mu t),$$

$$\text{amplitude} = \sqrt{R^2 + S^2} = \text{constant} = 1,$$

$$\phi = m \sin \mu t.$$

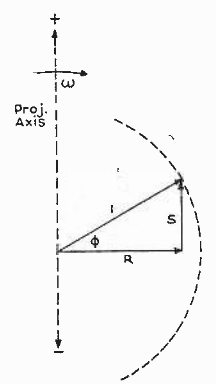


Fig. 5—Vector diagram for sinusoidal frequency modulation.

The polar diagram of the input signal becomes a circle, Fig. 5. The end point of the amplitude vector travels along a circle performing thereby a harmonic motion. If we plot ϕ versus μt we obtain a sine wave.

We now can discuss the results found for the output signal.

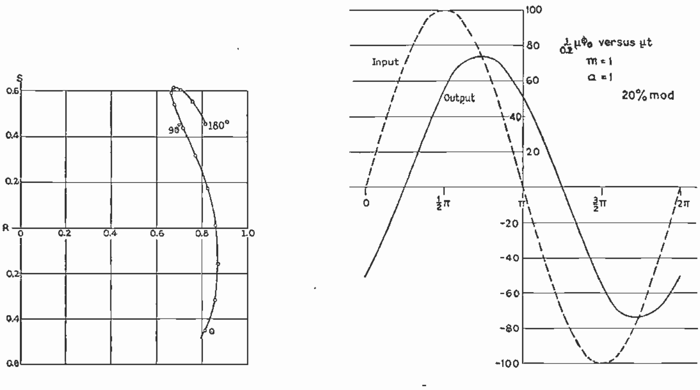
1. $m=1$	$\omega=2\pi$ 4,000,000
$a=1$	$\mu=2\pi$ 20,000
20% modulation	$\Delta\omega=2\pi$ 20,000

Polar diagram R versus S is shown in Fig. 6(a). The curve does not depart very much from a circle. ϕ_0 versus μt is shown in Fig. 6(b); this curve is approximately a sine wave.

In all ϕ_0 versus μt diagrams the magnitude $\mu\phi_0/k$ (where k is the per cent modulation) is always plotted versus μt in order to have all diagrams to the same scale for better comparison. (The dotted sections in the ϕ_0 curve indicate that during these intervals the amplitude will fall below the ten per cent safety limit.) The input diagram is shown dotted in Fig. 6(b); it would be the same for all following ϕ_0 diagrams.

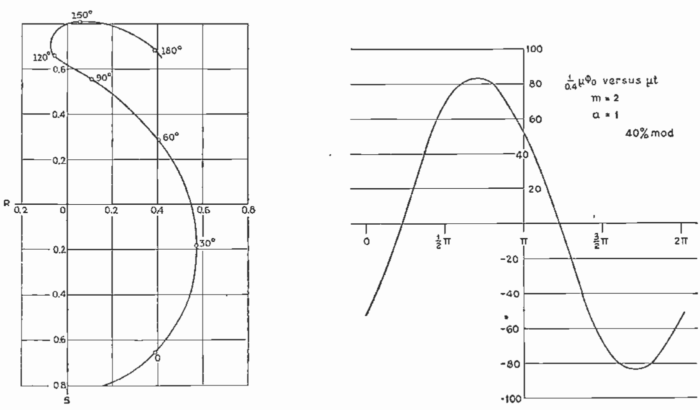
2. $m=2$ — ω and μ: same as before
   $a=1$ — $\Delta\omega = 2\pi \quad 40{,}000$
   40% modulation

Polar diagram: Fig. 7(a); ϕ_0 diagram: Fig. 7(b). The amplitude reduc-



(a) (b)

Fig. 6

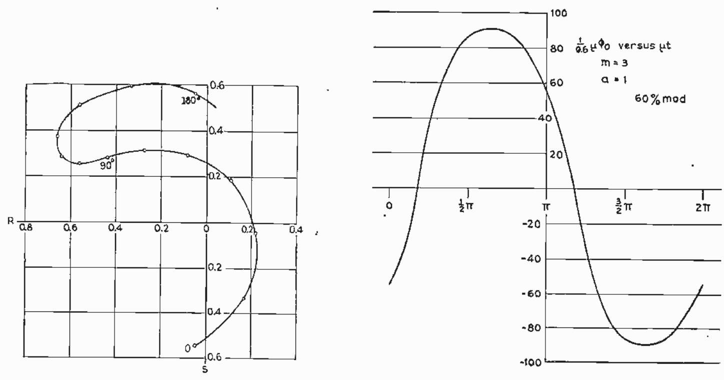


(a) (b)

Fig. 7

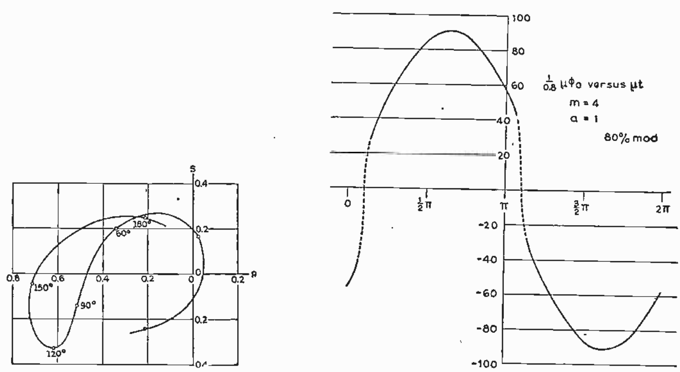
tion is already appreciable, while the ϕ_0 characteristic begins to show some distortion.

3. $m=3$ — 60% modulation
   $a=1$ — $\Delta\omega = 2\pi \quad 60{,}000$

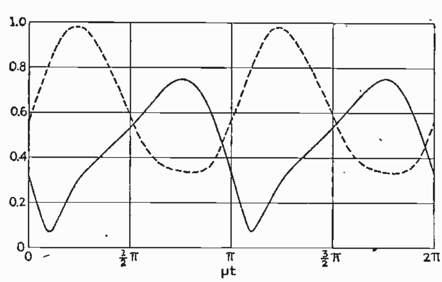


(a) (b)

Fig. 8



(a) (b)



(c)

Fig. 9

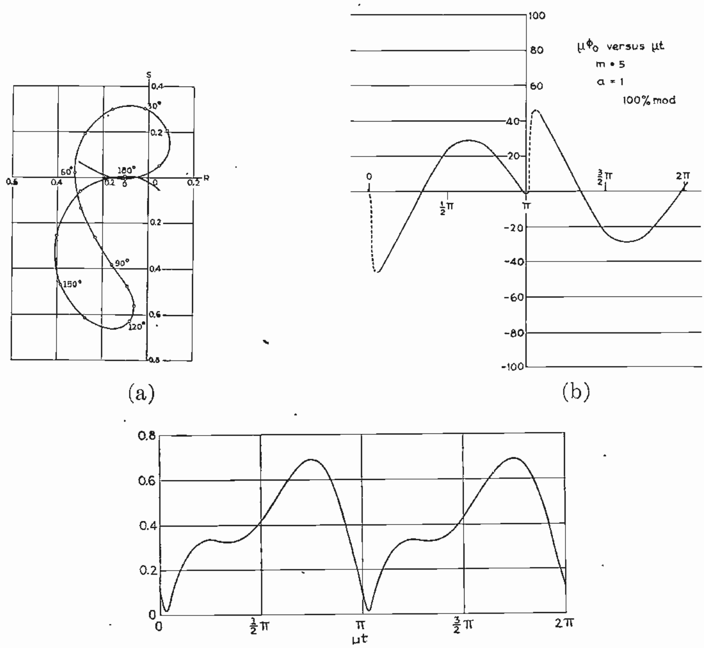
Figs. 8(a) and 8(b). Note how the spiral curls up, approaching the origin.

4. $m=4$ 80% modulation
$a=1$ $\Delta\omega=2\pi$ 80,000

Figs. 9(a), 9(b), and 9(c). The spiral approaches the origin closer over a short time interval, the amplitude falls below the safety limit of ten per cent from the normal value (Fig. 9(c)). The ϕ_0 curve begins to show appreciable nonlinear distortion.

5. $m=5$ 100% modulation
   $a=1$ $\mu=2\pi$ 20,000
   $\Delta\omega=2\pi$ 100,000

Figs. 10(a), 10(b), and 10(c). The spiral has curled up to such an extent as to pass on the other side of the origin. The amplitude falls for



(a)   (b)

(c)

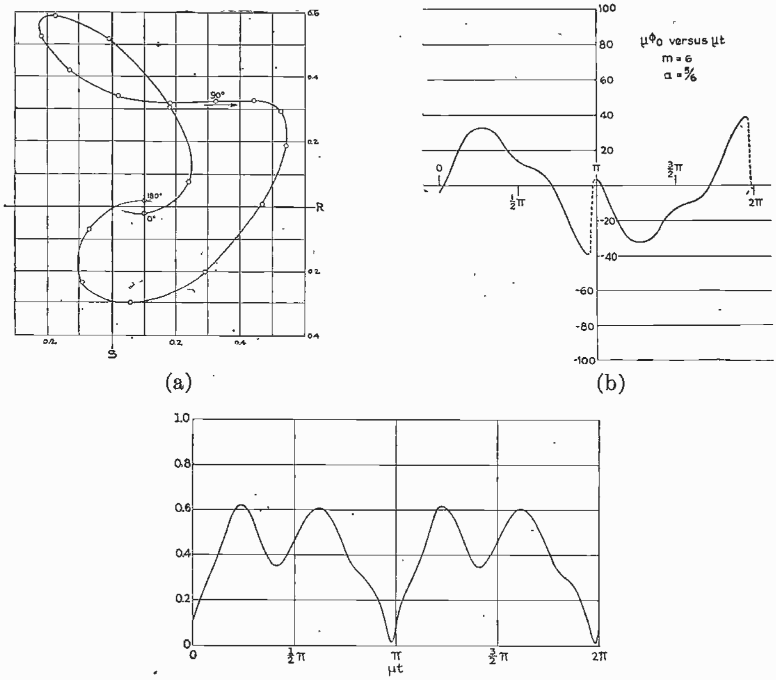
Fig. 10

a short interval below the ten per cent limit. The ϕ_0 curve is extremely distorted.

6. $m=6$ $\omega=2\pi$ 4,000,000
   $a=5/6$ $\mu=2\pi$ 16,667
   100% modulation $\Delta\omega=2\pi$ 100,000

Figs. 11(a), 11(b), and 11(c). The spiral passes very close to the origin, causing very severe distortions of the ϕ_0 curve.

7. $m=7$ $\mu=2\pi$ 14,280
   $a=5/7$ $\Delta\omega=2\pi$ 100,000
   100% modulation

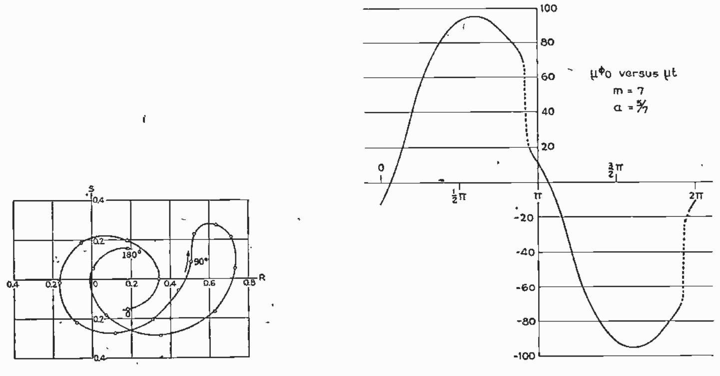


(a) (b) (c)

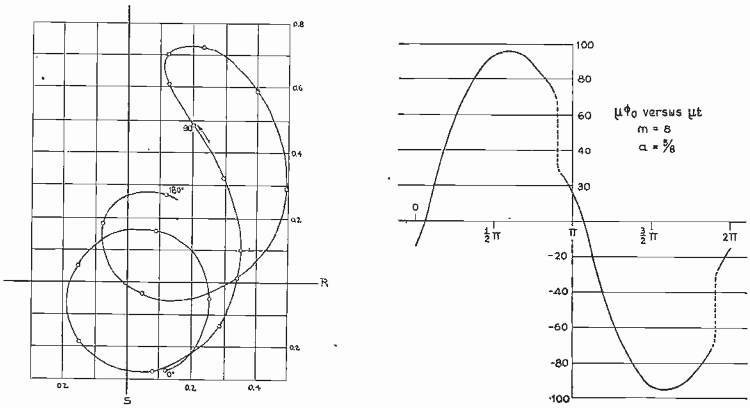
Fig. 11

Figs. 12(a) and 12(b). The spiral has retreated slightly from the origin, but the nonlinear distortion of ϕ_0 is still high.

8. $m=8$ $\quad\quad\quad\quad$ $\mu=2\pi \quad 12,500$
$a=5/8$ $\quad\quad\quad\quad$ $\Delta\omega=2\pi \quad 100,000$
100% modulation



(a) Fig. 12 (b)



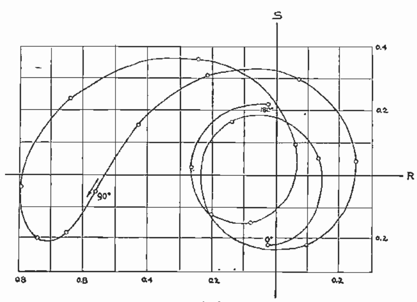
(a) Fig. 13 (b)

Figs. 13(a) and 13(b). ϕ_0 distortion somewhat less, but the spiral still intersects the ten per cent safety limit.

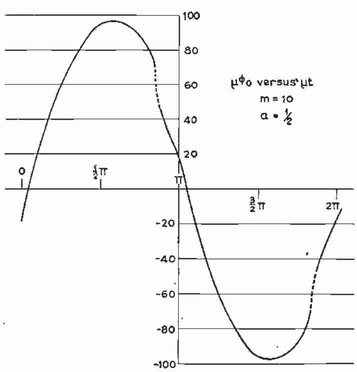
9. $m=10$ $\mu=2\pi$ 10,000

$a=1/2$ $\Delta\omega=2\pi$ 100,000

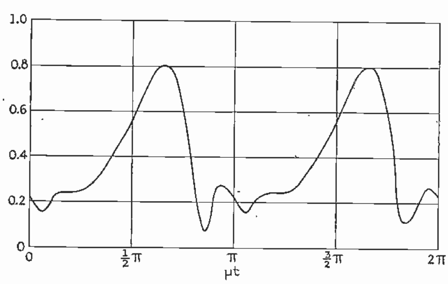
100% modulation



(a)



(b)



(c)

Fig. 14

Figs. 14(a), 14(b), and 14(c). Less ϕ_0 distortion, but the spiral still intersects the ten per cent safety limit.

10. $m=24$ $\qquad\qquad\qquad\qquad$ $\mu=2\pi \quad 4,170$
$a=1/5$ $\qquad\qquad\qquad\qquad$ $\Delta\omega=2\pi \quad 96,000$
100% modulation

Figs. 15(a), 15(b), and 15(c). The spiral has retreated sufficiently from the origin, such that the amplitude is always greater than the ten per

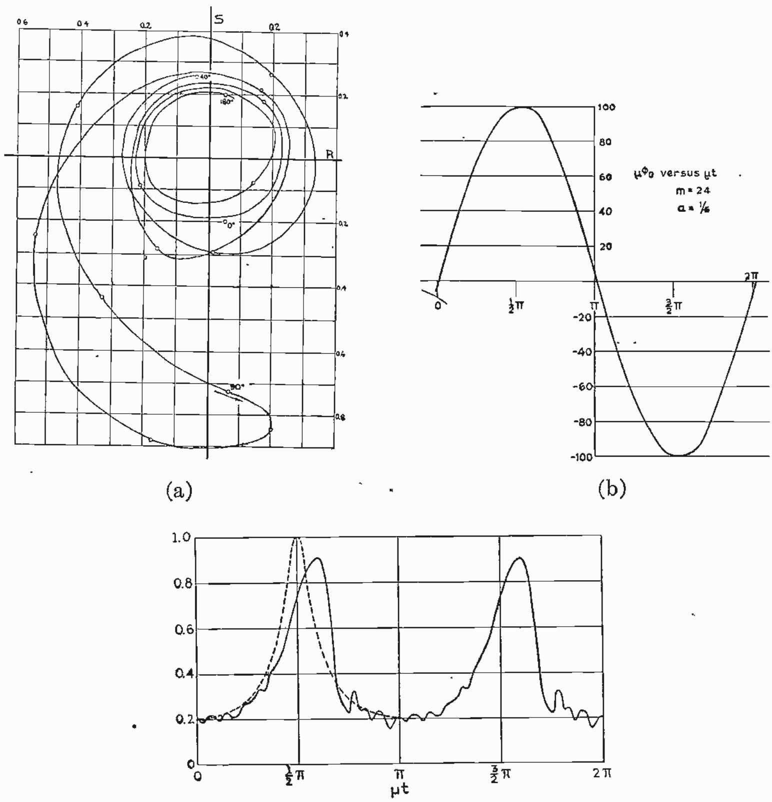


Fig. 15

cent limit. There is some ϕ_0 distortion left, but its amount is insignificant.

We note from these results that the tuned circuit has, for certain values of m and a, a very disturbing effect upon the frequency modulated signal, causing nonlinear distortions much more severe than would be encountered in a corresponding case with amplitude modulation. If the spiral diagram passes through or close by the origin, severe distortions of the output phase will occur; simultaneously the amplitude is liable to drop for a short time interval below the minimum limiter voltage, causing thereby additional disturbances, due to unsatisfactory limiter action. We can, at least in a qualitative way, mark off the "danger zone" by plotting a curve of m and a values, for which the spiral passes through the origin. From the relatively meager data we have, this curve can be drawn approximately. (Fig. 16.) Values of m and a corresponding to points below the line yield only small distortion for ϕ_0. As the line is approached, these distortions increase. For values of m and a, located on the line itself, the spiral passes through the origin, causing a "phase jump" of 180 degrees. Values of m and a located above the line, indicate a very highly distorted output.

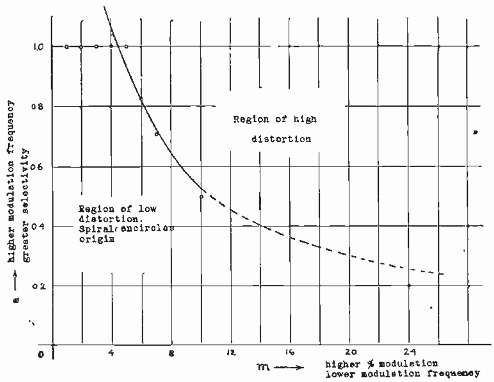


Fig. 16—Regions of high and low distortion of a frequency modulated signal, for one single lined circuit.

The largest value of m, which was considered in the data taken, was 24. It is not necessary to go to higher values of m, because for lower audio frequencies the dynamic effects of the tuned circuit become smaller. At very low audio frequency, amplitude and phase of the output can follow instantaneously those of the input. In other words, they can be computed from the steady-state amplitude and phase characteristics of the circuit. In Fig. 15(c) an amplitude characteristic computed in this manner is shown (dotted curve); it is seen that the $m=24$ amplitude characteristic already approaches the steady-state curve. The steady-state ϕ_0 characteristic was not drawn up, because it would be a sine wave. There is a slight second harmonic distortion in that characteristic, due to the curvature of the θ characteristic, Fig. 2(a). But the maximum value which θ can have is only $\pm\frac{1}{2}\pi$, while m is many times larger; consequently the superposition of the θ characteristic can only cause very minute nonlinear distortions.

For small values of m, say $m \leqq 1/2$, the output signal can be computed directly. In this case, the sine and cosine term on the right-hand side of (15) and (17), respectively, can be replaced by the first terms of their respective power series, yielding

$$R + a \frac{dR}{d(\mu t)} = 1$$

$$S + a \frac{dS}{d(\mu t)} = m \sin \mu t.$$

The steady-state solutions of these differential equations are

$$R = 1$$

$$S = \frac{m}{1 + a^2} (\sin \mu t - a \cos \mu t).$$

Because, from (11), $\tan \theta_1 = a$, we get

$$\phi_0 = \text{arc} \tan [m \cos^2 \theta_1 (\sin \mu t - a \cos \mu t)]$$

and

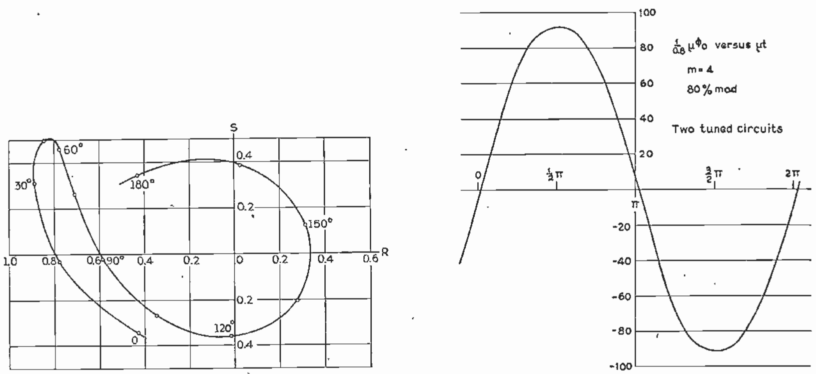
$$\omega_{\text{output}} = \frac{d\phi_0}{dt} = \frac{m\mu \cos \theta_1 (\cos (\mu t - \theta_1))}{1 + (m \cos \theta_1 \sin (\mu t - \theta_1))^2}$$

The squared term in the denominator is small in comparison to 1, hence

$$\omega_{\text{output}} = \mu m \cos \theta_1 \cos (\mu t - \theta_1). \tag{19}$$

This result is remarkable in so far as it is identical with the result obtained for amplitude modulation for a single tuned circuit of the same type: the audio amplitude is decreased by a factor $\cos \theta_1$ (where θ_1 is the phase shift between carrier and first side band) and the audio signal is delayed by a phase angle θ_1.

The nonlinear distortions in the output are due to the attenuation of the side-band amplitudes and due to the phase shifts which are not proportional to the frequency deviation. We will note from (5) and (8), if we put $s = 0$, that a transmission network whose transmission characteristic is perfectly uniform over the pass band, and whose phase characteristic is a linear function of frequency, cannot produce nonlinear distortions of the output. To prove this point, let us consider a network having two tuned circuits, like an ordinary intermediate-frequency transformer. For the example we assume critical coupling between the two circuits; i.e., the tightest coupling between the circuits which will give a single peak resonance curve. The transmission characteristic of this network will approach the ideal band-pass characteristic. The data are chosen such that at $\Delta\omega = 2\pi\ 100{,}000$ the same attenuation is obtained as for the single tuned circuit. The resulting



(a) Fig. 17 (b)

amplitude and phase are shown dotted in Fig. 2(a); these magnitudes are computed from the relation

$$\frac{E_2}{E_1} = +j\,\frac{\mu_0}{R_p}\,\frac{X_{m0}}{(p_1 + j2\delta)(p_2 + j2\delta) + k^2} \tag{20}$$

where,

$$p_1 = \frac{1}{Q_1} + \frac{X_{10}}{R_p} = \text{power factor of primary circuit}$$

$$p_2 = \frac{1}{Q_2} + \frac{X_{20}}{R_0} = \text{power factor of secondary circuit.}$$

Numerically, the values

$$p_1 + p_2 = 3.17 \cdot 10^{-2}$$

$$k^2 = 1/2(p_1^2 + p_2^2)$$

are chosen; the signal is an 80 per cent modulated signal with $m=4$. Equation (20) is now used in the same manner as (10) was used before. The results are presented in the polar diagram of Fig. 17(a), from which the ϕ_0 curve (Fig. 17(b)) and the amplitude curve (Fig. 9(c), dotted) are obtained. In comparison with the data for $m=4$, $a=1$, 80 per cent modulation resulting for a single tuned circuit, an over-all improvement has been obtained: the ϕ_0 curve shows practically pure sine wave form, while the minimum value of the amplitude is about four times higher. Thus, the band-pass type circuit yields a generally better performance, which is in accordance with expectations.

With this, we shall conclude the section on the effects of tuned circuits.

The results can be summarized as follows:

In *amplitude modulation* it will be sufficient for preventing nonlinear distortion, to have a transmission characteristic which, in phase and amplitude, is *symmetrical* with respect to the carrier frequency. If this condition holds, then curvature in the amplitude or phase characteristic will cause linear distortions only.

In *frequency modulation* symmetry of the transmission characteristic is not sufficient (except for large values of the modulation index m) Curvature of the phase characteristic will cause nonlinear audio distortions, while curvature of the amplitude characteristic may cause additional distortions if the amplitude happens to drop below the limiter threshold voltage. Band-pass type transmission characteristics whose phase and amplitude are linear functions of frequency over the required band should be used.

IV. Interference Between Two Frequency Modulated Signals

An interesting phenomenon in amplitude modulation is the "suppression of a weak modulated signal by a strong carrier" or, as the English termed it, the "demodulation" effect.[7] It takes place, if the carrier frequencies of the two signals differ by a relatively high frequency, say ten to twenty kilocycles provided the diode load circuit will still represent a substantial impedance at that frequency. If,

$$r = \text{ratio of } \frac{\text{unmodulated}}{\text{modulated}} \text{ carrier,}$$

and k is the percentage modulation of the undesired signal, then k_1, the audio output impressed upon the strong carrier, is

$$k_1 = 1/2 \frac{k}{r} \tag{21}$$

for large values of $r (r=3$ or greater).

[7] R. T. Beatty, "Apparent demodulation of a weak station by a stronger one," *Exp. Wireless and Wireless Eng.*, vol. 5, p. 300; June, (1928).
S. Butterworth, "Apparent demodulation of a weak station by a stronger one," *Wireless Eng. and Exp. Wireless*, vol. 6, p. 619; November, (1929).
F. M. Colebrook, "A further note on apparent demodulation," *Wireless Eng. and Exp. Wireless*, vol. 8, p. 409; August, (1931).
G. W. O. Howe, "Mutual demodulation and allied problems," *Wireless Eng. and Exp. Wireless*, vol. 8, p. 405; August, (1931).

We shall now investigate the corresponding case of interference for two frequency modulated signals. The desired signal shall be an unmodulated carrier for which we write

$$e_1 = A \sin \omega t.$$

The interfering signal shall be 100 per cent modulated with a low audio frequency, say 200 cycles. The frequency shift corresponding to 100 per cent modulation, shall be $\Delta\omega/2\pi$, which may be taken as 100,000 cycles. Thus we write for the interfering signal

$$e_2 = B \sin \left((\omega - s)t - m \cos \mu t + m \frac{\pi}{2} \right). \tag{22}$$

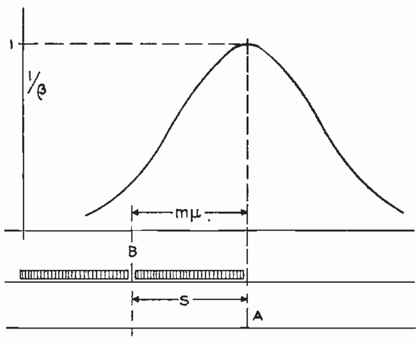


Fig. 18—Interference between two frequency modulated signals.

The instantaneous difference phase with respect to e_1 is

$$\Delta\phi = - st - m \cos \mu t + \frac{\pi}{2} m \tag{23}$$

and the instantaneous difference frequency

$$\Delta\omega' = - s + m\mu \sin \mu t. \tag{24}$$

The two carriers are spaced by a frequency interval of $s/2\pi$ cycles, for which we choose $s = \Delta\omega = m\mu$. In other words, we make the carrier spacing equal to the maximum frequency shift. We now tune the radio receiver to the carrier A. Fig. 18 is an illustration of the conditions which we shall consider. The curve $1/\beta$ represents the transmission characteristic of the radio receiver; β denotes the attenuation which is provided by the circuit.

According to the results of the preceding section we can study this case "statically," because m is large ($m = \Delta\omega/\mu = 500$) and μ relatively small. This means that B may be considered as a signal whose frequency changes $\pm 100{,}000$ cycles at a rate of 200 times per second, which rate of change the tuned circuits are capable of following instantaneously. At the time $\mu t=\pi/2$, $\Delta\omega'$ becomes zero; i.e., the frequency of B is equal to the frequency of A. If there is any interference upon A from B, it must originate during this time interval. It is therefore preferable to express (22), (23), and (24) in terms $\mu t-\pi/2$, by putting $\mu t=\pi/2+\delta$. By substitution into (23)

$$\Delta\phi = m(\sin\delta - \delta) = -m\frac{\delta^3}{6}\left(1 - \frac{\delta^2}{4.5} + \frac{\delta^4}{4\cdot 5\cdot 6\cdot 7}\right). \tag{25}$$

The instantaneous frequency f becomes

$$f = \frac{\Delta\omega}{2\pi} = -\frac{\Delta\omega}{2\pi}(1 - \cos\delta). \tag{26}$$

During the modulation cycle, B undergoes changes in amplitude and additional changes in phase due to the selective circuits. For these we assume three cascaded intermediate-frequency stages, each critically coupled for maximum band-pass effect, each giving an attenuation of 7.07, 7.07, and 2.0, respectively, at a frequency deviation of $\pm 100{,}000$ cycles from the frequency carrier A. Total attenuation and phase shift θ are as follows:

Frequency Deviation f from ω	Total Attenuation β	Total Phase Shift θ
0	1.00	0
20,000	1.08	+113°
40,000	2.29	+235°
60,000	8.70	+319°
80,000	31.50	+369°
100,000	100.0	+408°

The resulting signal can now be determined from the vector diagram of Fig. 19. We have, in this diagram, a projection axis rotating clockwise with the angular velocity ω, a stationary vector A of constant length, and a rotating vector of length B/β whose instantaneous phase position is $(\Delta\phi+\theta)$. β, $\Delta\phi$, and θ are functions of δ; i.e., of μt. The resulting signal which enters the input terminals of the limiter device, has the amplitude R and the instantaneous phase angle ξ. R must not fall below the limiter minimum input voltage; that means that the spiral L which is described by the end point of B/β must not intersect the little circle of radius T in Fig. 19.

The magnitude $d\xi/dt$ is proportional to the instantaneous frequency of R, this magnitude in turn is proportional to the rectified audio-frequency output. Hence $d\xi/dt$ is a measure for the interference superimposed by B upon A.

Next, we must define how the interference shall be measured. The fundamental frequency of the interference, if any, is μ. Let us assume that, if A is modulated, a frequency shift of $\pm 100{,}000$ cycles shall correspond to 100 per cent modulation; the same as we have assumed previously for B. Then if $d\xi/dt$ produces, say, a 50,000-cycle frequency variation of R, this would represent a 50 per cent interference, with $m=50{,}000/200=250$ radians.

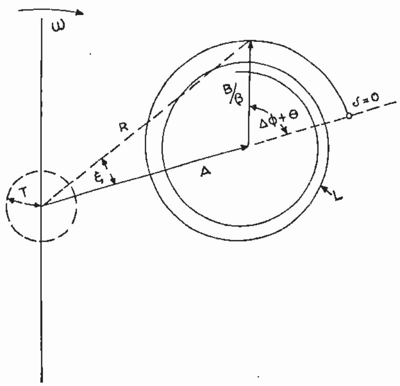


Fig. 19

Case 1. B smaller than A

Fig. 19 refers to this case. The spiral described by B lies externally to the center O. ξ is always smaller than $\frac{1}{2}\pi$ or 1.5 radians. The interference would be in the order of $1.5/500=0.3$ per cent.

Case 2. B equal to A

At $\delta=0$, A and B are in phase (equation (25)). As δ deviates from zero, B decreases in amplitude, describing the spiral L. $\xi_{\max}$ is still smaller than $\frac{1}{2}\pi$. There exists the possibility that the spiral L intersects the circle T, thereby causing some short duration amplitude disturbances. The interference is still in the same order as in Case 1; i.e., very small.

Case 3. B greater than A (Fig. 20).

We note that ξ is not very much different from $(\Delta\phi+\theta)$, as long as $A \ll B/\beta$.

Case 3 yields very appreciable interference from B upon A. We now assume for a numerical example the three cascaded stages referred to above and a carrier ratio of B:A of 6:1. We compute $\Delta\phi$ from (25), while θ is given in the above tabulation. B/β is also known from the tabulation. Next, Fig. 20 is drawn to scale and ξ determined from it, as a function of δ. $(\Delta\phi+\theta)$ is plotted versus δ in Fig. 21. In Fig. 22, both ξ and $(\Delta\phi+\theta)$ are plotted versus δ, whereby Fig. 22 is

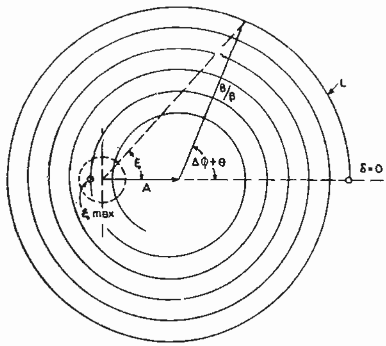


Fig. 20

an enlarged section of Fig. 21. As δ increases from zero up, it is seen that ξ follows the $(\Delta\phi+\theta)$ curve very closely, until, at about $\delta=63$ degrees it suddenly departs from it, following from there on an average value of ξ_{max}. This "break" occurs at the time at which B/β becomes

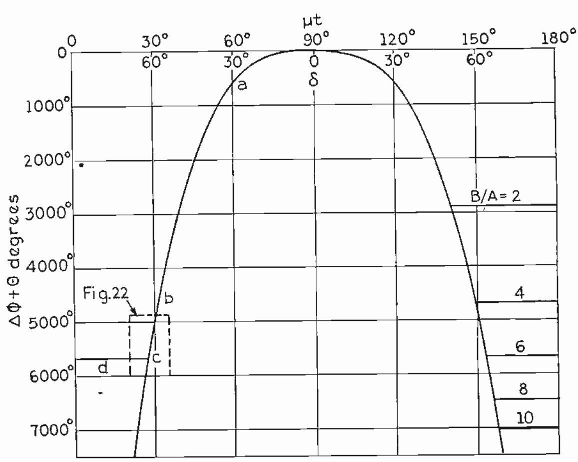


Fig. 21

smaller than A. ξ_{max} is equal to the value resulting for $(\Delta\phi+\theta)$ at the point (point ξ_{max} in Fig. 20) where

1. $(\Delta\phi+\theta)$ is an odd multiple of π, and
2. B/β is for the first (or last) time during one audio cycle greater than A.

It is evident that at this point also some amplitude disturbances will occur since A and B/β are nearly equal.

Inspection of Fig. 22 shows that the deviation of ξ from the $(\Delta\phi+\theta)$ and from the ξ_{max} curve is very small. The high-frequency variations

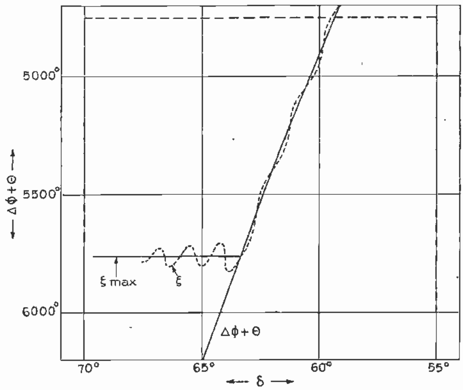


Fig. 22

present in ξ correspond to the instantaneous "beat" frequency, between A and B (equation (26)); this beat frequency is already supersonic at $\delta=60$ degrees. We can, therefore, for determining the low-frequency interference, replace the ξ curve by the line a, b, c, d, in Fig. 21; i.e., by $(\Delta\phi+\theta)$ and ξ_{max} respectively. The value of ξ_{max} is different

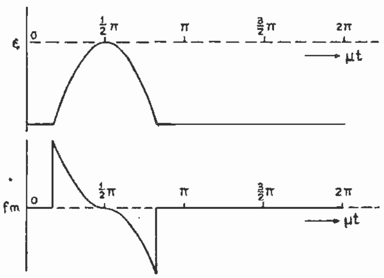


Fig. 23—Wave shape of phase and frequency disturbance, due to interference between A and B.

for different carrier ratios as shown in the right-hand side of Fig. 21, while the curve $(\Delta\phi+\theta)$ is, of course, independent of carrier ratio.

In Fig. 23, we have the approximate curve resulting for ξ plotted versus μt. The curve f_m, for which $f_m=d\xi/dt$, is the frequency modulation superimposed by the interfering signal B upon the carrier A. The numerical evaluation yields the following data for various carrier ratios:

		Amplitude Modulation 100% mod. of B		Frequency Modulation B modulated $\pm$ 100,000 cycles = 100%						Peak Value of f_m		
Ratio B/A	Ratio $B/\beta:A$	Modulation Suppression $1/2(k/r)$	% Modulation Impressed upon A	Fundamental Comp. of f_m Cycles	2nd Harmonic of f_m Cycles	3rd Harmonic of f_m Cycles	4th Harmonic of f_m Cycles	5th Harmonic of f_m Cycles	r-m-s Value (Cycles) of f_m (from first 5 components)	Cycles	%	% Modulation (r-m-s) Impressed upon A
1	2	3	4	5	6	7	8	9	10	11	12	13
1	1/100:1	0.005	0.005%	—	—	—	—	—	—	—	—	1%
2	2/100:1	0.010	0.020%	3900	6000	5150	2350	1120	9200	34,000	34%	9.2%
4	4/100:1	0.020	0.080%	7220	9700	6240	156	5000	14500	47,000	47%	14.5%
6	6/100:1	0.030	0.18%	9270	11830	6000	2270	6950	19000	52,500	52.5%	19.0%
8	8/100:1	0.040	0.32%	10750	13100	5650	3900	7900	19900	58,000	58%	19.9%
10	10/100:1	0.050	0.50%	11800	14000	5250	5100	8360	21400	60,000	60%	21.4%
12	12/100:1	0.060	0.72%	12970	14600	4700	6300	8800	22800	64,000	64%	22.8%

We have in column 1 the ratio of the two carrier signals at the receiver input terminals, and in column 2 their ratio at the limiter input terminals after having passed through the selective circuits. In columns 5 to 9, we have the harmonic components of the curve f_m, Fig. 24. Column 10 gives the root-mean-square value for f_m, resulting from the first five harmonic components (all figures given in cycles). Columns 10 and 12 give peak values of f_m. In column 13 we have the root-mean-square value of the f_m curve expressed in per cent, with 100,000 cycles having been taken as 100 per cent.

For reason of comparison, the corresponding values for amplitude modulation are given in columns 3 and 4. Column 3 refers to the "modulation suppression," equation (21), putting for r the ratios found in column 2. Column 4 gives figures resulting for the percentage modulation impressed by B upon A; this resulting percentage modulation is $\frac{1}{2}k/r^2$.

If one compares the figures in columns 12 and 13 with those of column 4, one will notice that, for the case considered above and illustrated in Fig. 18, the mutual interference between the two signals is about 100 times higher in frequency modulation than in amplitude modulation.

The reason for this excessive interference is immediately evident: it rests in the fact that during one part of the audio cycle the amplitude of B exceeds that of A. This is in accordance with the results of an earlier paper,[2] where we concluded that the receiver is always controlled by the signal having the greater amplitude. The selectivity curve $1/\beta$, which is provided to protect A, is of no avail because the frequency of

B becomes nearly equal to that of A during certain time intervals. A "point-pass" characteristic would of course shorten these time intervals and reduce the interference, but would distort the modulation of A. A sharp "band-pass" characteristic would make the interference worse by lengthening the time intervals during which B/β is greater than A, while on the other hand it would improve the modulation of A.

The remedy for eliminating the interference is also immediately evident. It is not an increase in selectivity, but an increase in the spacing between A and B. It can be simply formulated as

$$B/\beta < A \text{ during one audio cycle of } B.$$

As an example take the case of the three cascaded circuits considered above. If, in Fig. 18, we make the spacing $s = 180{,}000$ cycles, but maintain the frequency shift $m\mu/2\pi = 100{,}000$ cycles, then B/A may be any value up to 31.50 (see tabulation for β) before any noticeable interference can start. In case the receiver has a sharp band-pass characteristic which just accommodates a total band of 200,000 cycles, then the spacing s should be 200,000 cycles also.

Hence we note that the problem of interference between two frequency modulated signals is entirely different from that between two amplitude modulated signals. The helpful phenomenon of "demodulation" does not exist. Increasing selectivity alone does not help to reduce interference unless accompanied by sufficient channel spacing. This spacing should be at least equal to the total signal band width.

Acknowledgment

The writer takes pleasure in expressing his thanks to Dr. W. R. G. Baker and Mr. I. J. Kaar for their helpful interest in a subject as remote at present from practical application as the one above, to Dr. H. Poritsky of the Schenectady Works for valuable mathematical advice, and in particular to Professor Irven Travis of the Moore School of Electrical Engineering for his assistance in handling the work on the differential analyzer.

Proceedings of the Institute of Radio Engineers
Volume 25, Number 12 *December, 1937*

CHARACTERISTICS OF THE IONOSPHERE AT WASHINGTON, D.C., OCTOBER, 1937*,[1]

By

T. R. Gilliland, S. S. Kirby, N. Smith, and S. E. Reymer
(National Bureau of Standards, Washington, D. C.)

Fig. 1 shows the critical frequency and virtual height data for October, 1937. The critical frequencies of the F layer for the undisturbed days in October, 1937, exceeded those for October, 1936, by approximately the following amounts: noon f_{F_2}—1100 kilo-

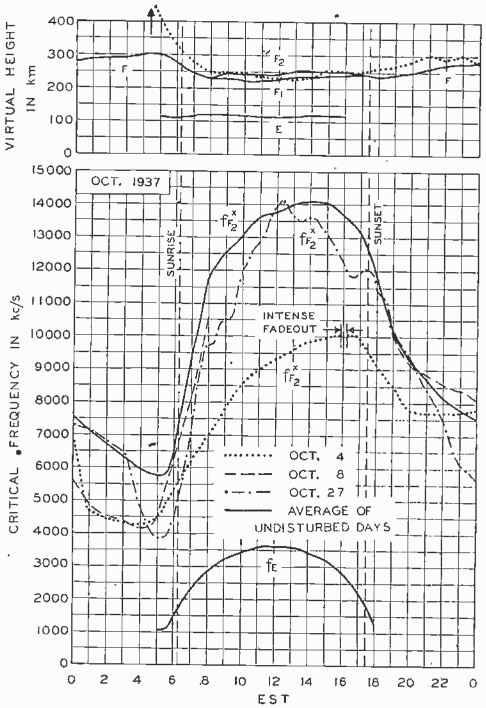


Fig. 1—Virtual heights and critical frequencies of the E, F_1, and F_2 layers of the ionosphere for October, 1937.

cycles, midnight f_F—1000 kilocycles, diurnal minimum (0520 local time) f_F—650 kilocycles. In October, 1937, the noon f_E was 100 kilocycles less than in October, 1936.

* Decimal classification: R113.61. Original manuscript received by the Institute, November 10, 1937.

[1] This is one of a series of reports on the characteristics of the ionosphere at Washington, D. C. For earlier publications on this subject see Proc. I.R.E., vol. 25, pp. 823–840; July, (1937), and a series of monthly reports beginning in Proc. I.R.E., vol. 25, pp. 1174–1191; September, (1937). Publication Approved by the Director of the National Bureau of Standards of the U.S. Department of Commerce.

About fifty per cent of the time at night during October, 1937, strong complex reflections were observed at frequencies much greater than $f_F{}^x$. These reflections provided a poor quality transmission at frequencies much above the maximum usable frequencies for F-layer transmission.

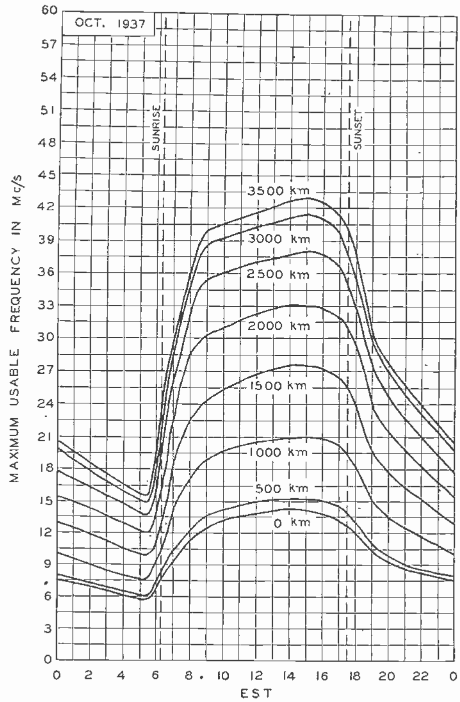


Fig. 2—Maximum usable frequencies for latitude of Washington average for October. Time to be used is local time where the wave is reflected from the layer.

TABLE I

Date 0000–2400 E.S.T.	h_F before sunrise km	Min. $f_{F_2}{}^x$ during day (before sunrise) kc	Max. $f_{F_2}{}^x$ during day (near noon) kc	Magnetic Character[1]	
				0000–1200 G.M.T.	1200–2400 G.M.T.
Oct. 4	452	4500	10,000	1.9	0.9
Oct. 26	348	6000	10,300	1.0	0.9
Oct. 27	336	3900	14,100	0.8	0.7
Oct. 8	404	4600	near normal	1.4	0.8
Oct. 10	382	4800	well above 10,000	1.5	0.8
Oct. 14	342	5600	near normal	0.4	0.2
Average of undisturbed days	294	5770	14,100	0.4	0.6

[1] American character figure, compiled by Department of Terrestrial Magnetism, Carnegie Institution of Washington, from data supplied by their two observatories and five observatories of the United States Coast and Geodetic Survey.

TABLE II

For 400 hours of observations between 1900 and 0700 E.S.T.								
Per cent	−30	−20	−10	−0	+0	+10	+20	+30
Number of hours	8	22	58	238	162	33	4	0
Disturbed hours (from days in Table I)	8	19	28	60	15	0	0	0
Undisturbed hours	0	3	30	178	147	33	4	0
For 44 hours of observations made on Wednesdays between 0800 and 1800 E.S.T.:								
Number of hours	0	0	2	22	20	0	0	0
Disturbed hours (for days in Table I)	0	0	2	8	1	0	0	0
Undisturbed hours	0	0	0	14	19	0	0	0

Out of 719 hours of observations during October strong sporadic-E reflections were present at 4400 kilocycles but not at 6200 kilocycles during six hours, at 6200 kilocycles but not at 7700 kilocycles during one hour and at 7700 kilocycles during one hour.

In Table I the October ionosphere storms are listed approximately in the order of their severity. Ionosphere storms are those disturbances of the ionosphere of the type usually associated with magnetic storms.

TABLE III

Date	Beginning of fade-out	Beginning of recovery	Recovery complete	Location of transmitter	Remarks	Minimum intensity
Oct. 2	1650	1714	1900	Ohio, D.C.		0.0
Oct. 2	1958	—	2006	Ohio		0.1
Oct. 2	2022	—	2108	Ohio, D.C.		0.2
Oct. 3	1432	1442	1452	Ohio, Mass., D.C.		0.0
Oct. 3	1500	1653	1725	Ohio, Mass., D.C.		0.0
Oct. 3	1743		1758	Ohio, D.C.		0.01
Oct. 3	1801	1815	1828	Ohio, D.C.		0.01
Oct. 3	1840	1906	1914	Ohio, Mass., D.C.		0.0
Oct. 3	2010	—	2020	Ohio, D.C.		0.2
Oct. 3	2153	2202	2210	Ohio, Mass., D.C.		0.05
Oct. 4	1341		1410	Ohio	also ionosphere storm	0.2
Oct. 4	1557		1640	Ohio	also ionosphere storm	0.3
Oct. 4	2048	2118	2136	Ohio, Mass., D.C.	also ionosphere storm	0.0
Oct. 5	1400	—	1413	Ohio, Mass.		0.1
Oct. 5	1530		1548	Ohio		0.1
Oct. 5	1623		1644	Ohio		0.1
Oct. 5	1705	1730	1755	Ohio, Mass., D.C.	ter. mag. pulse[1]	0.0
Oct. 5	1815		1835	Ohio		0.05
Oct. 5	2052	—	2100	Ohio, D.C.		0.2
Oct. 6	1406	1414	1430	Ohio, Mass., D.C.		0.0
Oct. 6	1437	1443	1448	Ohio, Mass.		0.0
Oct. 6	1656		1710	Ohio		0.1
Oct. 6	1812		1821	Ohio		0.05
Oct. 6	1922		1932	Ohio		0.1
Oct. 6	2023	2050	2135	Ohio, D.C.		0.0
Oct. 7	1531	1550	1700	Ohio		0.0
Oct. 8	1608	1726	1900	Ohio, Mass., D.C.	ter. mag. pulse ionosphere storm	0.0
Oct. 19	1641		1713	Ohio		0.1
Oct. 19	1918		2020	Ohio		0.5
Oct. 24	1751		1850	Ohio, D.C.		0.5
Oct. 27	1725		1840	Ohio		0.5
Oct. 28	1640		1700	Ohio, D.C.		0.2
Oct. 28	1730		1748	Ohio, D.C.		0.1
Oct. 28	1825		1852	Ohio, D.C.		0.1

[1] Terrestrial magnetic pulse observed on magnetograms from Cheltenham Observatory of United States Coast and Geodetic Survey.

Table II shows the number of hours f_{F^x} differed from the October average of the undisturbed days by more than the given percentages.

Sudden disturbances of the ionosphere at Washington during October were marked by the radio fade-outs listed in Table III.[1]

From September 27 to October 8 was a period of great fade-out activity. This was also a period of high daytime absorption of the medium high frequencies even at times when there were no fade-outs. Three fade-outs occurred during the severe ionosphere storm of October 4, and one fade-out occurred during the moderate disturbance of October 8.[2]

[1] All times G.M.T. Minimum intensities given in terms of transmissions from W8XAL, 6060 kilocycles, distance 650 kilometers.
[2] *Phys. Rev.*, vol. 51, p. 992, (1937).

CONTRIBUTORS TO THIS ISSUE

Bennon, Saul: Born August 9, 1914, at Philadelphia, Pennsylvania. Received A.B. degree, 1935; B.S. degree in electrical engineering, 1936; M.S. degree in electrical engineering, 1937, University of Pennsylvania. Graduate student, Westinghouse Electric and Manufacturing Company, 1937. Member, Phi Beta Kappa; associate member, Sigma Xi. Student member, American Institute of Electrical Engineers. Student Member, Institute of Radio Engineers, 1937.

Brunetti, Cledo: Born April 1, 1910, at Virginia, Minnesota. Received B.E.E. degree, 1932; Ph.D. degree, 1937, University of Minnesota. Teaching Fellow, department of electrical engineering, University of Minnesota, 1932–1936; instructor in electrical engineering, University of Minnesota, 1936–1937; instructor in electrical engineering, Lehigh University, 1937–1938. Member, Tau Beta Pi, Etta Kappa Nu, Sigma Xi. Nonmember, Institute of Radio Engineers.

Colwell, Robert C. See Proceedings for January, 1937.

Diamond, H.: Born 1900 at Quincy, Massachusetts. Received B.S. degree, Massachusetts Institute of Technology, 1922; M.S. degree, Lehigh University, 1925. Engaged in research work in mechanical engineering, General Electric Company, 1922–1923; instructor in electrical engineering, Lehigh University, 1923–1927; associate radio engineer, National Bureau of Standards, 1927–1928; radio engineer, 1929; senior radio engineer, 1930–1935; principal radio physicist, 1936 to date. In charge of development of radio aids to aviation, 1928–1934, and of ultra-high-frequency research, including radio meteorography, 1934 to date. Member, Washington Academy of Sciences and Washington Philosophical Society. Associate member, Institute of Radio Engineers, 1926; Member, 1930.

Dunmore, F. W.: Born January 24, 1891, at Haverhill, Massachusetts. Commercial radio operator, summers, 1911–1913; Received B.S. degree, Pennsylvania State College, 1915. Student engineering course, General Electric Company, 1915–1917; research department, American Radio and Research Corporation, 1918. Member of staff, National Bureau of Standards, specializing in research work on direction finders, directional transmission, ultra-high-radio-frequency transmitters and receivers, radio aids to air navigation, and radio meteorographs, 1918 to date; senior radio engineer, radio section, National Bureau of Standards, 1936. Nonmember, Institute of Radio Engineers.

Friend, Albert W.: Born January 24, 1910, at Morgantown, West Virginia. Received B.S.E.E. degree, 1932; M.S. degree in physics, West Virginia University, 1926. Transmission and distribution engineer, Ohio Power Company, 1933–1934; design and consulting engineer, WPHD, 1934; instructor in physics, West Virginia University, 1934–1937; assistant professor of physics, 1937. Member, Tau Beta Pi and Sigma Pi Sigma. Associate member, American Institute of Electrical Engineers. Associate member, Institute of Radio Engineers, 1934.

Gilliland, T. R.: See Proceedings for July, 1937.

Jansky, Karl G.: Born October 22, 1905, at Norman, Oklahoma. Received A.B. degree, 1927; M.A. degree, University of Wisconsin, 1936. Bell Telephone Laboratories, Inc., 1928 to date. Associate member, Institute of Radio Engineers, 1928; Member, 1934.

Kirby, S. S.: See PROCEEDINGS for July, 1937.

Reymer, S. E.: See PROCEEDINGS for July, 1937.

Roder, Hans: Born September 27, 1899, at Mengersreuth, Bavaria, Germany. Received M.S. degree in electrical engineering, Institute of Technology, Munich, 1923. Development and research of transmitting apparatus, Telefunken Gesellschaft für drahtlose Telegraphie, 1923–1929; radio transmitter department, General Electric Company, 1930–1933; in charge of advanced development, radio receiver engineering section, 1933–1937; general engineering laboratory, 1937. Member, Institute of Radio Engineers, 1929.

Salzberg, Bernard: Born July 22, 1907. Received E.E. degree, 1929; M.E.E. degree, Polytechnic Institute of Brooklyn, 1933. Brooklyn Edison Company, Inc., 1923–1925, summers, 1926–1927; Fada Radio Company, Inc., summer, 1928; RCA Communications, Inc., 1929–1931; research and development laboratory, RCA Radiotron Company, Inc., 1931–1935; Radiotron Division, RCA Manufacturing Company, Inc., 1935 to date. Junior member, Institute of Radio Engineers, 1925; Associate, 1930.

Smith, N.: See PROCEEDINGS for July, 1937.

Taylor, Paul B.: Born September 21, 1891, at Santa Monica, California. Received A.B. degree, 1913; M.A. degree, University of California, 1920; Ph.D. degree in physics, University of Pennsylvania, 1927. Assistant in mathematics, 1912–1914; assistant in physics, University of California, 1918. Air Service, 1919. Junior physicist, United States Bureau of Mines, 1919–1920; instructor in physics, Massachusetts Institute of Technology, 1920–1922; associate physicist, Bureau of Standards and Frankford Arsenal, 1922–1924; instructor in mathematics, Drexel Institute, 1925–1926. Physicist, Victor Talking Machine Company, 1927–1929; radio engineer, Westinghouse Electric and Manufacturing Company, 1929–1933; physicist, War Department, Aircraft Radio Laboratory Wright Field, 1935 to date. Member, American Institute of Electrical Engineers and Sigma Xi; associate member, Acoustical Society of America. Associate member, Institute of Radio Engineers, 1935.

PROCEEDINGS

OF

THE INSTITUTE OF RADIO ENGINEERS

(INCORPORATED)

VOLUME 25

1937

PUBLISHED MONTHLY BY

THE INSTITUTE OF RADIO ENGINEERS

(INC.)

330 WEST 42nd STREET, NEW YORK, N. Y

The Institute of Radio Engineers

OFFICERS AND BOARD OF DIRECTORS 1937

(Terms expire January 1, 1938, except as otherwise noted)

President

H. H. Beverage

Vice President

P. P. Eckersley

Treasurer
Melville Eastham

Secretary
Harold P. Westman

Editor
Alfred N. Goldsmith

Directors

Stuart Ballantine
Past President
T. A. M. Craven
L. C. F. Horle
C. M. Jansky, Jr.
C. B. Jolliffe
A. F. Murray
E. L. Nelson
Haraden Pratt
B. J. Thompson
L. E. Whittemore

Serving until January 1, 1939

E. H. Armstrong
Alfred N. Goldsmith
Virgil M. Graham

Alan Hazeltine, *Past President*

Serving until January 1, 1940

Ralph Bown
H. M. Turner

Board of Editors

Alfred N. Goldsmith, *Chairman*

R. R. Batcher
P. S. Carter
F. W. Grover
J. W. Horton
G. W. Pickard
B. E. Shackelford
K. S. Van Dyke
H. P. Westman, *ex officio*
L. P. Wheeler
L. E. Whittemore
William Wilson

Assistant Editor
H. M. Stote

Advertising Manager
J. D. Crawford

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

PROCEEDINGS. The PROCEEDINGS is the official publication of the Institute and in it are published all of the papers, discussions, and communications received from the membership which are accepted for publication by the Board of Editors. Copies are sent without additional charge to all members of the Institute. The subscription price to nonmembers is $10.00 per year, with an additional charge for postage where such is necessary.

RESPONSIBILITY. It is understood that the statements and opinions given in the PROCEEDINGS are views of the individual members to whom they are credited, and are not binding on the membership of the Institute as a whole. Papers submitted to the Institute for publication shall be regarded as no longer confidential.

REPRINTING PROCEEDINGS MATERIAL. The right to reprint portions or abstracts of the papers, discussions, or editorial notes in the PROCEEDINGS is granted on the express condition that specific reference shall be made to the source of such material. Diagrams and photographs published in the PROCEEDINGS may not be reproduced without making specific arrangements with the Institute through the Secretary.

MANUSCRIPTS. All manuscripts should be addressed to the Institute of Radio Engineers, 330 West 42nd Street, New York City. They will be examined by the Papers Committee and the Board of Editors to determine their suitability for publication in the PROCEEDINGS. Authors are advised as promptly as possible of the action taken, usually within two or three months. Manuscripts and illustrations will be destroyed immediately after publication of the paper unless the author requests their return. Information on the mechanical form in which manuscripts should be prepared may be obtained by addressing the secretary.

MAILING. Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage is provided for in the act of February 28, 1925, embodied in Paragraph 4, Section 412, P. L. and R., and authorization was granted on October 26, 1927.

Published monthly by

THE INSTITUTE OF RADIO ENGINEERS, INC.

Publication office, 450–454 Ahnaip St., Menasha, Wis.

BUSINESS, EDITORIAL, AND ADVERTISING OFFICES

Harold P. Westman, *Secretary*

330 West 42nd Street, New York, N. Y.

CONTENTS OF VOLUME 25*

1937

NUMBER 1; JANUARY, 1937

PART I

PART II

Technical Papers

NUMBER 2; FEBRUARY, 1937

PART I

* A cumulative index of the same type as this but covering the PROCEEDINGS from its start to the end of 1936 is available at $1.00 per copy.

Part II

Technical Papers

Number 3; March, 1937

Part I

Part II

Technical Papers

Number 4; April, 1937

Part I

Part II

Technical Papers

Number 5; May, 1937

Part I

Part II

Technical Papers

Number 6; June, 1937

Part I

Part II

Technical Papers

Number 7; July, 1937

Part I

PART II

Technical Papers

NUMBER 8; AUGUST, 1937

PART I

PART II

Technical Papers

NUMBER 9; SEPTEMBER, 1937

PART I

Part II

Technical Papers

Number 10; October, 1937

Part I

Part II

Technical Papers

Number 11; November, 1937

Part I

Part II

Technical Papers

Number 12; December, 1937

Part I

Part II

Technical Papers

AUTHOR INDEX

Numbers refer to the chronological list. **Bald-face** type indicates papers, light-face type indicates discussions, and *italics* refer to books and book reviews.

INDEX TO SUBJECTS

INCORRECT ADDRESSES

Listed below are the names and the last-known addresses of seventy members of the Institute whose correct addresses are unknown. It will be appreciated if anyone having information concerning the present addresses of the persons listed will communicate with the Secretary of the Institute.

Bacon, Frederic S., Jr.	194 Lafayette St., Salem, Mass.
Bailey, Homer M.	c/o M. V. Sawyer, Federal Barge Lines, Foot of North Market St., St. Louis, Mo.
Bard, Howard B., Jr.	245 Palasade Ave., Bridgeport, Conn.
Beane, E. A.	1000 North Harvey Ave., Oak Park, Ill.
Benzon, C. George	U.S. Dept. of Commerce, 4th Air Navigation District, P.O. Box 1689, Fort Worth, Texas.
Burghard, G. E.	520 E. 86th St., New York, N. Y.
Campbell, Walter H.	c/o Signal Officer, 3rd Corps Area, Hearst Tower Bldg., Baltimore, Md.
Chatterton, Lloyd N.	Department of Public Safety, Radio Station WRBH, Cleveland, Ohio.
Clark, Homer L.	331 W. Wayne St., Fort Wayne, Ind.
Daily, Harold C.	c/o Tropical Radio Telegraph Co., 1001-4th St., San Francisco, Calif.
Decker, Arthur L.	21st and Northwestern, Indianapolis, Ind.
Dillaplain, Vernon L.	Service Electric Company, Corydon, Iowa
Doyel, Lee C.	R.R. 2, Crawfordsville, Ind.
Dreyer, Harry W.	54 Windsor Place, Oceanside, N. Y.
Ekstrom, Emmett J.	145 E. 23rd St., New York, N. Y.
Epperson, Joseph B.	P. O. Box 972, Knoxville, Tenn.
Fillmore, F. A.	5728 Itaska St., St. Louis, Mo.
Foster, Arnot P.	67 Sheehan Ave., Cincinnati, Ohio.
Greene, Lloyd C.	Hotel Myles Standish, Boston, Mass.
Grove, William C.	1824 South Williams, Denver, Colo.
Haire, A. F.	1314 Valley Place, Anacostia, Washington, D. C.
Hamm, John L.	2826-6th St., N. E., Washington, D. C.
Hanson, Earl C.	1276 Bonnieview Ave., Lakewood, Ohio.
Jacobson, Larry	c/o National Geophysical Co., 1507 Tower Petroleum Bldg., Dallas, Tex.
Jefferson, Sidney	52 Cambridge St., London, W. 2, England
Jenny, Felice	Zenith S. A., Casella Postale 66, Monza, Italy.
Jensen, Jens O.	1134 S. Everglades Concourse, Miami Beach, Fla.
Johnson, Edgar S.	c/o S.S. American Trader, Pier 58, North River, New York, N. Y.
Johnson, George S.	209 So. Tulane, Albuquerque, N. M.
Johnston, Edward L.	5736 Midway Park, Chicago, Ill.
Kadri, Syed S.	International House, Chicago, Ill.
Marsden, Bernard	101 East Edsall Ave., Palisades Park, N. J.
Martlieu, C. H. O.	55 South St., Halifax, N. S., Canada.
Meastas, Ricordo	3040 E. Slawson Ave., Huntington Park, Calif.
Miller, Charles E.	358 So. Cochran Ave., Los Angeles, Calif.
Morris, F. Glenn, Jr.	Adams House B-45, Cambridge, Mass.
Muller, Conrad R.	Calif. Institute of Technology, Pasadena, Calif.
Neelon, W. J.	715 Amherst St., Buffalo, N. Y.
Neilson, Neil S.	234 W. Witherbee, Flint, Mich.
Neily, Joseph W.	55 Sunshine Road, Upper Darby, Pa.
Noe, Milford W.	5537 La Mirada, Apt. 200-A, Los Angeles, Calif.
Ong, Hian Y.	Marconi's College, Chelmsford, Essex, England.
Owens, William B.	652 W. Peachtree St., Atlanta, Ga.
Paterson, Jack	85 Baxter St., Buffalo, N. Y.
Perry, William A.	383 Kings Highway, Westport, Conn.
Petty, John M.	S.S. Dean Emery, c/o Standard Oil Co. of N. J., Marine Dept., 30 Rockefeller Plaza, New York, N. Y.
Pitzer, Robert L.	821 Summit Drive, So. Pasadena, Calif.
Powell, Albert E.	Southern Radio Corp., Cassilla 669, La Paz, Bolivia, South America.
Quinn, Clark E.	2025 Atkins Ave., Lakewood, Ohio.
Rainwater, Brown V.	Edson Hotel, Beaumont, Tex.
Rappaport, Morris	Fuller Gardens, Ossining, N. Y.
Reynolds, Charles B., Jr.	6905 Detroit Ave., Apt. 5, Cleveland, Ohio.
Sceales, Leslie B.	1515 W. Monroe St., Chicago, Ill.
Scofield, Robert W.	40-22 Lawrence St., Flushing, L. I., N. Y.
Sholkin, Arcady N.	R.C.A. Communications, Inc., 66 Broad St., New York, N. Y.

Smith, G. Gould	Tarver Ave., Lebanon, Tenn.
Snyder, Graves H.	U.S. Army Air Corps, Brooks Field, San Antonio, Tex.
Stuedle, Emry C.	4109 2nd Ave., Los Angeles, Calif.
Sumpter, Paul B.	102 Sunset Ave., No. Arlington, N. J.
Taylor, J. L.	140 Sanders Rd., Apt. 7, Buffalo, N. Y.
Tingey, Robert	45 Nevern Square, Earl's Court, S.W. 5, England.
Toy, Edward S.	412 E. Berry St., Fort Wayne, Ind.
Waller, James L., Jr.	414 N. Market St., Shawnee, Okla.
Weber, Rennie I.	2738 N. Moody Ave., Chicago, Ill.
Weil, William S., Jr.	6814 McCallum St., Philadelphia, Pa.
Weiss, Howard M.	2752 Hampshire Road, Cleveland Heights, Ohio.
Wendell, Edward N.	Les Laboratories L.M.T., 46, Avenue de Breteuil, Paris 7E, France.
Wiesner, Jerome B.	4710 Greenfield Ave., Dearborn, Mich.
Yoder Raymond S.	1549 Schreiber, Chicago, Ill.
Yokela, H. E.	Box 111, Inverness, Calif.

The stream-lined RCA 13-D! Comes with illuminated meter. Offers uniform frequency response, reliability of readings and convenience of use. Ideal for measurement or indicating purposes.

AN ELECTRONIC MASTER VOLUME INDICATOR

Provides wide range audio level readings with minimum circuit distortion, several meter speeds, internal calibrating voltage. Completely AC operated. Illuminated meter.

LABORATORIES or broadcasting stations can measure audio levels of lines or amplifiers, take characteristic curves, with the new RCA 13-D volume indicator. Calibrated directly in decibels, it's convenient, accurate, and has many entirely new features.

An important and unusual feature of the 13-D is its 2-stage isolating amplifier. This prevents the rectifier from causing distortion in measured circuits. Also steps up the range on the low end of sensitivity scale. Reads levels as low as minus 30 db.

The rapid rise and slow fall of the indicating meter simplifies reading on changing amplitudes yet provides a peak reading. Offers several speeds varying from that of an R.M.S. meter to that of a peak reading meter. Meter can be used for indicating plate currents of tubes. Instrument can be set to absolute readings. Full technical details on request.

SPECIFICATIONS:

Input Impedance: 500 or 20,000 ohms.
Frequency Response: ± ½ db. 30-10,000 cycles.
Range: −30 to +20 db.
Zero Level: 12½ or 6 M.W.
Power Supply: 105-130 volts, 60 cycles, 30 watts.
Dimensions: 7″ by 19″ by 7¾″.

RCA Special Purpose Tubes for every application

RCA Broadcast Equipment

RCA Mfg. Co., Inc., Camden, N. J. • A Service of the Radio Corporation of America

New York: 1270 Sixth Ave. Chicago: 589 E. Illinois St. Atlanta: 490 Peachtree St., N. E.
Dallas: 2211 Commerce St. San Francisco: 170 Ninth St. Hollywood: 1016 N. Sycamore Ave.

THESE RESISTORS WERE NOT PRACTICAL A YEAR AGO

7500 MEGOHM CONTINUOUSLY VARIABLE POTENTIOMETER

50,000 VOLT COMPACT RESISTOR (12" x 1⅛")

10,000 MEGOHM TO 100,000 MEGOHM METALLIZED FILAMENT RESISTOR

400 OHM, 200 WATT, 30 MEGACYCLE RESISTOR

9 CYCLE SALT WATER IMMERSION, WIRE WOUND RESISTOR

IRC
REG. U. S. PAT. OFF.
Initiative — Resourcefulness — Cooperation to advance the interests of the industry

Pioneering better resistors—special purpose resistors—resistors that have never been made before, is an important part of the work at Resistance Headquarters. Those listed are but a few of the types which, unobtainable a year ago, have since been designed and produced commercially by IRC in response to a specific need. Samples to your specifications gladly submitted. Write for Catalog of Standard IRC Resistor types.

INTERNATIONAL RESISTANCE COMPANY

401 NORTH BROAD STREET, PHILADELPHIA, PA.

Factories or Licensees in Canada, England, France, Germany, Italy, Denmark and Australia

MAKERS OF RESISTANCE UNITS OF MORE TYPES, IN MORE SHAPES, FOR MORE APPLICATIONS THAN ANY OTHER MANUFACTURER IN THE WORLD

"It's Good to Hear Your Voice"

THE tinkle of the telephone is a welcome sound in millions of homes. This day, the sun will shine brighter for some one because you called.

The telephone is important in the everyday affairs of life —vital in emergencies. But that is not the whole of its service. Its value grows because it helps to keep folks closer — makes this busy world a happier, cheerier place to live in. Friendship's path often follows the trail of the telephone wires.

Long Distance Rates Are Low

Your friends will be glad to hear your voice and you'll be surprised to see how little it costs to telephone Long Distance. Rates to most out-of-town points are lowest after 7 P.M. and all day Sunday. Then 3-minute station-to-station calls cost 35c for about 90 miles; 50c for about 150 miles; $1 for about 425 miles.

BELL TELEPHONE SYSTEM

AMERICAN TELEPHONE & TELEGRAPH CO. BELL SYSTEM AND ASSOCIATED COMPANIES

CAMPBELL'S
329 ADAMS AVENUE
SCRANTON, PA.

August, 7, 1937

Scranton Radio Supply Co.
809 Mulberry Street
Scranton, Pa.

Dear Mr. Mack:

May I take this opportunity to express to you my sincere appreciation for my outstanding success with Centralab products.

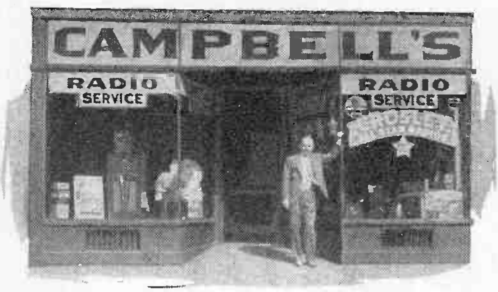
We consider quality of parts a major essential in rendering efficient radio service; we obtain this by using Centralab at no greater cost.

I have been, and am, consistently one hundred percent Centralab, and as I am conservative in my expressions, and having used hundreds of Centralab controls, resistors and switches, I highly recommend the use of Centralab products to other radio service men.

Yours sincerely,

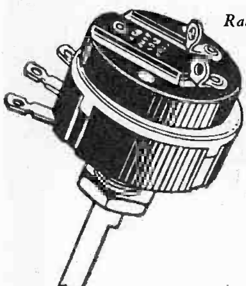
J. F. Campbell
RADIONEER

'100% Centralab'



Mr. J. F. Campbell
Radioneer of Scranton, Pa.

—thanks Scranton Radio Supply Co. for sending this on!



Servicemen . . . jobbers . . . experimenters . . . manufacturers . . . all voice a preference for the smooth, certain, satisfactory service that Centralab Controls offer. For "100%" satisfaction specify Centralab!

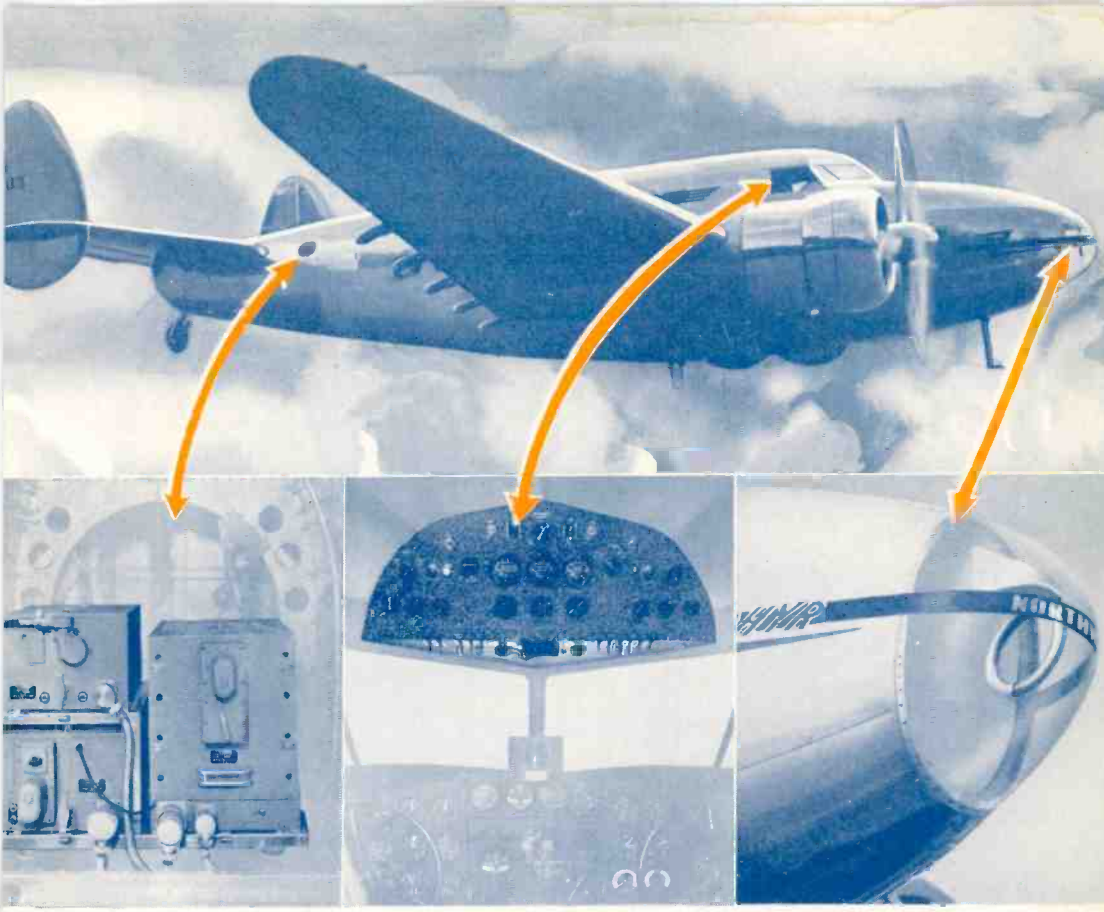
CENTRALAB Div. Globe-Union Mfg. Co., Milwaukee, Wis.

The standard radiohm offering accurate tapers—low noise level—longer life, and better power dissipation.

Centralab

BRITISH CENTRALAB, LTD.
Canterbury Rd., Kilburn
London, N.W. 6, England

FRENCH CENTRALAB CO.
118 Avenue Ledru-Rollin
Paris XI, France



UNIFIED RACK MOUNTING of equipment—Easily serviced—Utilizes Hydraulic shock mounts.

CENTRALIZED RADIO CONTROLS are a feature—Convenient in operation—Simplified installation.

BUILT-IN DIRECTION FINDER Mounted behind non-metallic nose—Reduces drag—Eliminates icing.

NORTHWEST AIRLINES

U.S. AIRMAIL EXPRESS — PASSENGER SERVICE

NEW SKY ZEPHYRS SCORE A FIRST!

Bendix Radio Equipment Used In First Commercial Radio Installation Engineered Simultaneously with the Airplane

Bendix engineers, in cooperation with the Northwest Airlines, assumed complete responsibility for every feature of the radio equipment installed in their Lockheed "14" transports. Conduit size and placement, junction boxes, terminal strips and a host of other details—all engineered by Bendix. Such refinements as dual volume controls, signal level meters and emergency transmitting antenna indicate the tangible results obtained by coordinated engineering. The radio units themselves reflect the extensive engineering and research facilities of Bendix. Reliability and performance are paramount—and they are insured by the excellence of workmanship which characterizes all Bendix equipment. Conservatively engineered, yet incorporating the "new" things in radio. For details of complete engineering service on any type of radio installation or information on specific equipment, address inquiries to—

BENDIX RADIO

BENDIX RADIO CORPORATION

CHICAGO • U.S.A.

COMMUNICATIONS TRANSMITTER

2 to 20 Megacycles . . . 10 Channels . . . 400 or 1000 Watts

The communications transmitter of the future made possible TODAY by the engineering skill and experience of BENDIX designers. To operating officials it affords greater operating and maintenance economies, low line charges for remote control and lowest costs on necessary channel changes. It also means a new high in reserve power, in simplified speedy operation, noiseless switching and accurate automatic tuning. Painstaking electrical assembly and mechanical construction provide a maximum of satisfactory service, flexibility and an operating economy amplified by lowest obsolescence per dollar cost.

1,000 WATTS OUTPUT . . . *on telephone or CW telegraph*

FREQUENCY RANGE . . . *1,500 to 20,000 Kcs.*

TEN CHANNELS . . . *local and/or remote control*

UNUSUAL TUBE ECONOMY

CHANNEL SWITCH . . . *simplified, rugged, quiet*

DIAL OPERATION . . . *over single telephone pair controls all channel selection and antenna switching functions.*

HIGH SPEECH FIDELITY and LOW HARMONIC CONTENT

FILTERED VENTILATION

CABINET . . . *sturdy, accessible, modern.*

BENDIX RADIO

BENDIX RADIO CORPORATION

CHICAGO • U.S.A.

Electrolytic Condensers
INVERTED MOUNTING UNITS IN CYLINDRICAL ALUMINUM CANS
500 Volts D.C. Peak • • • Type PG500
AEROVOX
HI-FARAD
WET ELECTROLYTIC CONDENSER
VOLTS
500
Type PG350
WET or DRY
Making *every* kind of condenser—electrolytic, paper, oil, mica, etc.—AEROVOX plays no favorites.
You can have that kind which best meets your precise needs.
Our engineers are always ready to collaborate in finding that *right* condenser for your job.
AEROVOX
CORPORATION
70 Washington St. : : Brooklyn, N. Y.
IN CANADA: AEROVOX CANADA, Limited Hamilton, Ont.

NOW READY . . .

A complete four-language glossary of terms used in radio engineering and television

DICTIONARY OF RADIO TERMINOLOGY IN THE ENGLISH, GERMAN, FRENCH, AND RUSSIAN LANGUAGES

BY A. S. LITVINENKO

EDITED BY PROF. V. I. BASHENOFF
(Member, I.R.E., and co-author of "Radio Beacons")

Wörterbuch der Englischen, Deutschen, Französischen und Russischen Funktechnischen Terminologie, VON A. S. Litvinenko HERAUSGEGEBEN VON Prof. V. I. Bashenoff

Dictionnaire de la Terminologie Technique de radioélectricité des Langues Anglaise, Allemande, Française et Russe, PAR A. S. Litvinenko, REDIGÉ PAR Prof. V. I. Bashenoff

АНГЛО-НЕМЕЦКО-ФРАНЦУЗСКО-РУССКИЙ СЛОВАРЬ РАДИОТЕХНИЧЕСКОЙ ТЕРМИНОЛОГИИ СОСТАВИЛ А. С. ЛИТВИНЕНКО ПОД РЕДАКЦИЕЙ проф В. И. БАЖЕНОВА

Published by ONTI NKTP, Moscow, 1937.
Bound in cloth boards. XLII + 559 pages.

PRICE $4.00, Postage Extra

This new reference book gives a systematic comparative review of modern radio and television terminology in the English, German, French, and Russian languages. In addition it includes a number of fundamental terms related to mathematics, acoustics, optics, generation and transformation of electric current, and the theory of transmission of alternating current over wires.

Translation from any one of the four languages into any one of the remaining three is readily accomplished without the use of a numerical index, due to the ingenious arrangement of terms employed. The dictionary is, therefore, equivalent to twelve ordinary two-language dictionaries.

All terms relating to radio engineering are drawn from material published in the language of the country of publication. Thus, in addition to being an invaluable aid in translating radio literature, the dictionary gives an accurate picture of the existing terminology.

ORDER FROM THE SOLE DISTRIBUTOR:

BOOKNIGA CORPORATION

IMPORTERS OF BOOKS AND PERIODICALS FROM THE U.S.S.R.

255 FIFTH AVENUE, NEW YORK, N.Y., U.S.A.

★ Self-Calibrating VACUUM TUBE VOLTMETER

★ **The most important advancement in circuit design for precision electrical instruments in recent years.**

Laboratories and engineers will use and immediately appreciate the significance of this remarkable instrument. Indispensable also in the servicing field for measuring electrical impulses either A.C. or D.C. of low magnitude, such as the carrier wave of signal circuits, and particularly for television work.

MODEL 1250

DEALER PRICE $36.67

The self-calibrating feature is automatic with the tube bridge circuit developed by Triplett engineers (Pat. Pending). The initial operation of adjusting the bridge at the zero level insures exact calibration independent of tube emission values or when replacing tubes.

Model 1250 is furnished with Triplett tilting type twin instrument. One instrument indicates when bridge is in balance. The other is a three range voltmeter with scales reading in peak A.C. and D.C. voltages. Ranges are 2.5, 10 and 50 volts.

Model 1250 is complete with all necessary accessories including 1-84, 1-6C6, 1-76. Case is metal with black wrinkle finish. Etched panels are silver, red and black. DEALER PRICE $36.67

Model 1251, Same as above, but with ranges of 3-15-75-300 volts. DEALER PRICE $47.67

THIS IS A TRIPLETT MASTER UNIT

MAIL THIS COUPON

TRIPLETT Precision ELECTRICAL INSTRUMENTS

The Triplett Electrical Instrument Co.
2112 Harmon Ave., Bluffton, Ohio

☐ Without obligation please send me complete information on Triplett Vacuum Tube Voltmeter.

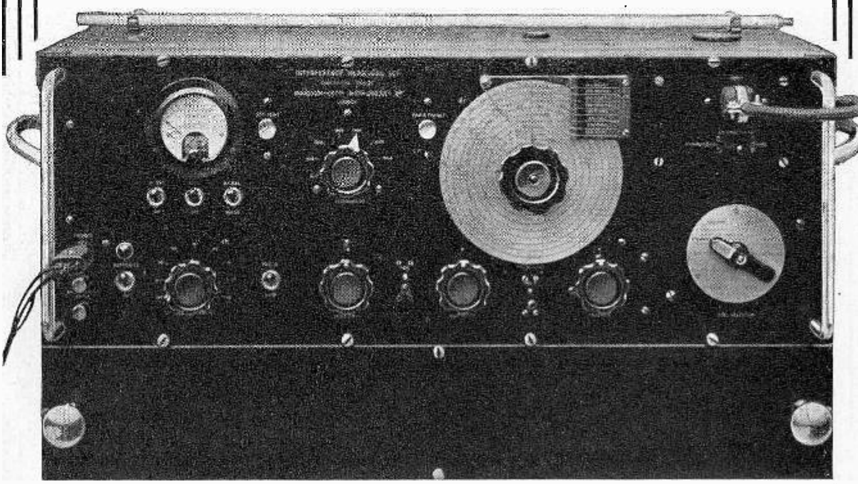
☐ I am also interested in ....

Name ....

Address ....

City .... State ....

INTERFERENCE MEASURING SET



TYPE T.F. 379

An instrument developed primarily for the measurement of R.F. interference, it is also suitable for most field strength determinations in the frequency range 150 kc.—25 Mc.

It comprises a superheterodyne receiver of very high gain, in which noise due to thermal agitation of free electrons in the input circuit is used as a constant signal. Calibrated attenuators in the R.F. and I.F. circuits are adjusted to reduce the signal under test to the same strength as the input circuit noise in the absence of any external signal.

BRIEF SPECIFICATION

Frequency Range: 150 kc.—25 Mc. Direct calibrated.

Sensitivity: (a) R.F. Voltages of 10 microvolts upward. (b) Field Strengths from 1 microvolt per meter upward.

Range of Attenuation: 30—120 dB relative to 1 microvolt.

Accuracy: 1/2 dB.

Acceptance Band: Approximately 9 kc.

Input Circuit Balance: 150 kc.—500 kc. better than 30 dB. 500 kc.—25 Mc. better than 20 dB.

Detector Time Constant: Charge 1 millisecond
Discharge 160 milliseconds.

Batteries: Self-contained accumulator and dry battery.

We are indebted to H.M. Post Office Research Station for their collaboration in the design of this instrument.

MARCONI-EKCO INSTRUMENTS LTD.

Electra House, Victoria Embankment, London, W.C. 2, England

The Institute of Radio Engineers

Incorporated

330 West 42nd Street, New York, N.Y.

APPLICATION FOR ASSOCIATE MEMBERSHIP

(Application forms for other grades of membership are obtainable from the Institute)

To the Board of Directors

Gentlemen:

I hereby make application for Associate membership in the Institute of Radio Engineers on the basis of my training and professional experience given herewith, and refer to the members named below who are personally familiar with my work.

I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I will be governed by the constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power, and if my membership shall be discontinued will return my membership badge.

....
(Sign with pen)

....
(Address for mail)

....
(Date) (City and State)

Sponsors:

(Signature of references not required here)

Mr. .... Mr. ....

Address .... Address ....

City and State .... City and State ....

Mr. ....

Address ....

City and State ....

The following extracts from the Constitution govern applications for admission to the Institute in the Associate grade:

ARTICLE II—MEMBERSHIP

Sec. 1: The membership of the Institute shall consist of: * * * (c) Associates, who shall be entitled to all the rights and privileges of the Institute except the right to hold any elective office specified in Article V. * * *

Sec. 4. An Associate shall be not less than tyenty-one years of age and shall be a person who is interested in and connected with the study or application of radio science or the radio arts.

ARTICLE III—ADMISSION AND EXPULSIONS

Sec. 2: * * * Applicants shall give references to members of the Institute as follows: * * * for the grade of Associate, to three Fellows, Members, or Associates; * * * Each application for admission * * * shall embody a full record of the general technical education of the applicant and of his professional career.

ARTICLE IV—ENTRANCE FEE AND DUES

Sec. 1: * * * Entrance fee for the Associate grade of membership is $3.00 and annual dues are $6.00.

ENTRANCE FEE SHOULD ACCOMPANY APPLICATION

(Typewriting preferred in filling in this form) No. ....

RECORD OF TRAINING AND PROFESSIONAL EXPERIENCE

Name ....

(Give full name, last name first)

Present Occupation ....

(Title and name of concern)

Business Address ....

Permanent Home Address ....

Place of Birth ....Date of Birth ....Age......

Education ....

Degree ....

(College) (Date received)

TRAINING AND PROFESSIONAL EXPERIENCE

DATES	

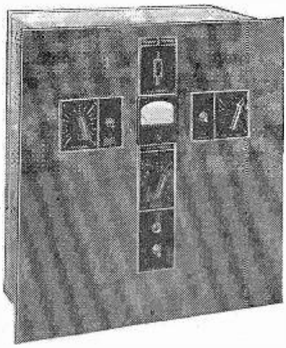
Record may be continued on other sheets of this size if space is insufficient.

Receipt Acknowledged ....Elected ....Deferred ....

Grade ....Advised of Election ....This Record Filed ....

They took our word

Even before the Western Electric 110A Program Amplifier was officially announced, orders began pouring in from broadcasters. They'd heard what *we* said it would do—and they bought on faith. Now they've been using it for some months—and some of them have written us what it has done for them. Today you need not take *our* word . . .



. . . you can take THEIRS!

WINS: "marked improvement in signal . . . certain dead-spots eliminated . . . both quality and volume improved."

WKBH: "signal at outer edge of service area noticeably improved . . . a great help in maintaining high program level without over modulation."

WOR: "areas where signal was hashed with monkey chatter now cleared considerably . . . 3 db audio increase has definitely aided in clearing this condition."

WTAG: "no difficulty in normal operation at level 3 to 4 db higher than previously used."

WAIM: "a very good investment . . . has increased fidelity of signal."

WDAE: "normal coverage increased 25% . . . quite possible to use 5 db of compression without any particular change in quality of transmission . . . never worry any more about any conceivable sort of line surge."

WISN: "Materially aids in maintaining higher average percentage of modulation . . . signal boosted between 3 and 4 db."

WMBD: "better signal to noise ratio."

KFYR: "average modulation percentage very much higher . . . interruptions due to high audio surges have ceased to exist . . . stations separated 10 KC can be tuned in without monkey chatter."

KXRO: "any station without it can hardly be called modern . . . makes it possible to broadcast most any voice, ballyhoo or shouting without spoiling effect."

WMBH: "unsolicited reports from localities and distances never or rarely heard from before, best prove the 110A is really doing its stuff."

WDAY: "unsolicited reports that we come-in much better . . . average modulation level about 3.5 db higher."

WJBO: "consider the 110A the outstanding development during the past 5 years . . . decided increase in signal . . . practical abolition of monkey chatter."

WHAM: "no fear of distortion from over modulation . . . will raise standards of any station which has one."

KQW: "has improved signal about 100% . . . much favorable comment from listeners and sponsors."

Western Electric Distributed by Graybar

Distributed by GRAYBAR Electric Company. In Canada: Northern Electric Co., Ltd.

BROADCASTING EQUIPMENT

MEN AVAILABLE

GRADUATE ENGINEER with 6 years of experience in tube manufacturer's laboratory wants work involving knowledge of tube characteristics and applications, either radio or industrial. Age 30, unmarried. New York-Philadelphia area preferred. Reply to Box No. 181, c/o The Institute of Radio Engineers, 330 West 42nd Street, New York, N.Y.

RADIO TELEPHONE OPERATOR— Desires location with progressive broadcast station that can place a new man. Age 27, unmarried, and free to go anywhere. High-school graduate with one year in residence at Dodge's Radio Institute. Reply to Box No. 182, c/o The Institute of Radio Engineers, 330 West 42nd Street, New York, N.Y.

ENGINEERING POSITIONS

Advertisements on this page are accepted from organizations interested in employing Institute members. Information on rates, preparation of material, and closing dates will be mailed on request.

THE INSTITUTE OF RADIO ENGINEERS
330 West 42nd Street
NEW YORK, N.Y.

ENGINEERING DIRECTORY

QUARTZ CRYSTALS

for Standard or Special
Radio Frequency Applications

Write for Catalog

BLILEY ELECTRIC CO.
230 Union Station Bldg.
ERIE, PENNA.

Cathode Ray Tubes and Associated Equipment For All Purposes

Write for Complete Technical Data

ALLEN B. DUMONT LABORATORIES
UPPER MONTCLAIR, N.J.

PRACTICAL RADIO ENGINEERING

One year Residence Course
Advanced Home Study Course
Combined Home Study-Residence Course

Write for details

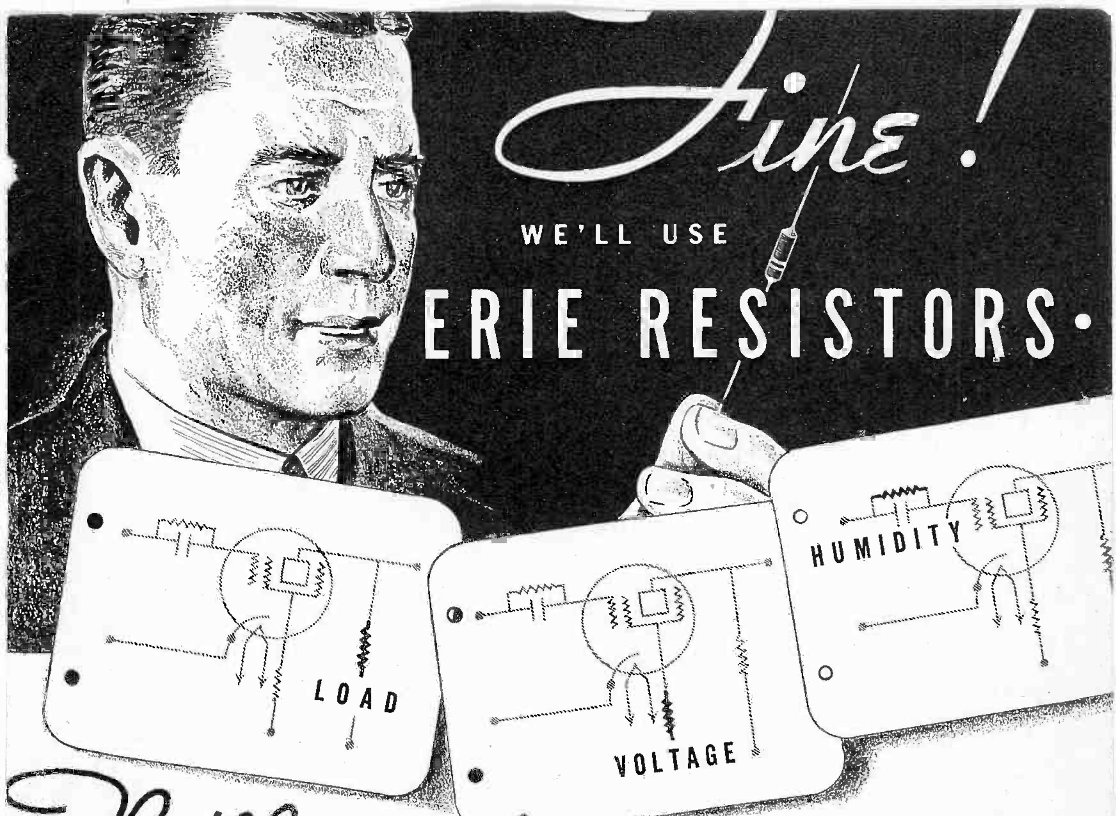
Capitol Radio Engineering Institute
Washington, D.C.

"Q"—Measurements

We specialize in equipment for the radio frequency measurement of "Q" (X/R) of coils, condensers and other radio components.

Write for Circular

BOONTON RADIO CORPORATION
BOONTON, NEW JERSEY



They'll retain their normal ratings

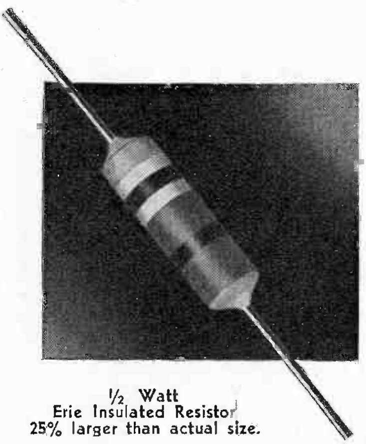
UNDER SEVERE CONDITIONS

"The fact that Erie Resistors more nearly retain their normal values under all sorts of operating conditions is a big help to us.

It means we can use one make of resistor throughout the complete assembly line. We won't have to worry about humidity in the grid leaks where there isn't much load, too much load in the plate circuits, or the effect of sudden voltage fluctuations in other circuit locations.

We certainly are glad we sent for those Erie Resistor samples* to test."

**Why not find out for yourself the excellent operating characteristics of Erie Insulated Resistors. A request on your letter-head will bring you a generous supply of samples to test in your own laboratory.*



½ Watt
Erie Insulated Resistor
25% larger than actual size.

CARBON RESISTORS AND SUPPRESSORS

ERIE RESISTOR CORPORATION

AUTOMATIC INJECTION MOLDING

TORONTO | ERIE, PENNSYLVANIA | LONDON

Ready now

for tomorrow's radio needs...

CORNELL-DUBILIER CAPACITORS

For Radio Receiving . . . Transmitting . . . Television . . . Power Amplification . . . Signal Systems . . . Power-Factor Correction . . . Telephony . . . Aircraft Instruments and all Industrial Purposes.

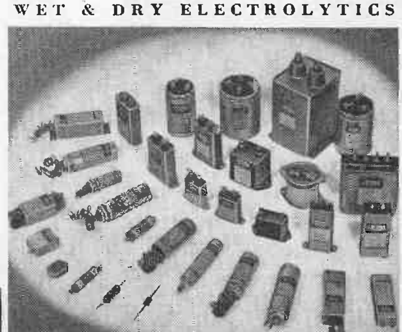
Designed with vision, laboratory proven, built to endure, Cornell-Dubilier, the World's Most Complete Line of Condensers—engineered to meet Tomorrow's radio needs *today*.

Step out ahead with C-D, for twenty-eight years the international symbol of dependability in capacitors.

World's Largest and Oldest Exclusive Manufacturer of Capacitors

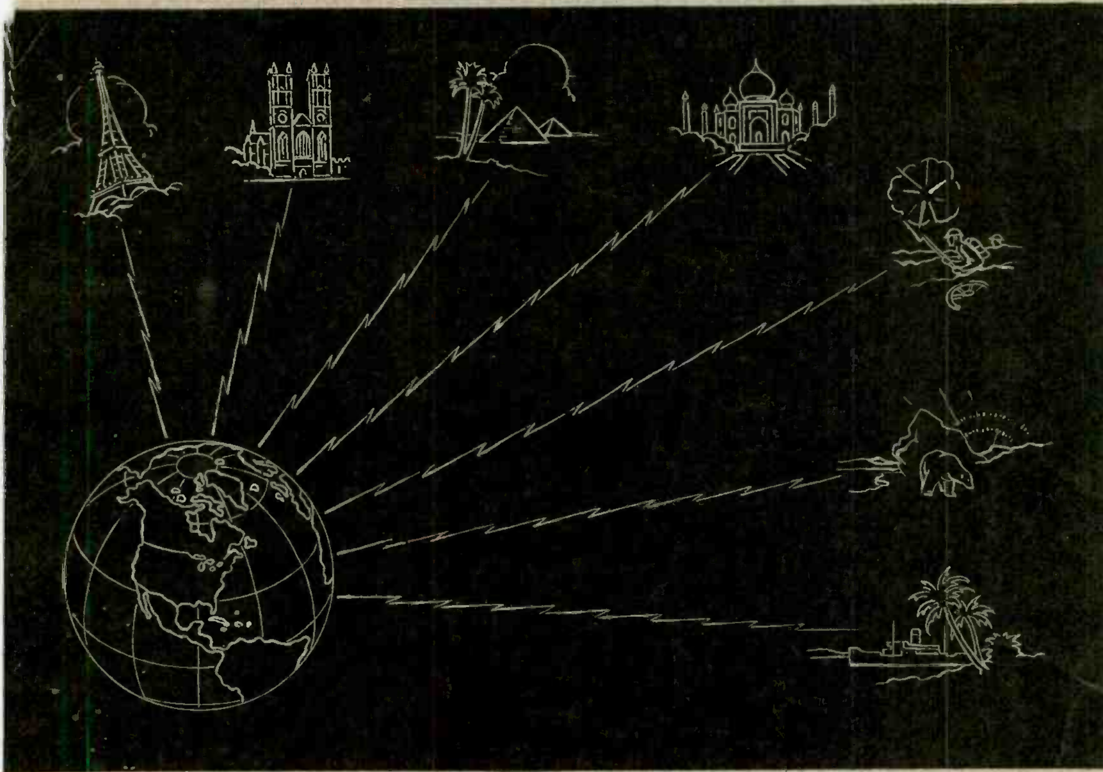
For complete information and technical data write to . . . 1000 Hamilton Boulevard, South Plainfield, N.J.

MICA · PAPER · DYKANOL
WET & DRY ELECTROLYTICS



CORNELL-DUBILIER
ELECTRIC CORPORATION
South Plainfield, New Jersey

NEWS ACROSS THE SEVEN SEAS



TO AND FROM every continent in the world messages are constantly being flashed across the seas via radio telegraph. News . . . business correspondence . . . private messages — all must reach their destination with minimum delay, maximum economy, and accurate transcription.

In commercial radio communication equipment Isolantite* ceramic insulators function with complete reliability . . . reducing dielectric losses . . . providing efficient insulation of transmission lines and antennae. Again, at the receiving end Isolantite's dependable insulating properties provide highest operating efficiency throughout the entire circuit.

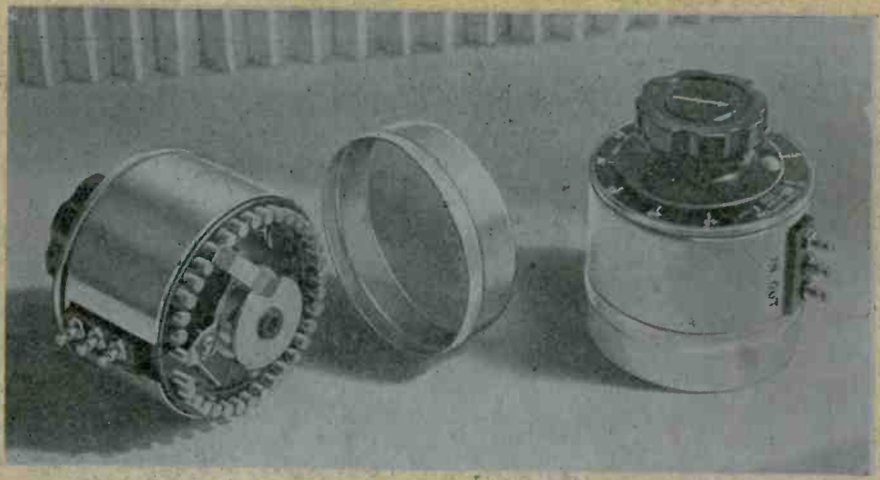
To all branches of the radio communications field Isolantite engineers offer the benefits of their long experience in the manufacture and application of ceramic insulators. If you are planning new installations of radio equipment you will find this experience of great practical value.

** Registered Trade-name for the products of Isolantite Inc.*

ISOLANTITE INC.

CERAMIC INSULATORS

Factory: Belleville, N. J. • Sales Office: 233 Broadway, New York, N. Y.



THE G-R 653 VOLUME CONTROL

- Lowest Noise Level
- Substantially Constant Impedance
- Linear Attenuation from 0 to 45 db
- Complete Cut-off
- Protected Windings
- Removable Dust Cover for Cleaning
- Finger Index Button and Arrow Indicators
- Improved Terminals
- [illegible]ur Sto[illegible] Terminal Impedances: 50, 2[illegible], 250 and [illegible]0 ohms.

Price: $12.50

Thousands of the Type 653 Volume Controls are in use. Why not try one?

● Write for Bulletin 205 for complete data

GENERAL RADIO COMPANY

30 STATE STREET, CAMBRIDGE, MASSACHUSETTS

New York **Los Angeles** **San Francisco**

GEORGE BANTA PUBLISHING COMPANY, MENASHA, WISCONSIN