VOLUME 22 SEPTEMBER, 1934 NUMBER 9

PROCEEDINGS

of

The Institute of Radio Engineers

Application Blank for Associate Membership on Page XIII

Institute of Radio Engineers
Forthcoming Meetings

DETROIT SECTION
September 21, 1934

LOS ANGELES SECTION
September 18, 1934

PHILADELPHIA SECTION
October 4, 1934

PITTSBURGH SECTION
October 16, 1934

WASHINGTON SECTION
September 10, 1934

PROCEEDINGS OF

The Institute of Radio Engineers

Volume 22 | September, 1934 | Number 9

CONTENTS

Part I

Part II

Technical Papers

The Institute of Radio Engineers

GENERAL INFORMATION

INSTITUTE. The Institute of Radio Engineers was formed in 1912 through the amalgamation of the Society of Wireless Telegraph Engineers and the Wireless Institute. Its headquarters were established in New York City and the membership has grown from less than fifty members at the start to several thousand.

AIMS AND OBJECTS. The Institute functions solely to advance the theory and practice of radio and allied branches of engineering and of the related arts and sciences, their application to human needs, and the maintenance of a high professional standing among its members. Among the methods of accomplishing this is the publication of papers, discussions, and communications of interest to the membership.

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Applications for transfer or election to the various grades of membership have been received from the persons listed below, and have been approved by the Admissions Committee. Members objecting to transfer or election of any of these applicants should communicate with the Secretary on or before September 29, 1934. These applications will be considered by the Board of Directors at its meeting on October 3, 1934.

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Committee Work

New York Program Committee

A meeting of the New York Program Committee was held on June 12 in the Institute office. Those present were: A. F. Van Dyck, chairman; Austin Bailey, J. L. Callahan, (representing H. H. Beverage) H. A. Chinn (representing E. K. Cohan), D. G. Fink (representing J. K. Henney), L. C. F. Horle, R. H. Langley, and H. P. Westman, secretary. Tentative programs were proposed for New York meetings for the balance of 1934 and the early part of 1935.

Radio Transmissions of Standard Frequencies

Standard frequency transmissions at 5000 kilocycles are made from the Bureau of Standards Station WWV at Beltsville, Md., every Tuesday, except on legal holidays, continuously from 12 noon to 2 P.M. and from 10 P.M. to midnight, Eastern Standard Time. The accuracy of the frequency of transmission is at all times better than one cycle per second (one in five million).

For the first five minutes the general call (CQ de WWV) and announcement of the frequency is transmitted, the frequency and call letters being given every ten minutes thereafter. The main portion of the transmissions consist of the continuous unkeyed carrier wave. Information on the utilization of these signals is given in a pamphlet obtainable from the Bureau of Standards.

The Bureau would appreciate reports on field intensity, fading characteristics, and suitability of the transmissions for frequency measurements. If field intensity measurement apparatus is not available, it is suggested that the following intensity designations be used: (1) hardly perceptible, unreadable; (2) weak, readable now and then; (3) fairly good, readable with difficulty; (4) good, readable; (5) very good, perfectly readable. Fading reports giving characteristics, such as time between peaks of signal intensity, are desired. Information as to the receiving equipment and antenna used is helpful. Reports on the use of these transmissions for all purposes would be appreciated, and communications should be addressed to the Bureau of Standards, Washington, D. C.

Institute Meetings

Boston Section

A meeting of the Boston Section was held on June 15th at Harvard University and presided over by E. L. Chaffee, chairman.

A paper on "A Quantitative Study of the Dynatron" was presented by F. M. Gager of the Department of Physics, Boston College and J. B. Russell, Jr., of the Department of Electrical Engineering, Columbia University.

The resignation of the Secretary, G. W. Kenrick, who is moving to Porto Rico, was accepted and R. G. Porter, Professor of Electrical Engineering at Northeastern University was elected his successor. The attendance totaled sixty-three.

Cincinnati Section

The May 15 meeting of the Cincinnati Section was held at the University of Cincinnati with R. E. Kolo presiding.

A paper on "Reduction of Interference from Automobiles" was presented by L. M. Perkins of the Allen Bradley Company. In it he presented a comprehensive discussion of automobile ignition systems and the types of interference which they create. He then outlined cures for radiation of an audio-frequency nature as well as for radio-frequency fields. The design of the radio receiver may be completely effective in reducing audio-frequency interference while the use of completely shielded ignition systems appears to offer the greatest possibility for the elimination of radio-frequency troubles. Lacking the completely shielded system, a satisfactory cure is found in the use of properly placed resistors and capacitors with care in the shielding of exposed wiring leading into the driver's compartment. Messrs. Cook, Felix, Kilgour, Osterbrock, and Tyzzer of the forty-three members and guests participated in the discussion.

Pittsburgh Section

On February 13 a joint meeting of the Pittsburgh Sections of the Institute and the American Institute of Electrical Engineers was held at the Westinghouse Electric and Manufacturing Company plant at East Pittsburgh. R. L. Kirk of the American Institute of Electrical Engineers presided and the attendance totaled 225.

J. Slepian of the Westinghouse Electric and Manufacturing Company presented a short introductory talk on the "Ignitron," the subject of two papers presented by members of the same organization. The first paper treated the "Ignitron as a Control Tube" and was presented by D. D. Knowles. This was followed with the presentation by L. R. Ludwig of the "Ignitron as a Rectifier." The principle of operation and wide application of these tubes were described in detail in the papers which were supplemented by laboratory demonstrations. A general discussion followed.

The April meeting of the section was held on the 17th at the Fort Pitt Hotel with Lee Sutherlin presiding and an attendance of thirty-seven.

A paper on "Crystal Microphones and Pick-Ups" was presented by C. M. Chorpening who is chief engineer of Astatic Laboratories at Youngstown, Ohio. The requirements for satisfactory microphones and pick-ups were outlined and several forms of these devices described in detail. In discussing phonograph pick-up devices, the differences in needle pressure between crystal and magnetic types and their effect on scratch noise were covered. Since no alignment between pole pieces is necessary in crystal units, dust and dirt have less effect on their performance and the voltage generated by a crystal reduces the need for high amplification. The paper was discussed by Messrs. Allen, Mag, Selcox, Wyckoff, and Williamson.

A Nominating Committee was appointed, and, in addition, a proposal to form an association of all engineering societies in the Pittsburgh area was discussed.

The annual meeting of the section was held on June 5 at the Hotel Kilkeary with Lee Sutherlin, chairman, presiding. Twenty-four members were present.

Messrs. Sutherlin and Krause outlined in detail plans under consideration for the formation of an organization comprising all the engineering and technical societies in the vicinity to promote coöperation and acquaintanceship between societies and the publication of a monthly bulletin listing all meetings. It would also make possible the holding of joint meetings to attract speakers of national importance.

Upon report of the Nominating Committee, the annual election was held with the following results: Chairman, C. K. Krause of the Duquesne Light Company; Vice Chairman, R. D. Wyckoff of the Gulf Research Laboratories; Secretary-Treasurer, H. V. Noble of the Gulf Research and Development Company.

Seattle Section

Howard Mason, chairman, presided at the May 4 meeting of the Seattle Section held at the University of Washington. Seventy-seven members and guests were present.

A paper on "Glass Work and Vacuum Technic in Electronics" was presented by Don McLennan, research engineer for Electrical Products Corporation. It consisted chiefly of the demonstration of the technic of blowing, forming, and degassing of ionized gas lighting tubes and mercury-vapor rectifiers. Elaborate equipment was set up to illustrate the author's explanations. During the intervals while the tubes were being degassed, R. G. Newberry of Electrical Products Corporation interested the audience with a skillfull demonstration of the glass blower's art. C. M. Lubcke of the same organization assisted Mr. McLennan in the manipulation of the vacuum pumps and other apparatus. By the time the speaker had finished, the three engineers had completed a neon tube and two rectifiers of the Thyratron type. A hot cathode type of lighting tube was exhibited for which were claimed higher efficiency and comparatively high intensity. There were also exhibited tubes containing various combinations of rare gases.

The paper was discussed by Messrs. Eastman, Hoard, Iverson, Libby, Robinson and Wooley.

Personal Mention

Victor J. Andrew is now commerical engineer with the National Union Radio Corporation, having left his former position with the Westinghouse Lamp Company.

Previously with H. Wall of Paris, France, A. C. Bernstein has joined the staff of Motorvox Company of Brooklyn, N. Y.

N. Bishop has joined the radio sales engineering department of the General Electric Company in Bridgeport, Conn.

L. D. Boji formerly with U. S. Radio and Television Corporation is now on the staff of General Household Utilities Company, Chicago, Ill.

Previously with N. V. Transformatoren and Apparatenfabriek, H. J. J. Bouman has joined the engineering department of N. V. Philips' Radio at Eindhoven, Holland.

G. S. Burros formerly with Leotone Radio Company has become an engineer for Fulton Radio Corporation.

P. F. Dugan, Lieutenant, U.S.N., has been transferred to Norfolk Navy Yard, Portsmouth, Va.

D. E. Foster formerly with General Household Utilities Company has joined the RCA License Laboratory staff in New York City.

O. T. Francis, Captain, U.S.M.C., has been transferred to the Marine Base at Quantico, Va.

W. H. Grosselfinger formerly with Ludington Air Lines has rejoined the radio laboratory of the Western Electric Company at Kearney, N. J.

Previously with RCA Victor Company, L. H. Junken has joined the General Electric staff at Schenectady, N. Y.

R. J. Keoch is now a radio engineer for Sears Roebuck and Company of Chicago having formerly been connected with the Stewart Warner Corporation.

M. M. Levy has established a consulting practice in New York City having formerly been with the Supercraft Products Corporation.

Previously with the Shanghai Radio Central Station, T. C. Loo has become chief of the engineering department of the Chinese Government Radio Administration at Shanghai, China.

Formerly with Emerson Radio and Phonograph Company, Walter Lyons has joined the staff of Hazeltine Service Corporation of New York City.

R. D. Martin is now in the firm of Edwards and Martin, consulting engineers in Detroit, Mich., having previously been chief engineer of KVPY, Spokane, Wash.

B. F. Miller is now transmission engineer for Warner Brothers First National Studio at Burbank, Calif., having formerly been with the Universal Pictures Corporation.

T. C. Ragan, Leiutenant, U.S.N., has been transferred to the Philadelphia Navy Yard.

W. H. Wenstrom, Lieutenant, U.S.A., has been transferred to Bolling Field, District of Columbia.

R. V. Williams, Lieutenant, U.S.A., is now located at Rockwell Field, Coronado, Calif.

Errata

Emrys Williams has brought to the attention of the editors the following errors which occurred in his paper "Audio-Frequency Measurement by the Electrically-Excited Monochord," which was published in the June, 1934, issue of the PROCEEDINGS.

Page 797, line 9 should read

$$\text{K.E.} = \frac{1}{2}\int_0^l \rho\left(\frac{dy}{dt}\right)^2 dx.$$

and not

$$\text{K.E.} = \frac{1}{2}\int_0^l \rho\left(\frac{dy}{dx}\right)^2 dx.$$

Page 797, equation (2) should read

$$\text{K.E.} = \sum_1^\infty \frac{1}{4}\rho l {\phi_n}^2$$

and not

$$\text{K.E.} = \sum_1^\infty \frac{1}{4}\rho l \phi_n^2.$$

Addenda

Paul W. Klipsch has brought to the attention of the editors the following citations which should be added to the bibliography of his paper "Suppression of Interlocking in First Detector Circuits," which appeared in the June, 1934, issue of the PROCEEDINGS.

(4) J. B. Dow, "Electron-coupled oscillator circuits," *QST*, p. 23, January, (1932).

(5) James J. Lamb, "Stabilizing superheterodyne performance," *QST*, p. 14, April, (1932).

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TECHNICAL PAPERS

THE NONLINEAR THEORY OF ELECTRIC OSCILLATIONS*

BY

BALTH. VAN DER POL
(Natuurkundig Laboratorium der N. V. Philips' Gloeilampenfabrieken, Eindhoven, Holland)

I. INTRODUCTION

RECENT research in some parts of the vast domain of radio science is characterized by an extensive study of many details which, as a rule, were formerly either generally considered to be of no great importance, or were not considered at all. Frequently it happens later on that these apparent details prove to be of fundamental importance for the understanding of the basic principles. One such instance is the nonlinear theory of electric oscillations, with which one is confronted whenever the characteristic surface of a tube $i_a = f(V_a, V_g)$ is not approximated by a plane, and the bends of the characteristic are therefore taken into account. Although the first researches in connection with our subject date back to 1920 and although the development of this theory has gradually continued ever since, recent years have shown a considerable increase of activity in this field by many research workers scattered all over the world, and a special international conference dedicated solely to discussions of the problems arising in the nonlinear oscillation theory was recently held in Paris, on January 28–30, 1933.

The results of the researches in this field are to be found in many scattered technical, physical, mathematical, and even biological journals. It might, therefore, not be out of place to give a connected account of the main lines of development of our subject.

From what follows it will appear that most of the typical oscillation phenomena considered can be investigated and explained with the aid of a simple antisymmetrical characteristic of the form $i = \alpha v - \gamma v^3$. On the one hand the presence in this equation of a term like βv^2, which is well known, to explain detection and modulation, is *not* essential for a clear insight into most of the many oscillation phenomena which we

* Decimal classification: R140. Original manuscript received by the Institute, May 15, 1934.

discuss below. On the other hand the introduction of the nonlinear term $-\gamma v^3$ through which our nonlinear treatment differs from the elementary and well-known linear consideration of oscillation phenomena has, in the course of fourteen years, proved to yield a great amount of information on and deeper insight into many oscillation phenomena which the radio worker almost daily encounters, and which cannot be understood on the basis of the elementary linear theory.

At the outset it must be noted that, although the formulation of the differential equations, which form the basis of our subject, is a rather simple matter, their solutions frequently involve a considerable amount of mathematical analysis, and in which for a rigorous treatment the methods of the planetary perturbation theory form a prominent part. Unfortunately our knowledge of the solution of nonlinear differential equations has not yet reached that transparent state which is so characteristic for linear equations (and it is doubtful whether this aim will ever be reached). Consequently the solution is often found in the form of a power series of some small quantity, and it is most difficult, if not impossible, to state the radius of convergence of this series.

It was, therefore, considered appropriate (and in this way we also follow the historical development) to treat our subject as regards the approximations used, from a more physical point of view without primarily laying stress on mathematical rigor. This has the advantage of yielding a simpler analysis and giving a better survey of the whole subject. Moreover, it not seldom happened that a later, more rigorous, mathematical treatment of the problem led to the same result as that obtained with the aid of more physical approximations. However, with a view to the importance of mathematical rigor, references to such later treatments will be given throughout in order to enable the more mathematically inclined reader to study the subject in greater detail.

The following cases will be treated. They are characterized by the differential equations which are shown in their simplest form.

$$v'' - \alpha(1 - v^2)v' + \omega^2 v = 0, \quad \left(\epsilon = \frac{\alpha}{\omega} \ll 1\right)$$

Triode oscillator with one degree of freedom. Free oscillations.

$$v'' - \alpha(1 - v^2 - \epsilon_2 v^4)v' + \omega^2 v = 0, \quad \left(\epsilon = \frac{\alpha}{\omega} \ll 1\right)$$

Same as before, but with two terms more in the approximation to the characteristic.

$$v_1'' - \alpha_1(1 - v_1^2)v_1' + \omega_1^2 v_1 + k_1\omega_1^2 v_2 = 0, \quad \left(\epsilon_1 = \frac{\alpha_1}{\omega_1} \ll 1\right)$$

$$v_2'' + \alpha_2 v_2' + \omega_2^2 v_2 + k_2\omega_2^2 v_1 = 0, \quad \left(\epsilon_2 = \frac{\alpha_2}{\omega_2} \ll 1\right)$$

Triode oscillator with two degrees of freedom (coupled circuits).

$$v'' - \alpha(1 - v^2)v' + \omega_0^2 v = \omega_1^2 E \sin \omega_1 t, \quad \left(\epsilon = \frac{\alpha}{\omega} \ll 1\right)$$

Triode oscillator with external electromotive force.

$$v'' - \alpha(1 - v^2)v' + \omega^2 v = 0, \quad \left(\epsilon = \frac{\alpha}{\omega} \gg 1\right)$$

Free relaxation oscillation.

$$v'' - \alpha(1 - v^2)v' + \omega_0^2 v = \omega_1^2 E \sin n\omega_1 t, \quad \left(\epsilon = \frac{\alpha}{\omega} \gg 1\right)$$

Relaxation oscillator with impressed electromotive force.

$$\omega_1^2 \approx \omega_0^2 \qquad \text{(frequency demultiplication).}$$

II. Differential Equation for the Free Oscillations of a Regenerative Triode Oscillator[1]

In order to obtain the differential equation for the triode oscillator in the simplest form we place the load resistance not in the L or C

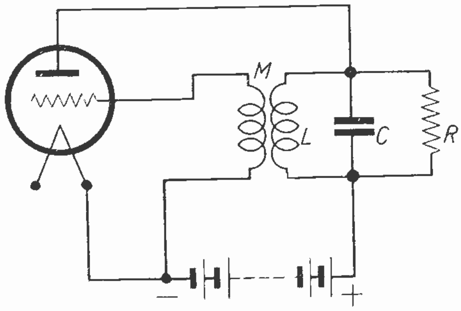

Fig. 1

branch of the oscillatory circuit but consider the load to be represented by a parallel resistance R. We thus obtain the circuit of Fig. 1.

Calling i_L, i_C, i_R the currents in the LCR branches respectively, v_a the anode potential, and i_a the anode current, we have

[1] Balth. van der Pol, "A theory of the amplitude of free and forced triode vibrations," *Radio Review*, vol. 1, pp. 701–754, (1920).

$$L\frac{di_L}{dt} = \frac{1}{C}\int i_C dt = Ri_R = E_a - v_a$$

$$M\frac{di_L}{dt} = v_g$$

and, as

$$i_a = i_L + i_C + i_R$$

we obtain

$$\frac{di_a}{dt} + \left(\frac{1}{L} + \frac{1}{R}\frac{d}{dt} + C\frac{d^2}{dt^2}\right)(v_a - E_a) = 0 \qquad (1)$$

where the grid currents have been neglected. We assume further i_a to be a function ϕ of the single variable $v_a + \mu v_g$, where μ is the amplification factor, so that

$$i_a = \phi(v_a + \mu v_g) = \phi\left\{v_a + \mu\frac{M}{L}(E_a - v_a)\right\}.$$

In the steady, though unstable, state, $v_{a_0} = E_a$. Hence the steady anode current when the system is not oscillating, is given by

$$i_{a_0} = \psi(v_{a_0}).$$

If v be the variable part of the plate potential, we have $v = v_a - v_{a_0}$ and hence the anode current is given by,

$$i_a = \phi(v_{a_0} - kv)$$

where,

$$k = \mu\frac{M}{L} - 1.$$

If, further, i be the instantaneous deviation of the anode current i_a from the unstable value i_{a_0}, we have

$$i = i_a - i_{a_0} = \phi(v_{a_0} - kv) - \phi(v_{a_0}),$$

which may be written

$$i = \psi(kv). \qquad (2)$$

Substitution of (2) in (1) leads to

$$C\frac{d^2v}{dt^2} + \frac{d}{dt}\left(\frac{v}{R} + \psi(kv)\right) + \frac{1}{L}v = 0. \qquad (3)$$

In the well-known linear theory we consider the characteristic to be given by the linear relation

$$i_a = \frac{1}{R_i}(v_a + \mu v_g)$$

and the substitution in (3) simplifies it to

$$C\frac{d^2v}{dt^2} + \left\{\frac{1}{R} - \frac{1}{R_i}\left(g\frac{M}{L} - 1\right)\right\}\frac{dv}{dt} + \frac{1}{L}v = 0. \tag{3a}$$

For sufficient retroaction, i.e.,

$$g\frac{M}{L} - 1 > \frac{R_i}{R}$$

the solution of (3a), however, is an exponentially increasing oscillation. In order to obtain an oscillation of constant amplitude, it is necessary to take into account the *curvature* of the characteristic, i.e., the non-linear terms in $i_a = \phi(v_a + \mu v_g)$.

We therefore approximate (2) over the domain of the characteristic in which the oscillation is to take place, by the third degree parabola:

$$i = \psi(kv) = -\alpha v + \beta v^2 + \gamma v^3,$$

and thus (3) becomes

$$C\frac{d^2v}{dt^2} + \frac{d}{dt}\left\{\left(\frac{1}{R} - \alpha\right)v + \beta v^2 + \gamma v^3\right\} + \frac{1}{L}v = 0. \tag{3b}$$

We further simplify (3b) by means of the following substitutions:

$$\frac{1}{C}\left(\alpha - \frac{1}{R}\right) = \alpha',$$

$$\frac{2\beta}{C} = \beta',$$

$$\frac{3\gamma}{C} = \gamma',$$

$$\frac{1}{LC} = \omega_0^2.$$

Thus (3b) can be written as

$$\frac{d^2v}{dt^2} - \left(\alpha' - \beta' v - \gamma' v^2\right)\frac{dv}{dt} + \omega_0^2 v = 0. \tag{3c}$$

Changing finally the scale of the time in such a way that

$$\omega_0 t = \tau$$

and the scale of v, such that

$$\frac{\gamma' v^2}{\alpha'} = v'^2.$$

while calling $\beta(\alpha\gamma)^{-1/2} = b$ and $\alpha'\omega_0^{-1} = \epsilon$, and omitting the dash with v', we obtain

$$\left| \frac{d^2v}{d\tau^2} - \epsilon(1 - bv - v^2)\frac{dv}{d\tau} + v = 0 \right|. \tag{I}$$

If, moreover, the triode is working on an inflection point of its characteristic, then $\beta' = b = 0$, and we have instead of (I) the simplified form:

$$\left| \frac{d^2v}{d\tau^2} - \epsilon(1 - v^2)\frac{dv}{d\tau} + v = 0 \right|. \tag{Ia}$$

So far the oscillatory circuit has been considered as being placed in the anode circuit of the tube. If it is placed in the grid circuit of, e.g., a tetrode or pentode where the influence of the anode potential on the anode current can be neglected, we again obtain[2] (I).

Moreover, a consideration of the dynatron oscillator[3] again leads to (I) so that this equation or (Ia) can be considered to be the *basic equation for the triode oscillator.*

III. Solution of (I)

Consider for simplicity the case where the triode is working on an inflection point of its characteristic, so that we are concerned with the simplified form (Ia). The physical interpretation of this equation is rather simple. ϵ being a positive constant, we notice that for small amplitudes (where v^2 can be neglected in comparison with unity) this equation reduces to

$$\frac{d^2v}{d\tau^2} - \epsilon\frac{dv}{d\tau} + v = 0,$$

representing an ordinary oscillatory circuit but with negative resistance and, as we shall suppose, with small logarithmic increment ($\epsilon \ll 1$) Hence oscillations will begin to build up exponentially but, as the amplitude increases, the nonlinear term $\epsilon v^2 dv/dt$ can no longer be neglected. The system therefore behaves as one with negative resistance for small amplitudes, but this resistance, which itself is a function of the ampli-

[2] Balth. van der Pol, Proc. I.R.E., vol. 17, p. 339; February, (1929).
[3] E. V. Appleton and Balth. van der Pol, *Phil. Mag.*, vol. 43, p. 177, (1922).

tude of the oscillations, changes its sign for larger amplitudes and thereupon becomes positive. Of course this is due to the bends in the characteristic. Hence an oscillation initially increasing will automatically be braked in such a way that a final stable and finite amplitude results.

This physical consideration suggests at once to try as a solution of (Ia) a cosinusoidal movement but with a slowly variable amplitude:

$$v = a(\tau)\cdot\cos\tau, \tag{4}$$

where the term slowly is to be understood to mean that during one cycle the percentile change of the amplitude is small, or

$$\frac{1}{a}\frac{da}{d\tau} \ll 1.$$

Moreover we know that the potential developing over the, assumed slightly damped, oscillatory circuit will be very nearly sinusoidal and hence we neglect higher harmonics. In the substitution of (4) into (Ia) we further want

$$v^3 = a^3\cos^3\tau = a^3(\tfrac{3}{4}\cos\tau + \tfrac{1}{4}\cos 3\tau).$$

Further, neglecting higher derivatives of a than the first, we have

$$\frac{dv}{d\tau} = -a\sin\tau + \frac{da}{d\tau}\cos\tau,$$

$$\frac{d^2v}{d\tau^2} = -2\frac{da}{d\tau}\sin\tau - a\cos\tau.$$

Also,

$$v^2\frac{dv}{d\tau} = \frac{1}{3}\frac{dv^3}{d\tau} \approx -\frac{1}{4}a^3\sin\tau$$

where, as this term in (Ia) occurs only multiplied by the small quantity ϵ, the term with $da^3/d\tau$ has also been neglected. The substitution of (4) into (Ia) therefore leads with the above approximation to

$$\left(-2\frac{da}{d\tau}\sin\tau - a\cos\tau\right) + \left(\epsilon a\sin\tau - \frac{\epsilon}{4}a^3\sin\tau\right) + a\cos\tau = 0$$

or to

$$\left\{\frac{da^2}{dt} - \epsilon\left(a^2 - \frac{1}{4}a^4\right)\right\}\sin\tau + (\alpha^2 - a^2)\cos\tau = 0. \tag{5}$$

That the coefficient of the $\cos\tau$ term in (5) automatically disappears is due to the fact that, with the approximations used, the frequency of the oscillation equals the natural frequency of the system. We are thus left with

$$\frac{da^2}{d\tau} - \epsilon(a^2 - \tfrac{1}{4}a^4) = 0. \tag{6}$$

this being the equation which determines the way in which the amplitude a builds up. Substituting temporarily in (6) $a^2 = x^{-1}$ it becomes

$$\frac{dx}{d\tau} + \epsilon(x - \tfrac{1}{4}) = 0$$

yielding,

$$x = \tfrac{1}{4}(1 + e^{-\epsilon(\tau - \tau_0)}),$$

or,

$$a^2 = \frac{4}{1 + e^{-\epsilon(\tau-\tau_0)}}$$

and hence,

$$v = \frac{2\cos\tau}{\sqrt{1 + e^{-\epsilon(\tau-\tau_0)}}}.$$

Finally returning to the original variables (v and t), we thus found as an approximate solution of (3b):

$$v = \frac{\sqrt{\dfrac{\alpha - \dfrac{1}{R}}{\frac{3}{4}\gamma}}}{\sqrt{1 + e^{-1/C(\alpha - 1/R)(t-t_0)}}}\cos(\omega_0 t + \varphi), \tag{7}$$

where an arbitrary phase constant φ has been introduced as one of the integration constants, the other being t_0. It can further be shown that, with the present approximations, if, instead of the shunt resistance R, a series resistance r were introduced in the oscillatory circuit, $1/R$ in (7) has to be replaced by Cr/L, and hence in this case we obtain instead of (7)

$$v = \frac{\sqrt{\dfrac{\alpha - \dfrac{Cr}{L}}{\frac{3}{4}\gamma}}}{\sqrt{1 + e^{-(\alpha/C - r/L)(t-t_0)}}}\cos(\omega_0 t + \varphi). \tag{7a}$$

Equations (7) and (7a) clearly show that the oscillation originally builds up like

$$e^{+1/2C(\alpha-1/R)t} \quad \text{or} \quad e^{+1/2(\alpha/C-r/L)t}$$

respectively, (as would also follow from a linear treatment), but that it finally reaches a stable amplitude given by

$$\sqrt{\frac{\alpha-\dfrac{1}{R}}{\frac{3}{4}\gamma}} \quad \text{or} \quad \sqrt{\frac{\alpha-\dfrac{Cr}{L}}{\frac{3}{4}\gamma}} \tag{8}$$

respectively, and thus our physical expectation is completely confirmed by an approximate analysis.

Returning to (Ia) another method will now be considered to find the frequency and the final steady amplitude. Assuming the solution to be periodic, we multiply (Ia) throughout by v and then integrate each term over a complete period. Denoting differentiation by dots we thus obtain

$$\overline{\ddot{v}v} - \epsilon\,\overline{v\dot{v}} + \tfrac{1}{3}\epsilon\,\overline{v^3\dot{v}} + \overline{v^2} = 0 \tag{9}$$

where a horizontal bar means the average over a complete period.

Now,

$$\overline{v\dot{v}} = \frac{1}{T}\int_0^T v\frac{dv}{d\tau}d\tau = \frac{1}{2T}\int_0^T \frac{dv^2}{d\tau}d\tau = \frac{1}{2T}\left[v^2\right]_{\tau=0}^{\tau=T} = 0$$

by the periodicity of v. Similarly the term $\frac{1}{3}\epsilon\,\overline{v^3\dot{v}}=0$. The term $\overline{\ddot{v}v}$ can be once integrated by parts and thus we obtain the simple relation

$$\overline{\dot{v}^2} = \overline{v^2}, \tag{10}$$

which is independent of the form of the characteristic.

Again we multiply (Ia) by $\int^\tau v d\tau$ and integrate over a complete period. In the same way we now obtain

$$\overline{v^2} = \tfrac{1}{3}\,\overline{v^4}. \tag{11}$$

Assuming v to be sinusoidal and of frequency ν, we have

$$v = a\cos\nu t.$$

On substitution in (10) we at once find in the reduced variables

$$\nu = 1$$

meaning that the oscillation frequency equals the proper frequency of the undamped circuit. Again on substituting in (11) we obtain

$$a^2\,\overline{\cos^2 t} = \tfrac{1}{3}a^4\,\overline{\cos^4 t}$$

or,

$$\frac{1}{2}a^2 = \frac{1}{3}a^4\frac{3}{8}$$

or,

$$a^2 = 4$$

and thus our former result is also verified as regards the stationary amplitude.

In connection with the question of frequency correction consider again (10), which, with the original variable $\omega t = \tau$, can be written

$$\overline{\left(\frac{dv}{dt}\right)^2} = \omega_0^2\,\overline{v^2}, \tag{10a}$$

which expression, as already remarked above, is independent of the form of the characteristic; it was only assumed in deriving (10) or (10a) that the system could oscillate at all.

If, instead of considering the fundamental only, we assume for v the form

$$v = \sum_{n=1}^{n=\infty} a_n \cos(n\omega t + \varphi)$$

where the angular frequency ω is still undetermined, we have

$$\overline{v^2} = \frac{1}{2}\sum_{n=1}^{n=\infty} a_n^2,$$

and,

$$\overline{\left(\frac{dv}{dt}\right)^2} = \frac{1}{2}\sum_{n=1}^{n=\infty} \omega^2 n^2 a_n^2.$$

Substitution in (10a) yields at once as an *exact* expression relating the amplitudes of the harmonics to the frequency:

$$\frac{\omega^2}{\omega_0^2} = \frac{\sum_{n=1}^{n=\infty} a_n^2}{\sum_{n=1}^{n=\infty} n^2 a_n^2}. \tag{12}$$

If, moreover,

$$a_1^2 \gg m^2 a_m^2 \qquad (m = 2, 3, \cdots)$$

(12) assumes the form

$$\frac{\omega^2 - \omega_0{}^2}{\omega_0{}^2} = - \sum_{n=2}^{n=\infty} (n^2 - 1) \frac{a_n{}^2}{a_1{}^2}$$

or, calling,

$$\omega - \omega_0 = \Delta\omega$$

we have

$$\frac{\Delta\omega}{\omega_0} = - \frac{1}{2}\left\{3 \frac{a_2{}^2}{a_1{}^2} + 8 \frac{a_3{}^2}{a_1{}^2} + 15 \frac{a_4{}^2}{a_1{}^2} + \cdots \right\} \tag{13}$$

giving the frequency correction $\Delta\omega$ as a function of the amplitudes of the harmonics. The frequency correction due to the flow of grid currents is here not considered as the latter were *ab initio* assumed to be zero.

IV. Extension of the Former Case to More Terms in the Development of the Characteristic

The solution (7) of (3b) found above shows, with the form of the characteristic there assumed, that the system is unstable when not oscillating and automatically the oscillations will build up until the final stationary state is reached. It is, however, well known that triode oscillators can be adjusted in such a way that the state of rest is stable, but that a slight impulse may start the oscillations, and then they build up to a stationary state. Thus, if one of the circuit parameters is altered slowly, a kind of oscillation hysteresis may present itself. This phenomenon can only be explained by taking into account at least *five* terms in representing the characteristic.[4]

In order to explain this phenomenon we must expound (2) as

$$i = \psi(kv) = \alpha v + \beta v^2 + \gamma v^3 + \delta v^4 + \epsilon v^5, \tag{14}$$

(where the sign of α has been changed), and hence we are led to consider instead of (3):

$$\frac{d^2v}{dt^2} + \frac{1}{CR} \frac{d}{dt} (v + Ri) + \frac{1}{CL} v = 0. \tag{15}$$

In order to solve this equation we multiply as before:

(1) with v and integrate over the complete period, and (2) with $\int^t v dt$ and integrate over the complete period.

[4] E. V. Appleton and Balth. van der Pol, *Phil. Mag.*, vol. 43, p. 177, (1922).

We thus obtain again as before

$$\overline{\left(\frac{dv}{dt}\right)^2} = \omega_0{}^2\overline{v^2}, \qquad \left(\omega_0{}^2 = \frac{1}{LC}\right)$$

but this time instead of (11) we have

$$\left(\alpha + \frac{1}{R}\right)\overline{v^2} + \gamma\overline{v^4} + \epsilon\overline{v^6} = 0. \tag{11a}$$

Substituting again $v = a \cos \omega t$ we find

$$\omega^2 = \omega_0{}^2$$

and,

$$\frac{1}{2}\left(\alpha + \frac{1}{R}\right)a^2 + \frac{3}{8}\gamma a^4 + \frac{5}{16}\epsilon a^6 = 0$$

yielding the following three roots

$$\left.\begin{aligned} a_1{}^2 &= 0 \\ a_{2,3}{}^2 &= -\frac{3}{5}\frac{\gamma}{\epsilon}\left\{1 \pm \sqrt{1 - \frac{40}{9}\frac{\left(\alpha + \frac{1}{R}\right)\epsilon}{\gamma^2}}\right\}, \end{aligned}\right\}$$

so that the signs of $\alpha + 1/R$, γ, and ϵ determine the possibilities (a^2 must be positive) of these amplitudes. In the paper referred to, also the stabilities of the three amplitudes $a_{1,2,3}$ are considered, and it is shown there, that *two* possible and stable amplitudes exist (one of which is zero) when $\alpha + 1/R$ and ϵ are positive but γ is negative. Moreover, the root must obviously be real. With these conditions the effective resistance of the system is positive for small oscillations, so that the oscillation does *not* build up automatically from zero. An impulse may, however, bring the representative point in a region on the characteristic, where the resistance is negative, and then the oscillation builds up to such an amplitude that this is finally limited by the positive ϵ term. A full experimental confirmation of these phenomena is given in the paper referred to.

In the above analysis a resistance R was again considered to shunt the oscillatory system. When, however, instead, a series resistance r is present, we can again replace R^{-1} by CrL^{-1}.

V. Further Consideration of the Solutions of (3)

Equations (3) or (15) can be written in the form

$$\frac{d^2v}{dt^2} + \frac{1}{CR}\frac{d}{dt}\chi(v) + \omega_0{}^2 v = 0, \tag{16}$$

where,

$$\chi(v) = v + Ri. \tag{17}$$

Calling again $\omega_0 t = \tau$ and $1/\omega Cr = \epsilon$ (16) can be written,

$$\frac{d^2v}{d\tau^2} + \epsilon\frac{d}{dt}\chi(v) + v = 0 \tag{18}$$

where again we assume

$$\epsilon \ll 1.$$

Using the more rigorous methods of the astronomical perturbation theory, a series development in the small quantity ϵ for the solution of (16) was given by E. V. Appleton and W. Greaves,[5] and W. Greaves.[6] In the latter paper also the stability of the solutions obtained is considered. It is interesting to note that the results with regard to the amplitude of the fundamental and the frequency correction thus obtained coincide with the following much simpler treatment.

With the above notation, we found already from (16)

$$\overline{\left(\frac{dv}{dt}\right)^2} = \omega_0{}^2\ \overline{v^2}. \tag{10a}$$

Also, as before, multiplication of (16) with $\int^t vdt$ and integration gives

$$\epsilon\overline{v\chi(v)} = 0. \tag{19}$$

As (19) is multiplied by the small quantity ϵ, a first approximation to v, which can then be taken as

$$v = a \cos t$$

is sufficient for insertion in (19) and thus (19) at once yields as equation for the possible amplitudes

$$\frac{1}{2\pi}\int_0^{2\pi} a\cos\tau\cdot\chi(a\cos\tau)d\tau = 0. \tag{20}$$

[5] E. V. Appleton and W. Greaves, *Phil. Mag.*, vol. 45, p. 401, (1923).
[6] W. Greaves, *Proc. Roy. Soc.* (London), A, vol. 103, p. 516, (1923); *Proc. Cambridge Phil. Soc.*, vol. 22, p. 16, (1923).

Thus, e.g., if $\chi(v)$ has the form

$$\chi(v) = -\sin v \tag{21}$$

(20) becomes

$$\frac{1}{2\pi}\int_0^{2\pi} \cos\tau \cdot \sin(a\cos\tau)d\tau = 0$$

or,

$$J_1(a) = 0, \tag{22}$$

being the Bessel function of order unity and argument[7] a.

Thus the roots of (22) represent the possible stationary amplitudes for a characteristic given by (21).

As regards the frequency correction expressed as a function of the amplitudes of the harmonics a_n, this was already obtained above in the form (13):

$$\frac{\Delta\omega}{\omega_0} = -\frac{1}{2a_1^2}(3a_2^2 + 8a_3^2 + 15a_4^2 + \cdots) \tag{13}$$

and this confirms the results arrived at in the papers quoted.

This formula was also the subject of a recent investigation by Groskowski,[8] who, however, derived (13) in a different way. Moreover, (13) is only an approximation, whereas in deriving

$$\frac{\omega^2}{\omega_0^2} = \frac{\sum_{n=1}^{n=\infty} a_n^2}{\sum_{n=1}^{n=\infty} n^2 a_n^2} \tag{12}$$

no approximations have been used; it is only assumed that the series in the denominator converges. It is further interesting to note that (12) holds independently of the form of the characteristic $\chi(v)$, as long as this is such that the system can oscillate with a stationary amplitude.

VI. Triode Oscillator With two Degrees of Freedom

We now turn our attention to a system with two degrees of freedom. In order to obtain our differential equations in the simplest form, we consider two capacity coupled circuits, one of which contains a retro-

[7] Derived in a different way by E. V. Appleton and W. Greaves, *loc. cit.*

[8] J. Groskowski, Proc. I.R.E., vol. 21, p. 958; July, (1933); *see also* Y. Rocard, *Comptes Rendus del' Acad. des Sciences* (Paris), vol. 194, p. 1325, (1932).

active triode (Fig. 2). With the same notations as used above we now obtain the two simultaneous differential equations:[9]

$$\left.\begin{aligned} \frac{d^2v_1}{dt^2} - \alpha_1(1 - v_1^2)\frac{dv_1}{dt} + \omega_1^2 v_1 + k_1\omega_1^2 v_2 = 0, \\ \frac{d^2v_2}{dt^2} + \alpha_2\frac{dv_2}{dt} + \omega_2^2 v_2 + k_2\omega_2^2 v_1 = 0, \end{aligned}\right\} \quad (23)$$

where, as before, we again assume

$$\alpha_1 \ll \omega_1,$$

$$\alpha_2 \ll \omega_2,$$

and where $k_1k_2 = k^2$ is the coupling coefficient.

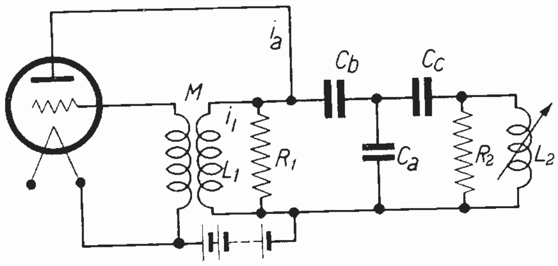

Fig. 2

It is the term

$$\alpha v_1^2 \frac{dv_1}{dt}$$

in the first equation, which represents the nonlinearity of the characteristic. The coupling is further assumed to be so tight that the influence of the damping terms on the two natural frequencies $\omega_{\rm I}$ and $\omega_{\rm II}$ of the system may be ignored. The latter are therefore given by

$$\omega_{\rm I,II}^2 = \tfrac{1}{2}(\omega_1^2 + \omega_2^2) \pm \tfrac{1}{2}\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k^2\omega_1^2\omega_2^2}.$$

In the paper referred to (23) is solved by eliminating v_2 from (23) and trying as solution for v,

$$v_1 = a(t)\sin\omega_{\rm I}t + b(t)\sin(\omega_{\rm II}t + \lambda),$$

so that in this general solution both frequencies $\omega_{\rm I}$ and $\omega_{\rm II}$ are assumed to be present simultaneously. Due, however, to the nonlinearity of our system these two oscillations interact. This interaction is brought about by the dv_1^3/dt term, which can be approximated to (retaining the two frequencies $\omega_{\rm I}$ and $\omega_{\rm II}$ only) by

[9] Balth. van der Pol, *Phil. Mag.*, vol. 43, p. 700, (1922).

$$v^3 = \tfrac{3}{4}a(a^2 + 2b^2)\sin\omega_{\mathrm{I}}t + \tfrac{3}{4}b(b^2 + 2a^2)\sin(\omega_{\mathrm{II}}t + \lambda). \quad (24)$$

We thus get as equations determining the building up of the two oscillations

$$\left.\begin{aligned}\frac{da^2}{dt} &= E_{\mathrm{I}}a^2\{a_0^2 - (a^2 + 2b^2)\}\\ \frac{db^2}{dt} &= E_{\mathrm{II}}b^2\{b_0^2 - (b^2 + 2a^2)\}\end{aligned}\right\} \quad (25)$$

where,

$$E_{\mathrm{I}} = \frac{3}{4}\gamma\frac{\omega_{\mathrm{I}}^2}{\omega_1^2}\cdot\frac{\omega_{\mathrm{I}}^2 - \omega_2^2}{\omega_{\mathrm{I}}^2 - \omega_{\mathrm{II}}^2}$$

$$E_{\mathrm{II}} = \frac{3}{4}\gamma\frac{\omega_{\mathrm{II}}^2}{\omega_1^2}\cdot\frac{\omega_2^2 - \omega_{\mathrm{II}}^2}{\omega_{\mathrm{I}}^2 - \omega_{\mathrm{II}}^2}$$

so that both E_{I} and E_{II} are positive, and further a_0^2 and b_0^2 are written for

$$a_0^2 = \frac{\alpha_1}{\frac{3}{4}\gamma} - \frac{\alpha_2}{\frac{3}{4}\gamma}\cdot\frac{(\omega_{\mathrm{I}}^2 - \omega_1^2)^2}{k^2\omega_2^4},$$

$$b_0^2 = \frac{\alpha_1}{\frac{3}{4}\gamma} - \frac{\alpha_2}{\frac{3}{4}\gamma}\cdot\frac{(\omega_{\mathrm{II}}^2 - \omega_1^2)^2}{k^2\omega_2^4}.$$

a_0^2 thus represents the square of the stationary amplitude which would be obtained when only an oscillation of frequency ω_{I} was present. Similarly b_0 is the stationary amplitude which would be attained if the system oscillated only with the frequency ω_{II}. However, a_0 and b_0 are not the only "possible" stationary amplitudes as follows from (25) by putting

$$\frac{da^2}{dt^2} = \frac{db^2}{dt} = 0.$$

We thus have in general for the "possible" stationary amplitudes a_s and b_s the two equations:

$$\left.\begin{aligned}a_s^2(a_0^2 - a_s^2 - 2b_s^2) &= 0\\ b_s^2(b_0^2 - b_s^2 - 2a_s^2) &= 0\end{aligned}\right\}$$

the four sets of solutions of which are

(I)	$a_s^2 = 0,$	$b_s^2 = 0,$
(II)	$a_s^2 = \frac{1}{3}(2b_0^2 - a_0^2),$	$b_s^2 = \frac{1}{3}(2a_0^2 - b_0^2),$
(III)	$a_s^2 = a_0^2,$	$b_s^2 = 0,$
(IV)	$a_s^2 = 0,$	$b_s^2 = b_0^2.$

By "possible" is here meant a real solution for the amplitudes. But whether they occur in practice depends on their stability.

A further investigation into the conditions of stability of these four solutions is given in the paper referred to. It is there shown, that with sufficient retroaction solution (I) is unstable and therefore the system cannot remain at rest. Solution (II), where both oscillations would be present simultaneously, is shown also to be unstable. It is further proved that solution (III)

$$a_s^2 = a_0^2, \qquad b_s^2 = 0$$

is only stable as long as the circuit parameters are such that

$$a_0^2 > \tfrac{1}{2}b_0^2, \tag{26}$$

and similarly, solution (IV)

$$a_s^2 = 0, \qquad b_s^2 = b_0^2$$

is only stable as long as

$$b_0^2 > \tfrac{1}{2}a_0^2. \tag{27}$$

As the two conditions (26) and (27) overlap, it is possible that for a given set of parameters of the circuit either (III) or (IV) may occur.

The obvious consequence is, therefore, that the mode of oscillation which will actually occur depends on the way in which the particular set of parameters is reached, and a very pronounced oscillation hysteresis may occur, as is also well known in practice. The explanation of this hysteresis can, however, only be given on the basis of a nonlinear theory such as explained above, because this typically involves the interaction of one oscillation by another, as the principle of superposition is no more valid in nonlinear systems. In fact it was shown[9] that, with the cubic characteristic here considered, the building up of one oscillation suppresses the other. This is exemplified by Fig. 3 (reproduced from reference (9)), where the amplitude a^2 is seen to build up while at the same time it is suppressing b^2.

It is of interest to note that our equations (25) determining the real quantities a^2 and b^2 and which were first derived in 1921, are exactly equal to those occurring in a now famous problem of parasitology, where the coexistence is investigated of two species, a host population and a parasite population. This problem was investigated by A. J. Lotka[10] and by V. Volterra.[11] The number of individuals in the two

[10] A. J. Lotka, "Elements of Physical Biology," Baltimore, (1925).
[11] V. Volterra, "Leçons sur la Théorie Mathématique de la Lutte pour la Vie," Paris, (1931).

species there replaces the positive variables a^2 and b^2 of our equations (25).

From a mathematical point of view the annihilation of one species by another is analogous to the problem considered above, where one oscillation, while building up, suppresses the other (see Fig. 3). This analogy will, however, not be further pursued here.

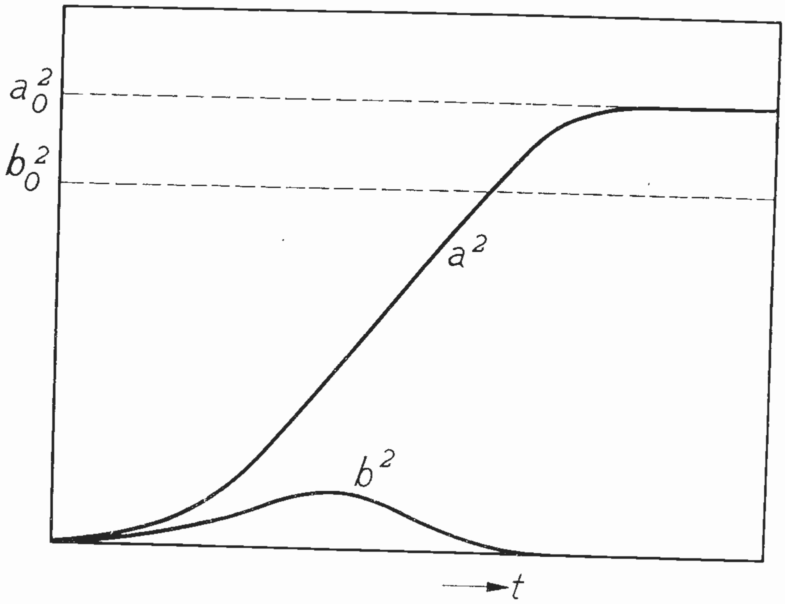

Fig. 3

VII. Triode Oscillator with External Electromotive Force

When an external electromotive force of frequency ω_1 is acting on a triode oscillator of frequency ω_0, a second member must be given to the representing differential equation. Our equations (3) or (16) must therefore be replaced by

$$\frac{d^2v}{dt^2} + \frac{1}{CR}\frac{d}{dt}\chi(v) + \omega_0{}^2v = \omega_1{}^2E \sin \omega_1 t. \tag{28}$$

If further, as was done before, the characteristic is represented by a third degree parabola (28) can be written

$$\frac{d^2v}{dt^2} - \frac{d}{dt}(\alpha v - \gamma v^3) + \omega_0{}^2v = \omega_1{}^2E \sin \omega_1 t. \tag{29}$$

As before it is again the nonlinear term

$$\gamma \frac{dv^3}{dt}$$

which explains the many interesting phenomena which present themselves with these circumstances. In this paragraph we assume again as before: $\alpha \ll \omega$.

This equation was fully considered by the author in 1924.[12]

We first investigate what happens when ω_1 is near ω_0. To this end we try as a solution

$$v = b_1 \sin \omega_1 t + b_2 \cos \omega_1 t \tag{30}$$

where b_1 and b_2 are supposed to be slowly varying functions of the time, such that

$$\frac{db_1}{dt} \ll \omega_1 b_1,$$

$$\frac{db_2}{dt} \ll \omega_1 b_2.$$

Solution (30) allows for the possibility of the presence of free oscillations, by the assumption that both b's are functions of t. In the term v^3 we only retain the fundamental frequency such that

$$v^3 = \tfrac{3}{4} b^2 (b_1 \sin \omega_1 t + b_2 \cos \omega_1 t)$$

where for the square of the total amplitude has been written

$$b^2 = b_1^2 + b_2^2.$$

Substitution of (30) and (31) into (29), neglecting small quantities and separately equating to zero the terms containing $\sin \omega_1 t$ and $\cos \omega_1 t$, yields

$$\left.\begin{aligned} 2\frac{db_1}{dt} + zb_2 - \alpha b_1\left(1 - \frac{b^2}{a_0^2}\right) &= 0, \\ 2\frac{db_2}{dt} - zb_1 - \alpha b_2\left(1 - \frac{b^2}{a_0^2}\right) &= \omega_1 E, \end{aligned}\right\} \tag{32}$$

where,

$$z = \frac{\omega_0^2 - \omega_1}{\omega_1} \approx 2(\omega_0 - \omega_1),$$

and,

$$a_0^2 = \frac{\alpha}{\tfrac{3}{4}\gamma}$$

represents, as before, the amplitude of the free stationary oscillation.

[12] Balth. van der Pol, *Tijdschr. Ned. Rad. Gen.*, October, (1924), (in Dutch). An English translation appeared in *Phil. Mag.*, vol. 3, p. 65, (1927).

Let us consider temporarily what the result (32) means for the linear case, in which $\gamma=0$. Equation (32) can then at once be integrated and gives

$$\left.\begin{aligned} b_1 &= e^{(\alpha/2)t}\left(C_1 \sin \frac{z}{2}t + C_2 \cos \frac{z}{2}t\right) - \frac{z\omega_1 E}{z^2+\alpha^2}, \\ b_2 &= e^{(\alpha/2)t}\left(-C_1 \cos \frac{z}{2}t + C_2 \sin \frac{z}{2}t\right) - \frac{\alpha\omega E}{z^2+\alpha^2}. \end{aligned}\right\}$$

b_1 and b_2 are therefore of a slow periodicity. By substitution of (33) into (30) we obtain

$$v=C_3e^{(\alpha/2)t}\sin(\omega_0 t+\varphi)+\frac{\omega_1 E}{\sqrt{(\omega_0-\omega_1)^2+\alpha^2}}\sin\left(\omega_1 t+\tan^{-1}\frac{2\alpha}{\omega_0-\omega_1}\right)$$

which expression agrees with the exact *linear* solution of (29), if we neglect the frequency correction to which the free oscillation is subjected due to the resistance. Hence the *free* oscillations are represented by the slow periodic part of the b's. If in the nonlinear case the external electromotive force $E=0$, (32) leads again to (7), so that also this special case is verified.

Returning again to the solution of the general problem, we find that a particular solution of (32) is given by

$$\frac{db_1}{dt} = \frac{db_2}{dt} = 0,$$

so that then (32) gives

$$b^2\left\{z^2+\alpha^2\left(1-\frac{b^2}{a_0{}^2}\right)^2\right\} = \omega_1{}^2E^2, \qquad (33)$$

giving in general *three* different solutions for b^2.

Although the idea of free and forced oscillations has lost its rigid mathematical *foundations* in nonlinear systems, it will physically be sufficiently clear when we interpret (33) as representing the case where only "forced" oscillations are present, both b's being independent of the time. The free oscillations which are represented by periodic parts of the b's are here completely absent. The circumstances under which this condition obtains are determined by a further stability investigation which is given in the paper cited. It is there found that the solution (33) with "forced" oscillations only is stable as long as

$$\left.\begin{aligned} &b^2 > \tfrac{1}{2}a_0^2, \\ &\alpha^2\left(1 - \frac{3b^2}{a_0^2}\right)\left(1 - \frac{b^2}{a_0^2}\right) + z^2 > 0 \end{aligned}\right\}. \qquad (34)$$

The resonance curves represented by (33) are plotted in Fig. 4 for seven different values of $E' = \omega_1^2 E/a_0$. The parts which are stable are drawn in full line while the unstable parts are shown dotted. The ordinates represent b^2/a_0^2 while the abscissas represent the amount of

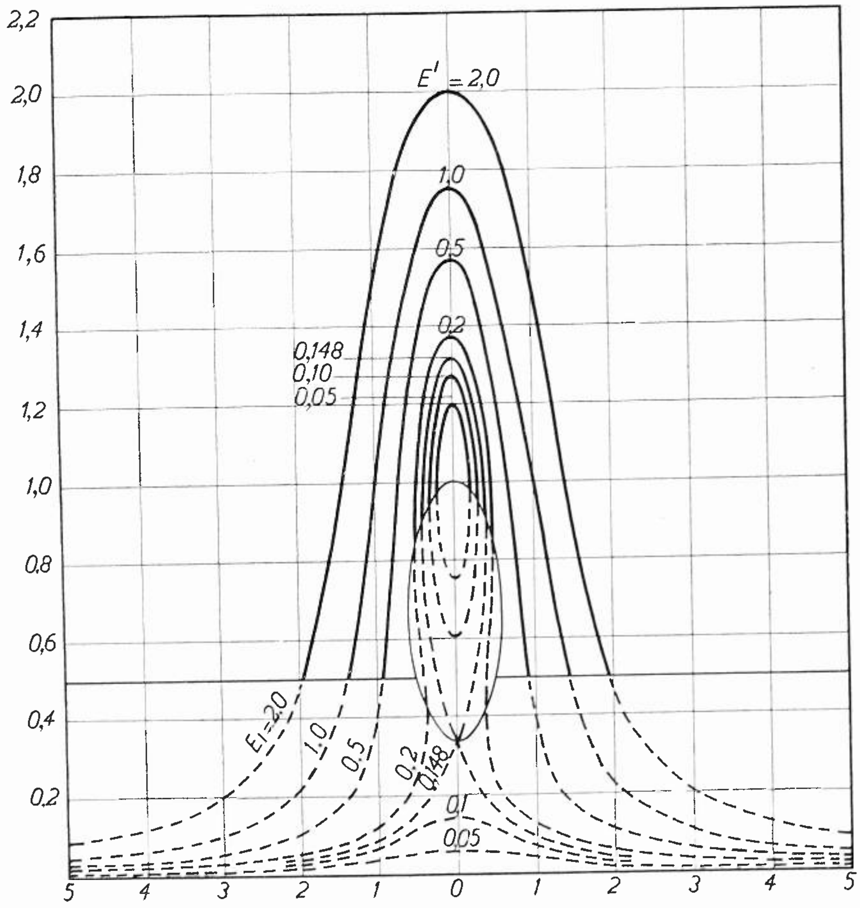

Fig. 4

detuning z/α. This figure clearly shows that for a strong signal the resonance curves have more or less the usual shape whereas for a small electromotive force the curves are slightly warped. Again the limits of stability as determined by (34) are represented by thin lines.

We still have to consider the disturbance occurring outside the limits of stability of $b^2 = \text{const}$. Under these circumstances the "free" oscillation will be present. It follows simply from (33) and (34) that for a strong external electromotive force the amount of detuning necessary for the free oscillation just to set in is given by

$$\frac{\omega_0 - \omega_1}{\omega_1} = \pm \frac{E}{a_0\sqrt{2}}. \tag{35}$$

This relation was experimentally found confirmed by E. V. Appleton.[13]

Returning to (33) we can investigate the resulting amplitude b if the system is in tune with the impressed electromotive force, i.e., for

$$z = 0$$

and when, moreover, the system is brought to the threshold of oscillation, i.e., for

$$\alpha = 0.$$

The amplitude of the resulting signal then follows from (33) as[14]

$$b = \sqrt[3]{\frac{\omega_0 E}{\frac{3}{4}\gamma}}. \tag{36}$$

Hence, with these conditions the resulting amplitude is proportional to the cube root of the amplitude of the impressed electromotive force. This explains the fact that an automatic limiter action presents itself with reception with a critical retroaction. Of course it is here assumed that the triode system shows no hysteresis in its retroaction such as considered in IV. When, moreover, we are only concerned with small amplitudes, the representation of the characteristic by three terms only can be identified by the first three terms of a Taylor development. As before the β term is here again of minor importance and hence it can be said that under these circumstances the developing amplitude follows from (36) to be inversely proportional to the cube root of the third differential coefficient of the characteristic.

Returning to the general case (29) and in order to find an approximate expression for the case where apart from the "forced" oscillation also the "free" oscillation is present, we now try the solution

$$v = a \sin(\omega_0 t + s) + b \sin(\omega_1 t + \lambda).$$

The approximate expression for v^3 now becomes, similar to (24):

$$v = \tfrac{3}{4}a(a^2 + 2b^2)\sin(\omega_0 t + s) + \tfrac{3}{4}b(b^2 + 2a^2)\sin(\omega_1 t + \lambda). \tag{37}$$

Substitution in (29) yields

[13] E. V. Appleton, *Proc. Cambridge Phil. Soc.*, vol. 21, p. 231, (1923).
[14] Balth. van der Pol, Proc. I.R.E., vol. 17, p. 339; February, (1929).

$$\left.\begin{aligned} \omega_0 - \omega_0 &= 0, & \text{(a)} \\ a\left(1 - \frac{a^2 + 2b^2}{a_0^2}\right) &= 0, & \text{(b)} \\ b^2\left\{z^2 + \alpha^2\left(1 - \frac{b^2 + 2a^2}{a_0^2}\right)^2\right\} &= \omega^2E^2. & \text{(c)} \end{aligned}\right\} \quad (38)$$

Equation (38b) resolves itself into

$$a = 0 \quad (39)$$

and,

$$1 - \frac{a^2 + 2b^2}{a_0^2} = 0. \quad (40)$$

Equation (39) represents again the suppression of the "free" by the "forced" oscillation, which was the case already considered above, for then (38c) is reduced to (33).

The other solution (40) gives

$$a^2 = a_0^2 - 2b^2, \quad (41)$$

and according to (34) belongs to the solution where the "free" oscillation is present.

According to (38c) the amplitude of the "forced" oscillation is then given by

$$b^2\left\{z^2 + \alpha^2\left(1 - \frac{3b^2}{a_0^2}\right)^2\right\} = \omega_1^2E^2, \quad (42)$$

which differs by the factor 3 in the term $3b^2/a_0^2$ from the similar term in (33). Hence the apparent resistance of the system to the "forced" oscillation is increased due to the presence of the "free" oscillation. Moreover, it follows from (41) that, due to the presence of the "forced" oscillation, the free oscillation cannot develop to its final free amplitude a_0^2 but only to $a_0^2 - 2b^2$, from which may be concluded that, as soon as the "forced" amplitude reaches the value

$$b^2 = \tfrac{1}{2}a_0^2$$

no room is left for the "free" oscillation to build up, or, in other words, the "free" oscillation is then suppressed by the "forced" one. This agrees with the limit of stability as given by (34).

The results obtained above enable us to extend Fig. 4 to Fig. 5. In the latter figure the total disturbance

$$\frac{a^2 + b^2}{a_0{}^2}$$

is plotted instead of $b^2/a_0{}^2$ only, as was the case in Fig. 4, again as a function of the detuning z/α. In the region where the free frequency is suppressed ($a^2 = 0$) the two figures coincide. Moreover, far outside resonance the curves approach the horizontal line given by the

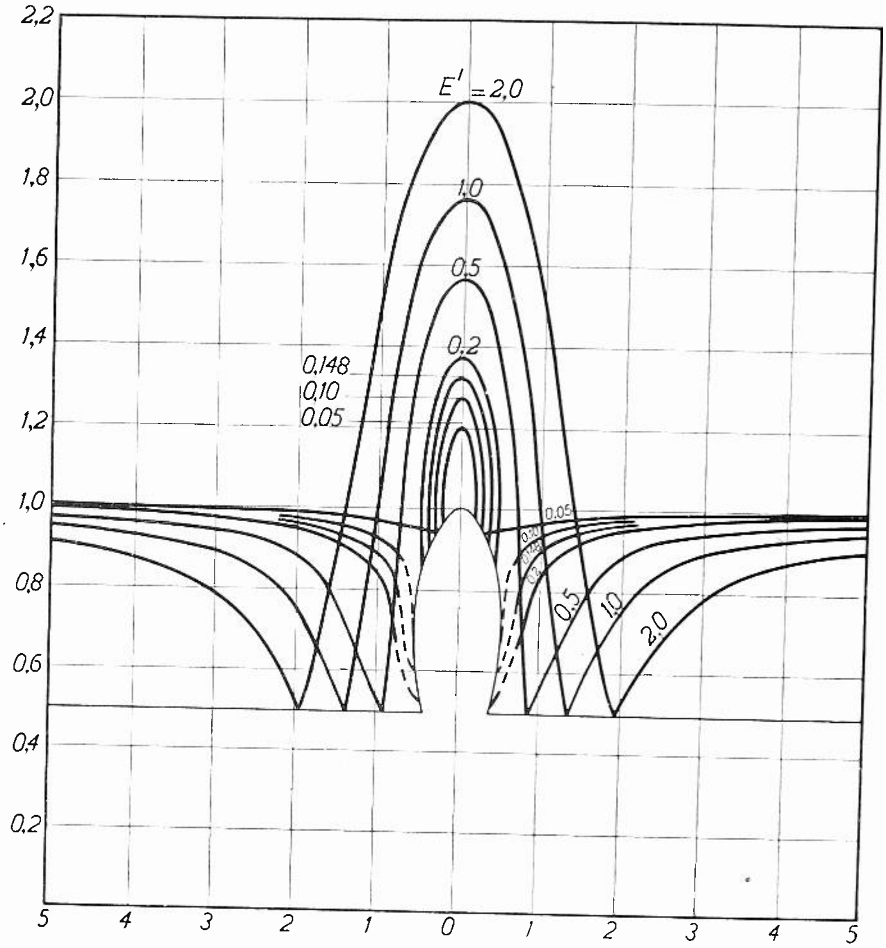

Fig. 5

ordinate unity. In these regions the "free" oscillation is only present and practically undisturbed by the small "forced" one. In Fig. 5 some parts are dotted, meaning that our two solutions (30) and (42) do not completely fit in some parts of the curves. This gap has been discussed by Andronow and Witt,[15] who showed that a closer investigation of the solutions of (32), contrary to an opinion expressed by Ollendorf,[16] revealed the fact that the effect discussed above, and which may be called *automatic synchronization*, i.e., the suppression of

[15] A. Andronow and A. Witt, *Archiv für Elektrotech.*, vol. 24, p. 99, (1930).
[16] F. Ollendorf, *Archiv für Elektrotech.*, vol. 16, p. 280, (1926).

the "free" oscillation by the "forced" one, is always present, although in a small region round the resonance, however small the impressed electromotive force may be chosen. This phenomenon of automatic synchronization was known already to Huygens who observed that two clocks hung on the same wall could synchronize.

VIII. Relaxation Oscillations

The equation which so far formed the basis of our considerations was (3) or (18), or, written in a somewhat specialized form:

$$\frac{d^2v}{d\tau^2} - \epsilon(1 - v^2)\frac{dv}{d\tau} + v = 0. \tag{Ia}$$

In the cases considered so far the parameter ϵ was taken small compared with unity:

$$\epsilon = \frac{\alpha}{\omega} \ll 1. \tag{43}$$

The condition (43) simply means that, as long as the oscillation has a small amplitude its (initial) logarithmic increment is small, i.e., the relative change of the amplitude during one oscillation is small compared with unity. This condition (43) is usually amply fulfilled in ordinary triode oscillators.

It is of considerable interest also to investigate the sequence of events when instead of (43) we have the opposite condition

$$\epsilon \gg 1. \tag{44}$$

This case was fully treated in 1926.[17] Consider first for the moment small amplitudes only, i.e., momentarily ignoring the term $\epsilon v^2 dv/d\tau$ in (Ia); it will be clear that our equation then concerns a system with a negative resistance of such a value that the system becomes aperiodic. It is obvious that for such a system $v = 0$ is not a stable solution. Hence it cannot remain at rest in the zero position. But when in the general case (Ia) the deviation increases and $v^2 > 1$ the coefficient of the second term becomes positive indicating a positive resistance and therefore after some time a reduction of the deviation with time. Moreover, the limiting values $v = \pm 1$ are no solutions of (Ia), and hence we may expect the solution of (Ia) still to be periodic even with the condition (44).

[17] Balth. van der Pol, *Physica*, vol. 6, p. 154, (1926); *Tidjschr. Ned. Rad. Gen.*, vol. 3, p. 25, (1926); vol. 4, p. 94, (1927); *Phil. Mag.*, vol. 2, p. 978, (1926); *Jahr. der draht. Tel. und Tel.*, vol. 28, p. 178, (1926); vol. 29, p. 114, (1927); *Exp. Wire.*, vol. 3, p. 338, (1926).

A formal analytical solution of (Ia) with the condition (43) has so far not been found but a partially analytical, partially graphical solution can be obtained. As the method is quite general, we shall at the same time investigate with it the case $\epsilon \ll 1$ also.

For, writing

$$\frac{dv}{d\tau} = y,$$

we have

$$\frac{d^2v}{d\tau^2} = \frac{dy}{dv}\frac{dv}{d\tau} = y\frac{dy}{dv} \tag{45}$$

and hence the order of (Ia) can be reduced and we obtain

$$y\frac{dy}{dv} - \epsilon(1 - v^2)y + v = 0.$$

Now (45) can graphically be solved with the aid of the isocline method.[18] Thus, a relation between $y = dv/d\tau$ and v is obtained.

The latter can be further graphically integrated and the results are depicted in Fig. 6 for three different values of ϵ, viz.,

$$\epsilon = 0.1,\ 1,\ \text{and}\ 10.$$

The top curve of this figure therefore represents the case of $\epsilon \ll 1$ which was fully considered already in II and III, and it confirms the approximate solution given there. It represents the slow building up of an approximately sinusoidal oscillation. $\epsilon = 1$ represents an intermediate case, while the curve for $\epsilon = 10$, which is of special interest, depicts the case considered in this paragraph, viz., (44).

Fig. 6 further clearly shows the fact that a gradual and continuous change exists between solutions of (Ia) for $\epsilon \ll 1$ and for $\epsilon \gg 1$. In fact these two solutions can be considered to be the limiting cases of the solutions of (Ia). The first limiting case ($\epsilon \ll 1$) represents the sinusoidal oscillations studied above, but the second limiting case ($\epsilon \gg 1$) represents a type of periodic phenomenon, which possess very special properties and are found in so many physical and technical instances that it was considered appropriate to give them a special name; therefore, we named them *relaxation oscillations* for a reason which will appear below.

The third curve of Fig. 6 is a typical relaxation oscillation. Contrary to the case $\epsilon \ll 1$ where the building-up process covers many oscil-

[18] Another form of graphical solution has been given by A. Liénard, *Rev. Gen. de l'Élec.*, vol. 23, pp. 901, 946, (1928).

lations, the relaxation oscillation is seen to attain its final stationary amplitude after only one oscillation. This steady state is further characterized by a very marked departure from the sinusoidal form. It is seen that, for the reduced units of (Ia), the amplitude alters very slowly from the value 2 to the value 1 and then it very suddenly drops to the value -2. Next we observe a very gradual increase from the value -2 to the value -1 and again a sudden jump to the value $+2$. And then this process repeats itself indefinitely. Obviously this form of oscillation contains many higher harmonics of considerable amplitude.

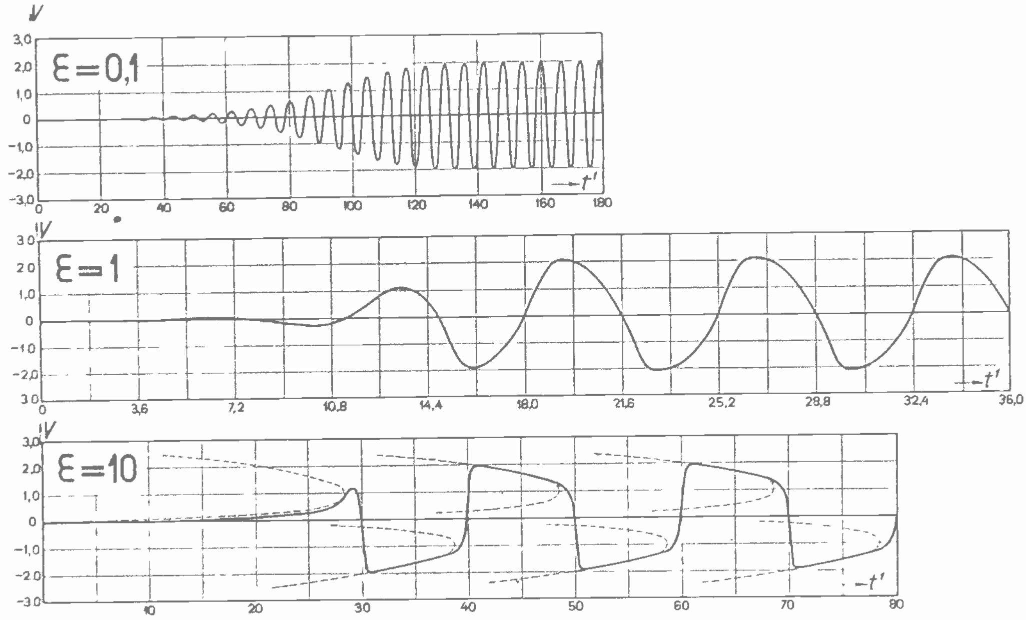

Fig. 6

Characteristic for a relaxation oscillation is the fact that during the greater part of the period the phenomenon has a typical aperiodic or asymptotic behavior (such as shown by the well-known intermitted discharges of a neon tube shunted by a capacitance and fed by a battery through a resistance), and thereupon suddenly the system becomes unstable and the disturbance jumps rather discontinuously to another value, and then the same discharge phenomenon repeats itself over again, etc., so that a relaxation oscillation has the character of an ever repeating discharge phenomenon. Therefore, as also will be shown below, the time period is given by a relaxation time such as CR or Lr^{-1}, and hence the name.

We showed[17] that an approximate solution of (Ia) with the condition (43) during a part of the period can be obtained by ignoring the first term of (Ia). This solution is easily found to be

$$\log v^2 - v^2 = \frac{2\tau}{\epsilon} + \text{const.} \tag{46}$$

and this function for different values of const. is drawn as dotted lines in the case ($\epsilon = 10$) of Fig. 6. This solution is seen to give a good approximation to the actual solution, e.g., from $v = 2$ to $v = 1$. Hence the time period T of the relaxation oscillation as defined by (Ia) is, with the reduced time units of (Ia), approximately given by

$$T = \left(\log 2 - \frac{3}{2}\right)\epsilon \approx 1.61\epsilon \tag{47}$$

and hence, going back to the original time $t = \tau\omega^{-1}$ we find for the time period T_{rel} of a relaxation oscillation:

$$T_{\text{rel}} \approx 1.61 \frac{\alpha}{\omega^2}. \tag{48}$$

A typical technical example of a system producing relaxation oscillations is the multivibrator of Abraham and Bloch[19] which is a two-stage RC amplifier with the output side connected to the input side. If we assume the two stages to oscillate in opposite phase and call the grid leaks R, and the coupling capacities C and assume again the (i_a, v_g) characteristic in the vicinity of the working point to be given by a cubic parabola, the representative equation is of the form (Ia) and the condition (44) is certainly fulfilled. Thus we find from (48) for the time period:

$$T_{\text{rel}} \approx \text{const. } RC$$

where the const. is of the order unity. It is therefore again a typical relaxation time which determines the time period of this relaxation oscillation, and moreover the phenomenon is characterized by an ever repeating condenser discharge.

Another typical example of a relaxation oscillation is produced by a separately excited motor fed by a series dynamo which is revolving at a constant speed. It is well known[20] that the motor will first revolve in one direction, then in the other, then in the first again, etc. The motor can also be replaced by a large capacity and the current through this capacity, as was the case with the current through the motor, will alternate from one sense to the other. The differential equation governing this phenomenon is again of the form (Ia) and also the condition

[19] H. Abraham and E. Bloch, *Ann. der Phys.*, vol. 12, p. 237, (1919).
[20] *See*, e.g., M. Janet, *Ann. des P.T.T.*, vol. 14, p. 1193, (1926).

(44) is fulfilled so that here again we have another typical example of a relaxation oscillation. For comparison in Fig. 7 three curves are given. The first is an oscillogram taken with the series dynamo feeding a separately excited motor, the second is the solution of (Ia) with $\epsilon \gg 1$,

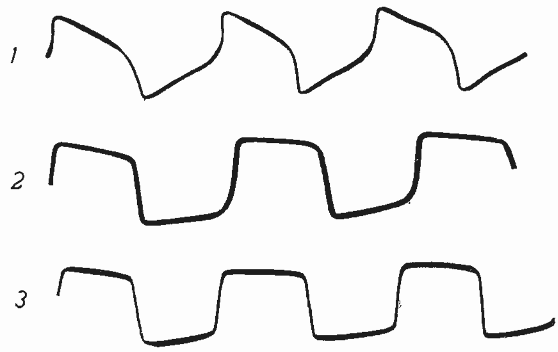

Fig. 7

and the third is an oscillogram of a multivibrator. The great similarity of these three curves clearly illustrates the fact that the equation (Ia) represents both phenomena.

Another experiment which clearly shows the gradual transition from a sinusoidal oscillation ($\epsilon \ll 1$) to a relaxation oscillation ($\epsilon \gg 1$) is

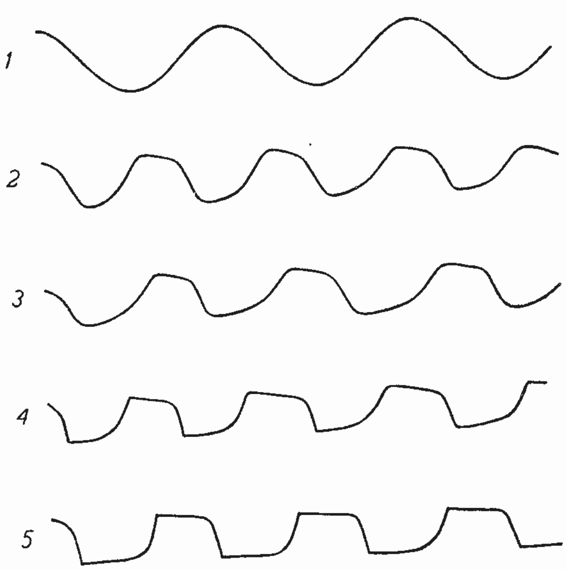

Fig. 8

provided when an LC circuit is shunted to a dynatron. For big values of C where the oscillations are just maintained, the oscillation is typically of the sinusoidal type. A gradual reduction of the value of C gradually changes the form of the oscillation till at the moment where C approaches zero the oscillation (as is also shown by theory) becomes typically of the relaxation type. Fig. 8 gives a series of oscillograms thus obtained where the successive curves correspond to decreasing values of the capacity C. It is clearly seen how the oscillation is first of the sinusoidal type, then gradually changes its form and finally obtains the form of a relaxation oscillation.

IX. Systems Producing Relaxation Oscillations under the Influence of an Impressed Periodic Electromotive Force

In Section VII we considered a triode oscillator with the condition $\epsilon=\alpha/\omega\ll 1$ under the influence of an impressed periodic force. It was there shown that, near resonance, only an oscillation of the period of the impressed electromotive force was present in the system. We saw that the "free" oscillation of frequency ω_0 could be suppressed by the "forced" oscillation of frequency ω_1. This, however, only occurred in a small region near resonance, where also the amplitude rose to a high maximum. Now experiment has shown that a similar phenomenon presents itself when a relaxation oscillation ($\epsilon\gg 1$) is brought under the influence of an impressed periodic force.

However, there are two very marked differences. First, the frequency region over which this automatic synchronization presents itself is very considerably wider in the case of a relaxation oscillation and even may attain a width of the order of an octave, and second, the phenomenon of resonance is practically absent in the case of relaxation oscillations, so that an external electromotive force cannot appreciably influence the *amplitude* of the relaxation oscillation but can easily influence its *time period*. Moreover it was found experimentally[21] that when the frequency ω_1 of the external electromotive force was increased beyond this region of automatic synchronization, the frequency of the relaxation oscillation suddenly jumped to half the frequency of the electromotive force ($\frac{1}{2}\omega_1$) so that here an automatic synchronization produced itself but on a subharmonic of the impressed electromotive force. Increasing ω_1 still further first left the relaxation synchronized on $\frac{1}{2}\omega_1$, and then suddenly its frequency jumped to the next subharmonic $\frac{1}{3}\omega_1$, etc. Thus the phenomenon of *frequency demultiplication* presented itself. Regions of automatic synchronization were found with a demultiplication as far as 200:1. Similar phenomena were also obtained in the case of sinusoidal oscillations by Koga.[22]

The frequency regions over which automatic synchronization is present is, however, much smaller than in the case of relaxation oscil-

[21] Balth. van der Pol and J. van der Mark, *Nature*, vol. 10, September (1927).

[22] I. Koga, Proc. I.R.E., vol. 15, p. 669; August, (1927).

lations and also with sinusoidal systems this frequency demultiplication cannot be obtained for such big ratios of frequency as in the case of relaxation oscillations.

This principle of frequency demultiplication has found numerous applications in precision measurements of frequency.[23]

Obviously the equation representing these phenomena is of the form

$$\left.\begin{aligned} \frac{d^2v}{dt^2} - \alpha(1 - v^2)\frac{dv}{dt} + \omega_0{}^2 v &= \omega_1{}^2 E \sin n\omega_1 t, \\ \omega_1{}^2 &\approx \omega_0{}^2, \\ \epsilon &= \frac{\alpha}{\omega} \gg 1. \end{aligned}\right\}. \tag{49}$$

Though the analytical investigation of the solutions of this equation with the given conditions has hardly begun, one instance of frequency demultiplication which we owe to Mr. de Vries of this laboratory may be cited. The expression,

$$v = 2 \cos \omega_0 t,$$

namely, solves exactly the differential equation

$$\frac{d^2v}{dt^2} - \alpha(1 - v^2)\frac{dv}{dt} + \omega^2{}_0 v = -\omega_0{}^2 E \sin 3\omega_0 t$$

if the amplitude of the impressed electromotive force is given by

$$E = \frac{2\alpha}{\omega} = 2\epsilon.$$

Returning to a general consideration of relaxation oscillations many more instances of these oscillations can be cited, e.g., the beating of the heart with its many anomalies showing ab origine frequency demultiplication is typical in this respect.[24] Also it seems that many aerodynamic phenomena associated with eddies are of this nature. Even the periodic reoccurrence of economical crises and epidemics may possibly follow similar laws.[25]

[23] *See*, e.g., L. M. Hull and J. K. Clapp, PROC. I.R.E., vol. 17, p. 252; February, (1929).

[24] Balth. van der Pol and J. van der Mark, *L'Onde Électrique*, vol. 7, p. 365, (1928).

[25] L. Hamburger, "Indices du Mouvement des Affairs," Institut de Statistique de l'Université de Paris vol. 9, Janvier, (1931).

Further the periodical electrical disturbances generated in a nerve by a constant stimulus are of the nature of relaxation oscillations.[26]

In general, it appears that many periodic phenomena for which an explanation on the basis of sinusoidal oscillations fails to give a satisfactory explanation can better be studied from the point of view of relaxation oscillations.

X. Recent Investigations

The above paragraphs give a connected account of many oscillation phenomena which present themselves to the radio worker and which can only be understood on the basis of a nonlinear theory. As has been remarked in the introduction, the approximations introduced were mostly of a physical nature and often they not only gave a formal solution to the problem but also provided us with a deeper physical insight into the phenomena under investigation.

Recently solutions of some of the differential equations treated above but with the aid of the methods of the perturbation theory as developed by Henri Poincaré in his *Mécanique Céleste* I have been obtained by several workers mostly in the Soviet Republic.

Mandelstam and Papalexi give some very general theoretical considerations in *Zeit. für Phys.*, vol. 73, p. 223, (1931), while Kryloff and Bogoliuboff's contributions are to be found in the *Comptes Rendus*, vol. 194, pp. 957, 1064, 1119, (1932). The main problem treated concerns the case of frequency demultiplication in sinusoidal systems ($\epsilon \ll 1$). Quite recently a whole volume of the *Zeitschrift für technische Physik* of the Soviet Union, Band 4, Heft 1, of over 230 pages (published in Moscow) in the Russian language was devoted to theoretical and experimental investigations of problems closely related with those treated here.

We further mention two very clear expositions of the nonlinear problem of the generation of electric oscillations by Ph. le Corbeiller, "Les Systèmes Autoentretenus et les Oscillations de Relaxation," Hermann et Cie, Paris, (1931); "Le Mécanisme de la Production des Oscillations," *Ann. des P.T.T.* vol. 21, p. 697, (1932), which have materially contributed towards a better understanding of many phenomena concerning electric oscillations.

Finally a bibliography is appended.

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NOTE ON THE SYNCHRONIZATION OF BROADCAST STATIONS WJZ AND WBAL*

By
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(Bureau of Standards, Washington, D.C.)

The field intensities of the broadcast stations WJZ, Bound Brook, N.J., and WBAL, Baltimore, Md., were recorded continuously throughout a number of 24-hour periods in 1932 and 1933 at the Bureau of Standards receiving station at Meadows, Md., (latitude 38°48′32″N, longitude 76°52′40″W) near Washington, D.C. The method used for recording the field intensity is described elsewhere.[1] The two stations are synchronized on 760 kilocycles on alternate days during the daytime, and similarly at night. The synchronization is accomplished by means of an audio-frequency current transmitted to each station over a wire line and multiplied to the radio frequency of the station at the transmitter.

Fig. 1. illustrates the type of record obtained, showing that the fading is of a radically different character when the stations are synchronized than when not. On the lower record may be seen the comparatively steady ground wave from WJZ during the daytime, having the constant intensity of about 200 microvolts per meter. On the upper record, during the daytime, the two stations are synchronized but fading occurs. This fading is due to interference of the two ground waves, the intensities of which are about 200 and 600 microvolts per meter, so that the maxima and minima are 800 and 400 microvolts per meter, i.e., the sum and difference of the two fields.

Fig. 2 shows this same interference phenomenon for two hours during the daytime, with the recorder operating three times as fast for half the time in order to resolve the fading. It may be seen that the average period of the fading is about one minute; it was observed on a receiver with automatic volume control that this slow fading did not introduce any serious distortion into the received modulation where the ratio of the intensities of the two ground waves was three to one. It is believed that no serious distortion would be introduced into the received modulation by the synchronization fading in that part of the primary service area of the two stations where the ratio of the intensi-

* Decimal classification: R550. Original manuscript received by the Institute, April 16, 1934. Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] RP597. K. A. Norton and S. E. Reymer, "A continuous recorder of radio field intensities," *Bur. Stan. Jour. Res.*, vol. 11, September, 1933.

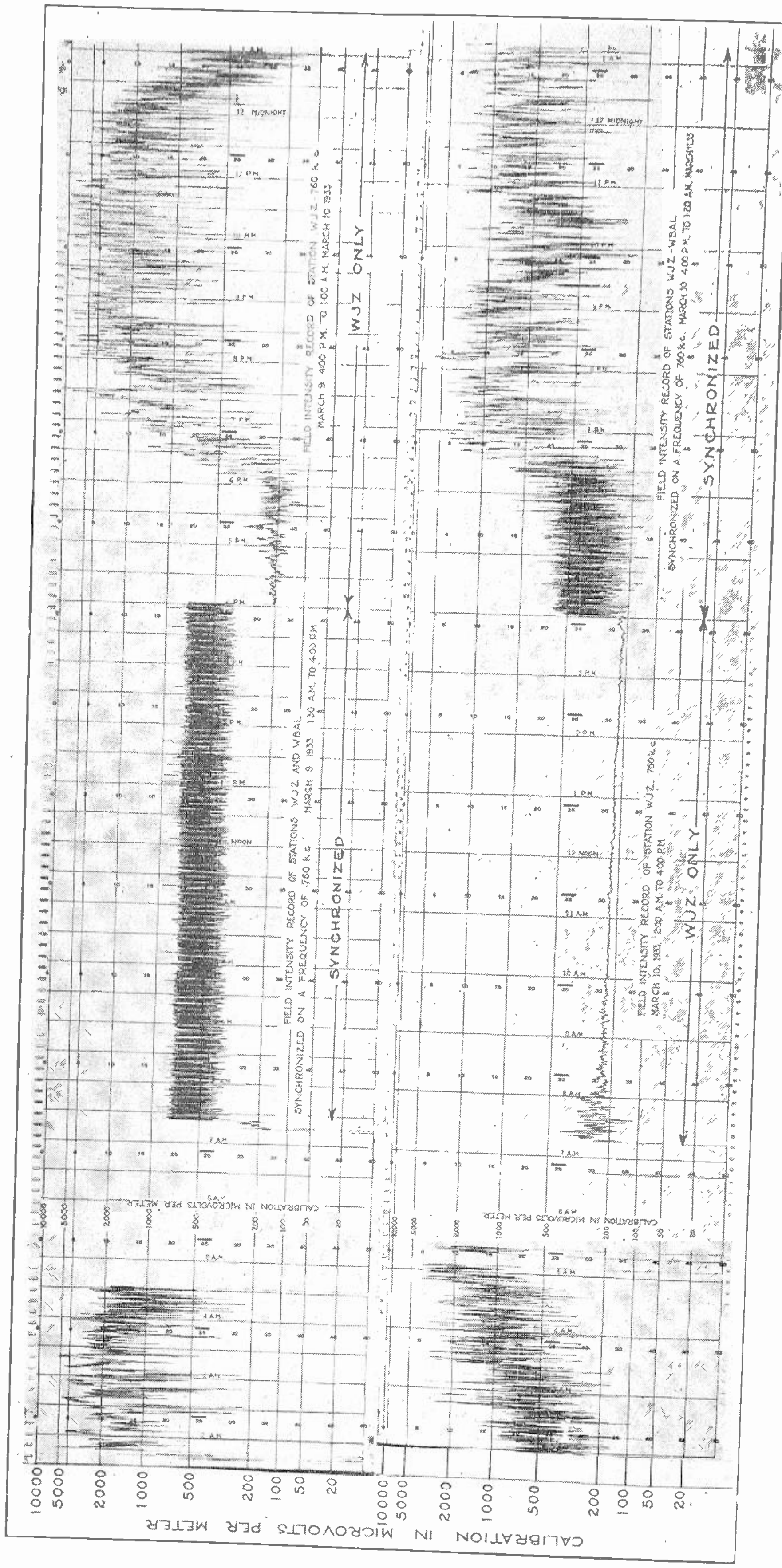
FIELD INTENSITY RECORD OF STATIONS WJZ AND WBAL
SYNCHRONIZED ON A FREQUENCY OF 760 k.c. MARCH 9 1933 1:30 A.M. TO 4:00 P.M.
SYNCHRONIZED
FIELD INTENSITY RECORD OF STATION WJZ 760 k.c.
MARCH 9 4:00 P.M. TO 10:0 A.M. MARCH 10 1933
WJZ ONLY
FIELD INTENSITY RECORD OF STATION WJZ 760 k.c.
MARCH 10, 1933, 2:00 A.M. TO 4:00 P.M.
WJZ ONLY
FIELD INTENSITY RECORD OF STATIONS WJZ-WBAL
SYNCHRONIZED ON A FREQUENCY OF 760 k.c. MARCH 10 4:00 P.M. TO 1:20 A.M. MARCH 11 33
SYNCHRONIZED
CALIBRATION IN MICROVOLTS PER METER
10000
5000
2000
1000
500
200
100
50
20

Fig. 1

ties of their two ground waves is about two to one or greater. The Federal Radio Commission states[2] that a ratio of at least four to one in the intensities of the radio waves from two synchronized stations is necessary in order to prevent modulation distortion; this latter ratio is based on the average receiver in use; e.g., a receiver without automatic volume control.

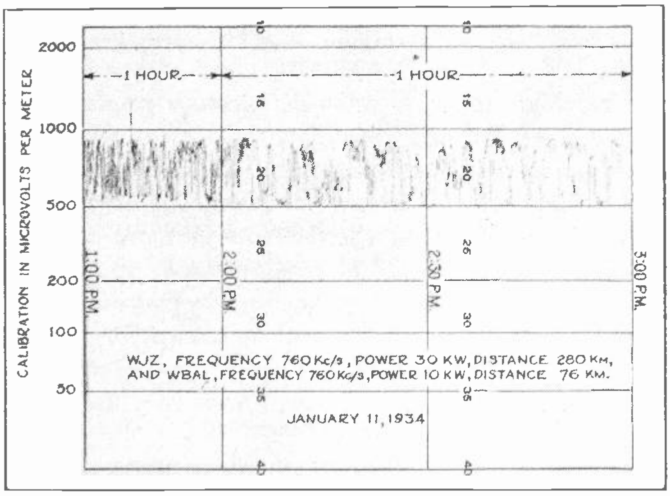

Fig. 2

In Fig. 1, the upper record is of WJZ alone after 4 P.M. and illustrates the type of fading observed for this frequency at this distance when the station operates alone. It may be seen that the peak field intensities reach 10 millivolts per meter. The fading here may be compared with the fading shown on the lower record for the same evening period when the two stations are synchronized.

[2] Seventh Annual Report of the Federal Radio Commission, page 20.

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A "SHORT-CUT" METHOD FOR CALCULATION OF HARMONIC DISTORTION IN WAVE MODULATION*

By

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Summary—*Precalculation of harmonic distortion produced by vacuum tubes used in all stages of a class B audio amplifier, for a great variety of operating conditions, is an important item in the work of the radio design engineer. A graphial method for this and similar calculations, which gives considerable saving of time has been developed. It is applicable to all symmetrical periodic curves containing harmonic components plotted to the sine of the fundamental frequency, sin ωt. The procedure consists in connecting the ends of such a curve by a straight line and measuring the ordinate differences between the curve and the chord for five definite values of abscissas. Simple expressions allow, then, for a rapid calculation of harmonic components up to 11th order. The method has proved to be very useful in class B modulator design.*

Elimination of distortion from radio signals is one of the important items in designing radio transmitters. In connection with this, the calculation of the amount of harmonics from theoretical or experimental modulation curves plays an essential rôle in the work of the radio designer. The significance of this problem is especially emphasized by the restrictions imposed on radio transmitters by the Federal Radio Commission, according to which the sum of all audio harmonics present in a modulated radiation must not exceed 10 per cent. On the other hand, a careful study of the shape of modulation waves becomes indispensable with the introduction of high efficiency plate modulators, which fall under the definition of class B audio amplifiers. In this case the limit of the load assigned to each individual tube in the modulator stage is essentially established by the allowable distortion.

High efficiency, high power plate modulators deserve particular attention because they allow for more economical utilization of vacuum tubes than do the old class A Heising plate modulators and more recent circuits employing class B radio amplifiers. Class B plate modulators were strongly advocated and first actually introduced into commercial practice by the transmitter engineers of the Westinghouse

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Electric and Manufacturing Company.[1] Practical realization of these modulators depends inherently on the proper design of large audio transformers covering the frequency range from 30 to 10,000 cycles. Two transformers of this type with the rating of 40 kilovolt-amperes were designed by J. F. Peters, consulting engineer with this company, and have been used for about two years with excellent results in KDKA broadcast transmitters. With this experience in hand, several other much larger transformers were quite recently designed for the new 500-kilowatt Crosley transmitter, now in the process of installation.

The general solution of the problem of measuring harmonic distortion is fundamentally given by the Fourier series for any periodic curve. But the numerical determination of the coefficients in a Fourier series by successive integrations is tedious and takes too much time, if many curves are to be analyzed. To make the task easier, this work has been ingeniously mechanized in the so-called "Harmonic Analyzer Machines."[2] The important objection to all integrating devices is that they are not always within easy reach of the designer.

The task of the radio designer can be simplified by considering the specific properties of the operating characteristic curves which are to be analyzed. Undoubtedly, any simplification of this problem is welcome, because often the designer has to explore the tube charts over the entire field of all feasible operating conditions for the tube, or for the complete transmitter, in order to choose the best from among a dozen, or even dozens of curves. This is the reason for the frequent appearance of methods for simplified harmonic analysis in current radio literature.

The following graphical method, which can be described as "Ordinate-Difference Harmonic Analysis" has been developed in connection with a comparative study of several existing types of water-cooled tubes used as class B audio amplifiers. It is felt that a great saving of time was accomplished as a result of its application on more than 150 curves. The basic features of this method are:

(1) The modulation curves need not be replotted in any way; they are used in their conventional form, familiar to every radio engineer, and taken directly from the transconductance chart of the tube.

(2) No planimetric measurement of curves, or calculation of their areas is required.

[1] J. A. Hutcheson, Proc. I.R.E., vol. 21, p. 953; July, (1933). For application of a similar scheme to receivers, see article by L. E. Barton, Proc. I.R.E., vol. 19, p. 1131; July, (1931).

[2] For example, L. W. Chubb, *Electric Journal*, p. 91, (1914).

(3) The method involves ordinate differences between the actual and the ideal undistorted output curves, instead of considering the total length of ordinates of the curve which is to be analyzed. This makes the calculation easier both numerically and graphically, and results in greater accuracy.

(4) From the measured values of the differences of five selected ordinates, the amplitudes of the odd harmonics up to the 11th order can be calculated within a few minutes.

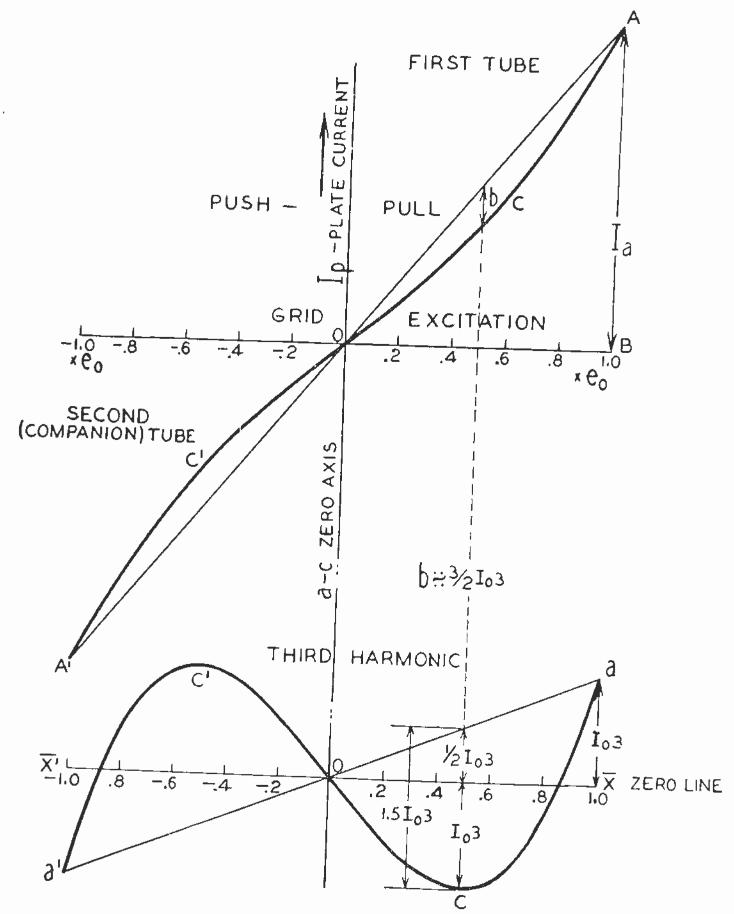

Fig. 1—Modulation characteristic of class B audio amplifier.

A conventional modulation tube characteristic, *OCA*, taken directly from the tube transconductance chart, is shown in Fig. 1. It represents one half of the symmetrical audio-output curve, *ACOC'A'*, for two companion tubes used in push-pull, the two tubes working through an audio transformer into a bank of oscillators, or class C radio amplifiers, the output of which is to be modulated.

The characteristic of Fig. 1 indicates the variation of the output current or the plate voltage proportional to it, as a function of the alternating input (or grid) voltage, having its alternating-current zero value near the cut-off or no-signal point at a given operating plate voltage. In the figure this point is designated by "O." The study of distortion must be made under the assumption that the input voltage is strictly sinusoidal in shape. A single tube, or the entire audio amplifier can be subjected to such a study.

The variable abscissas of the modulation curve are given by

$$x = e_g = e_0 \sin \omega t = e_0 \sin \alpha \tag{1}$$

or putting $e_0 = 1$, $x = e_g = \sin \omega t$ where e_g is the alternating grid voltage. Thus, the abscissas are proportional to *the sine of the time angle* of the fundamental frequency, $x = \sin \omega t$; *not to the angle itself*, nor to the time, as is the case with conventional methods of harmonic analysis. The horizontal projection of the modulation curve is proportional to the input voltage amplitude, e_0; the end ordinates of the curve give the amplitude of the output current, I_a.

For distortionless amplification it is required that the instantaneous values of the output current, I, be strictly proportional to those of the input voltage, e_g. In such a case the relation exists:

$$y = I = I_a \sin \omega t = I_a \sin \alpha \tag{2}$$

or,

$$y = I = I_a x.$$

This is the equation of a straight line. Hence, *in the absence of distortion the modulation characteristic must be a straight line.* If it is not, harmonics are present in the output wave. Thus, by simply connecting the ends of a modulation characteristic of a tube by a straight line (OA in Fig. 1) one can see at once whether the output wave is distorted or not. Naturally, the more a modulation curve departs from the straight line, the greater is the distortion. This departure is generally due to the combined effect of several harmonics present in the output wave. Our problem is to show how the amplitudes of these harmonics as well as the true amplitude of the fundamental frequency can be calculated directly from the shape of the modulation curve given by $y = f(x)$, plotted as a *function of the sine of the fundamental frequency*, $x = \sin \omega t$.

Before outlining the proposed method, we must note that in the condition of class B audio amplifiers, working always in push-pull, *only sine components of odd harmonics are possible.* Indeed, the symmetry of a complete modulation curve such as $ACOC'A'$ (Fig. 1) with respect to the operating or no-signal point, O, excludes cosine components of all even harmonics. Moreover, the "single-pathness" of modulation, or the identity of the modulation path on the upward and downward swing in each tube, which is witnessed by a single-line curve, rather than a loop, precludes the possibility of sine components of even and of cosine components of odd harmonics. This can be easily deduced from the Fourier series. But it can even better be seen directly from the graphs of these harmonics: The graphs of all cosine components of even harmonics lie entirely on one side of the chord spanning their ends; the other two kinds of excluded harmonics have loop-shaped graphs.

In order to explain the proposed method, let us first assume that distortion of the modulation curve shown in Fig. 1 is due exclusively to the presence of the third harmonic component:

$$I_3 = I_{03} \sin 3\omega t \tag{3}$$

the amplitude of which, I_{03}, is to be determined from the curve. We may imagine the total output wave decomposed into: (1) A fundamental frequency component with the amplitude, I_a, which corresponds to the straight line AOA', and (2) a distortion component pictured by the departure of the actual curve $ACOC'A'$ from the straight line, AOA'. The amplitude of the third harmonic, I_{03}, can easily be found from a direct comparison of the ordinate differences between the curved modulation line and its chord to the corresponding ordinates of the graph of the third harmonic. In Fig. 1 the line XOX' represents the abscissa axis on which $x = \sin \omega t$, the sine of fundamental frequency, is plotted; these abscissas are, therefore, identical with those of the modulation curve of the same Fig. 1. The curve, $acoc'a'$ in Fig. 1 is the graph of (3) with $x = \sin \omega t$ as independent variable. It can be plotted either directly by calculating I_3 for each value of ωt, or by developing (3) into the more obvious form:

$$I_3 = I_{03} \sin 3\omega t = I_{03}(3 \sin \omega t - 4 \sin^3 \omega t)$$

or,

$$I_3 = I_{03}(3x - 4x^3). \tag{4}$$

One can see that the I_3 curve has "overhanging" ends, and in this shape cannot be correlated with the actual modulation curve $ACOC'A'$ of Fig. 1, the ends of which are connected by a chord. Let us, therefore, also connect the ends of the harmonic graph by a chord, aoa'. Now we can measure the vertical distances between points of the graph and its chord and compare them to the ordinate differences on the actual modulation curve $ACOC'A'$. It is quite evident from a comparison of the two curves that in the assumed case of third harmonic only, its amplitude is equal to two thirds of the ordinate difference at the midpoint of the modulation curve for a single tube:

$$I_{03} = 2/3b.$$

It may be emphasized here that the inclination of the chord with respect to the X-axis does not play any rôle in this calculation as long as ordinate differences only are considered.

A similar train of arguments can be applied to the 5th, 7th and further harmonics. The graphs of all odd harmonics, up to the 11th, are given in Fig. 2. For the sake of simplification, only one half of each entirely symmetrical graph is drawn. Each graph is referred to a horizontal X-axis of its own with values of $x = \sin \omega t$ identically plotted on all of them. The chords connecting the ends of each curve are also shown, and the distances between the chords and the corresponding points of the curves for several specific values of x are indicated. These ordinates form the basis of the proposed harmonic calculation.

In a general case, some, or all of these harmonics can be present in the modulation curve. Then, the ordinate differences at any point are equal to the algebraic sum of the effect of all component harmonics. On the other hand, from the graphs of harmonics we know what relation each component bears to its amplitude, for any x. Hence, calculation of their respective amplitudes is an easy matter; one has to set up and solve as many equations as there are components to be considered. Each equation combines ordinates of all components for a definite value of $x = \sin \omega t$, which may be arbitrarily chosen. With graphs of Fig. 2 on hand, one can readily decide which points should be preferred for compiling the simplest equations. Generally, these will be the abscissas, for which the chord-to-curve distances are zero for some components, or maxima, but not necessarily so.

Regarding the highest order of harmonics, in which the radio designer may be interested, one can state that the modulation or audio amplification curves seldom contain harmonics higher than of the 9th or 11th order. Ordinarily, even the last two are but very little pronounced. However, very simple expressions for calculation of the amplitudes up to the 11th harmonic can be derived by measuring ordinate differences for

$$x = 0.309;\ 0.5;\ 0.707;\ 0.809;\ 0.866.$$

These are the sines of the respective angles of fundamental frequency: 18°; 30°; 45°; 54°; 60°.

Combining the values of the corresponding ordinates, one may first write the equations:

For

$$x = 0.309 \qquad a = 1.118 I_{03} + 0.691 I_{05} + 1.118 I_{07}$$

$$x = 0.5 \qquad b = 1.5 I_{03} - 1.5 I_{09}$$

$$x = 0.707 \qquad c = 1.414 I_{03} - 1.414 I_{05} + 1.414 I_{11}$$

$$x = 0.809 \qquad d = 1.118 I_{03} - 1.809 I_{05} + 1.118 I_{07}$$

$$x = 0.866 \qquad f = 0.866 I_{03} - 1.732 I_{05} + 1.732 I_{07} - 0.866 I_{09}$$

Here, the symbols a, b, c, d, and f designate the ordinate differences between the actual and the ideal modulation curves to be obtained directly from the modulation curve being studied.

Solving these equations with respect to five unknown harmonic amplitudes, one arrives at the following expressions:

$$\begin{aligned} I_{05} &= 0.4(a - d) \\ I_{03} &= 0.4475(a + d) + b/3 - 0.578f - 0.5I_{05} \\ I_{07} &= 0.4475(a + d) - I_{03} + 0.5I_{05} \\ I_{09} &= I_{03} - \tfrac{2}{3}b \\ I_{011} &= 0.707c - I_{03} + I_{05}. \end{aligned} \tag{6}$$

In using these expressions on actual modulation curves encountered in practice, one will usually avoid the writing of too many minus signs, if the ordinate differences a, b, c, d, and f are called positive for points on the modulation curve lying below the corresponding point on the chord, and vice versa. One may note that with this designation positive harmonics cross the zero axis in the direction opposite to the fundamental wave, as is shown in Fig. 2.

The true amplitude of the fundamental frequency is equal to the observed modulation amplitude I_a (Fig. 1) only in the case of no distortion. If distortion is present, the true amplitude will have, generally, a different value. It can be calculated from the expression:

$$I_{01} = I_a - I_{03} + I_{05} - I_{07} + I_{09} - I_{011}. \tag{7}$$

This can be seen directly from the graphs of harmonics shown in Fig. 2. Indeed, the ordinates of harmonic curves measured from their respective chords include: (1) The ordinates of the harmonics from the respective zero lines, and (2) the ordinates of the chords from the same axis. These chords, being straight lines, represent components of the fundamental frequency with amplitudes equal to their respective end ordinates. Therefore, if one determines the amplitude of a harmonic, by measuring the ordinate differences, one simultaneously records an additional component of fundamental frequency. The amplitude of this component must be, then, added algebraically to the apparent amplitude I_a, each with its proper sign. From the same Fig. 2 it is clear that the end ordinate of each chord is equal in magnitude to the amplitude of the corresponding harmonic, and that they alternate in sign.

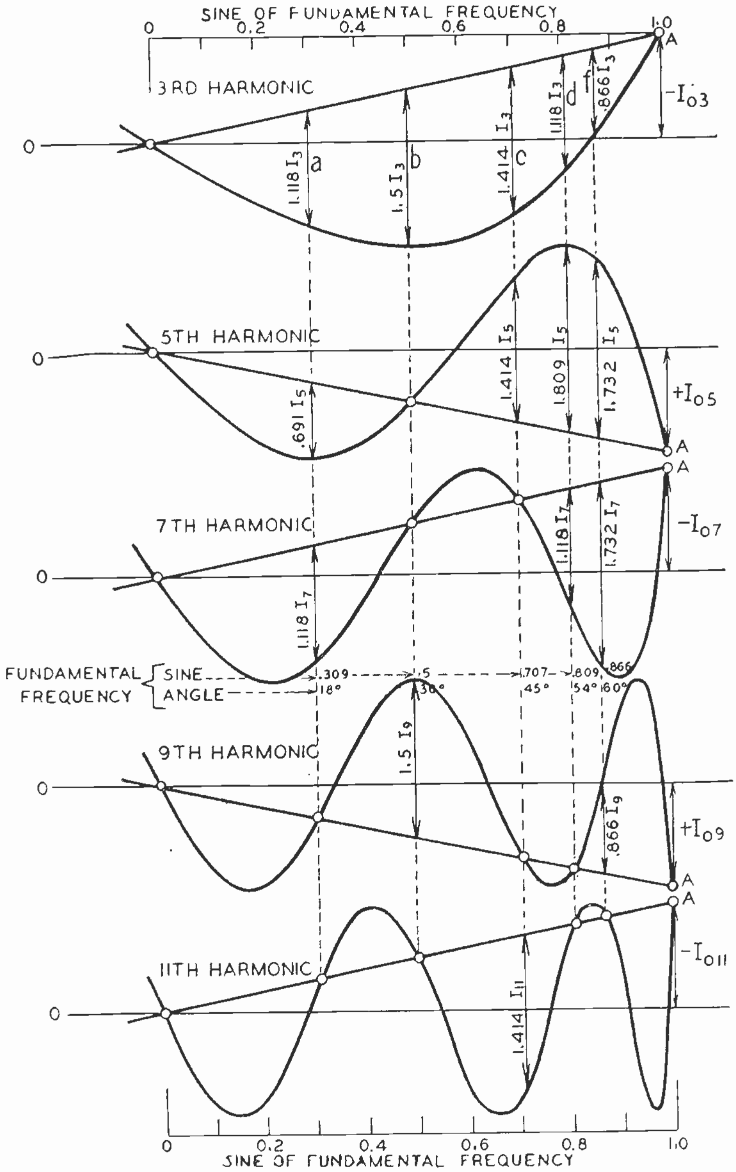

Fig. 2—Harmonic component relations.

Practical Example of Graphical Analysis of Modulation Curves

A modulation curve, *OCBA*, taken from a practical case is plotted in Fig. 3. It belongs to a UV-863 tube with the operating plate voltage, $E_p = 10{,}000$ volts and the load resistance, $R_L = 1680$ ohms.

Calculation of Distortion. (a) First, let us assume that during the active half cycle the grid of the tube swings from -100 to $+700$ volts, or that the crest voltage of the grid excitation is 800 volts. Applying the outlined method of calculation, one will connect the ends of the operating curve by a straight line, OA, and measure the ordinate differences for the values of grid excitation amounting to 0.309; 0.5; 0.707; 0.809; 0.866 of its amplitude e_0. One may note that all measure-

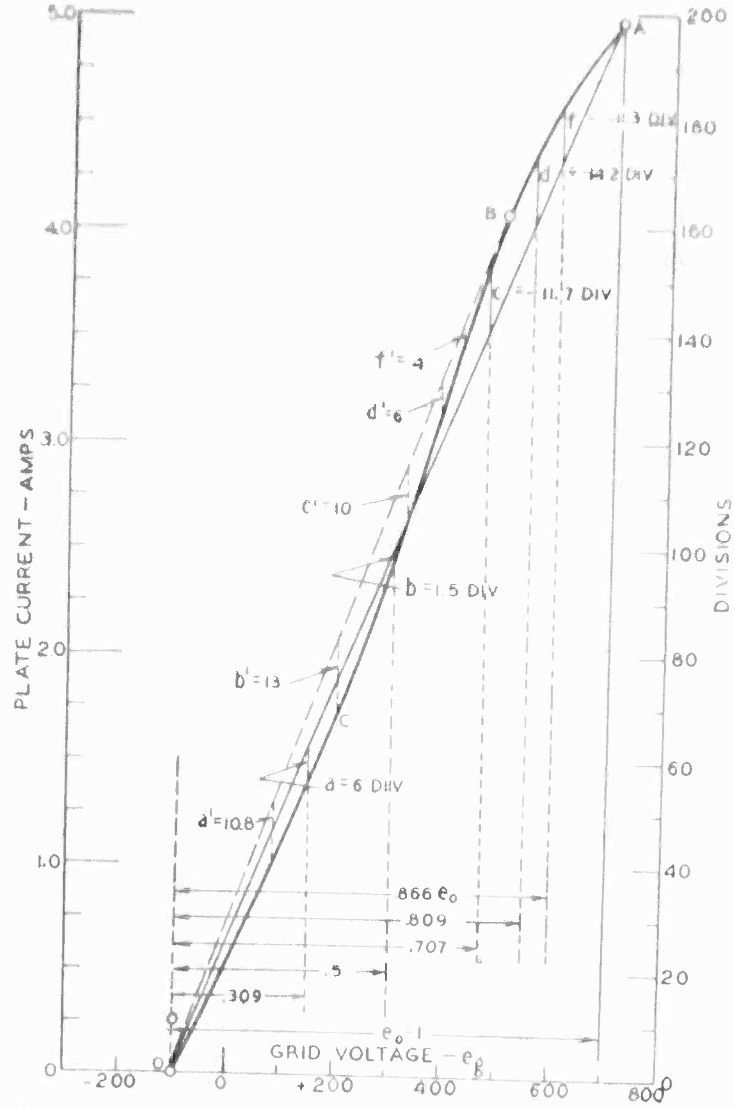

Fig. 3—Typical modulation curve; calculation of distortion.

ments and calculations can be easier carried out if numbered small divisions of cross-section paper (not reproduced in the drawing), instead of volts and amperes, are employed. This makes the calculation independent of the current and voltage scales.

The measured ordinate differences are:

$$a = 6;\ b = 1.5;\ c = 11.7;\ d = -14.2;\ f = -11.3.$$

By application of (6) one will find:

$I_{03} = -0.66$	0.32% distortion	
$I_{05} = 8.04$	3.96	"
$I_{07} = 1.02$	0.52	"
$I_{09} = -1.65$	0.81	"
$I_{011} = 0.43$	0.21	"
Total	5.82%	"

The true amplitude of fundamental frequency, I_{01}, is found by application of (7):

$$I_{01} = 198 - 0.66 + 8.04 - 1.02 - 1.65 - 0.43 = 203.6$$

Note: The true amplitude is here greater than the apparent amplitude, $I_a = 198$, due to the S-shaped curve revealing the presence of the 5th harmonic.

The ratio of the harmonic amplitudes to the fundamental gives their respective percentage of distortion and is indicated in the second column of the above Table.

(b) As another instance of calculation, let us limit the grid swing to +500 volts (point B on the curve). This reduces the alternating-current amplitude to 600 volts and corresponds to 75 per cent modulation, if case (a) is considered as 100 per cent modulation. The new ordinate differences between the same curve and the chord, OB, yield the values:

$$a = 10.8;\ b = 13;\ c = 10;\ d = 6;\ f = 4.$$

The calculated harmonic and fundamental amplitudes and distortion are:

$I_{03} = 8.58$	5.51% distortion	
$I_{05} = 1.92$	1.23	"
$I_{07} = -0.10$	0.06	"
$I_{09} = -0.12$	0.07	"
$I_{011} = 0.41$	0.26	"
$I_{01} = 155.41$	Total 7.13%	

This time, the true fundamental amplitude is smaller than the apparent amplitude, $I_a = 162.5$.

Calculation of Power Output. The power delivered by the modulator tubes is consumed in the load resistance, R_L, which can be considered as a pure ohmic resistance. Hence, each harmonic component delivers to R_L power of its own, and the total output is proportional to the sum of the squares of the fundamental and harmonic amplitudes. With total distortion not exceeding 10 per cent, the share of harmonics in total power output cannot be more than 1 per cent of the fundamental frequency output (usually it is much less). Therefore, it may be neglected, and only the fundamental frequency output is to be considered. This is expressed by

$$P_0 = 1/4 I_{01}^2 \cdot R_L \cdot s^2.$$

Here s is the number of amperes per division of cross-section paper, or the current scale. With this, the power output for the above two cases will be calculated as:

(a) $P_0 = (203.6 \times 0.025)^2 \times 1680 = 10.7$ kw

(b) $P_0 = (155.4 \times 0.025)^2 \times 1680 = 6.34$ kw.

Calculation of the Average Plate Current, I_{av}. In absence of distortion, one has

$$I_{av} = 1/\pi \times I_{01} \times s.$$

Each harmonic contributes to the value of I_{av} only by a single half cycle, because all the rest of half cycles, being alternately positive and negative, neutralize the effect of each other. Therefore, the influence of the harmonics is, generally, very small. Nevertheless, it can be considered by writing:

$$I_{av} = s/\pi \times (I_{01} - 1/3 I_{03} - 1/5 I_{05} - 1/7 I_{07} - 1/9 I_{09} - 1/11 I_{011})$$

For the two cases of the above example I_{av} is:

(a) $I_{av} = 1.63$ amperes

(b) $I_{av} = 1.21$ amperes.

Power Input, Plate Dissipation, and Plate Efficiency. Power input, P_i; plate dissipation, P_h; and efficiency, η, can be calculated from the familiar expressions:

$$P_i = I_{av} \times E_p$$

$$P_h = P_i - P_0$$

$$\eta = P_0/P_i.$$

For our two cases they are:

(a) $P_i = 16.3$ kw; $P_h = 5.6$ kw; $\eta = 65.6\%$

(b) $P_i = 12.1$ kw; $P_h = 5.8$ kw; $\eta = 52.4\%$

Conclusion. The outlined method allows for rapid calculation of distortion of modulation curves. It has been worked out in connection with class B audio amplifiers, but is not necessarily limited to this particular case of engineering practice. Whenever a curve, plotted against sine of the fundamental frequency, possesses the property of symmetry with respect to the zero and 90-degree phase, and it is known (from the general shape of the curve) that harmonics of higher than the 11th order can be neglected, the method is applicable without alteration. If desired, it can also be carried further and harmonics of higher than the 11th order considered, although the resulting equations will be more involved. Moreover, suitable formulas can be developed for other kinds of periodic curves; for instance, for curves containing, in addition to the odd, the even harmonics of lower orders.

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REPORT OF IONOSPHERE INVESTIGATIONS AT THE HUANCAYO MAGNETIC OBSERVATORY (PERU) DURING 1933*

By

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Summary—*Equipment for ionosphere investigation was placed in operation at the Huancayo Magnetic Observatory of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington during 1933. The equipment designed for multifrequency operation by manual methods is described. Three layers identified with the E, F_1, and F_2 layers at Washington, D. C., are found. The F_1 layer appears to be formed by a separation from a general F region during the morning rather than direct ionization of a separate layer. Two reflection components are found near the maximum ionization of the F_1 and F_2 layers, reaching critical values at different frequencies. This difference corresponds very closely to the separation calculated for the effect of magneto-ionic double refraction due to the earth's magnetic field. From these data further information of the actual ionization conditions is obtained. The maximum ionization of the F_1 layer is found to reach 2.5×10^5 electrons per cubic centimeter, while the density of heavier ions is not greater than about 10^7 per cubic centimeter. The maximum ionization of the F_2 layer reaches 8×10^5 to 9×10^5 electrons per cubic centimeter and on some occasions exceeds 1.1×10^6 electrons per cubic centimeter. A dip in the critical frequency of the F_2 layer usually occurs in the morning in the summer, which may be related to the appearance of the F_1 layer. If this dip in the diurnal critical frequency characteristic is due to absorption rather than ion limitation, the ionization may be higher than indicated during these periods. The F_2 critical frequency appears to be about 2000 to 3000 kilocycles higher at Huancayo than at Washington for the corresponding local summer season.*

I. Introduction

The study of the electrical and physical structure of the upper atmosphere presents a problem of importance to the physicist and engineer. Because of its close relation, not only to the phenomena of radio transmission but to many other fields of physics, investigations of the ionosphere are being carried on by a number of workers. These investigations are of particular interest to the magnetician because of the relation of the phenomena of this region in the atmosphere to magnetic changes. The suggestion of this relation was first published by Balfour-Stewart in 1882 and further considered by Schuster, Chapman, Gunn and others in some detail, in the study of the diurnal variation and of the irregular disturbances in the earth's magnetic field.

* Decimal classification: R113.61. Original manuscript received by the Institute, February 15, 1934.

The development of methods, equipment, and technique was first undertaken by Breit and Tuve[1] in Washington and by Appleton and Barnett[2] in London, independently and by different methods, in order to confirm the existence of an ionized region in the upper atmosphere and to study its characteristics.

This early work has been followed by a perfection of the experimental methods and a rapid extension of experimental and theoretical knowledge by a number of workers. A definite need for continual experimental work at various places on the earth, necessary to the expansion of the theory, however, has become apparent. The Department of Terrestrial Magnetism of the Carnegie Institution of Washington, therefore, has equipped its magnetic observatories at Huancayo in Peru (12° 02.′7 south, 75° 20.′4 west) and at Watheroo in Western Australia (30° 19.′1 south, 115° 52.′6 east) with apparatus necessary to make ionosphere measurements over an extended period. It is the purpose of this paper to report and discuss the early measurements made by H. W. Wells at the Huancayo Magnetic Observatory.

II. Methods and Equipment

All of the methods available for such studies depend upon the transmission of electromagnetic waves of radio frequency. These waves are reflected or refracted from the ionized regions. Because of the dispersive character of the ionosphere as a refractive medium for such waves, the depth of penetration of the transmitted wave into the ionosphere increases with frequency. The various levels of the ionosphere may then be studied, depending upon the character of ionization, by varying the frequency of the transmitted signal. The reflected or refracted waves bear certain "earmarks" from which the character of the ionized region may be determined.

The general method of observation is that of Breit and Tuve.[1] A short pulse of a few ten-thousandths of a second in duration is transmitted and simultaneously a deflection is shown on the oscillograph screen. The reflections, arriving later, are observed farther along the screen depending upon the angular velocity of a rotating mirror, so that the time retardation or virtual height and character of each reflection can be observed.

The equipment in use (see Figs. 1 and 2) at the two observatories of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington was planned in collaboration[3] with the Radio Sec-

[1] *Phys. Rev.*, vol. 28, pp. 554–575, (1926).
[2] *Proc. Roy. Soc.*, A, vol. 109, p. 621, (1925).
[3] S. S. Kirby, L. V. Berkner, and D. M. Stuart, *Bur. Stan. Jour. Res.*, vol. 12, pp. 15–51, (1934); Proc. I.R.E., vol. 22, pp. 481–521; April, (1934).

tion of the United States Bureau of Standards upon the design which they have used for manual measurements for a number of years. The detailed design and construction of the equipment were under the supervision of C. Huff of the instrument shop of the Department of

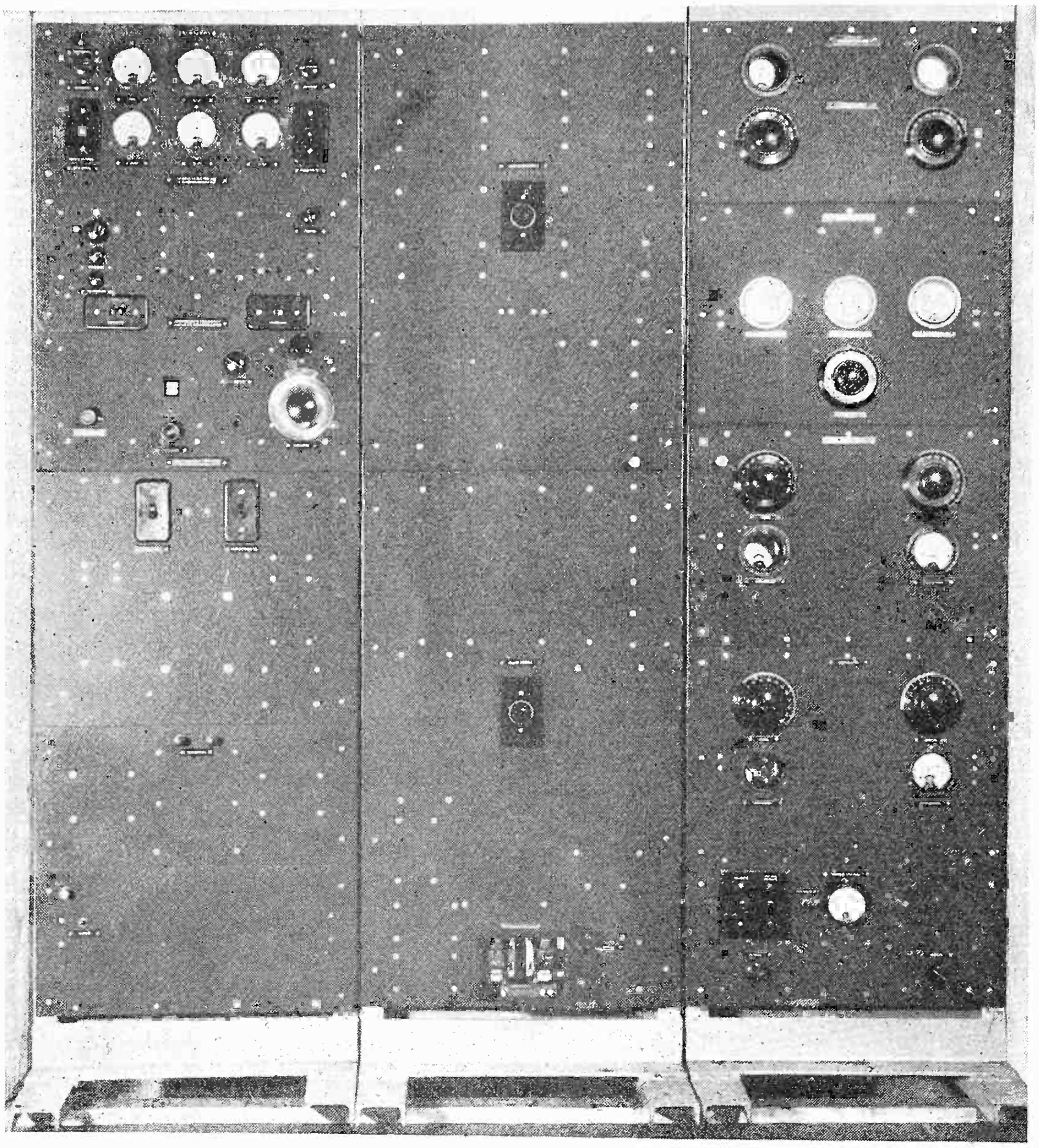

Fig. 1—Manually operated multifrequency radio receiver and transmitter at Huancayo Magnetic Observatory, Peru.

Terrestrial Magnetism and include features found desirable in the maintenance of such equipment operating at isolated observatories. It is assembled of individual units in panel form, mounted on relay racks for accessibility, and is arranged for easy modification with changing requirements such as may be encountered in problems of continuous recording.

This equipment consists of a radio transmitter, receiver, and direct measuring oscillograph. An engine generator unit furnishes power to the equipment. The transmitter consists of a lightly loaded oscillator, an intermediate, and a power amplifier, in which either the intermediate amplifier or oscillator can be pulsed. The multicircuit transmitter limits the frequency band of the emitted wave to the fundamental and such side bands as are necessary to the formation of a sufficiently short pulse. Pulse distortion and scattering due to frequency dispersion,

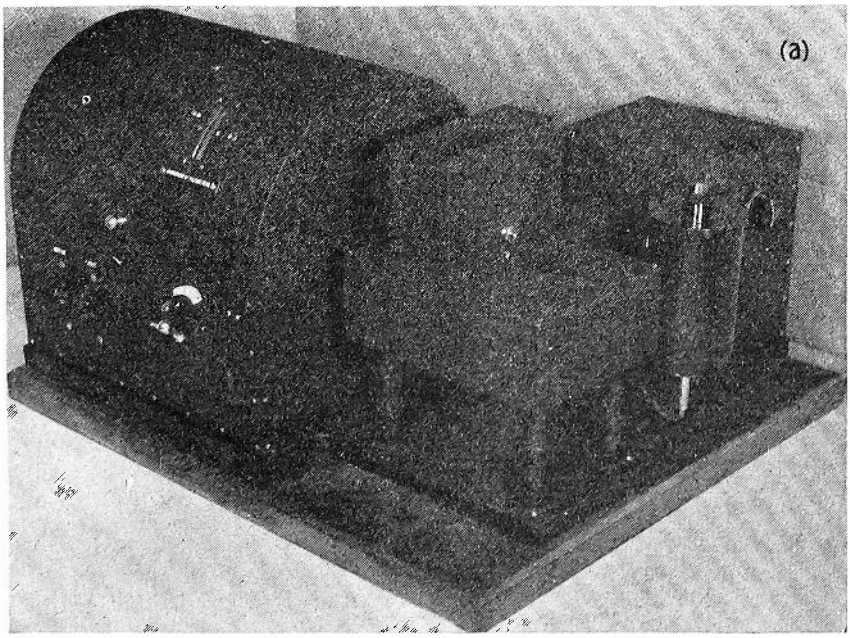

Fig. 2—(a) Direct measuring oscillograph for ionosphere virtual heights with screen for manual measurements (the screen may be replaced with special camera for continuous recording).

which is especially noticeable at critical frequencies, is thus largely avoided, and interference is reduced. The receiver is a superheterodyne with the intermediate amplifier tuned as sharply as is consistent with the necessary sharpness of the pulse. Excessive sharpness of tuning is avoided because of spurious lengthening of the pulse due to lack of proper circuit damping. A linear detector drives a direct-current amplifier which is arranged in the form of a bridge, of which the oscillograph galvanometer element is one leg. The galvanometer element is normally under an electrical bias, so that during deflection it is accelerated electrically, during both its rise and fall. With this arrangement, the element executes a forced oscillation of a period determined by circuit constants of the receiver. The oscillograph consists of a small synchronous motor driving a mirror which distributes the light beam from the galvanometer to the screen. The power for this unit is obtained from a constant frequency, tuning fork controlled source, which also pulses the transmitter. As a result, the reflection pattern is stationary on the screen. A micrometer screw calibrated directly in kilometers meshes a helical gear which rotates the frame of the motor so that each reflection is measured directly as it is brought to a reference line. The addition of a camera with a film moving slowly along the axis of the mirror allows the continuous recording of virtual height.[4]

The transmitter and receiver are located within easy reach of the observer, so that measurements can be made rapidly on a succession

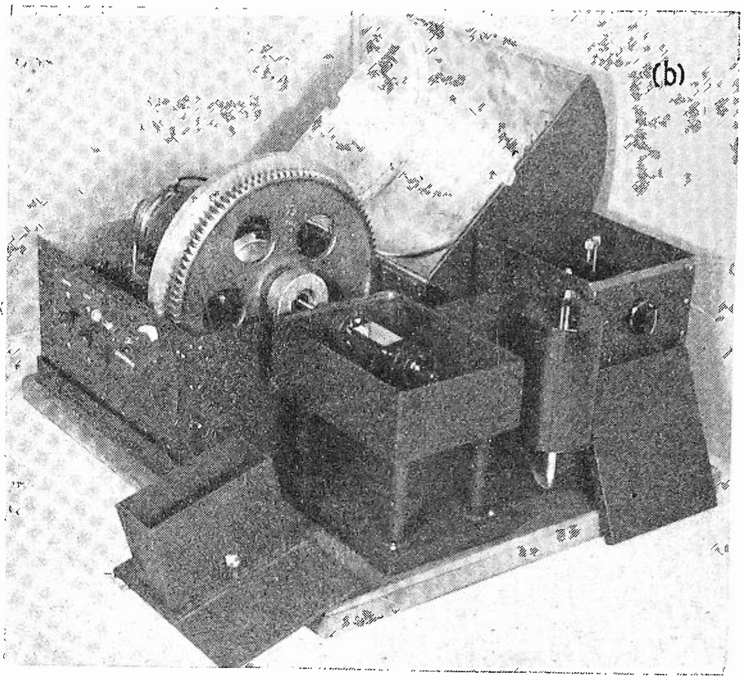

Fig. 2—(b) Same with covers opened and screen removed.

of frequencies. In this way certain features of the ionosphere can be observed within a very short time interval. Under these conditions the waves reach the ionosphere at normal incidence so that the ionization necessary to cause refraction or reflection can be computed.

III. Underlying Theory

Because of the recent accumulation of data by observers located in the temperate zones, particularly at Washington and at London, and the accompanying development of the theory, it appeared profitable to direct early experiments at the Huancayo Magnetic Observatory

[4] T. R. Gilliland and G. W. Kenrick, *Bur. Stan. Jour. Res.*, vol. 7, pp. 783–789, (1931).

along certain definite lines, namely, (1) to examine the general characteristics of the ionosphere near the equator with a view to comparing the layer conditions with those found in the temperate zones in order to determine the world-wide characteristics, and (2) to study the relation of the earth's magnetic field to magneto-ionic double refraction because of the important conclusions resulting from such experiments.

The earth's field at the Huancayo Magnetic Observatory is approximately 0.296 gauss at the surface as compared to about 0.571 gauss at Washington and to 0.468 gauss at London at which two places most of the previous studies have been made. Therefore, before considering the experimental data, it appears desirable to discuss briefly some of the theoretical aspects of the problem leading to this program.

It has been shown that an electromagnetic wave traveling in an ionized medium will be bent because of a reduction of the refractive index corresponding to an increase in phase velocity. When dissipation is neglected, this may be expressed by[5,6] (see appendix)

$$\mu = \sqrt{1 - Ne^2/[\pi(f'')^2 m + a]} \qquad (1)$$

where μ = refractive index, N = number of ions of charge e and mass m per cm^3, and f'' = frequency.

The constant a was introduced by Lorentz to take care of local polarization-effects in the medium and its value lies between 0 and 1/3. Because of the uncertainty[7,8,9] of its value, a matter still somewhat controversial, it is here neglected. Its inclusion as 1/3 would only affect absolute ionization estimates by a constant value of 50 per cent.

Because of the much greater mass of the lightest singly charged molecules, about 10^4 times as many of such ions as electrons would be necessary to cause effects similar in certain respects. The effects of refraction, therefore, are usually attributed to electrons.

At vertical incidence the ray must be bent through 90 degrees to be returned and according to Snell's law $\mu = \sin i/\sin r = 0$ for this condition. Simultaneous measurements at normal and slightly greater angles of incidence have shown such a treatment to be valid. Considering that N is any arbitrary value for a given layer, at sufficiently low frequencies $\mu = 0$ for this layer and the waves are returned. As the frequency is increased a frequency will be reached such that $\mu > 0$ and the

[5] P. O. Pedersen, "Propagation of Radio Waves," chap. VI, (1927).
[6] E. V. Appleton, *Jour. I.E.E.* (London), vol. 71, pp. 642–650, (1932).
[7] D. R. Hartree, *Cambridge Proc. Phil. Soc.*, vol. 27, pp. 143–162, (1931); also *Nature*, vol. 132, pp. 929–930, (1933).
[8] L. Tonks, *Nature*, vol. 132, p. 101, (1933).
[9] K. A. Norton, *Nature*, vol. 132, p. 676, (1933); also C. G. Darwin, *Nature*, vol. 133, p. 62, (1934).

wave will penetrate this layer to reach a higher layer with higher ionization or will completely penetrate the ionosphere.

At the frequency at which the wave just penetrates μ is very nearly zero for a considerable portion of the path and it has been shown that, as this frequency is approached, the group velocity is greatly reduced and the reflections are subject to considerable delays.[10] (For a dispersive medium of this type the group velocity $w \doteq \mu c$ where w is group velocity.) The frequency at which these long retardations occur is termed the critical frequency of the particular layer and is taken as a measure of the maximum ionization of the layer according to

$$N = [\pi m(f_c'')^2]/e^2 \tag{2}$$

where f_c'' is the critical frequency. For purposes of observation the critical frequency is defined as the frequency at which the change of virtual height with respect to increasing frequency becomes a maximum.

Early investigators recognized that the earth's magnetic field must also affect the velocity of propagation. A detailed discussion of this effect has been given by Nichols and Schelleng[11] and by Taylor and Hulburt.[12] In 1927 the general equations for the velocity of propagation at any angle to the magnetic field were developed independently by Breit[13] and by Appleton.[6]

Breit showed that

$$u(pu^2 + qu + r) = 0$$

where $u = V^2/c^2$, $p = (1+\gamma)[(1+\alpha)^2 - \beta^2]$, $\alpha = 4\pi Ne^2/m[\omega_H^2 - \omega^2]$, $\beta = 4\pi Ne^2\omega_H/m\omega[\omega_H^2 - \omega^2]$, and $\gamma = -4\pi Ne^2/m\omega^2$. (It is to be noted that Breit's symbol ω' is replaced here by the symbol ω_H which is more generally used in this connection.) Writing $\mu = \sqrt{1/u}$ we have a quadratic in which p must be zero for $\mu = 0$. Then

$$(1 + \gamma)[(1 + \alpha)^2 - \beta^2] = 0$$

which is fulfilled by

$$(1 + \gamma) = 0 \tag{3}$$

and by

$$(1 + \alpha) = \pm \beta. \tag{4}$$

[10] T. R. Gilliland, G. W. Kenrick, and K. A. Norton, Proc. I.R.E., vol. 20, pp. 286–309; February, (1932).

[11] *Bell Sys. Tech. Jour.*, vol. 4, pp. 215–234, (1925).

[12] *Phys. Rev.*, vol. 27, pp. 189–215, (1926).

[13] Proc. I.R.E., vol. 15, pp. 709–723; August, (1927).

Substituting $f_0^2 = 4\pi Ne^2/(2\pi)^2 m = Ne^2/\pi m$, $f_H = \omega_H/2\pi = He/2\pi mc$, and $f_H/f' = s$, where f' is the value of f in (4), we have the necessary conditions fulfilled by two values of f

$$f = f_0 = f'' \tag{3a}$$

$$f = f_0/\sqrt{1 \pm s} = f' \tag{4a}$$

where f'' is the frequency of the ordinary ray (shown to be unaffected by the earth's magnetic field) returned from an ionization N, and f' is the frequency of the extraordinary ray (whose velocity is affected by the earth's magnetic field) which will be returned from the identical ionization N.

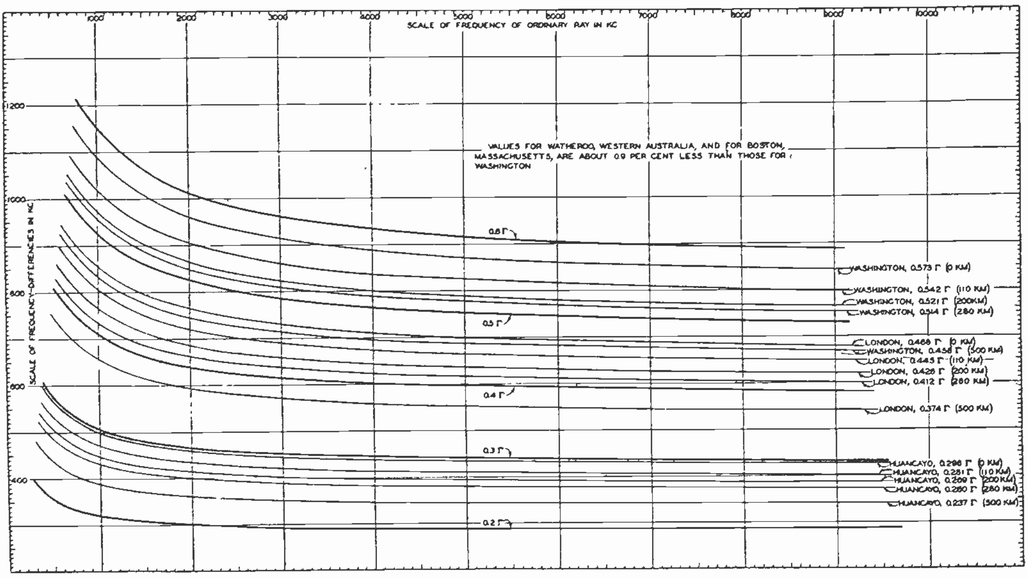

Fig. 3—Calculated frequency difference, neglecting dissipation and heavy ions of the ordinary ray and the extraordinary ray returned from same electron density, and therefore the same point in the ionosphere, the ionization being given in terms of the frequency for ordinary ray. [Values of earth's total magnetic field are given in Γ = c.g.s. unit (electromagnetic).]

It is apparent that the extraordinary ray can have only one polarization at a given height. It therefore will be returned upon reaching the first value of ionization for $\mu = 0$ given by $f' = f_0/\sqrt{1 - s}$, and corresponding to the root giving the largest reduction in refractive index. The root given by the positive sign will not be reached and therefore is not found physically.[3] When the frequency is such that the extraordinary ray just penetrates a layer, its virtual height will be greater than the virtual height for the ordinary ray because of the larger influence of the lower layer on the group retardation of the extraordinary ray. This is illustrated in Fig. 4 between 4600 and 5000 kilocycles. As the frequency is increased, the influence of the lower layer upon both rays becomes negligible, and if the ion gradient is sharp the rays will be returned superimposed. If the gradient is low, the ordinary ray will give the greatest virtual height due to its greater penetration. This is shown in Fig. 4 above 5000 kilocycles. From this it can be seen that the two rays will be returned from a given point in the layer at different frequencies, the frequency difference being given by

$$f' - f'' = f_0\left[(1/\sqrt{1 - s}) - 1\right].$$

This relation is plotted in Fig. 3, using the value of e/m for the electron. As an approximation, the total intensity of the earth's magnetic field is taken to decrease as the cube of the distance from the center of the earth, and on this basis the change due to height has been calculated. It is readily seen that for any singly charged molecule the mass is so great as to render f_H almost entirely negligible, and the two rays will always be superimposed giving effectively only one ray. For large values of f the value of $(f' - f'')$ rapidly approaches constancy, namely,

$$f' - f'' = f_H/2 \tag{5}$$

with an error of only about 8 per cent when $f'' = 3f_H$.

At the maximum ionization of a layer, two critical frequencies will be found, one corresponding to each velocity, and separated in frequency according to the relation given, this separation approaching a constant value at high frequencies for a given height.

IV. Experimental Data

The experimental data thus far obtained at Huancayo are entirely confined to daytime, and hence the conclusions in this paper relate to daytime only.

Investigations below 3500 kilocycles show that during the daytime the reflections are almost always returned from virtual heights of about 90 to 110 kilometers, identifying this layer with the E layer found in the temperate zones. So far not many determinations of the critical frequency for the E layer have been made. A few such determinations have not shown a sharp definition and this phase of the problem now awaits more detailed investigation.

A graph showing virtual-height—frequency curves for frequencies above 3500 kilocycles for four runs selected as typical near the September equinox for various daylight hours is shown in Fig. 4. Though there is a considerable variation in the details of such curves, certain general characteristics are at once apparent. The virtual heights appear to be quite stable through two frequency ranges (roughly 3500 to

3900 kilocycles or slightly higher, and 5000 to 6000 kilocycles or higher apparently representing definite layers). These are separated by a frequency range in which the virtual heights are quite variable with respect to frequency. A more detailed study of the variation of virtual heights for the latter frequency range for October 17, 1933, at various times of the day is shown in Fig. 5. Such studies show certain definite characteristics are associated with this range, namely:

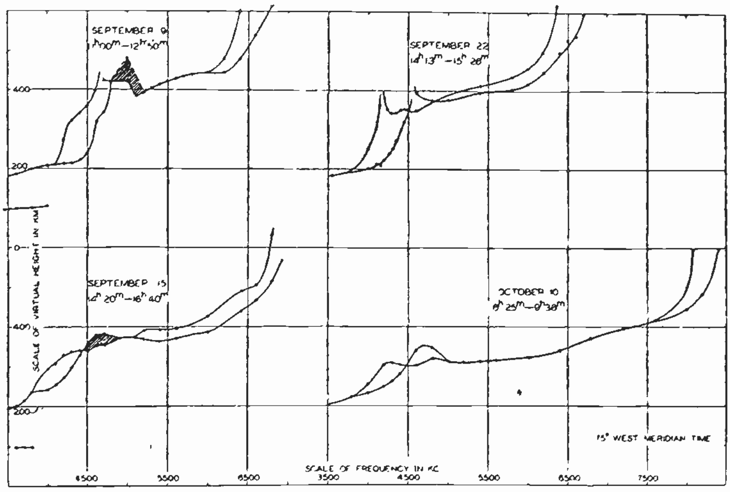

Fig. 4—Typical virtual height frequency runs above 3500 kilocycles near September equinox, Huancayo Magnetic Observatory, September and October, 1933.

(1) There is splitting of the reflection into two components.

(2) The rate of change of virtual height with respect to frequency is most marked near noon, the whole effect gradually appearing in the morning and disappearing in the afternoon.

(3) The two components appear to be subject to nearly the same virtual height fluctuations at different frequencies. When the rate of change of virtual height with frequency is large, the frequency separation of the two components at a given height is nearly constant.

(4) This splitting first appears at a frequency of about 3800 kilocycles in the morning during October, the frequency at which the splitting occurs increasing toward noon and decreasing in the afternoon, the effect finally disappearing between 3600 and 3800 kilocycles.

From these data it appears that the rapid changes in virtual heights in this frequency range represent the critical frequencies for a definite layer. The lowest virtual heights for this layer, during the period in which it exists, fall between limits of 180 and 210 kilometers. It therefore can be identified with the F_1 layer observed at Washington, D.C.[14,15]

The diurnal variation of this critical frequency taken from Fig. 5 is shown in Fig. 6 for both components. It is seen from these figures that in the morning the critical effect appears to build up out of the

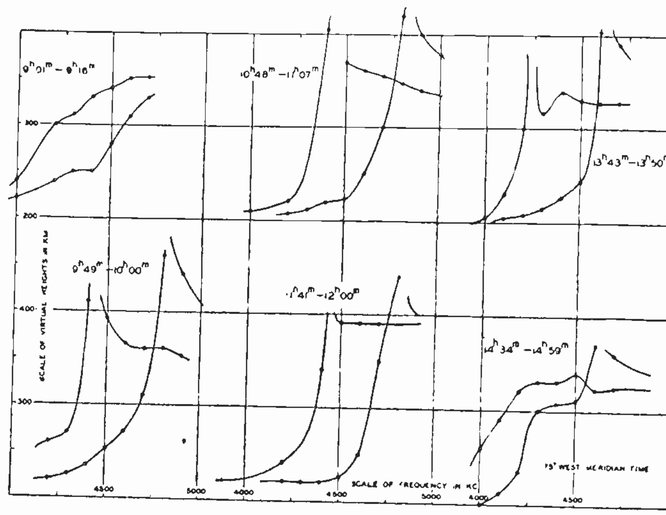

Fig. 5—Diurnal changes in character of virtual height variation at F_1 critical frequency, Huancayo Magnetic Observatory, October 17, 1933.

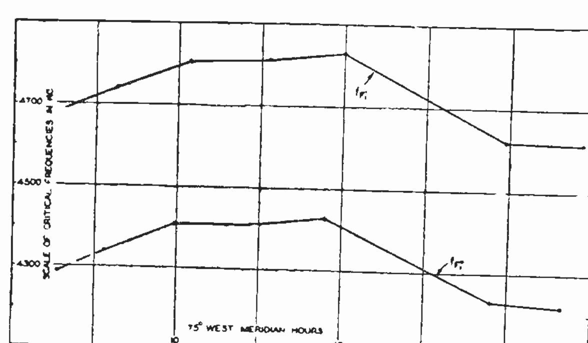

Fig. 6—Diurnal changes of F_1 critical frequency, Huancayo Magnetic Observatory, October 17, 1933.

F region. As the critical frequency becomes observable the accompanying retardations are noticed to increase somewhat. During the afternoon this critical effect gradually diminishes again, indicating that the F region is returning to a more evenly ionized condition. During this period the critical frequency decreases somewhat.

14 See footnote (3) and T. R. Gilliland, *Bur. Stan. Jour. Res.*, vol. 11, pp. 561–566, (1933).

15 S. S. Kirby, L. V. Berkner, T. R. Gilliland, and K. A. Norton, *Bur. Stan. Jour. Res.*, vol. 11, pp. 829–845, (1933).

The appearance and disappearance of this layer are interpreted as predominantly the effect of a separating of the general F region into two layers, an F_1 layer and an F_2 layer, at high angles of incidence of the sun's rays, and of a merging of these layers into one at low incidence of the sun's rays, rather than a simple matter of ionization and recombination in a separate layer. This effect of separation for the F_1 layer at Washington has been discussed in papers[3] recently published and it is found that for similar angles of incidence of the sun's rays the critical frequencies at the two locations are about the same.

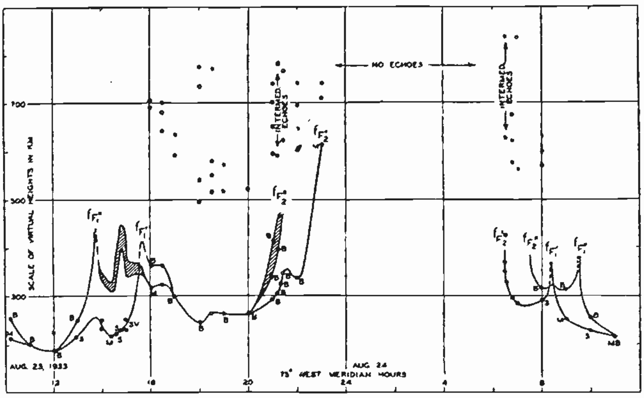

Fig. 7—Day's record of virtual heights on 4000 kilocycles showing effect of F_1 and F_2 critical frequencies as they vary through recorded frequency with changing ionization, Huancayo Magnetic Observatory, August 23–24, 1933.

Because of the change in the F_1 critical frequency after the effect appears, it is possible to observe the details of the effect by continuous observation at a frequency through which the critical frequency will pass during increasing or decreasing ionization. Such a study is shown in Fig. 7 for continuous observation on 4000 kilocycles. The rise in virtual height as the F_1 critical frequency becomes coincident with the recorded frequency for both components can be clearly observed. Such studies together with very careful height-frequency runs give justification for representing the discontinuities in Figs. 4 and 5.

The nature of the reflections in the next higher frequency range having fairly stable virtual height characteristics associate themselves with the F_2 layer reflections found in the temperate zones. Because of the large day-to-day variations of the F_2 critical frequency, any conclusions in this respect must await a lengthy series of data. The results of four days are shown in Figs. 8(a), 8(b), 8(c), and 8(d). The rapid variation of the F_2 critical frequency can best be illustrated by observation of the ordinary ray on a single frequency near the F_2 critical frequency such as shown in Fig. 9. It appears from measurements so far that the critical frequency usually shows a definite decrease

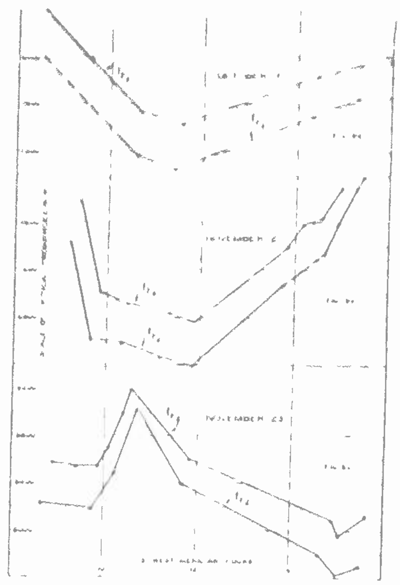

Fig. 8—(a), (b), and (c) Diurnal changes of F_2 critical frequency, Huancayo Magnetic Observatory, October and November, 1933.

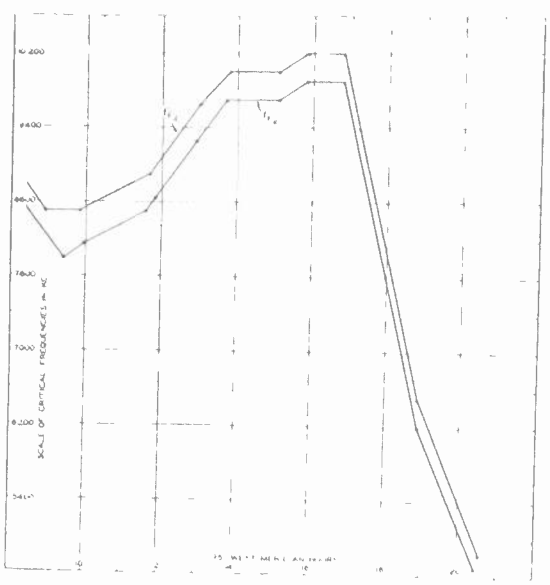

Fig. 8—(d) Diurnal changes of F_2 critical frequency, Huancayo Magnetic Observatory, December 18, 1933.

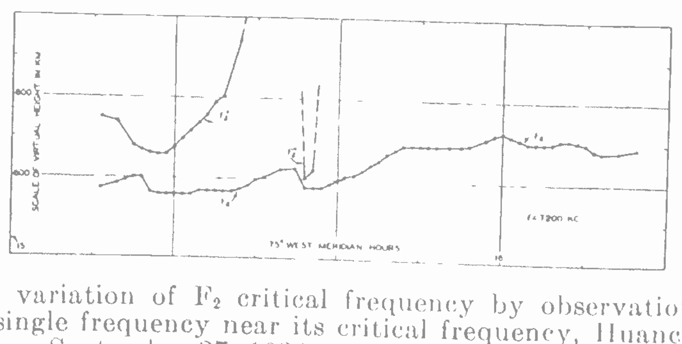

Fig. 9—Rapid variation of F_2 critical frequency by observation of ordinary ray on a single frequency near its critical frequency, Huancayo Magnetic Observatory, September 27, 1933.

before noon similar to the decrease found at Washington[3] during the summer.

V. Discussion of Experimental Data

The frequency separation for the two rays from the F_1 layer at their critical frequencies is of very great interest. The study of a large number of observations indicates that this frequency difference is very close to 390 kilocycles at 4100 kilocycles for the lower component. A precise determination of this difference is difficult without continuously recording, multifrequency equipment because of the change in critical frequency during the time required to complete the measurements, but from data such as illustrated in Fig. 5 quite accurate determinations can be made during periods in which the change is small. It is seen from Fig. 3 that this value corresponds to within the accuracy of the method to the separation calculated from the strength of the earth's magnetic field at a height of about 200 kilometers at Huancayo in this frequency range.

In the data obtained at Washington,[3] London,[16] and Deal (New Jersey),[17] it has been found that the frequency separation of these two rays for this layer (between 700 and 800 kilocycles) is very close to the calculated value based on the intensity of the earth's magnetic field at these locations as corrected for the assumed height of this layer.

The remarkable agreement of these data with the predicted results, together with the results of the polarization experiments of Appleton,[18] Green,[19] Ratcliffe,[20] and White,[21] leave no doubt that these two components are due to magneto-ionic double refraction caused by the earth's magnetic field. The calculations given in Section III show that the particles causing these effects must be electrons as the separation of the two components would be negligible for heavier ions.

It must be observed that, while the refraction of the ordinary ray might be due in part to the existence of a great predominance of heavier ions over electrons, this predominance would result in a lesser separation of these two frequencies than is observed. The separation when heavy ions are included is (see appendix).

$$f' - f'' = f_0\{\sqrt{[1/(1-s)] + r} - \sqrt{1+r}\} \tag{6}$$

[16] E. V. Appleton and G. Builder, *Proc. Phys. Soc.*, vol. 45, pp. 208–220, (1933).
[17] J. P. Schafer and W. M. Goodall, paper presented before Washington Section, URSI, April, 1933 (not published).
[18] *Proc. Roy. Soc.*, A, vol. 117, pp. 576–588, (1928).
[19] Radio Research Board (Melbourne), Report No. 2, (1932).
[20] *Phil. Mag.*, vol. 16, pp. 125–144, (1933).
[21] *Phil. Mag.*, vol. 16, pp. 423–440, (1933).

where r is the ratio $(N_1/m_1)/(N/m)$, in which N, N_1, m, and m_1 are the numbers and masses of the electrons and ions, respectively. Assuming, for purposes of calculation, that $f'=4500$ kilocycles and $f_H=750$ kilocycles, so that $s=1/6$, values as given in Table I are calculated.

TABLE I

r	f_0	$(f'-f'')$	f''
	kc	kc	kc
1.0	3020	211	4289
0.1	3945	363	4137
0.01	4090	389	4111
0.0	4106	394	4106

If the mass of the heavier ions is in the order of 10^4 times the mass of the electrons, then $r=1$ for $N_1=10^4N$. From the table above it can be seen that if N_1 is more than 10^2N, this reduction would be apparent in the observations. It is further shown by the table that for 10^2N ions the number of electrons calculated directly from $f''=f_0$ will not be seriously in error.

Before reaching any conclusions with regard to the limiting numbers of heavier ions in the F_1 layer, it is desirable to examine the initial assumptions. From Fig. 3 we see that any assumption with regard to the actual height of the layer will not materially affect the results. The height of the layer cannot be less than the height of the E layer which is fairly well established at about 100 kilometers because of the broad range of frequencies the reflections of which are returned from that height. Furthermore, the fact that the virtual height frequency curves for the F_1 layer are fairly constant over a band of several hundred kilocycles lends weight to the idea that the lowest virtual heights are not much affected by the E layer and that actual height through this band is not much less than the virtual height.

From the equations of Appleton[6] it is seen that if dissipation is not negligible, it may affect differently the frequency at which the two rays become critical when μ becomes very small. The discontinuities in the curves show that the absorption must become great under these conditions.

However, calculations by Epstein[22] indicate that the effect of dissipation is to reduce the frequency separation of the two components because of a greater reduction in the critical frequency of the extraordinary ray. This is in the same direction as the effect of heavier ions and therefore should be observable. In addition, changes due to dissipation should be a function of frequency. However, the observed

[22] *Proc. Nat. Acad. Sci.*, vol. 16, pp. 37–45 and 627–637, (1930).

frequency separations of the two components for the F_1 layer agree with the values computed through the observed range on the assumption that the effects of dissipation may be neglected. Furthermore, points at which the frequency separation of the two components can be measured, as indicated by the amplitudes returned, are not subject to greatly increased absorption.

It seems reasonable, under the conditions noted, to assume that dissipation does not affect the results. From these considerations it may be concluded that the maximum number of electrons in the F_1 layer is about 2.5×10^5 and that the maximum number of ions of mass about 10^4 times as great as the mass of an electron cannot greatly exceed 10^7.

The two components from the F_2 layer are often returned separately over a wide frequency range. This shows that the ion gradient of that portion of the F_2 layer not rendered "invisible" by the lower layers is low. The form of the graph plotting virtual height against frequency indicates a continually decreasing ion gradient with respect to height and leads to the conclusion that the F_2 layer is thick with a considerable vertical extent between the lower part of the layer and the height of maximum ionization. The virtual height of the lower part of the layer is ordinarily between 300 and 400 kilometers.

In considering the two components from the F_2 layer at the F_2 critical frequency, much less can be said about their frequency separation. The following factors must be considered:

(1) It is difficult to estimate the height of the maximum ionization for the F_2 layer on the same basis as for the F_1 layer because of the sloping characteristic of the graph of virtual height against frequency.

(2) At times one component is entirely missing.

(3) The continual rapid variation of the critical frequency of the F_2 layer, as illustrated in Fig. 9, makes the time interval during measurement of greater importance.

(4) It is not yet certain that the critical frequency observed by the rapid change in virtual height and by the disappearance of all echoes is a "real" critical frequency as defined for the lower layers[15] because of the possibility of reflectional limitation due to absorption.

An estimate of the frequency separation from a large number of observations leads to an average figure of about 340 kilocycles. If penetration actually occurs, this value of frequency separation, which is appreciably lower than the value determined from the F_1 layer, might arise from one or more of a number of causes: (*a*) An actual height of the maximum ionization for this layer of about 400–500 kilometers due to the appreciable decrease in the earth's total magnetic field at these heights, as shown in Fig. 3; (*b*) a much lower actual height of the maximum ionization, with the layer containing from 100 to 1000 times as many heavier ions as there are electrons; or (*c*) the effect of dissipation upon the wave near the critical frequency in this layer due to low ion gradients. A study of these factors together with the form of reflection splitting leads to the opinion that the cause (*a*) plays perhaps the most important part in this effect.

From the maximum values of F_2 critical frequency obtained, it may be estimated that the ionization reaches a value of at least 8×10^5 and 9×10^5 electrons during the morning and evening. If the midday dip in critical frequency is due to absorption rather than ion limitation, as has been suggested, the ionization of the F_2 layer may be very much higher than indicated. This is suggested by the F_2 critical frequency characteristic for November 23, 1933, shown in Fig. 8 in which the ionization reached 1.1×10^6 electrons per cubic centimeter during a period when, for some reason, the usual dip did not occur. The data also show that the F_2 critical frequency for summer at Huancayo is somewhat higher (from 2000 to 3000 kilocycles higher than observed) at Washington[3] during the summer.

With regard to this perhaps unexpected and peculiar dip in the F_2 critical frequency, as illustrated in Fig. 8, one point of importance should be considered. A study of results obtained at Huancayo and at Washington[3] shows that this effect is associated with angles of incidence of the sun's rays near the normal. At the same time, the formation of the F_1 layer from the F_2 layer takes place only under the same conditions. From the results obtained at Washington it can be seen that during the winter midday, when no F_1 layer is apparent, the dip in F_2 critical frequency does not occur.

It has been noticed on a few occasions that when the F_1 layer is indistinct, the midday dip in F_2 layer critical frequency is less marked. It seems possible from these somewhat meager data that there may be a direct relation between dip in the F_2 critical frequency and the appearance of the F_1 layer. Further discussion in this regard must await the collection of additional data. Because of the relation to terrestrial magnetism of such layer movements as seem to be indicated, further investigation of this possibility may be expected to forward our understanding of certain magnetic phenomena, particularly concerning the drift current theory of the diurnal variations of the earth's magnetic field as suggested by Chapman.[23]

The fact that, at least during certain seasons of the year, the F_2 critical frequency usually reaches definite maxima in the morning and in the afternoon is especially worthy of attention. Such a marked maximum as shown in Fig. 8(d) near sunset, where the ratio of values of $(f_{F2}'')^2$ at late afternoon to those at noon exceeds 1.4 to 1, does not at once seem consistent with a maximum ionization varying with direct ultra-violet radiation from the sun as is the case for the lower layers. It would seem that some more complex phenomenon is occurring, perhaps such as has been discussed by Kirby, Berkner, and Stuart[3] at Washington. Further, it is difficult to account for such rapid changes in F_2 critical frequency as are shown to occur, for example as in Fig. 8(d), on the basis of recombination alone. The occurrence of such perhaps unexpected phenomena offers an additional possibility for a more complete understanding of this layer when additional data become available.

VI. Summary

(1) Three major daytime ionosphere layers are usually found at Huancayo which can be identified as the E, F_1, and F_2 layers found in the temperate zones.

(2) The virtual heights of the lower limits of these layers ordinarily fall between 90 and 110 kilometers for the E layer, 180 and 210 kilometers for the F_1 layer, and 300 and 400 kilometers for the F_2 layer on the basis of observations so far made.

(3) The F_1 layer appears to separate from a general F region as the incidence of the sun's rays increases in the morning and to merge with the F_2 layer to form a general F region as the incidence of the sun's rays decreases in the afternoon.

(4) At the maximum ionization of the F_1 layer, two separate reflections are found to reach critical values for separate frequencies. These two frequencies are separated by an amount corresponding to the separation predicted from the theory of magneto-ionic double refraction due to the earth's magnetic field.

(5) From this information the upper limit of the ratio of heavier ions to electrons possible in the F_1 layer is calculated; the maximum number of electrons is about 2.5×10^5 per cubic centimeter (neglecting local polarization of the medium) and the heavier ions are not in excess of about 10^7 per cubic centimeter.

(6) The character of the graphs, when virtual height is plotted

[23] *Proc. Roy. Soc.*, A, vol. 122, pp. 369–386, (1929).

against frequency for the F_2 layer, indicates a thick layer with a lower ion gradient with respect to height than for the lower layers, the ion gradient decreasing with height.

(7) The maximum electron density reached in the F_2 layer is not less than 8×10^5 to 9×10^5 per cubic centimeter and on certain occasions reaches values above 10^6 per cubic centimeter; if absorption rather than ion limitation causes disappearance of F_2 layer reflections, the maximum may even be higher.

(8) Two components are usually found at the critical frequency for the F_2 layer but a number of undetermined factors prevent a detailed study of this effect.

(9) Some relation appears to exist between the appearance of the F_1 layer and the dip in the F_2 critical frequency, but this is not certain because of insufficient data.

(10) The F_2 critical frequency at Huancayo during the summer is from 2000 to 3000 kilocycles higher than at Washington during the summer.

ACKNOWLEDGMENT

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VII. APPENDIX

The general solution of the equations of motion of a wave propagated in an ionized medium for any angle with respect to the magnetic field was first given by Breit.[13] In order to determine the effect of heavier ions on the experimental results, these equations have been recalculated, including the displacement currents due to the heavier ions. The method of Breit's solution has been followed in detail. The effect of local polarization of the medium and dissipation has been neglected.

Assuming the direction of the earth's magnetic field H (it is to be noted that here H is used to denote the earth's total magnetic field and not in its general usage among magneticians to denote the horizontal component) to be coincident with the axis of z, the equations of motion of a particle of charge e and mass m, acted upon by an electric force E, become

$$\begin{aligned} m\ddot{x} &= e[E_x + (1/c)\dot{y}H] \\ m\ddot{y} &= e[E_y - (1/c)\dot{x}H] \\ m\ddot{z} &= eE_z. \end{aligned} \tag{1}$$

If N is the number of particles of charge e and $\mathcal{H}$ is the varying magnetic field associated with the electromagnetic wave, then

$$\text{curl } \mathcal{H} = (1/c)[\delta E/\delta t + 4\pi Ne(\dot{x}, \dot{y}, \dot{z})] \tag{2a}$$

$$\text{curl } E = -(1/c)(\delta \mathcal{H}/\delta t) \tag{2b}$$

where $4\pi Ne(\dot{x}, \dot{y}, \dot{z})$ is the current in the medium due to the presence of ions or electrons. Assuming the time variation of the field to be proportional to $\epsilon^{i\omega t}$ and solving (1)

$$\begin{aligned} -\omega^2 x - i\omega y(He/mc) &= eE_x/m \\ -\omega^2 y + i\omega x(He/mc) &= eE_y/m \\ -\omega^2 z = eE_z/m. \end{aligned} \tag{3}$$

Setting $(He/mc) = \omega_H$ and solving (3) for x, y, and z

$$\begin{aligned} x &= [(-E_x\omega + iE_y\omega_H)/(\omega^2 - \omega_H{}^2)](e/m\omega) \\ y &= [(-iE_x\omega_H - E_y\omega)/(\omega^2 - \omega_H{}^2)](e/m\omega) \\ z &= -eE_z/m\omega^2. \end{aligned} \tag{4}$$

Differentiating to obtain the particle velocities

$$\begin{aligned} \dot{x} &= (e/m)[(-iE_x\omega - E_y\omega_H)/(\omega^2 - \omega_H{}^2)] \\ \dot{y} &= (e/m)[(+E_x\omega_H - iE_y\omega)/(\omega^2 - \omega_H{}^2)] \\ \dot{z} &= (e/m)[-iE_z/\omega]. \end{aligned}$$

For singly charged ions, the mass will be of the order of 10^4 times the mass of the electron and $\omega_H \rightarrow 0$ so that factors containing ω_H in $(\dot{x}, \dot{y}, \dot{z})$ can be neglected for heavy ions. Letting N be number of electrons of mass m, and N_1 the number of ions of mass m_1 substitution in 2(a) gives

$$\begin{aligned} (\delta\mathcal{H}_z/\delta_y - \delta\mathcal{H}_y/\delta_z) = 1/c\{ & i\omega E_x \\ & + (4\pi Ne^2/m)[(-iE_x\omega - E_y\omega_H)/(\omega^2 - \omega_H{}^2)] \\ & + (4\pi N_1 e^2/m_1)(-iE_x\omega/\omega^2)\} \end{aligned}$$

$$(\delta \mathcal{H}_x/\delta z - \delta \mathcal{H}_z/\delta x) = 1/c\{i\omega E_y + (4\pi N e^2/m)[(E_x\omega_H - iE_y\omega)/(\omega^2 - \omega_H^2)] + (4\pi N_1 e^2/m_1)(-iE_y\omega/\omega^2)\}$$

$$(\delta \mathcal{H}_y/\delta x - \delta \mathcal{H}_x/\delta y) = 1/c[i\omega E_z + (4\pi N e^2/m)(-iE_z\omega/\omega^2) + (4\pi N_1 e^2/m_1)(-iE_z\omega/\omega^2)].$$

Substituting $(\delta/\delta x, \delta/\delta y, \delta/\delta z) = (-i\omega/V)(l, m, n)$, where l, m, and n are the direction cosines of normal to wave front and V is the phase velocity, and $f=\omega/2\pi$, $f_H=\omega_H/2\pi$, $f_0^2=Ne^2/\pi m$, and $F_0^2=N_1e^2/\pi m_1$, equations (2) become

$$\left.\begin{aligned}
(m\mathcal{H}_z - n\mathcal{H}_y) &= -(V/c)\{E_x - (f_0^2/f)[(E_xf - iE_yf_H)/(f^2 - f_H^2)] - (F^2{}_0/f^2)E_x\}\\
(n\mathcal{H}_x - l\mathcal{H}_z) &= -(V/c)\{E_y - (f_0^2/f)[(iE_xf_H + E_yf)/(f^2 - f_H^2)] - (F_0^2/f^2)E_y\}\\
(l\mathcal{H}_y - m\mathcal{H}_x) &= -(V/c)[E_z - (f_0^2/f^2)E_z - (F_0^2/f^2)E_z]\\
mE_z - nE_y &= (V/c)\mathcal{H}_x\\
nE_x - lE_y &= (V/c)\mathcal{H}_y\\
lE_y - mE_x &= (V/c)\mathcal{H}_z
\end{aligned}\right\}. \quad (5)$$

Solving equations (5) simultaneously for E_x, E_y, E_z and adding and subtracting l^2E_x, m^2E_y, n^2E_z from first terms of equations, respectively

$$\left.\begin{aligned}
-E_x + l(lE_x + mE_y + nE_z) &= -(V^2/c^2)\{E_x[1 - f_0^2/(f^2 - f_H^2) - F_0^2/f^2] + [iE_yf_0^2/(f^2 - f_H^2)](f_H/f)\}\\
-E_y + m(lE_x + mE_y + nE_z) &= -(V^2/c^2)\{E_y[1 - f_0^2/(f^2 - f_H^2) - F_0^2/f^2] - iE_x[f_0^2/(f^2 - f_H^2)](f_H/f)\}\\
-E_z + n(lE_x + mE_y + nE_z) &= -(V^2/c^2)[E_z(1 - f_0^2/f^2 - F_0^2/f^2)]
\end{aligned}\right\}. \quad (6)$$

For $\mu=0$, fulfilled by $V^2/c^2=\infty$

$$\left.\begin{aligned}
\{1 - [f_0^2/(f^2 - f_H^2)] - F_0^2/f^2\}E_x + \{f_Hf_0^2/[f(f^2 - f_H^2)]\}iE_y &= 0\\
\{1 - [f_0^2/(f^2 - f_H^2)] - F_0^2/f^2\}E_y - \{f_Hf_0^2/[f(f^2 - f_H^2)]\}iE_x &= 0\\
(1 - f_0^2/f^2 - F_0^2/f^2)E_z &= 0
\end{aligned}\right\}. \quad (7)$$

Solving simultaneously to eliminate E_x, E_y, E_z, for E_x, E_y, and $E_z \neq 0$

$$1 - f_0^2/f^2 - F_0^2/f^2 = 0 \quad (8)$$

$$1 - [f_0^2/(f^2 - f_H^2)] - F_0^2/f^2 = \pm [f_0^2/(f^2 - f_H^2)][f_H/f]. \quad (9)$$

Let r be the ratio $(N_1/m_1)/(N/m)$ of heavier ions to electrons so that $F_0^2 = rf_0^2$ and let the ratio of $f_H/f' = s$ where f' is the value of f in (9)

$$f^2 - f_0^2(1 + r) = 0 \quad (10)$$

and,

$$f^2(1 \mp s) - f_0^2[1 + r(1 \mp s)] = 0 \quad (11)$$

$$f = + f_0\sqrt{1 + r} = f'' \quad (12)$$

and,

$$f = + f_0\sqrt{[1/(1 \mp s)] + r} = f' \quad (13)$$

using positive (+) signs only, as frequency can only be positive.

These equations show the effect of heavier ions on the two rays due to magneto-ionic double refraction. For $r \ll 1$ (8) and (9) become

$$f'' = f_0 \quad (14)$$

$$f' = f_0\sqrt{1/(1 \mp s)} \quad (15)$$

in which the negative (−) sign only has significance as only the solution having the greatest reduction in refractive index can exist. This result is then identical with that of Breit shown earlier.

These equations show that at a given height in the layer two rays will be returned at different frequencies because of the earth's magnetic field, and that the frequency separation of these two frequencies will be independent of the angle of propagation with respect to the earth's field.

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MEASUREMENTS OF ELECTRICAL STATE OF UPPER STRATOSPHERE IN POLAR REGIONS (KENNELLY-HEAVISIDE LAYER)*

By

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Summary—*In this paper are described the results of wireless observations made in connection with the International Polar Year 1932–1933 at Moormansk (latitude 68°56′ N; longitude 33°05′ E) in the U.S.S.R. This work was done by the Leningrad Section of the Institute for Scientific Research of the People's Commissariat for Communication in common with the Central Geophysical Observatory under the direction of the author. A special system with two 150-watt tubes was designed in order to send out short pulses of 20-kilowatt peak power. This was accomplished by using a condenser charged to high tension by a rectifier. By means of a rotary spark gap this condenser was discharged fifty times per second through the plate circuit of the tube oscillator, in which short oscillations of great power were thus produced. The rest of the time the condenser was not connected to the oscillator and the charge was gradually stored up. Thanks to this method it was possible to carry out experiments under expeditionary conditions, using but a small power.*

The observations were made with a cathode ray tube, the current of a small alternator, driven on the same shaft as the transmitter discharger, causing a circular motion of the spot on the tube. The transmitter and receiver were separated by a distance of three kilometers and linked by wire.

Records are given of measurements on several days of June, July, and August, 1933, as well as a classification of observed phenomena. Reflections came from both E *and* F *layers and also from the intermediate region. The lower* F*-layer boundary appeared usually at a height of 220–250 kilometers, but often rose to 300 kilometers and sometimes even to 500 kilometers. The lower* E*-layer reflections generally occurred at a height of above 100 kilometers. The* E*-layer reflections were usually rather weak, often disappearing during the day, but were nearly always present around midnight.*

Strong reflections came from regions above the lower boundaries of the layers and especially from the F *layer, where their number rose to a score or two and could not be counted. The layer height generally changed but little during one day, but one day's results differed greatly from another's. In some cases the* E*-layer reflections appeared by day and disappeared at night; in others, on the contrary, they could not be found by day, but were present at night. In general, however, records for times around noon and midnight showed very similar conditions. All sorts of changes were observed during the intermediate hours.*

There were times of complete cessation of echoes, which occasionally lasted for some minutes, and sometimes several hours. Very disturbed conditions frequently were followed by sudden cessations of echoes on all of the available wavelengths from 75 to 150 meters. The echo was absent but for a short time, and then the whole picture of reflections was completely repeated.

* Decimal classification: R113.61. Original manuscript received by the Institute, January 26, 1934.

A classification of observed echoes is given corresponding to the character of visibility on the screen of the cathode ray tube. The author puts forward the assumption that there is a separate absorbing layer or region at a height less than 65 kilometers.

Apparatus

THERE is given here a description of Kennelly-Heaviside layer height measurements which were carried out at Moormansk in connection with the second International Polar Year. As the experiments had to be done under expeditional conditions and there would be no powerful station in the polar region, a special apparatus was designed in order to send out short signals of great power, without consuming much of the mean (effective) power. The circuit diagram of this equipment is shown in Fig. 1.

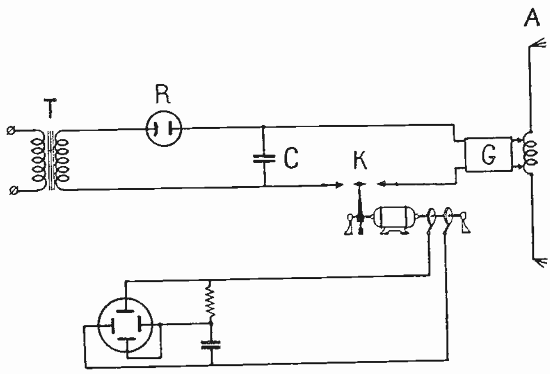


Fig. 1—Circuit diagram of transmitter.

The transformer T is fed from a 1000-cycle alternator. At the secondary terminals of this transformer there is a voltage of 20,000 volts, which charges a small condenser C through a rectifier R. When the contact K is closed, the condenser C is connected to the circuit of the generator G at the points to which the plate voltage is ordinarily connected.

The contact point K is a rotary spark gap, making 50 cycles per second. Each time that the rotating pole of this spark gap approaches close enough to the stationary pole, a spark is produced and the condenser C discharges through the generator G. At this moment the aerial A connected to the generator G emits a group of waves. The duration of such a group may easily be regulated by the value of the capacity of C.

The generator G makes use of two tubes, having a nominal power of 150 watts and 3000 volts on the plate. During the moment the condenser C, charged to a tension of 20 to 30 kilovolts, is coupled and the circuit closed, the tubes are strongly forced and produce a power of 10 to 20 kilowatts, which may be controlled by the instruments.

However the mean power is found to be of the order of 100 watts or so, as a short operating period of 0.0002 second is followed by a quiescent interval of 0.02 second. In order to produce regular sparking the spark gap had to be illuminated by a small arc. The rhythm of signals sent out under such conditions was maintained with great accuracy.

A small alternator, giving 50 cycles per second, was fixed on the same shaft as the motor driving the spark gap. The current from this alternator was transmitted by wire to the receiving station, which was situated at a distance of 3 kilometers from the transmitting station. The signal and echo were observed visually by means of a cathode ray tube of the Ardenne type; the alternating current was used to give a circular motion to the spot of the tube. The receiver used was a superheterodyne with one stage of high frequency, two stages of intermediate frequency, and one low-frequency resistance stage. The antennas consisted of single horizontal wires, their axes being disposed on approximately the same line.

Character of Observed Phenomena

A pulse produced by the described method is characterized by the quick rise of amplitude and its exponential drop. Therefore the luminous ring, when observed on the screen of the cathode ray tube, is seen to be cut off abruptly the moment the signal is received, because of the very quick projection of the spot. The exponential return of the spot is always very clearly seen, whereas the line corresponding to the time of rise of the oscillation amplitude often remains invisible. Sometimes this picture may also be observed during the first echo. Such an abrupt cut-off of the luminous line is of great help for accurate measurements.

A quite different picture is caused by repeated echoes. This is due to the transient phenomenon in a medium and especially to the fact that many echoes are superimposed on the same curve. It is often the case that the first echo traces a line on which both the rise and fall of the amplitude are equally well defined.

The amplitude of echoes changes very often, and generally speaking, very seldom remains constant for many minutes. The echoes were generally of lower amplitude than the direct signal but there were cases when they were stronger.

Fig. 2 is an example of an oscillogram taken for one echo with an amplitude less than that of the direct signal. In Fig. 3 may be seen besides the first echo, a second of smaller amplitude. The echo in Fig. 4 is of a more complicated form with several peaks.

Occasionally during undisturbed conditions it was possible to take photographs of the patterns. The rest of the time the echo picture on the screen was of very erratic character, with extremely rapid changes, separate elements of the picture moving rapidly. Although the character of the echoes themselves as well as of their motion varied con-

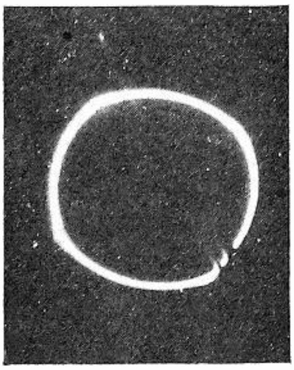


Fig. 2—Oscillogram of echo with amplitude less than that of direct signal.

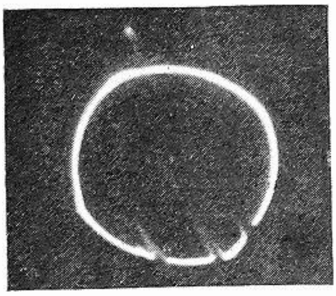


Fig. 3—Oscillogram of two echoes.

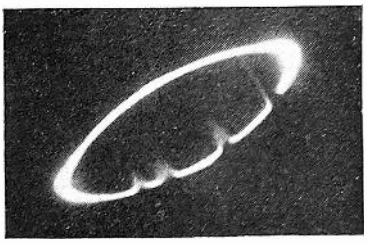


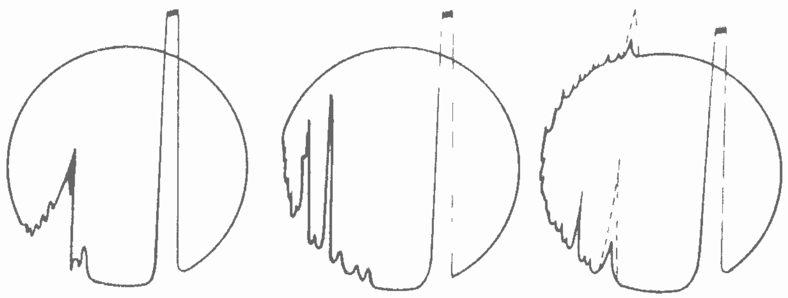
Fig. 4—Oscillogram of complex echo with several peaks.

siderably from day to day they may be classified in the following general way:

(1) *Steady simple echo.* Consists of separate peaks widely separated and independent of one another. Amplitude changes gradually (smoothly and slowly) or remains constant for several seconds or more. The height of reflection is constant for long periods.

(2) *Steady multiple echo.* Is composed of several peaks, with small intervals between them or of several groups of such peaks. Each group forms a separate system. Peaks of one system have the same character of changes. These changes take place periodically and are very similar in character to fading, their period being of the order of one or several seconds. All peaks of a given group undergo changes of a more or less synchronous character, but for the most part these take place at different relative phases.

A successive rise and fall of two peaks in the same group may be observed frequently. In this case, when one peak is growing, the other is diminishing; it disappears completely at the minimum and is nearly as great as the direct signal at the maximum.



Figs. 5, 6, and 7—Unsteady, oscillating and moving echo, as seen on the screen of the cathode ray tube.

(3) *Unsteady, oscillating, and moving echo.* Consists of many peaks, some of which are higher and relatively more steady. Between are many small peaks that constantly change their amplitude, rising and again disappearing quickly. The flashing of these peaks and jags gives an impression of rapid motion, which is sometimes perceived visually as a regular movement along the ring. For the most part this motion seems to be oscillatory or occasionally unilateral (one-sided).

The region occupied by peaks corresponds to heights up to 1500 kilometers and more. The number of peaks is so great that it cannot be counted, but it is at any rate no less than several score. Sometimes the whole ring is lifted up, as separate peaks are reciprocally superimposed.

The picture was usually of an erratic character and could not be photographed. Manually made drawings of this sort of echoes on the screen of the cathode ray tube are shown in Figs. 5, 6, and 7.

(4) *Irregular echo.* Has the aspect of separate peaks and groups, appearing without any order whatsoever, and lasting only for a very short time.

Method of Recording

The frequency with which the short pulses were sent out during these observations was about 50 pulses per second. To record the observed picture, sheets of white paper were used in which round holes 5 centimeters in diameter were cut. This sheet was arranged on the screen of the cathode ray tube in such a way as to make the luminous ring fit the rim of the hole over its circumference. The echo positions, their intensity, and character were marked with a pencil by lines designating the place of echo on the tube, and by different supplementary symbols to fix its character and behavior. Such marks were made each time any change took place in the picture. Sometimes this occurred every two to three minutes; at other times there were intervals of ten to fifteen minutes. In addition to this graphic tracing, a detailed description was included in a separate table for the time corresponding to the period of observation. By means of these records it has been possible, when working up the material, to trace the gradual transition of one picture into another. The graphs showing the variation of conditions with time were plotted on the basis of these records. In the calculation of layer height the velocity of propagation was taken equal to that of light.

This method of recording is rather difficult, and requires considerable concentration on the part of the observer. However it has some advantages when compared with automatic recording, as the latter gives but integral effects. Moreover many characteristic features and especially phenomena of very short duration (or of unsteady character) are not registered by automatic recording.

The work of recording during the series of tests described was skilfully performed by Engineers W. S. Fedookowitch and G. I. Tchernjawski.

Remarks Concerning Graphs Given Below

The records are illustrated in graphs, in which Greenwich Mean Time and Local Mean Time at Moormansk are indicated along the axis of abscissas. It must be noted that during the first period of measurements up to July 15th the sun remained over the horizon the whole time. At the end of August, when the last observations took place, the sun was below the horizon for eight hours. "Midnight" and "noon" as referred to in the following discussion denote the lowest and highest positions of the sun; i.e., they correspond to 2400 and 1200. local time, regardless of whether the sun was below or over the horizon. The "virtual heights" are plotted along the axis of ordinates without correction for the change of propagation velocity in the ionized medium.

Here a very important remark must be taken into account. There is no doubt but that only a portion of the echoes observed arrive from a vertical direction, or in other words actually determine the height of reflection. In all other cases the echo retardation depends upon the distance of the reflecting point, which may be in any direction from the observer. Until directive aerials are used for this sort of measurements, the results obtained as regards the height of layers cannot be considered as authentic. Therefore the points indicating the reflection height on the graphs, and the term "reflection height" in the text, must to a certain degree be accepted conditionally. This conditionality enables a comparison to be drawn between the general character of phenomena observed in polar regions and results of observations in temperate latitudes already published by other authors.

The observations were not made continuously; during intervals the behavior of other wavelengths was investigated. In order to indicate the periods of observations a heavy broken line has been drawn at the bottom of the graphs. Observations took place during the periods designated by this line. The absence of the line means that no observations were made. Separate echoes are marked by points. Points surrounded by circles correspond to very strong echoes, the amplitudes of which were nearly as great as those of the main signal. The connection of points by vertical lines means that the intervening peaks belonged to one group or had a common base, preventing the identification of separate groups. The vertical rippled lines on some of the graphs show the presence of moving echoes. The shadowed regions are those of weak echoes, which were not fixed by points because of their unsteadiness. Lines connecting points along the time axes are traced on the basis of detailed records and show the relation between different positions (in time) of one peak or group. Dotted lines indicate suggested relation between the points.

Results of Measurements

In rare cases only, the daily course of phenomena was similar to that observed in temperate latitudes. As is well known, in temperate latitudes the E layer is found to exist during daytime and disappears at night, whereas the F layer region is gradually rising during the hours of darkness. At Moormansk this sort of phenomena was observed, for instance, on June 24–25, when the sun remained over the horizon at midnight. During later periods of measurement in August when the sun dropped below the horizon at night there were days when no similar results were obtained. The graph for June 24–25 is shown in Fig. 8. A wavelength of 110 meters was used. The lower layer is seen to be at a height of from 110 to 160 kilometers, greater heights predominating. This layer is present during the "day" time, and after 0700 local time ionization becomes so pronounced that it is able to screen the upper layer.

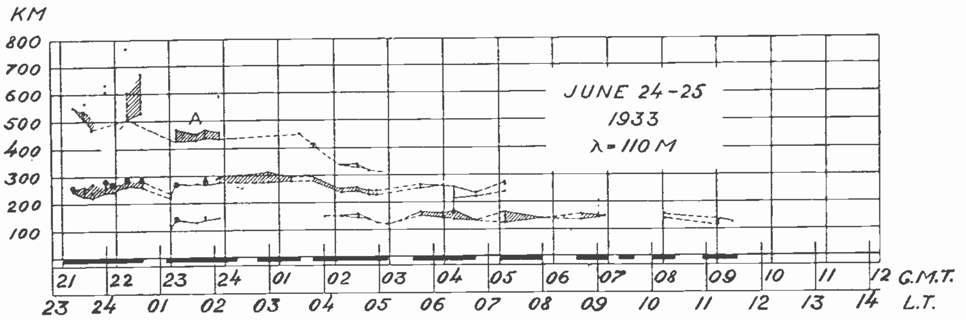


Fig. 8—Observations on June 24–25, showing the presence of the E layer by day and around midnight. The height of the F layer is somewhat less by day than at night.

The appearance of the lower layer after midnight, which is seen in the graph of Fig. 8, is often observed in temperate latitudes. In Moormansk this phenomena had a regular character and at least several weak reflections from the lower layer were always present around midnight. Its appearance at this time did not in any noticeable way affect the reflections from the upper layer. An example of this for

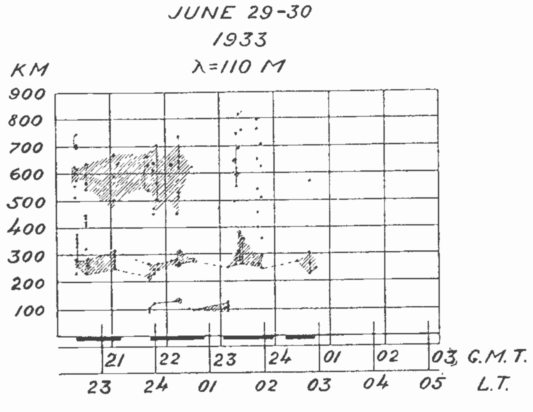


Fig. 9—Observations on June 29–30, wavelength 110 meters.

June 29–30 is shown in Fig. 9 on a wavelength of 110 meters. Reflections from the F layer in Fig. 8 correspond to a virtual height ranging from 220 to 310 kilometers. This height remains nearly constant for the whole day. Most of the reflections from heights of 425 to 650 kilometers are probably due to repeated echoes. That however cannot be said definitely of the group *A* in Fig. 8.

Fig. 10 (June 28-29, wavelength 110 meters) illustrates an example of similar variations in reflections reversed in point of time, as if the day and night pictures were transposed. For the most part however the character of reflections at noon and midnight on the same day was very similar. Rapid changes in the state of layers more often occur at intermediate hours between midnight and noon.

The independence of regions E and F was very distinctly seen during the whole period of measurements. The reflections from the lower region appeared generally at a height above 100 kilometers. Very seldom were echoes observed corresponding only to the smaller heights. The greatest decrease of height was observed on June 21-22, when reflections from 70 and even 65 kilometers were noted around midnight.

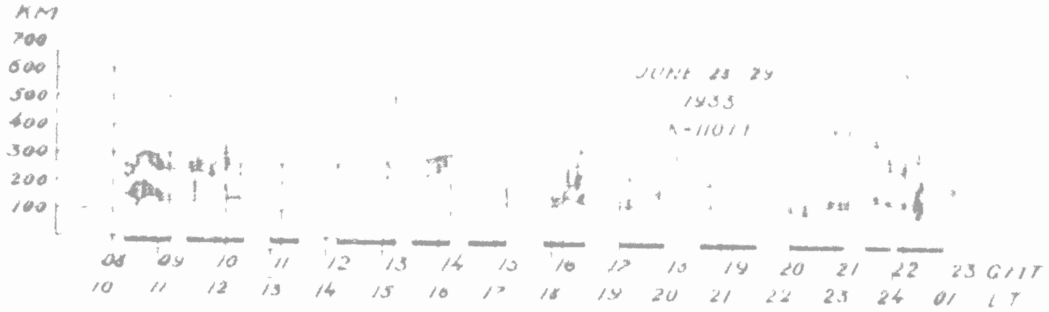


Fig. 10—Observations on June 28-29, wavelength 110 meters, showing the presence of the F layer by day and the E layer at night.

It might be expected that the ionization of the E layer is most intense when it is at low levels. This however has not been confirmed by observations. First of all, at times when the layer was at low levels no increase of absorption was noticed and there were as many reflections from the upper layer as before. In the second place, when the E layer was at a low level, as in Fig. 9, its height and the duration of time it was present were generally less steady. The intermediate Appleton layer, which is assumed to be at a height of 130-180 kilometers, did never appear clearly. Records of observations on June 23-24 (wavelength 110 meters), as shown in Fig. 11, are an exception. On this day the E layer was at a height about 100 kilometers from 1130 till 1630. Echoes were very weak during all this time, except for momentary increases. About 1630 two new peaks appeared, corresponding to heights of 165 and 180 kilometers and then the reflections from the lower layer ceased. During the next hour reflections came from the region above 150 kilometers, steady peaks lasting for the entire time with a great many moving echoes between them. This region gradually dropped to a height of 100 kilometers, and the gradual transition of the lower peak to this height was clearly observed. After 1800 local time a steady echo appeared from 130–140 kilometers, lasting for a long time. The record for June 27, wavelength 110 meters, Fig. 12, shows the virtual height of the layer decreasing from 200 to 150 kilometers and less.

The lower boundary of the F layer was generally found at a height

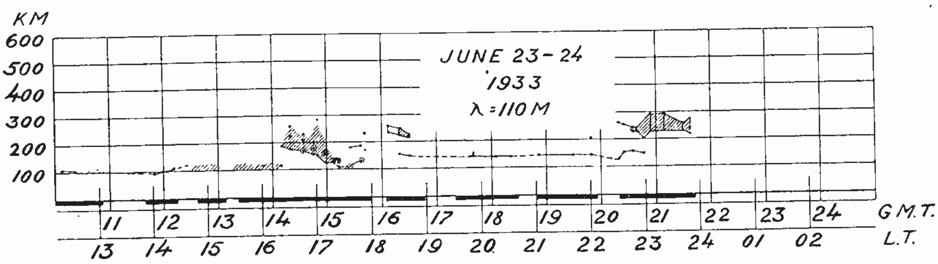


Fig. 11—Observations on June 23–24, wavelength 110 meters, showing the drop of layer from a height of 180 to 100 kilometers.

of 200 to 400 kilometers, and occasionally at 500 kilometers. This height changed but little during one day, but the variability of the mean height from day to day was very great. A general tendency of height change was noted in both the E and F layers. On days when the height of the E layer was less than usual the lower boundary of the F layer was also found at lower heights. Many reflections always ap-

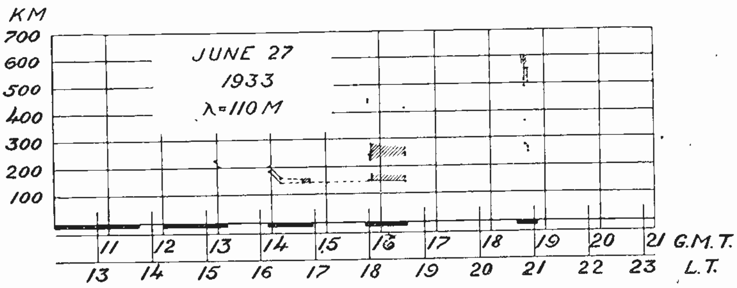


Fig. 12—Observations on June 27, showing the drop of layer from a height of 200 to 140 kilometers.

peared from the region above the lower boundary. Reflections from this region were very often accompanied by many separate peaks and jags, which moved continually and rapidly, forming the above-mentioned moving echo.

A most variable picture was observed on July 19–20 with a wavelength of 75 meters, separate strong peaks being visible over nearly half of the luminous ring of the cathode ray tube. The results of these observations are plotted in Fig. 13. The character of the peaks moving on the screen was such that it seemed to the observer as if all the peaks had a general motion. This general motion was for the most part such as to give the impression that the whole mass of layer was alternately falling and rising. In other cases the entire picture seemed to have a unilateral (one-sided) right- or left-handed motion; i.e., as if some continuous upward or downward motion were taking place in the reflecting region. There is some question as to the reality of this motion since it is continuously accompanied by rapid changes in amplitude of all the peaks and jags. It may be noted only as a subjective impression.

Complete cessation of echoes was observed from time to time during the whole period of measurement; even with the greatest ampli-

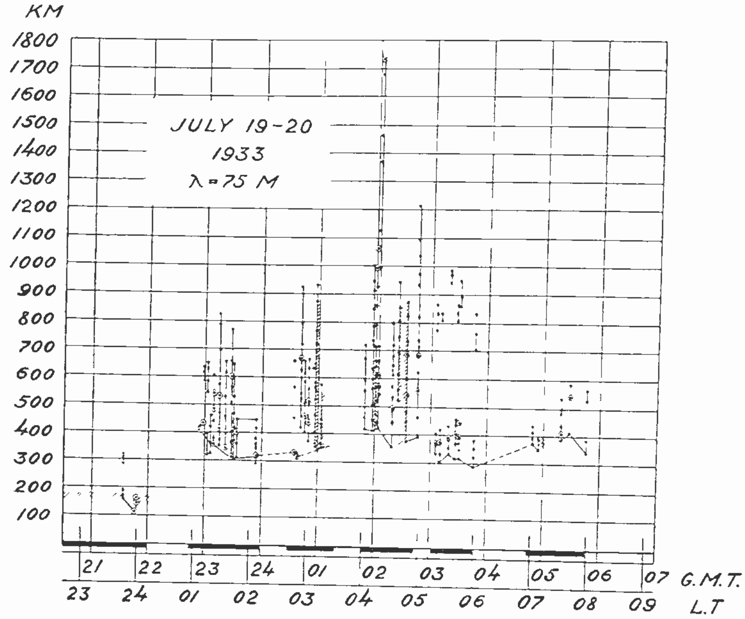


Fig. 13—Observations on July 19–20, wavelength 75 meters, showing an unusually great number of reflections.

fication possible no echoes could then be detected. Such a disappearance of echoes was sometimes preceded by a gradual fading of all the echoes, and occasionally the echoes ceased abruptly. These intervals were of different duration. For instance, on July 19–20 (and also other days), the echoes on many occasions disappeared for one or several minutes. It is very characteristic that upon reappearance after a short interval, the picture on the cathode ray tube is the same as before. If there were steady peaks, which are easy to trace, they would all appear again in their former positions. Echoes disappeared and reappeared on all of the available wavelengths from 75 to 150 meters simultaneously, at least during those times when such a comparison was undertaken. Thus the cessation of echoes evidently does not depend on any changes observed in the E and F layers. The factor suppressing the echoes is probably situated at a height below 65 kilometers.

On many occasions the absence of echoes lasted for several hours and sometimes occupied the greater part of time the observation took place. This phenomenon was observed for the most part in August, whereas in June there were but few days when echoes were weak.

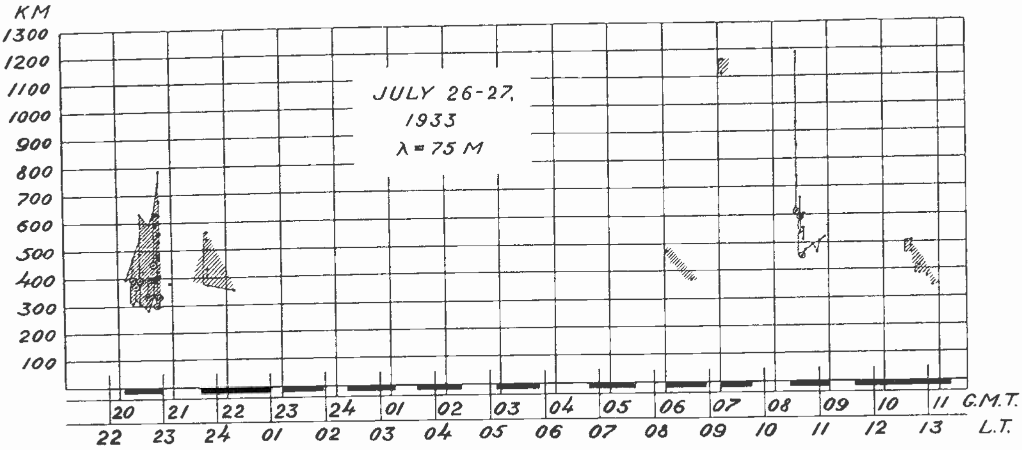


Fig. 14—Observations on July 26–27, wavelength 75 meters, showing disappearance of echoes for periods of long duration.

Comparison of this phenomenon with magnetic activity shows the tendency of echoes to cease during magnetic storms. In Fig. 14 is given the record for July 26–27 with a wavelength 75 meters and in Fig. 15 a magnetogram for *D* (declination), taken at Kandalaksha on the same day. The magnetogram shows an intense magnetic storm especially

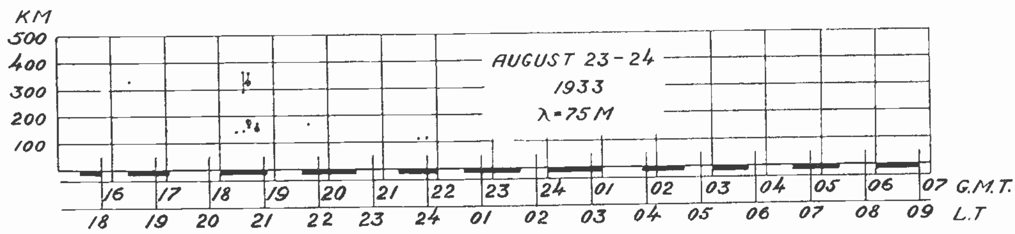


Fig. 16—Observations on August 23–24, wavelength 75 meters, showing absence of echo for nearly the whole time.

between 2300 and 0600 G.M.T. There were no echoes during this period, but both before and after the magnetic storm they were present. During the magnetic storm on August 23–24 there were no echoes for the major part of the time. The wavelength used was 75 meters. In Fig. 16 is shown a record for this day, and in Fig. 17 a magnetogram, taken at Kandalaksha on the same day.

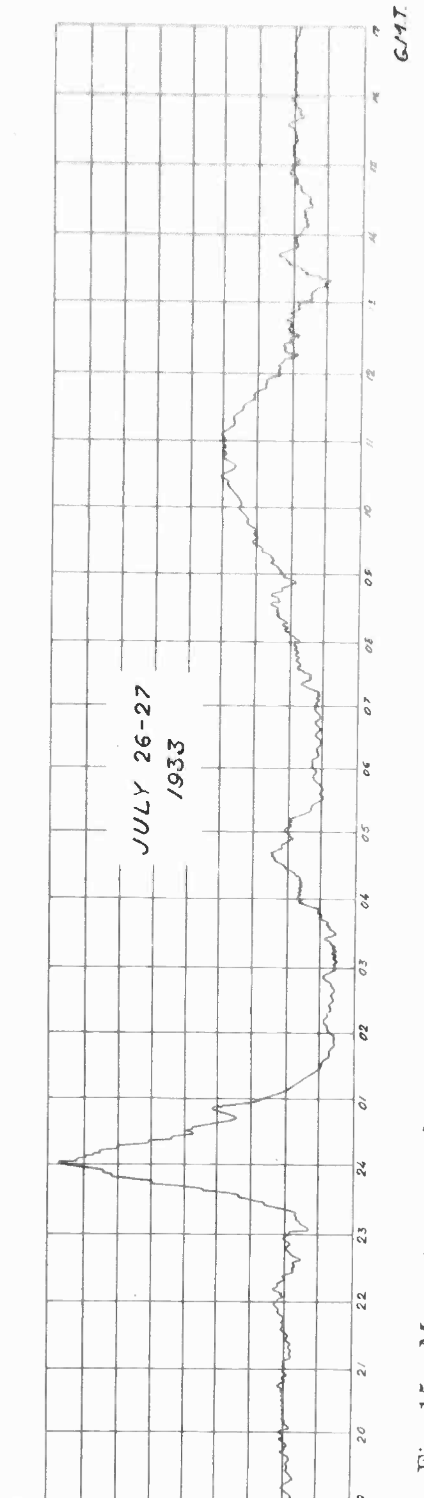


Fig. 15—Magnetogram, showing change of D, taken on July 26–27 at Kandalaksha (latitude 67°08′ N; longitude 32°26′ E) in the U.S.S.R.

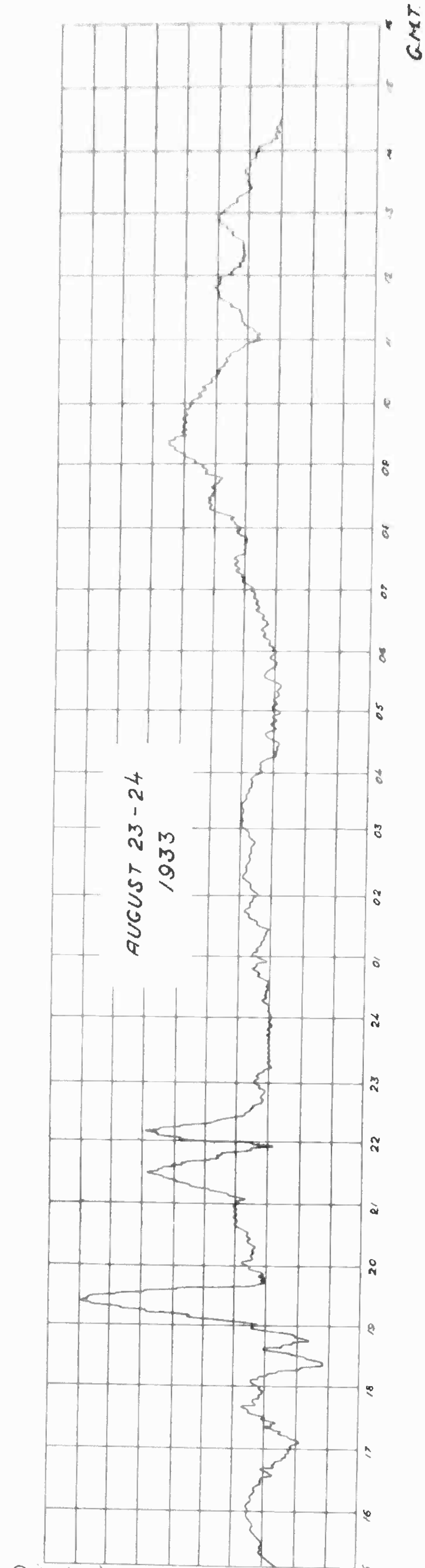


Fig. 17—Magnetogram, showing change of D, taken on August 23–24 at Kandalaksha (latitude 67°08′ N; longitude 32°26′ E) in the U.S.S.R.

Fig. 18 shows many short as well as long intervals of echo cessation on August 9–10, the wavelength used being 75 meters. The magnetic field was absolutely quiet on this day, except around 1900 G.M.T. when a disturbance of small intensity was recorded.

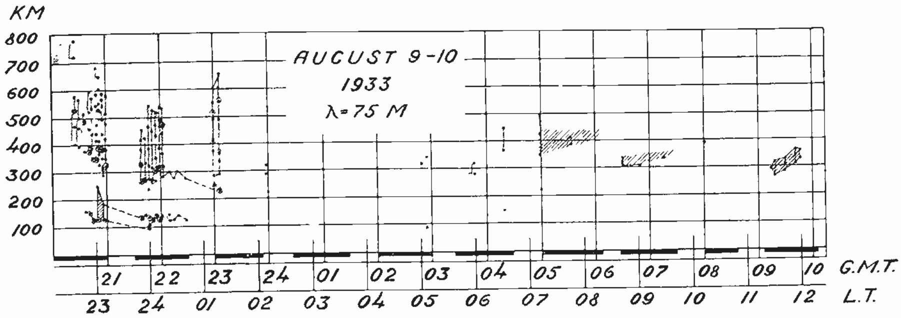


Fig. 18—Observations on August 9–10, wavelength 75 meters, showing long and short intervals of echo cessations. The magnetic field was quiet on this day, except for 1900 G.M.T.

As already mentioned above, many echo intervals of short duration were noted on July 19–20. Fig. 19 is a magnetogram for this day.

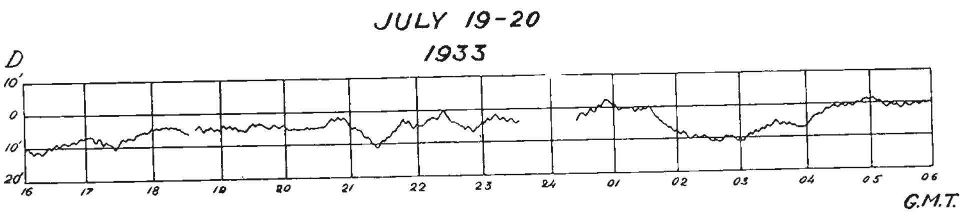


Fig. 19—Magnetogram, showing change of D, taken on July, 1920, at Kandalaksha (latitude 67°08′ N; longitude 32°26′ E) in the U.S.S.R. Corresponds to echo observations shown in Fig. 13.

Conclusion

The following conclusions and suggestions are based upon the experimental results obtained:

(a) In polar regions during the period of summer solstice and for some time after, both of the main reflecting layers E and F of the ionosphere are found to exist.

The E layer is in general less active than in temperate latitudes and therefore but seldom capable of screening the F layer. It is mostly in evidence for wavelengths of 75 to 110 meters around midnight and occasionally during the day.

(b) The daily variations of ionization are found to be in some cases similar to those in temperate latitudes, whereas in others they were rather of opposite character. Pictures for noon and midnight were nearly always alike, but differed from those for intermediate hours.

(c) Very complex reflections from the upper region are due to the stratified or undulatory structure of the ionosphere. Rapid motion is found to exist in this layer.

(d) No increase of the shielding effect of the E layer nor any changes in absorption have been observed at times when this layer dropped to a height of 65 kilometers. This seems to indicate that in this case such a falling of the E layer is due to changes in distribution of the gas pressure at great heights and corresponds to a deep barometric minimum of the upper atmosphere.

(e) Periods of complete cessation of echoes have been observed, which lasted sometimes for several hours. Sometimes, however, the echoes were absent for very short periods lasting only about a minute or so. The picture of reflections before and after such a short-period absence of echoes was found to be the same.

Such observations permit a suggestion that this disappearance of echoes is due to some factor having the character of a screen placed between the observer and the reflecting layer at an intermediate height; or it may be said that a separate absorbing layer is produced at times below the E layer, at a height probably less than 65 kilometers.

The character of this phenomenon offers some basis for an explanation of the structure of this layer; it may be composed of separate moving masses, shielding the Kennelly-Heaviside layer (as does a cloud, when shading the sun), or it may be produced by some variable agent, and is able to appear and disappear very rapidly.

Some further light might be thrown on this question by comparing moments of echo cessations at two points not very far apart.

(f) No correlation was found between the changes taking place in the E and F layers and the presence or absence of the absorbing layer. Therefore the absorbing layer must be considered as an independent formation, quite apart from the E layer (not its organic continuation) and due to other agencies than the E and F layers.

(g) There is undoubtedly direct correlation between the phenomenon of echo cessation and magnetic activity.

(h) The difficulty of maintaining continuous wireless communication over high latitudes caused by magnetic storms may be partly attributed to the existence of the absorbing layer.

Proceedings of the Institute of Radio Engineers
Volume 22, Number 9 *September, 1934*

DISCUSSION ON "OPTIMUM OPERATING CONDITIONS FOR CLASS C AMPLIFIERS"*

W. L. Everitt

L. B. Hallman, Jr.:[1] In the beginning of his useful and very instructive paper, Dr. Everitt states that it is customary to speak of any amplifier where plate current flows for less than half a cycle as a class C amplifier. He therefore speaks of his analysis as dealing with class C amplifier operation when it is concerned, principally, with the type of operation wherein the power output is a linear function of the grid voltage squared. Such an amplifier is usually referred to as a linear amplifier.

To speak of any amplifier where the plate current flows for less than 180 degrees as a class C amplifier would lead to confusion. Unless definitions were further modified to include two types of class C amplifiers it would be impossible to tell just what type of operation one referred to when an amplifier was spoken of simply as a class C amplifier. Especially, is this true in the light of Dr. Everitt's recent work.

I feel that it would greatly simplify matters if workers in this field adhered rigidly to the present Institute definitions in this regard. Thus, why not always speak of a class B amplifier as one wherein the power output is a linear function of the grid-input voltage squared? This would leave the definition of the class C amplifier to include any amplifier operating in such a manner that the power output is a linear function of the plate voltage squared. The class A amplifier, then, becomes a special case of the class B amplifier wherein the plate-current wave form is the same as that of the input-grid voltage.

In practical work the designer is primarily interested in whether the output power will be linear with respect to grid or plate voltage squared. Consequently, our definitions would be most useful if stated in terms of these basic relations.

W. L. Everitt:[2] While it true that a portion of the paper is devoted to operation where the power output is a linear function of the grid voltage squared, this was only incidental to the analysis and not its primary purpose. No present Institute definition applies to this particular linear amplifier, since the definition of class B operation specifically states that the current is to flow in half-wave pulses. For this reason the term class B′ was suggested for this linear amplifier operating during less than half the cycle.

Radio-frequency power amplifiers frequently are used to amplify an unmodulated continuous wave. When they are not themselves modulated the present Institute definitions are inadequate. Common usage has permitted application of the term class C to such an amplifier when plate current flows during less than half a cycle.

In the operation of amplifiers with a linear relation between plate voltage squared and power output (class C), it will be found that it is necessary to provide sufficient grid excitation to operate on the flat portion of the radio-frequency grid-voltage—tank-current curve. If the direct plate-voltage—plate-current curve is also linear, as it should be to provide a linear load during modulation, then the efficiency is nearly constant as the plate voltage is varied. If the output

* Proc. I.R.E., vol. 22, p. 152; February, (1934).
[1] Montgomery Broadcasting Company, Montgomery, Ala.
[2] Department of Electrical Engineering, Ohio State University, Columbus, Ohio.

and loss are known for the unmodulated operation they can be determined for the modulated condition because with a constant efficiency the input and loss will both increase by the percentage that the output increases. The increase in output for the modulated wave can be computed from the percentage of energy which will reside in the side band.

The optimum operating conditions which may be obtained from Fig. 12 and Table IV of the paper, show how to obtain the maximum output with a given loss when the grid voltage is large enough to operate the tube on the flat position of the grid-voltage—tank-current curve. The experimental curves given in Figs. 13 and 14, while they were drawn with the grid voltage as abscissa, simply show that the computed values occur when the excitation is such that the maximum grid voltage equals the minimum plate voltage, and this is also the point of maximum efficiency.

It has been called to my attention that the values of *A* and *B* in Table III are interchanged. This is also true of the legend for Figs. 13 and 14. The grid return in Fig. 3 should be connected to ground instead of to the center top of the filament transformer.

Proceedings of the Institute of Radio Engineers
Volume 22, Number 9 *Spetember, 1934*

BOOK REVIEWS

Applied Acoustics, by H. F. Olson and Frank Massa. Published by P. Blakiston's Son & Co., Philadelphia, Pa. 430 pages. Price $4.50.

This book gives an excellent picture of current practice in the field of electro-acoustics, in which both of the authors are well known, being particularly fortunate in its comparisons of the results of theory and practice.

The fundamental equations of sound are first developed in terms of the vector calculus. Then the analogy between electrical, mechanical, and acoustic systems is stated (unfortunately not with sufficient care to point out the pitfalls which exist) and the equations of complete dynamical systems are developed in some detail. Concepts of acoustic and mechanical impedance are examined and expressions for their value derived for the important cases. The fundamental acoustic measurements are critically described. The next portion of the book (almost 200 pages) is given over to a very complete description of microphones, telephone receivers, and loud speakers, with a critical discussion of methods of calibration and fields of use of the various types. (There is here an understandable overstressing of the importance of the ribbon microphone with the development of which the authors have been intimately associated.) Excellent chapters on measurements upon dynamical systems and of noise are included. Also summaries of the recent discoveries and developments in the fields of architectural and physiological acoustics are given.

The bibliography is good, the indexing only fair. The minor errors of style and context usual in first editions are present.

In much of the book in the authors' own words "Only the important results will be shown and proper references will be given for those interested in further study." This tendency limits the usefulness of the book as a text, as does also the complete absence of problems. The book however is an excellent *guide* to the study of electro-acoustics for the highly trained electrical engineer and could be used in graduate study if supplemented by much outside material or if only present practice were of interest.

For reference the book gives the best balanced picture of recent developments in the field of electro-acoustics which I have seen, and I am very glad to have it in my library.

*Knox McIlwain

Magnetic Materials at Radio Frequencies. A critical survey of present knowledge, by F. M. Colebrook. Radio Research Special Report No. 14 of the Department of Scientific and Industrial Research. Published by His Majesty's Stationery Office, London, 22 pages. Price 6d.

The use of magnetic materials for the cores of coils to be employed at radio frequencies has the practical advantages of giving low ratios of resistance to inductance in a given space, together with the possibility of effective screening and good coupling between windings. To avoid excessive iron losses at the high

* Moore School of Electrical Engineering, Philadelphia, Pennsylvania.

frequencies in question, cores composed of iron powder, mixed with an insulating binder, are common.

As the title suggests, this report summarizes what is known regarding the magnetic characteristics and energy losses of such cores, particularly at frequencies higher than 100 kilocycles.

In addition to the complexities inherent in magnetic relations in iron, in general, the high frequencies involved and the presence of an imperfect dielectric in the magnetic field give rise to the further complications of skin effect, magnetic viscosity, and dielectric losses.

The author has given a thorough but concise summary of the results of experimental work along these various lines, making clear what may be regarded as established facts, and presenting the evidence on points where further research is necessary.

An extensive bibliography is appended.

*F. W. GROVER

Report of the Radio Research Board for the period of January, 1932, to 30th September, 1933. To the Lords of the Committee of the Privy Council for Scientific and Industrial Research. Published by His Majesty's Stationery Office. 137 pages. Price 2s. 6d. net.

Since the last Report, March, 1929, the Committee has been reorganized and the Wireless Division of the National Physical Laboratory (corresponding to the Bureau of Standards, U. S.) and the Radio Research Station, Slough, have been amalgamated into a new Radio Division Department National Physical Laboratory.

The subjects of investigation reported are: The Travel of Wireless Waves, Experiments in the Arctic Circle, How the Heaviside and Appleton Layers are Produced, Wireless and Thunderstorms, Fading Caused by Two Circularly Polarized Waves, Trans-Atlantic Telephony, Direction Finding, Soil Resistance, and Wobble-Free Transmitters.

The report gives a rather brief discussion of the researches and their findings. In a review of this kind it is impossible to give more than a list of researches as above.

The report indicates that the Radio Research Board is doing efficient research work.

†R. R. RAMSEY

The National Physical Laboratory—Report for the year 1933. Published by His Majesty's Stationery Office for the Department of Scientific and Industrial Research, 1934. 264 pages. Price 13s.

This report contains a general résumé of the activities of the National Physical Laboratory of England, an establishment which corresponds in a general way to the National Bureau of Standards in the United States. The report describes in rather complete outline the work of the following departments: Physics, Electricity, Radio, Metrology, Engineering, Metallurgy, Aerodynamics, and the William Froude Laboratory (which is involved in testing ship models).

The Superintendent of the Radio Department of the National Physical Laboratory is Dr. R. A. Watson Watt. The work of his department consists al-

* Union College, Schnectady, New York.
† Department of Physics, Indiana University, Bloomington, Indiana.

most exclusively of projects carried out for the Radio Research Board and close contact is maintained with the work of Professor Appleton at the Laboratory of Kings College, London, and also with the radio research work at Cavendish Laboratory, Cambridge.

This report for 1933 is rather unusual for the detail which it gives of the projects and methods which are being employed in research work on propagation of waves, direction finding, atmospherics, and certain types of radio apparatus and materials. Any worker in these fields will find this report very helpful for reference, not only in connection with the work of the Radio Department but in connection with the Electricity Department, of which the report is also quite complete.

A complete list of the papers published by the Laboratory in 1932 and 1933 is included in the report. One of the outstanding papers is entitled, "Applications of the Cathode Ray Oscillograph in Radio Research." Copies of the papers listed can be obtained from His Majesty's Stationery Office, London.

*L. E. Whittemore

Vector Conversion Chart by O. W. Walter, Published by the Engineering Publishing Company, Philadelphia.

A combined rectangular and polar chart conveniently arranged for obtaining graphically the solution of vector relations with sufficient accuracy for most engineering purposes. Near 0 and 90 degrees it is more difficult to use because of the small angle between the radius vector and the coördinate lines. On account of its application to many types of problems it is a very useful chart.

†H. M. Turner

Short-Wave Wireless Communication, by A. W. Ladner and C. R. Stoner. Published by John Wiley & Sons, Inc. 440 Fourth Ave., New York, N. Y. 384 pages. Price, $3.75.

The book commences with an interesting chapter on the history of the development of short waves.

A section on propagation presents very clearly some of the most widely accepted concepts of this very complex subject.

A section on modulation deals briefly with some of the practical problems such as coincidental phase modulation, and single side-band working.

The balance of the book deals mostly with the details of practical equipment used in modern practice. The aspects of constant frequency oscillators, modulation circuits, high-frequency feeders, antenna systems, problems of reception, commercial receivers, and transmitters are discussed.

A final chapter covers briefly, the ultra-short-wave field, including some of the late work of Marconi and Mathieu in Italy.

The descriptive material deals considerably with the practices of Marconi's Wireless Telegraph Company, which in itself makes the book of unusual interest to communications engineers.

Most of the technical information is drawn from papers that have been presented before various engineering societies of international scope. Many selected references are given.

‡H. O. Peterson

* American Telephone and Telegraph Company, New York City.
† Yale University, New Haven, Connecticut.
‡ RCA Communications Inc., Riverhead, L. I. New York.

CONTRIBUTORS TO THIS ISSUE

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* Paper published in July, 1934, issue of the PROCEEDINGS.

Norton, K. A.: See Proceedings for February, 1934.

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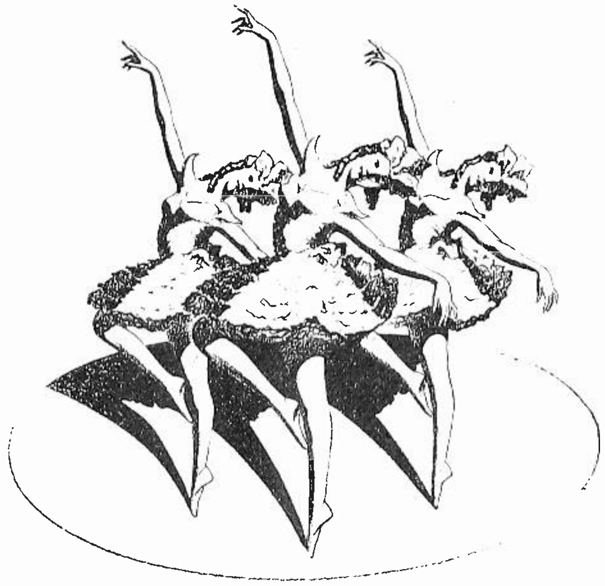

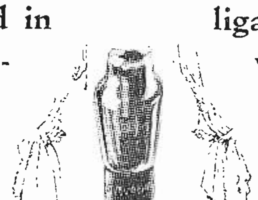

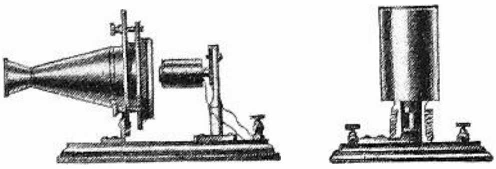

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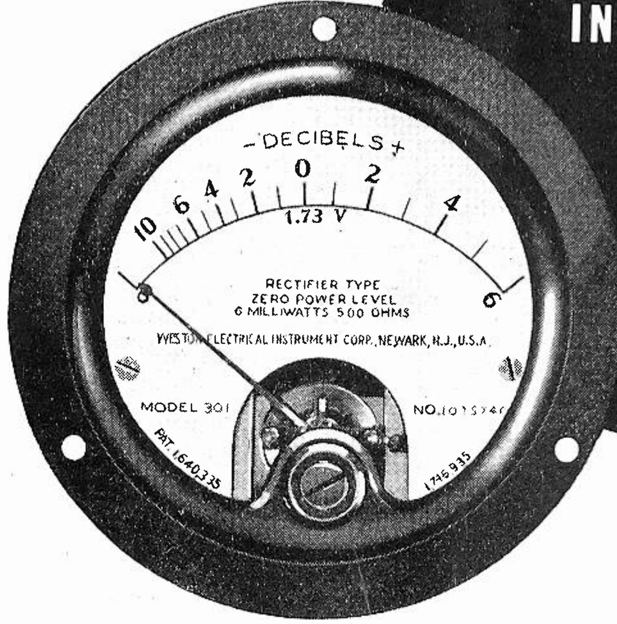

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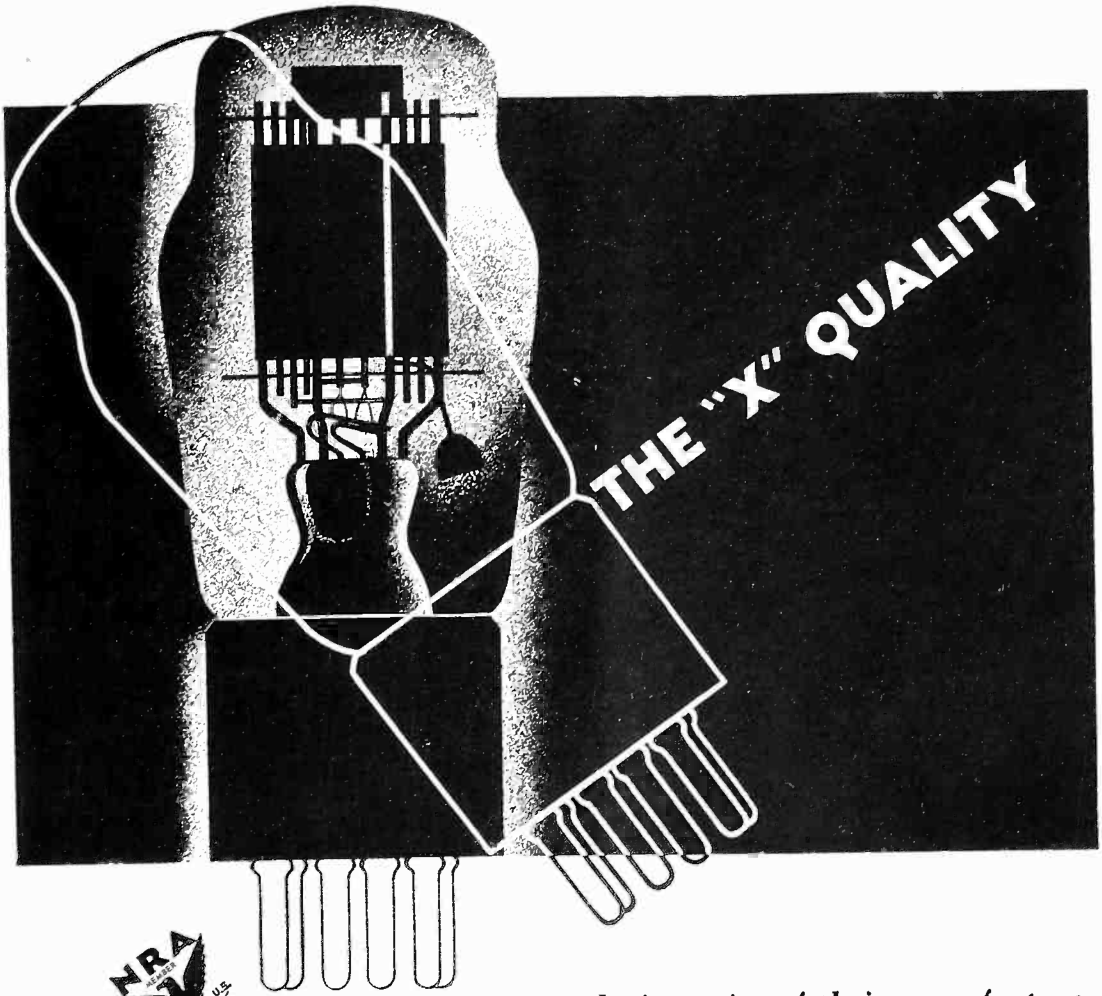

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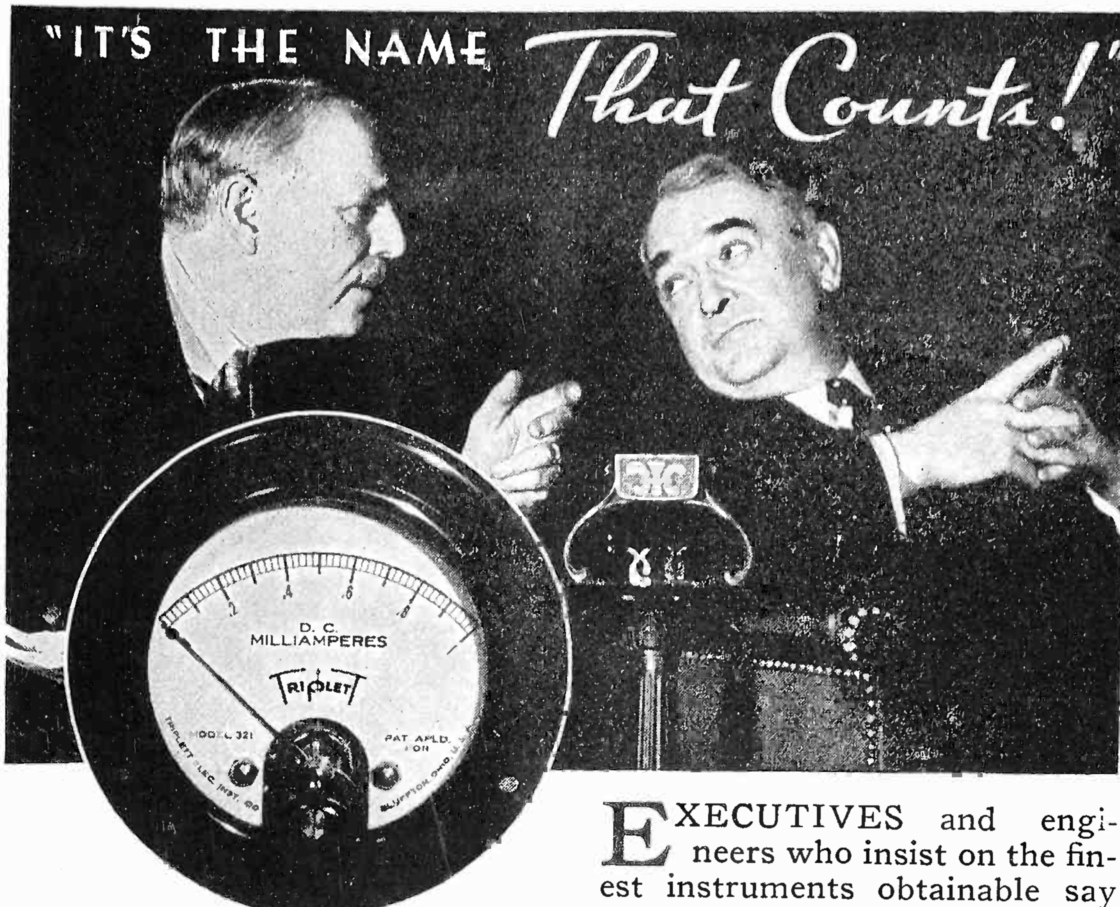

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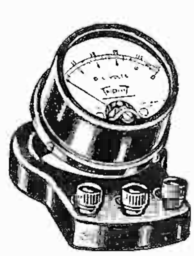

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Gentlemen:

I hereby make application for Associate membership in the Institute of Radio Engineers on the basis of my training and professional experience given herewith, and refer to the members named below who are personally familiar with my work.

I certify that the statements made in the record of my training and professional experience are correct, and agree if elected, that I will be governed by the constitution of the Institute as long as I continue a member. Furthermore I agree to promote the objects of the Institute so far as shall be in my power, and if my membership shall be discontinued will return my membership badge.

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Sec. 1: The membership of the Institute shall consist of: * * * (c) Associates, who shall be entitled to all the rights and privileges of the Institute except the right to hold any elective office specified in Article V. * * *

Sec. 4. An Associate shall be not less than twenty-one years of age and shall be a person who is interested in and connected with the study or application of radio science or the radio arts.

ARTICLE III—ADMISSION AND EXPULSIONS

Sec. 2: * * * Applicants shall give references to members of the Institute as follows: * * * for the grade of Associate, to three Fellows, Members, or Associates; * * * Each application for admission * * * shall embody a full record of the general technical education of the applicant and of his professional career.

ARTICLE IV—ENTRANCE FEE AND DUES

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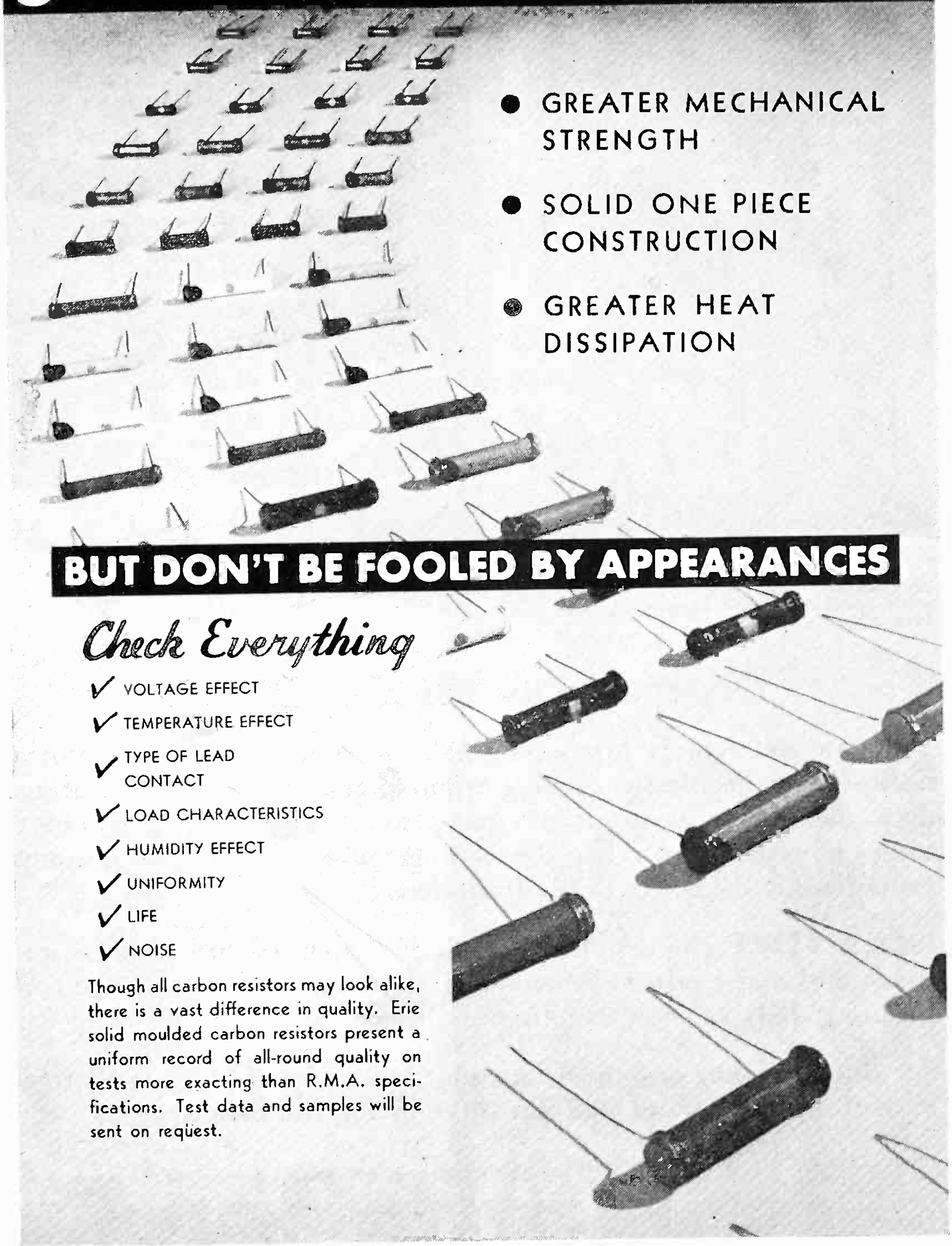

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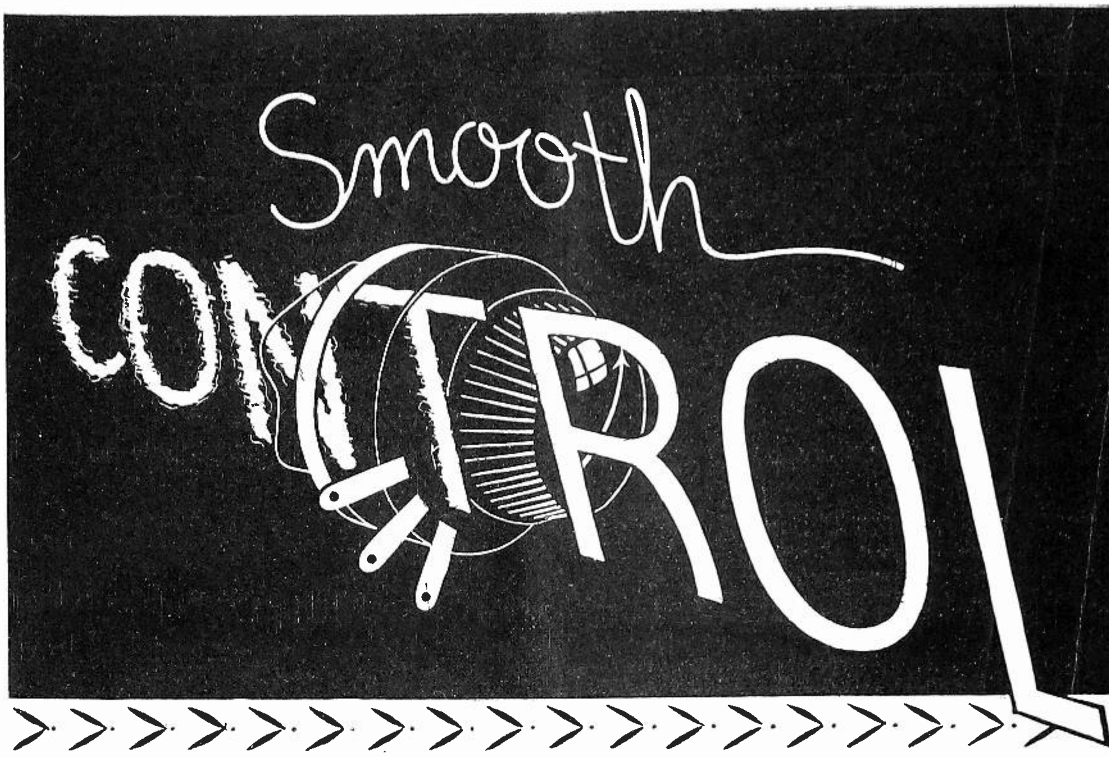
RADIO AND SCIENTIFIC LITERATURE

Alphabetical Index to Advertisements



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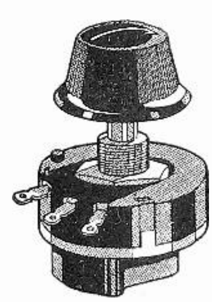

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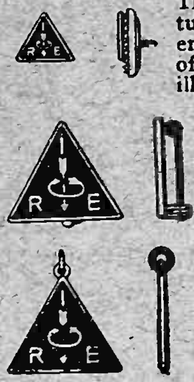

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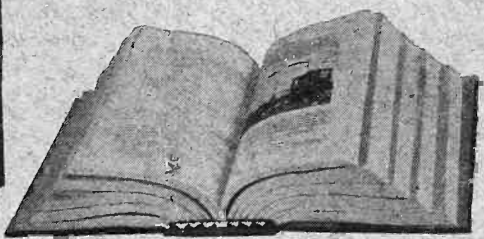
BINDERS



Binders are available for those who desire to protect their copies of the PROCEEDINGS with stiff covers. Each binder will accommodate the twelve monthly issues published during the year. These binders are of blue Spanish Grain Fabricoid with gold lettering, and will serve either as temporary transfers or as permanent binders. They are so constructed that each individual copy of the PROCEEDINGS will lie flat when the pages are turned. Copies can be removed from the binder in a few seconds and are not damaged by their insertion. They are priced at $1.50 each, and the member's name or PROCEEDINGS Volume Number will be stamped upon the binder in gold for $0.50 additional.

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Complete volumes of back issues of the PROCEEDINGS are available in unbound form for the years 1918, 1920, 1921, 1922, 1923, and 1926 at $6.75 per year (six issues). Also complete unbound for 1931, 1932, and 1933 at $7.50 per year. A list of the available miscellaneous copies published from 1913 on may be obtained from the Secretary. Single copies published prior to 1927 are sold to members at $1.13 each, and those issued during and after 1927 are $0.75 each. Foreign postage on single copies is $0.10 per copy, making $0.60 on unbound volumes before 1927.

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TWENTY-TWO-YEAR INDEX

An Index of the PROCEEDINGS for the years 1909-1930, inclusive, is available at $1.00 per copy. It is extensively cross indexed. The earlier references are to the PROCEEDINGS of the Wireless Institute; the PROCEEDINGS of the Institute of Radio Engineers has been published since 1913.

When ordering any of the above, send remittance with order to the Secretary, Institute of Radio Engineers, 330 West 42nd Street, New York City.

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