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EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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THE INSTITUTE OF RADIO ENGINEERS
announces with regret the death of

Morris N. Liebmann

Mr. Liebmann was a Westerner, a graduate of the University of Nebraska, and served with a Western regiment in the Spanish-American War. He came to New York several years later.

In 1901, he joined Company I of the old Twenty-third Regiment, of Brooklyn, rising in rank until he became its Commander. He served as Captain on the Mexican border in June, 1916.

He was Vice-President and Secretary of a large New York company manufacturing scientific and engineering equipment. He was a member of The Institute of Radio Engineers.

In May, 1917, he was commissioned Lieutenant-Colonel, and was repeatedly commended, at Spartanburg and elsewhere, for the excellent showing made by the men under his command.

On August 8, 1918, Lieutenant-Colonel Liebmann was killed in action while leading his men in a charge at the front in Flanders.

He is remembered among his many friends in the radio field as an able and indefatigable worker and a man of loyal and attractive personality.

FEASIBILITY OF THE LOW ANTENNA IN RADIO TELEGRAPHY *

By

Edward Bennett

(Professor of Electrical Engineering, University of Wisconsin)

Contents

I. Considerations Applying to Both Transmitting and Receiving Antennas

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* Received by the Editor, September 7, 1917.

1. THE PRACTICE IN THE CONSTRUCTION OF HIGH POWER RADIO STATIONS

The present practice in the construction of high power radio telegraph stations is to mount the conductors constituting the upper capacity area of the antenna at the greatest elevation above the ground which is deemed to be mechanically feasible. For example, the Government Naval Stations recently erected at Darien in the Canal Zone and at San Diego, Honolulu, and the Philippine Islands, have the capacity areas in the form of horizontal triangular shaped platforms of wire suspended at an elevation of approximately 150 meters (500 feet) above the ground. The wires are suspended from three steel towers each 183 meters (600 feet) high, and the towers themselves are placed at the vertices of a triangle which is approximately equilateral, with the sides 320 meters (1,100 feet) in length. The antennas of the stations at Nauen (near Berlin) and at Tuckerton (near Atlantic City) are carried to even greater heights. These stations have antennas of the umbrella type supported from insulated towers 305 meters (990 feet) and 252 meters (850 feet) high, respectively. It will be noted that the mean radius of the capacity area in the Naval stations above referred to is approximately equal to the height above the ground at which the capacity area is mounted.

In a bulletin entitled, "High versus Low Antennas in Radio Telegraphy and Telephony"[1] it is shown that if the capacity area of the antenna of a radio telegraph station consists of an extended horizontal network of wires mounted above a conducting plane, and if the mean radius of the capacity area is two or more times as great as any height above the ground at which it is feasible to mount the capacity area, then the electric and magnetic forces at a great distance from such a radiator are practically independent of the height of the network above the ground, provided the frequency of oscillation and the operating voltage from the network to ground are kept the same for the different mounting heights. That is to say, an extended network of wires charged

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[1] "High Versus Low Antennas in Radio Telegraphy and Telephony," by Edward Bennett, "Bulletin of the University of Wisconsin," number 810, Engineering Series, Volume 8, number 4, pages 179–248, September, 1916.

The conclusions arrived at in this bulletin with reference to the properties of the low antenna are based purely upon a mathematical analysis of the case, and not upon actual experiments between stations with low antennas. Since the publication of the bulletin, the writer has been advised that an experimental trial of a network of wires mounted at a low elevation was made by R. A. Fessenden in 1908. The manner in which the antenna was constructed, and the tests conducted, or the results obtained (save that they were not promising) have not been disclosed.

to a given voltage and allowed to discharge to earth thru an inductance tuned to give a frequency of say 100,000 cycles per second, sets up the same electric and magnetic forces at distant points whether mounted 10 feet (3 m.) or 200 feet (60 m.) above the ground. In these two cases the rate of radiation (in kilowatts) from the two antennas is the same but the initial store of energy in the case of the 10-foot (3 m.) mounting height is about 15 times as great as in the case of the 200-foot (60 m.) mounting height. Therefore, the oscillation in the former case is much more persistent than in the latter; in fact, the oscillation becomes so persistent for low mounting heights that the power condensers and coupled circuits at present required in spark systems of radio telegraphy may be dispensed with and a simple series circuit comprising capacity area, tuning inductance and spark gap, may be used. Such a circuit has the merit of oscillating at a single frequency, whereas the coupled circuits have two frequencies of oscillation.

In a comparison of the receiving properties of two such stations with networks at different elevations it is shown that the above stations are ultimately able to abstract energy at the same rate from passing electromagnetic waves, provided these waves are persistent and not rapidly damped,—the plane of the wave front of the advancing waves being assumed to be normal to the surface of the earth. The high antenna is shown to abstract energy at a greater rate than the low antenna during the initial stages (first few swings) of the oscillation. The high antenna will, therefore, respond much more readily to highly damped waves than will the low antenna. This means, of course, that when receiving undamped or slightly damped waves, the high antenna will be subject to greater interference from atmospheric disturbances than will the low antenna. To reduce this interference in the case of stations with high antennas, additional capacity is used in the "interference preventer" circuits. In other words, the low antenna is to be regarded as the equivalent of a high antenna and "interference preventer" combined.

The summary of the bulletin above referred to contains the statement—"It is feasible to construct a radiator with a capacity area at a very moderate elevation which will have a radiation figure of merit equal to or greater than the values which are at present attained by mounting the capacity area at a great elevation in long-distance radio stations."

The disclosure that the low antenna mounted above a *conducting plane* will initially radiate energy at the same rate as the elevated antenna, and that it will *ultimately* be able to abstract energy from impinging waves at the same rate as the elevated antenna is by no means a demonstration of the feasibility of the low antenna. Its feasibility hinges upon a score of other considerations such as the following:

(a) the relative effects produced by resistance losses in the earth in the vicinity of antennas of the two forms—high and low.

(b) the relative response of antennas of the two forms to oscillations for which they are tuned and to disturbing sources, such as strays and detuned stations.

(c) the feasibility of obtaining from generators of undamped waves the large currents required by the low antenna.

(d) at low frequencies, the time constant of the receiving antenna may be so great that a dot or a dash is completed before the oscillation in the receiving antenna has built up to its full value.

This paper presents calculations and data relating to the feasibility of the low antenna. It deals mainly with those wasteful antenna and earth resistances which are common to the use of the low antenna both in sending and receiving. It is the purpose to discuss the other aspects of the feasibility of the low antenna in subsequent publications under the following headings:

II. THE LOW ANTENNA FOR RECEIVING PURPOSES
III. THE LOW ANTENNA AS A RADIATOR
IV. MECHANICAL CONSIDERATIONS

2. DISTINCTION BETWEEN THE "LOW" ANTENNA AND THE "GROUND" ANTENNA

Before proceeding, it may be well to distinguish between two types of antenna, namely, the *Marconi* antennas of Figure 1, 2, and 3, and the *ground*, or *Hertzian* antenna, of Figure 4,—and between three forms of the Marconi antennas, namely, the low power Marconi antenna of Figure 1, the high power *elevated* Marconi antenna of Figure 2, and the high power *low* Marconi antenna of Figure 3. The features of these types and forms are shown in the figures. By the high power *elevated* antenna is meant an antenna in which the radius of the capacity area and the elevation of the capacity area above the ground are about equal in value. By the high power *low* antenna is meant an antenna in which the elevation of the capacity area is equal to 1-10th or less of the radius of the capacity area.

The Marconi antennas (for sending and receiving purposes) are most effective if mounted over a plane of infinite conductivity, while the *ground* antenna (which apparently was first investigated by Kiebitz[2]) would not function at all as a receiver if mounted adjacent to such a plane. The *low* antenna of Figure 3 should not be confused with the *ground* antenna of Figure 4.

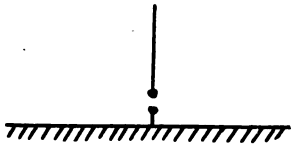

FIGURE 1—Marconi Antenna of Low Power

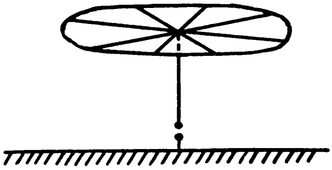

FIGURE 2—"Elevated" Marconi Antenna of High Power

With the low antenna it is advisable to use as the lower capacity area a network of wires mounted a few feet above the earth's surface, because low resistance in the lower capacity area is absolutely essential to the success of the low antenna On the other hand, high conductivity in the surface layers of the earth would render the ground antenna inoperative. The ground antenna is effective only if the electromagnetic wave front is tilted forward by reason of losses occurring in the earth. This paper deals only with the relative merits of the Figure 2 and Figure 3 forms of the Marconi antennas, and not at all with the ground antenna.

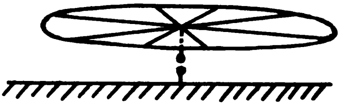

FIGURE 3—"Low" Marconi Antenna of High Power

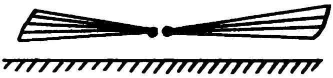

FIGURE 4—Hertzian "Ground" Antenna

3. ILLUSTRATIVE LOW AND ELEVATED ANTENNAS

In order that the comparison between the low and the elevated antenna may be as specific as possible, the proportions and

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[2] F. Kiebitz, "Jahrbuch der drahtlosen Telegraphie," 1912, Volume 5, page 360; Volume 6, pages 1, 554.

constants of an extended horizontal antenna having a height of 10 meters (33 feet) and a radiation figure of merit equal to that of the antenna of the United States Government Naval Station at Darien in the Canal Zone will now be computed. The treatment which follows will then be illustrated by comparing the properties of the "10-meter Darien-equivalent" antenna with the corresponding properties of the Darien antenna.

By the "radiation figure of merit" of an extended antenna is meant the product obtained by multiplying the capacity of the elevated network of the antenna by its height above the lower capacity area—generally the surface of the earth. Two extended antennas having the same figure of merit and located upon a plane of infinite conductivity radiate energy at the same rate if they are operated at the same frequency[3] and at the same voltage between the networks and earth. When in *full oscillation* as receiving antennas, they abstract energy at the same rate from sustained waves, provided that in each case the ohmic resistance of the antenna is made equal to its radiation resistance.

The constants[4] of the Darien antenna are as follows:

Capacity of network to earth	0.01 microfarads
Effective height of network	146. meters
Figure of merit of antenna	1.46 microfarad-meters

Therefore the constants of the 10-meter Darien-equivalent antenna will be:

Height of network	10. meters
Capacity of network to earth	0.146 microfarads
Area of network (approximately)	165000. sq meters
Radius of circular network	229 meters

The network area and radius are first approximations only, calculated upon the assumption that the wires of the network

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[3] The statement that the radiating and absorbing powers of a given extended horizontal platform antenna (having a radius equal to two or more times its mounting height) are independent of the mounting height, is applicable only to the usual condition of operation in radio telegraphy. The usual practice is to have much of the inductance in the tail of the antenna, and to operate at a frequency much lower than the natural frequency of the unloaded antenna. If the inductance in the tail of an antenna is made so low and the operating frequency so high that there is at any instant a large difference in potential between points of the extended capacity area, the case becomes so complex as to transcend present powers of analysis. The above conclusions are only approximations in such a case.

[4] For a description of the Darien Station, see Lieutenant R. S. Crenshaw, "The Darien Radio Station of the U. S. Navy," "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," 1916, Volume 4, page 35. Also L. W. Austin, "Experiments at U. S. Naval Radio Station, Darien, Canal Zone," "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," 1916, Volume 4, page 251.

are so close that the network may be treated as a conducting sheet or plate. The spacing of the wires of the network will be discussed later.

4. Electrical Constants of the High and Low Antenna Circuits

Before attempting to discuss such features of the antenna as the spacing of the wires, conductor resistance, earth resistance, etc., it is helpful to consider the electrical constants of hypothetical antenna circuits free of all resistance save the radiation resistance. Table I is a compilation of the electrical constants of the hypothetical Darien and the hypothetical 10-meter Darien-equivalent antennas for the frequencies corresponding to wave lengths of 2,500 and 10,000 meters. In calculating these constants, the antenna circuits have been regarded as series circuits containing segregated inductance and capacity and a resistance equal to the radiation resistance.

TABLE I

ELECTRICAL CONSTANTS OF HYPOTHETICAL ANTENNA CIRCUITS CONTAINING NO RESISTANCE SAVE THE RADIATION RESISTANCE

	Varies as	Darien Antenna $C=0.01$ μf. $h=146$ meters		10-meter Darien-equivalent Antenna $C=0.146$ μf. $h=10$ meters	
Wave length (meters)		2,500	10,000	2,500	10,000
Frequency (cycles per sec.)		120,000	30,000	120,000	30,000
Inductance (μ henrys)	$h a^{-1} f^{-2}$	176	2,820	12.1	193
Capacity reactance (ohms)	$h a^{-1} f^{-1}$	132	530	9.08	36.3
Critical resistance (ohms)	$h a^{-1} f^{-1}$	265	1,060	18.16	72.6
Radiation resistance (ohms)	$h^2 a^0 f^2$	5.37	0.336	0.0252	0.00158
Current at 100 r. m. s. kv. (amp)	$h^{-1} a f$	754	188	11,020	2,755
Power radiated at 100 kv. (kw.)	$h^0 a^2 f^4$	3,080	12	3,080	12
Time constant (seconds)	$h^{-1} a^{-1} f^{-4}$	0.000066	0.0168	0.00095	0.244
Time constant (periods)	$h^{-1} a^{-1} f^{-3}$	7.87	504	114	7,320
Resistance ratio	$h^{-1} a^{-1} f^{-3}$	49.4	3,160	718	46,000

Referring to Table I,

Line 3 shows the inductance necessary to make the antenna circuit reasonant to the wave lengths and frequencies tabulated in lines 1 and 2.

Line 4 shows the capacity reactance to these frequencies. The inductive reactance is, of course, substantially equal to the capacity reactance.

Line 5 shows the critical resistance R_c of the circuit,—the resistance which would render the circuit just non-oscillatory. The critical resistance is substantially equal to twice the reactance.

Line 6 gives the radiation resistance R_r of the antennas as computed from the formula,

$$R_r = \frac{160\,\pi^2 h^2 f^2}{s^2}$$

The efficiency of the antenna as a radiator is equal to the ratio of its radiation resistance to its total resistance. The low value to which the radiation resistance of the 10-meter antenna falls is striking,—0.00158 ohm for the wave length of 10,000 meters.

Line 7 gives the r. m. s. value of the current which would be observed in the tail of the antenna with a sustained voltage of 100 r. m. s. kilovolts from the capacity network to earth.

Line 8 gives the rate of radiation in kilowatts from the antenna at the above current and voltage. It will be noted that a sustained voltage of 100 r. m. s. kv. between antenna and earth at a frequency of 120,000 cycles per second would mean a rate of radiation of 3,080 k.w. This means that the voltages attained at this frequency under sustained wave operation are far lower than 100 kv.

Lines 9 and 10 show the *time constants* of the circuits, expressed in seconds and in periods respectively. In the case of a receiving antenna, the time constant is the interval of time required, after the alternating voltage is first impressed upon the circuit, for the current and the condenser voltage to build up to 73.2 per cent. of their final or full oscillating values. It is to be noted that the time constant of the hypothetical 10-meter antenna at a frequency of 30,000 cycles is 0.24 seconds. In such a circuit the oscillation would not have sufficient time during a Morse dot interval to build up to its full value, since the approximate length of the dot interval is only 0.05 seconds.

Line 11 gives the "*resistance ratio*" of the hypothetical antennas. By the *resistance ratio* of a series oscillatory circuit is meant the ratio of the *critical resistance* of the circuit to its *actual* resistance. This ratio is equal to twice the ratio of the reactance (at the natural frequency of the circuit) of the inductance, or of the condenser, to the actual resistance of the circuit. Therefore under sustained wave operation in either a receiving or radiating antenna, the ratio of the condenser voltage (voltage from network to ground) to the induced voltage is equal to one-half of the resistance ratio. The high values this ratio attains for the 10-meter antenna should be noted. For example, at a frequency of 30,000 cycles, the condenser voltage would build up to a value equal to 23,000 times the induced voltage.

The column headed "*Varies as*" shows the manner in which the values of the quantities listed above vary with the height and area of the antenna, and with the frequency of oscillation. For example, the time constant (expressed in seconds) of an extended antenna varies inversely as the first power of the height, the first power of the area, and the fourth power of the frequency.

Reference will be made to this table in succeeding sections.

5 POWER LOSSES IN THE ANTENNA CIRCUIT

The efficiency of the antenna circuit as a radiator of electromagnetic energy is the ratio of the power radiated to the total power expenditure in the antenna in radiation, wire and earth resistance, etc. The power radiated from a given extended antenna *at a given frequency* is directly proportional to the square of the current measured in the tail of the antenna. Since the power radiated is proportional to the square of the current, it is convenient to introduce the quantity known as the *radiation resistance*, R_r, of the antenna—a fictitious resistance (not a constant, but a variable having a value proportional to the square of the frequency) of such a value that the product $I^2 R_r$ is equal to the power radiated. It is then convenient to compute the *equivalent resistances* of all other sources of loss for comparison with the radiation resistance. By the *equivalent resistance* of such a source of loss as the ionization of the air around the antenna wires at high voltages is meant a fictitious resistance of such a value that the product obtained by multiplying this resistance by the square of the antenna current will give the power expended as the result of ionization. For a given antenna, these fictitious resistances are not constant, but they are variables the values of which depend on the frequency, and in some cases on the voltage impressed upon the insulating medium in which the loss occurs. The sum of these resistances may be termed the *wasteful*, or *dissipative* resistance, R_w, of the antenna, while the sum of the *radiation* resistance plus the wasteful resistance may be termed the antenna resistance, R_a. If the equivalent resistances of all sources of loss are so expressed, the efficiency of an antenna circuit as a radiator of electromagnetic energy is the ratio of its radiation resistance to its antenna resistance.

When used as a receiver, the energy abstracted from the passing waves by the antenna circuit is expended partly in the detector, partly in the other resistances, and is partly re-radiated. When receiving sustained oscillations, the power expenditure in the detector is a maximum if the equivalent resistance of the detector is made equal to the sum of the radiation resistance plus the wasteful resistance of the antenna. With such an adjustment, the power abstracted by an antenna of given dimensions from sustained waves of a given frequency and intensity and delivered to the detector is inversely proportional to the antenna resistance. Since the radiation resistance of an antenna of given form and dimensions is inherent in the form and dimensions, it follows that the efficiency of a given antenna as a receiver is 100 per cent. if the equivalent resistance of the detector is made equal to the radiation resistance and if the wasteful resistance is made negligibly small in comparison with the radiation resistance. If the wasteful resistance is not negligibly small, the efficiency of the antenna as a receiver is equal to the ratio of the radiation resistance to the total antenna resistance (exclusive of the detector).

Thus in either case, whether used as a radiator or as a receiver, the efficiency of an antenna circuit is equal to the ratio of the radiation resistance to the sum of the radiation plus the other antenna circuit resistances. There are, then, the following resistances to be considered.

R_r representing the radiation resistance of the antenna
R_w representing the wasteful resistance of the antenna
R_d representing the equivalent resistance of the detector (reduced to the antenna circuit)
$R_a=(R_r+R_w)$ representing the antenna resistance
$R_t=(R_r+R_w+R_d)$ representing the total resistance

The regions of power expenditure in the antenna have been listed in Table II. The computed or estimated equivalent antenna resistances of some of these regions of loss have been given in this table for the 10-meter Darien-equivalent antenna at frequencies of 120,000 and 30,000 cycles per second. The methods used in arriving at these equivalent resistances are discussed in succeeding paragraphs.

TABLE II

EQUIVALENT ANTENNA RESISTANCES OF THE TEN-METER DARIEN-EQUIVALENT ANTENNA

(Resistances are in ohms)

Frequency (cycles per second)	120,000	30,000
Wave length (meters)	2,500	10,000
Region of power expenditure		
Common to sending and receiving		
1. Radiation resistance	0.0252	0.00158
2. Conductor resistance—upper network	0.0034	0.0017
3. Conductor resistance—lower network	0.0034	0.0017
4. Earth resistance from surface to buried wires	0.0007	0.0007
5. Earth resistance from choking effect	0.0016	0.0001
6. Extra-peripheral earth resistance	0.0012	0.0006
7. Grass resistance	0.003	0.004
8. Insulator dielectric hysteresis	———	———
9. Insulator wet weather leakage	———	———
10. Supporting poles or structures	avoidable	avoidable
Additional regions when receiving		
11r Tuning inductance and secondary	———	———
12r Detector	———	———
Additional regions when sending		
11. Ionization losses around wires	avoidable	avoidable
12. Ionization losses at insulators	———	———
13. Tuning inductance	———	———
14. Spark or	———	———
14a Poulsen arc generator, and radio frequency transformer or	———	———
14b Radio frequency alternator, and radio frequency transformer	———	———

6. SPACING OF THE ANTENNA WIRES

The radiation resistance of the low antenna is so low that ordinary methods of grounding the antenna may easily lead to a power expenditure in the earth resistance far in excess of the power radiated. An extended low antenna will therefore necessarily contain a lower as well as an upper network of wires. In some cases the lower network may be buried in the earth at a depth of 0.2 meter or more. In other cases it may be advisable to mount the lower network at an elevation of two or three meters above the surface of the earth, in order to avoid the earth losses discussed later. In either case, the lower network should not only be co-extensive with the upper network, but it should extend beyond the periphery of the upper network to a distance of 30 meters (100 feet) or more, and it should be thoroly grounded at many points of its periphery.

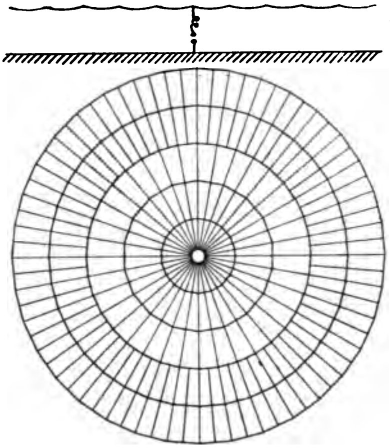

FIGURE 5

The conductors constituting either network will necessarily issue from common points in a radial manner, either as the spokes of a wheel as in Figure 5 or as the ribs of a fan. The diameter of the conductor used in the networks will be determined by the requirement of mechanical strength. To meet this requirement, it is assumed in the following calculations that a hard drawn copper wire (or a copper clad steel-core wire) having a diameter of 4.1 mm. (0.16 inch) (number 6 B. and S. gauge) will be used in networks above the ground and a copper wire having a diameter of 3.3 mm. (0.13 inch) (number 8 B. and S. gauge) in buried networks.

The greater the mean distance between the wires of a network, the less will be the weight of the copper required to obtain a given capacity, and the less will be the mechanical difficulties of suspension. On the other hand, the less the distance between the conductors, the less will be the ohmic resistance of the network, the smaller will be the ground area required to obtain a given capacity, and the higher will be the sending voltage between network and earth which may be used without incurring ionization losses in the air around the conductors.

The effect of the spacing of the wires upon the steady-state capacity to earth of a horizontal harp of given area made up of 4.1 mm. (0.16 inch) wires stretched parallel to each other at an elevation of 10 meters above the ground is shown in Figure 6.

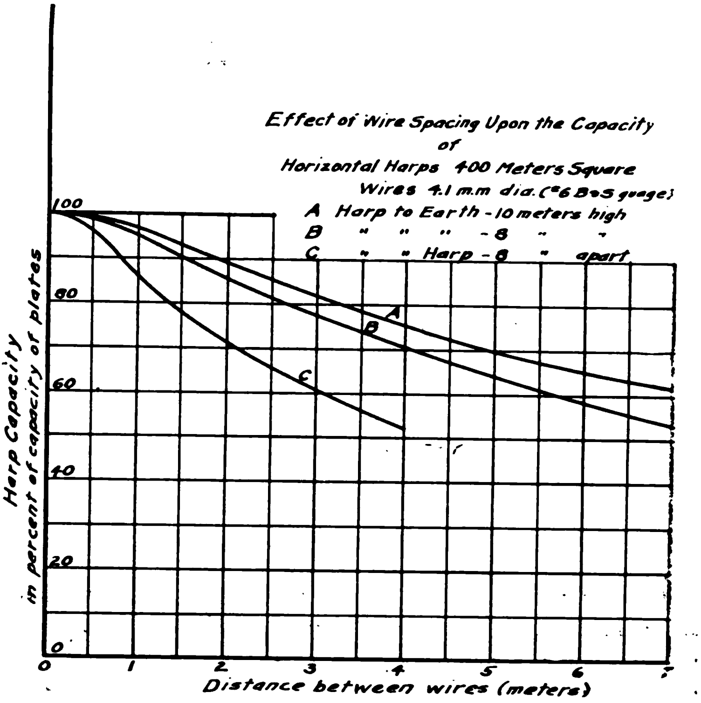

FIGURE 6

The points for this curve have been computed by Kelvin's method of images for a square harp 400 meters (1,350 feet) on a side. The computations are based upon the assumption that the charge per unit length of wire is uniform over the entire harp. As a matter of fact, the charge per unit length of wire is somewhat greater near the edges of the harp than near its center, but the error in the curves resulting from the assumption of uniform distribution is very slight indeed. The capacities of the harps with different wire spacings have been plotted in per cent. of the capacity which the same area would have if it were made up of a continuous sheet, or plate, of conducting material. Figure 6 also contains a similar curve for the capacity to earth of a harp 8 meters (25 feet) above the earth, and also for the capacity between two parallel wire harps mounted 8 meters apart in space.

No ionization losses in the air surrounding the conductors are to be expected if the electric intensity or potential gradient at the surface of the conductors during the peak of the voltage wave does not exceed 30 peak kilovolts per cm. Figure 7 shows for different spacings of 4.1 mm. (0.16 inch) wire the root-mean-square value of the voltage between the harp and earth which will give rise to a gradient of 30 kilovolts per cm. at the surface of the wire. Since corona does not form around polished 4.1 mm. wires except at gradients 67 per cent. greater than 30 kv. per cm., these voltages may, somewhat arbitrarily, be designated as the "safe non-ionizing sending voltages."

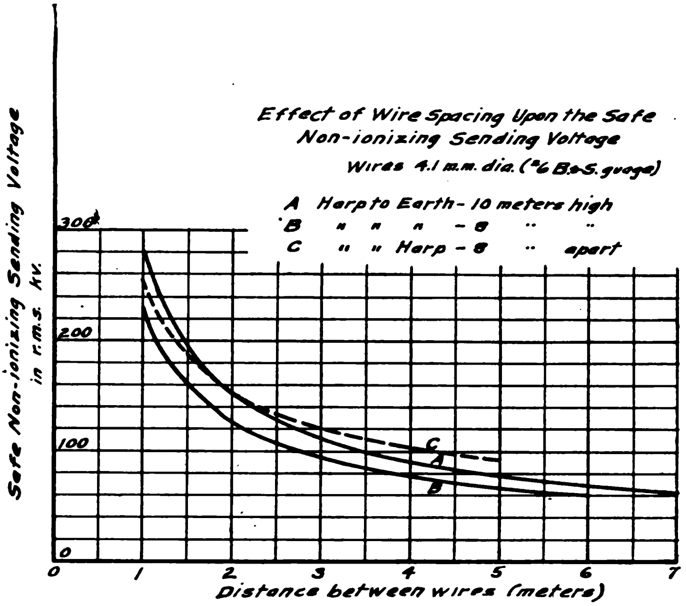

FIGURE 7

The voltages plotted in Figure 7 may be obtained as follows: The electric displacement or the electrostatic flux density, D, at the surface of the wire under a gradient, F, of 30,000 volts per cm. is

$D = pF = 30{,}000\, p = 30{,}000\ (8.84 \times 10^{-14})$ coulombs per sq. cm.

(p represents the permittivity of air)

The quantity of electricity Q per centimeter length of 4.1 mm. wire at this flux density is

$$Q = 0.41\,\pi\, D = 3.42 \times 10^{-9} \text{ coulombs}$$

The voltage to earth necessary to cause this charge per centimeter length of wire may readily be obtained from the capacity curves plotted in Figure 6.

An inspection of Figure 6 shows that the capacity of the 10-meter Darien-equivalent antenna is less than the capacity between plates of the same dimensions by only 4 per cent. if the wires are spaced 1 meter apart, by 11 per cent. with a spacing of 2 meters (6 feet 7 inches), 18 per cent. at 3 meters (9 feet 10 inches), and 25 per cent. at 4 meters (13 feet 1 inch). Considering the effect of the wire spacing upon the capacity of the antenna, a spacing of at least 3 or 4 meters is preferable if land is cheap.

An antenna designed to be used mainly at the lower frequencies (20,000 to 50,000 cycles) will, for two reasons, require a closer spacing of the antenna wires than if it were designed to be used at the high frequencies.

The first reason for a closer spacing of wires in low frequency stations is that high operating voltages must be used in order to radiate any great amount of power at the low frequencies. For example, the rate of radiation from the Darien antenna, and from the 10-meter Darien-equivalent antenna, at a frequency of 30,000 cycles is only 12 kw. at 100 r. m. s. kv. (See Table I). The higher the operating voltage, the closer must be the spacing of the antenna wires if ionization losses at the conductors are to be avoided. An inspection of Figure 7 shows that for a working voltage of 100 r. m. s. kv. the distance between wires need not be less than 3.5 meters (12 feet). In stations operated from Poulsen arcs at the higher frequencies (120,000 cycles), the sending voltage will not exceed 40 r. m. s. kv., since the rate of radiation of the Darien antenna at this frequency and at 100 r. m. s. kv. is 3,080 kw.—a rate 8 times as great as the capacity of any radio frequency generating plant at present in use. For a voltage of 40 kv. a wire spacing of 10 meters would be sufficient to avoid ionization losses around the wires.

The second reason for the closer spacing of wires in low frequency stations is the low radiation resistance of the antenna at the low frequencies. At $1/n$th the frequency, the radiation resistance is $1/n^2$ as great. From this it follows that to obtain the same antenna efficiency at two different frequencies (either in sending or receiving), the conductor resistance for the low frequency antenna should be made $1/n^2$ as great as that of the high frequency antenna. This lower antenna resistance can in practice be obtained only by using more wires spaced closer together.

7. RESISTANCE OF ANTENNA WIRES

With the wires of the capacity area arranged as in Figure 5, namely, with 270 copper wires of 4.1 mm. (0.16 inch) diameter (number 6 B. and S. gauge) extending in a radial manner from a common point to a circle having a diameter of 280 meters (918 feet), and with 540 wires extending radially from this circle to a circle having a diameter of 460 meters (1,508 feet), the direct current resistance of the network from the common point to the center of load is approximately 0.0006 ohms. The alternating current resistance of 4.1 mm. (0.16 inch) copper wire at frequencies of 30,000 and 120,000 cycles is 2.9 and 5.6 times its d. c. resistance. Therefore the a. c. resistance of the network will be 0.0017 and 0.0034 ohms to the two frequencies. At 30,000 cycles the resistance of the conductors in the upper network is substantially equal to the radiation resistance, while at 120,000 cycles the conductor resistance is only 14 per cent. of the radiation resistance. The conductors of the lower network will be regarded as arranged to have substantially the same resistance as the upper. For example, if buried, the lower network might consist of 360 copper wires of 3.3 mm. (0.13 inch) diameter (number 8 B. and S. gauge), extending in a radial manner to a circle having a diameter of 280 meters (918 feet) with 720 wires extending from this circle to a circle having a diameter of 520 meters (1,705 feet.)

8. LOSSES IN THE EARTH

If the lower network is buried beneath the surface of the earth, or is laid upon the surface of the earth, the damping losses due to the flow of current in the earth may be considered under three headings.

The specific resistance of the earth is so high in comparison with that of copper that the current which flows radially inward and outward from the ground end of the antenna tail is carried mainly by the buried network. Conduction currents flow from the earth's surface (at which the lines of displacement from the upper network terminate) and converge upon the conductors of the buried network in the manner shown in Figure 8. That resistance which when multipled by the square of the antenna current will give an I^2R product equal to the power expenditure due to the currents pictured in Figure 8, will be termed the "*earth resistance from surface to buried conductors.*" This is the first resistance to be considered.

Altho the current which flows out radially from the foot of the antenna tail is carried mainly by the buried network, still a part of the current streams out thru the earth itself, being confined to a surface layer the depth of which varies from about one meter (3.28 feet) for sea water at a frequency of 120,000 cycles to 100 meters (328 feet) for earth of fairly high resistivity at a frequency of 30,000 cycles. While the conductance of the earth is in parallel with the conductance of the buried network and would at first sight seem to give rise to a smaller loss than if the network alone carried the current, yet paradoxically, the result is just the opposite. The effective resistance of the two conductors in parallel—the buried wires and the conducting earth—is greater than the resistance of the wires alone. The explanation of this paradox is that the concentration of the current in the buried wires leads to the distribution of magnetic flux about the wires pictured in Figure 9. As a result, the voltage consumed by inductance is greater in the copper conductor than in like filaments of earth at some distance from the conductor. This means that the high resistance earth filaments at some distance from the wire are forced to carry a greater current than they would carry with a continuous electromotive force impressed, and the resultant loss is greater than if the copper conductors carried all the current. That resistance which when multiplied by the square of the antenna current will give an I^2R product equal to the power expenditure due to the earth currents just pictured, will be termed the "*earth resistance from the choking effect.*" This is the second resistance to be considered. Both this resistance and the *earth resistance from the surface to buried conductors* may be eliminated by mounting the lower network at a height of 1 to 3 meters (3 to 10 feet) above the surface of the earth.

It is not feasible to extend the lower network much beyond the periphery of the upper. Lines of displacement originating on the upper network terminate, as illustrated in Figure 10, upon the surface of the earth at points beyond the limits of the lower network. From the termination of the lines of displacement upon the earth's surface, conduction currents flow in the surface skin of the earth, converging in radial lines upon the grounded periphery of the lower network. That resistance which when multiplied by the square of the antenna current will give an I^2R product equal to the power expenditure caused by these currents, will be termed the "*extra-peripheral earth resistance.*" This is the third resistance to be considered.

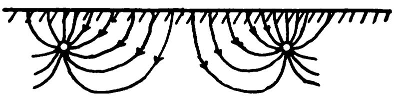

FIGURE 8—Current from Earth's Surface to Buried Wires

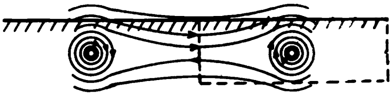

FIGURE 9—Distribution of Magnetic Lines About the Buried Wires

Wires and current flow are perpendicular to the plane at the paper

Earth resistance from surface to buried wires. The resistivity of the earth in which the network is buried will lie between the limits 1,000 ohm-cms. and 100,000 ohm-cms. In the subsequent calculations a resistivity of 10,000 ohm-cms. will be assumed.

With the 137,000 meters (449,400 feet) of 3.3 mm. (0.13 inch) (number 8 B. and S. gauge) wire buried at a depth of 0.25 meter (0.82 feet), and with an average spacing of approximately 1.5 meters (4.9 feet) between wires, the computed resistance from the surface of the earth to the buried conductors is of the order of 0.0007 ohms. This resistance is equal to 44 per cent. and 3 per cent. of the radiation resistances at 30,000 and 120,000 cycles respectively. When the ground is frozen a resistance much higher than the value above computed may be expected.

Earth resistance from the choking effect. As pointed out above, the concentration of the current in the buried wires leads to the distribution of magnetic lines of force pictured in Figure 9. As a result, the voltage consumed by inductance is greater in the case of a filament of the copper wire than in the case of a parallel filament of earth at some distance from the wire. In other words the "choking action" of the flux which encircles the copper wire but does not encircle the earth filament forces the earth filament of high resistivity to carry more current than would be determined by the ratio of the conductances of the two filaments.

The ratio of the current density in the film of earth on the surface of the copper wire to the current density in the surface film of copper will equal the ratio of the conductivity of earth to the conductivity of copper. For earth of the conductivity previously assumed, this ratio is 10^{-4} to (5.5×10^{5}) or 1 to (5.5×10^{9}).

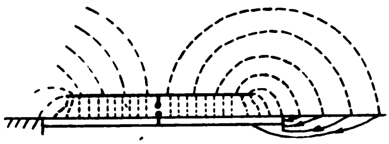

Figure 10—Current Causing Extra-peripheral Loss

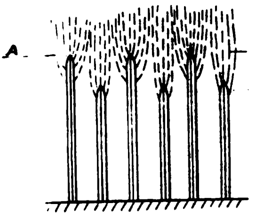

Figure 11—Lines of Displacement from Upper Portions of Grass

As the film of earth under consideration is taken farther and farther from the surface of the copper wire, the current density increases with the distance in the manner shown in Table III. This table has been computed for a 3.3 mm. (0.13 inch) wire and an earth resistivity of 10,000 ohm-cms. at a frequency of 30,000 cycles per second. There is some uncertainty as to the correctness of the values given for the current densities at distances greater than 5 centimeters (2 inches) from the center of the wire.

An extreme upper limit for the magnitude of the I^2R loss occasioned by these earth currents may be rapidly arrived at in the following way: Assume that the current density in the entire block of earth enclosed by the dashed lines in Figure 9 is 30 times as great as the actual current density in the earth adjacent to the wire. This block of earth, containing a number 8 wire at its center, is 1.5 meters (4.9 feet) wide by 0.5 meters (1.6 feet) deep. For the above current density the current carried by the earth works out to be 0.15 of one per cent. of the current carried by the wire, and the loss in the earth works out to be 6 per cent. as great as the loss in the wire. Since the conductor resistance of the lower network is 0.0017 ohms, the *earth resistance from the choking effect* at a frequency of 30,000 cycles is 0.0001 ohms, or it is 6 per cent. as great as the radiation resistance. At 120,000 cycles the current flowing in the earth will be four times as great as at 30,000 cycles, or it will be 0.6 of one per cent. of the current flowing in the wire. The equivalent resistance will be 16 times as large, or 0.0016 ohms. This resistance is 6 per cent. as great as the radiation resistance at 120,000 cycles. It is interesting to note that the higher the conductivity of the earth (within the limits encountered in practice), the greater will be the loss now under discussion. For example, if the resistivity of the earth is 1,000 ohm-cms., the loss and the resistances will be ten times as great as the values estimated above for a resistivity of 10,000 ohm-cms.

TABLE III

CURRENT DENSITY IN THE EARTH IN TERMS OF THE DENSITY IN THE EARTH AT THE SURFACE OF THE WIRE

Distance from center of wire		Current density
r	0.163 cm.	*D*
2 *r*	0.326 cm.	4.5 *D*
3 *r*	0.489 cm.	6.6 *D*
4 *r*	0.652 cm.	8.2 *D*
5 *r*	0.815 cm.	9.4 *D*
6 *r*	0.98 cm.	10.3 *D*
8 *r*	1.30 cm.	11.9 *D*
10 *r*	1.63 cm.	13.1 *D*
20 *r*	3.26 cm.	16.8 *D*
40 *r*	6.72 cm.	20.6 *D*
60 *r*	9.8 cm.	22.7 *D*
100 *r*	16.3 cm.	25.5 *D*
200 *r*	32.6 cm.	29. *D*
400 *r*	67.2 cm.	33. *D*
600 *r*	98. cm.	35. *D*

r represents the radius of the wire

D represents the current density in the earth at the surface of the wire

Extra-peripheral earth resistance. An estimate of the magnitude of this resistance for the 10-meter (33-foot) Darien-equivalent antenna at a frequency of 30,000 cycles may be arrived at in the following manner:

1. Let the lower network, which may either be buried or mounted above the surface of the earth, be assumed to extend 30 meters (98 feet) beyond the upper network, and to be thoroly grounded at frequent intervals around its circular periphery.

2. Imagine the surface of the earth beyond the grounded periphery to be divided into narrow circular zones for a distance of a quarter wave length (2,500 meters) (8,200 feet) beyond the periphery. Let the steady state surface density of charge over each of these zones corresponding to a given uniform surface density of charge q upon the upper network be calculated. This calculation may be made by introducing the image with reference to the earth's surface of the charge upon the upper network, and then by an arithmetical integration determining the electric intensity set up at the mid point of each zone by the two distributed charges. The error in the final result occasioned by the assumption of uniform surface density on the upper network is very slight in comparison with the errors which may be due to the assumption of steady state conditions.

3. Let the assumption be made that when the antenna is oscillating the current which flows radially inward across any zone toward the periphery of the grounded network is the current necessary to supply the computed surface density of charge beyond the zone in question.

4. Let the current crossing each zone be so computed; and let the $I^2 R$ loss due to the flow of current across each zone be then computed on the basis that these currents flow in a surface layer or skin of the earth whose depth is taken as

$$\frac{5033}{\sqrt{\gamma f}} \text{ centimeters.}$$

γ represents the conductivity of the earth.

5. Let the power expenditure in all the zones be summed up, and a resistance be then computed which multiplied by the square of the antenna current will give an $I^2 R$ product equal to this power expenditure.

In the manner thus outlined, I have obtained 0.0006 ohms as the estimated equivalent resistance of the losses beyond the periphery of the grounded network at a frequency of 30,000 cycles. It will be noted that this treatment is far from rigorous. The problem is not susceptible of rigorous treatment, and it is extremely difficult to visualize the state of affairs in the vicinity of an *actual radiating antenna.* In the judgment of the writer, the extra-peripheral earth resistance will not exceed the value estimated above by more than 100 per cent., and may possibly be somewhat less than the estimate. The extra-peripheral resistance estimated above is 38 per cent. as great as the radiation resistance. At 120,000 cycles the extra-peripheral resistance may be put at something less than twice the above value—twice since the skin thickness of the earth at 120,000 cycles is only one-half as great as at 30,000 cycles. A resistance of 0.0012 ohm at 120,000 cycles is an extra-peripheral resistance of only 5 per cent. of the radiation resistance at that frequency.

9. LOSSES IN THE GRASS

Of greater magnitude than the power losses in the earth is the power loss which may be occasioned by conduction currents flowing in grass or vegetation growing under the antenna. This loss cannot be experimentally determined, but its order of magnitude may be estimated for the case in which the antenna is mounted over a meadow or lawn. The conditions in such a case are crudely illustrated in Figure 11. Very little of the displacement terminates upon the surface of the earth, but the lines of displacement from the upper network terminate upon the upper portions of the blades of grass in the manner illustrated in the figure. Conduction currents flow from these upper portions thro the blades of grass to the earth. The loss under consideration is the I^2R loss caused by these conduction currents.

As a justification of these statements, let us compare the condensive reactance and the ohmic resistance from the earth to a plane *AB*, Figure 11, touching the longer blades of grass. Consider a square meter (10.7 square feet) of the lawn under an antenna. The grass may be assumed to be 5 cm. (2 inches) deep. The condensive reactance from the surface of the earth to the plane *AB* one meter square and 5 cm. distant at a frequency of 30,000 cycles is 30,000 ohms. For the purpose of this calculation it may be assumed that at intervals of two centimeters (0.8 inch) a blade of grass extends about one centimeter (0.4 inch) above the blades of the next shorter group of blades, as illustrated in the figure. Ordinary lawn grass blades, 0.3 cm. (0.12 inch) wide by 0.018 cm. (0.007 inch) thick, have a resistance of about 1,000,000 ohms per cm. (0.4 inch) of length. If all the conduction current were carried by the longer blades, of which there are 2,500 per sq. m., (10.7 square feet), the ohmic resistance per square meter would be 2,000 ohms. Since the condensive reactance is 15 times as great as this ohmic resistance, this justifies the statement that the lines of displacement terminate mainly upon the upper portions of the blades of grass, and not upon the earth's surface.

The resistance of 2,000 ohms per square meter has been arrived at upon the assumption that only the longer blades of grass carry the conduction current. Of course some of the lines of displacement terminate on the shorter blades, and the grass instead of standing vertically upright, as in the illustration, is greatly matted. Lines of displacement spring from blade to blade, and near the surface of the earth many more blades take part in the conduction of current. Under these conditions, any estimate of the resistance may be considerably in error. A resistance of 2,000 ohms per sq. m. is probably an upper limit. I would estimate the resistance to be about one-third of this or 700 ohms per sq. m. As the total area under the 10-meter (33 feet) Darien-equivalent antenna is 165,000 meters (1,760,000 square feet), the equivalent resistance of the grass losses is estimated to be of the order of 0.004 ohm. This resistance is equal to 2.5 times the radiation resistance. That is, at 30,000 cycles 2.5 kw. will be expended in heating the grass per kilowatt radiated.

At 120,000 cycles the condensive reactance per square meter (10.7 square feet) is one-quarter of 30,000 ohms, or 7,500 ohms. This means that displacement will take place more readily from blade to blade and the shorter blades will take a greater part in conveying the conduction current in the lower depths of the grass. The grass resistance will, therefore, be somewhat lower than at 30,000 cycles, possibly about 0.003 ohms. This is of the order of 12 per cent. of the radiation resistance. The grass loss may be eliminated by mounting the lower network at an elevation of 1 to 3 meters (3.3 to 9.9 feet) above the ground.

10. Pole or Steel Structure Losses

If wooden poles are used to support the antenna, the conduction currents which will flow in the poles may occasion a loss as high as 10 kw. per pole under a sustained voltage of 100 r. m. s. k.v. It will be absolutely necessary to cover wooden poles with galvanized iron or copper netting. This will make the loss inappreciable.

Steel supporting structures will have a resistance of the order of 0.1 ohm. Any conduction currents the structures may carry will cause an inappreciable loss.

11. Insulator Losses—Dielectric Hysteresis and Wet Weather Leakage

The writer is not in possession of data for estimating the magnitude of these losses. They will necessarily be greater in the case of the low antenna than in the case of the high, because it is necessary to support the low antenna at a greater number of points.

12. CONCLUSIONS

From Table II, showing the computed resistances of the 10-meter (33-foot) Darien-equivalent antenna, it may be seen that at the higher frequency of 120,000 cycles the wasteful resistance of the antenna (exclusive of the equivalent resistance of the insulators, the ionization losses and the tuning inductance) is only 53 per cent. of its radiation resistance. At the frequency of 30,000 cycles, the wasteful resistance is 5.5 times the radiation resistance. This means that as an absorber of electromagnetic energy from sustained waves, the efficiency of the low antenna (neglecting for the present the losses noted above) is 65 per cent. at 120,000 cycles and 15 per cent. at 30,000 cycles.

It should be noted, however, that if the lower network is not buried but is mounted several meters above the earth, the grass losses and all the earth losses save the "extra-peripheral earth loss" are eliminated. Under these conditions the efficiency of the 10-meter Darien-equivalent antenna at 120,000 and 30,000 cycles becomes 76 per cent. and 28 per cent. respectively.

The resistances of the existing elevated high power radio antenna have not been worked out in the detailed manner exhibited in Table II, but the best stations are reported to have wasteful resistances not less than 0.6 ohms. Assuming the wasteful resistance of the Darien antenna to be 0.6 ohms, the efficiency of the Darien antenna as an absorber of energy from sustained waves is 90 per cent. at 120,000 cycles and 36 per cent. at 30,000 cycles.

These efficiencies have been compiled in Table IV.

TABLE IV

COMPUTED EFFICIENCIES OF HIGH AND LOW ANTENNA (EXCLUSIVE OF INSULATOR, IONIZATION, AND TUNING INDUCTANCE LOSSES)

Frequency	120,000	30,000
Darien-equivalent antenna		
with lower network buried	65 per cent	15 per cent.
with lower network unburied	76 per cent	28 per cent.
Darien antenna	90 per cent	36 per cent.

The comparison of efficiencies is somewhat unfavorable to the low antenna at the lower frequencies. The relative merit of high and low antennas is determined not alone by the radiating and absorbing efficiencies of the two types, but also by considerations having to do with the power generating devices and the selective reception of signals under the conditions of commercial operation. For example: the function of a receiving antenna is not simply "to deliver to a detecting device energy abstracted from impinging waves." Its function is "to *selectively* abstract energy from impinging waves and to deliver it to the detecting device." That is to say, if antenna *H* is able to deliver to the detector twice as much energy from the *correspondant* station as antenna *L*, it does not follow that antenna *H* is superior to antenna *L*, since antenna *H* may at the same time deliver to its detector—not twice—but ten times as much energy from "*strays*" or *interferent stations* as does antenna *L*.

The ultimate comparison of the relative merits of two antennae for receiving purposes *under the conditions to be met in commercial operation* must be a comparison of the amounts of energy delivered to the detectors from the desired correspondant when the strengths of the interfering signals or noises have been reduced to the same intensity in the two cases. As previously stated, it is the purpose to discuss these other aspects of feasibility in subsequent papers.

SUMMARY: This paper supplements a previous publication entitled "High versus Low Antennas in Radio Telegraphy," in which it is shown that if the antenna of a radio telegraph station consists of an extended horizontal network of wires mounted above a highly conducting plane, and if the mean radius of the capacity area is two or more times as great as any height above the plane at which it is feasible to mount the network, then the rate of radiation from the antenna at a given voltage and frequency, and the rate at which the antenna will ultimately be able to abstract energy from impinging (sustained) waves, are both independent of the mounting height.

The conclusions in the previous paper are the result of a mathematical analysis for the hypothetical case in which the antenna is mounted over a highly conducting plane. The present paper deals mainly with those wasteful antenna and earth resistances which are common to the use of the low antenna both in sending and receiving. The distinction between the "low" antenna and the "ground" antenna is pointed out. The electrical constants of an antenna having the same radiation figure of merit as the Darien (Canal Zone) antenna, but mounted at an elevation of only 10 meters (33 feet), are contrasted with those of the Darien antenna. The wasteful resistances of this 10-meter Darien-equivalent are then computed. Of these losses, that in the vegetation growing under the antenna is found to be the most serious. The efficiency of the 10-meter Darien-equivalent antenna, with its lower capacity area not buried but mounted above the ground, is computed to be of the order of 76 per cent. for frequencies of 120,000 cycles per sec. and 28 per cent. for 30,000 cycles. These efficiencies are slightly lower than those reported for existing high-power elevated antenna.

APPENDIX A

ELECTROMAGNETIC RADIATION

A compilation* of the expressions applying to stations in which the antenna is a horizontal network the radius of which is large (ten times) in comparison with its mounting height, but small (one-tenth) in comparison with the wave length at the operating frequency. The stations are assumed to be on an infinitely extended plane of infinite conductivity.

A_o, A_1	represent the areas in sq. cm. of the sending and receiving networks
h_o, h_1	represent the heights in cm. of the sending and receiving networks
Q	represents the maximum charge (in coulombs) on the upper network at the sending station
r	represents the distance in centimeters to the receiving station
f	represents the frequency
p	represents the permittivity of air $=8.84\times10^{-14}$ coulomb-volt-cm.
s	represents the velocity of propagation $=3.0\times10^{10}$ cm. per sec.

NOTE: All values of current, voltage, and electric and magnetic intensity are r. m. s. values.

Sending station values

Current (amperes)	$I_s=\sqrt{2}\pi f Q$	(1)
Voltage to earth (volts)	$E_s=\dfrac{Q h_o}{\sqrt{2}\, p A_o}$	(2)
Rate of radiation (watts-hemisphere)	$P=320\pi^4 Q^2 h_o^2 f^4 s^{-2}$	(3)
Rate of radiation (watts-hemisphere)	$=160\pi^2 h_o^2 f^2 s^{-2} I_s^2$	(4)
Rate of radiation (watts-hemisphere)	$=\dfrac{4}{90}\pi^2 A_o^2 f^4 s^{-4} E_s^2$	(5)

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*Compiled from Bulletin 810, University of Wisconsin, "High Versus Low Antennas in Radio Telegraphy."

Radiation resistance (hemisphere-ohms)	$R_r = 160\,\pi^2 f^2 h_o^2 s^{-2}$	(6)
Critical resistance (ohms)	$R_c = 2\sqrt{\frac{L}{C}} = \frac{1}{\pi f C} = \frac{h_o}{\pi f p A_o}$	(7)
Logarithmic decrement (due to radiation)	$\delta = 2\pi \frac{R_r}{R_c} = \frac{8}{3}\pi^3 f^3 h_o A_o s^{-3}$	(8)
Time constant (in periods)	$T_c = \frac{3\,s^3}{8\,\pi^3 f^3 h_o A_o}$	(9)
Resistance ratio R_c/R_r	$R_c/R_r = 2\,\pi\,T_c = \frac{3\,s^3}{4\pi^2 f^3 h_o A_o}$	(10)

At distant points

Electric intensity (volts per cm.)	$F = 120\sqrt{2}\,\pi^2 Q\, h_o f^2 (s\,r)^{-1} \sin\theta$	(11)
Magnetic intensity (amp-turns per cm.)	$H = \sqrt{2}\,\pi Q\, h_o f^2 (s\,r)^{-1} \sin\theta$	(12)
Power flow—average value (watts per sq. cm.)	$P_1 = 240\,\pi^3 Q^2 h_o^2 f^4 (s\,r)^{-2} \sin^2\theta$	(13)
Power flow—average value	$= 120\,\pi\, h_o^2 f^2 I_s^2 (s\,r)^{-2} \sin^2\theta$	(14)
Power flow—average value	$= \frac{\pi A_o^2 f^4 E_s^2}{30\,s^4 r^2} \sin^2\theta$	(15)

Receiving station values

Induced voltage (volts)	$E = \frac{120\sqrt{2}\,\pi^2 f^2 h_o h_1 Q}{s\,r} = \frac{120\pi f h_o h_1 I_s}{s\,r} = \frac{2\pi f^2 A_o h_1 E_s}{s^2\,r}$	(16)
Radiation resistance	$R_r = \frac{160\,\pi^2 f^2 h_1^2}{s^2}$	(6)

Final values when receiving sustained waves with radiation resistance alone in the antenna:

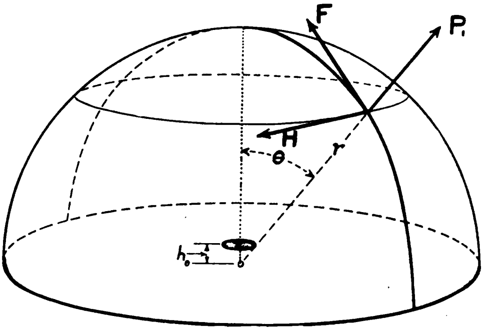
Current (amperes)	$I = \frac{3\,s\,h_o Q}{2\sqrt{2} r h_1} = \frac{3\,s\,h_o I_s}{4\pi f r h_1} = \frac{A_o E_s}{80\pi\, r\, h_1}$	(17)
Condenser voltage	$E = \frac{90\,s^2 h_o Q}{\sqrt{2} r f A_1} = \frac{45\,s^2 h_o I_s}{\pi f^2 r A_1} = \frac{3\,s\,A_o E_s}{4\,\pi f r A_1}$	(18)
Power re-radiated (watts-hemisphere)	$P = \frac{180\,\pi^2 f^2 h_o^2 Q^2}{r^2} = \frac{90 h_o^2 I_s^2}{r^2} = \frac{f^2 A_o^2 E_s^2}{40\,r^2 s^2}$	(19)

Time constant (in periods) $T_c = \dfrac{3\,s^3}{8\,\pi^3 f^3 A_1 h_1}$ . . . . (20)

$\dfrac{\text{Final condenser voltage}}{\text{Induced voltage}} = \pi\, T_c = \dfrac{3\,s^3}{8\,\pi^2 f^3 A_1 h_1}$ . . . . (21)

In order to abstract and utilize the maximum power possible from the impinging electromagnetic waves, the detector resistance R_d should be made equal to the sum of the radiation resistance plus the wasteful resistances of the antenna circuit. If the wasteful resistances are negligibly small in comparison with the radiation resistance, then for maximum power utilization the detector resistance should be made equal to the radiation resistance. Under these conditions—

The final value of the current equals ½ of the above value.
The final value of the condenser voltage equals ½ of the above value.
The final value of the power utilized equals ¼ of the above value.
The final value of the power re-radiated equals ¼ of the above value.



Relation Between Electric and Magnetic Intensities

DISCUSSION

L. W. Austin (by letter): The success of Professor Bennett's plan seems to depend entirely on his ability to keep down the wasteful resistance of his antenna. I am afraid that this will prove more difficult than his calculations indicate. The minimum resistance of the best modern high power antennas, exclusive of the inductance and radiation resistances, lies between one and two-and-one-half ohms, and it seems improbable that even the most elaborate counterpoise system can ever reduce this much below one-half ohm. The problem of a further reduction to less than one-hundreth ohm will certainly be a difficult one.

L. L. Israel: If I were an engineer designing a transmitting station with the low antenna described to-night, I would be somewhat troubled by the design of the loading inductances for carrying from 1,000 to 10,000 amperes, in spite of the fact that the inductance would be somewhat less than is usual with high antennas. The difficulties in the way of placing coil conductors so that they are out of the intense magnetic fields are considerable.

The next difficulty that comes to mind is to get rid of the losses due to the magnetic fields in the antenna itself. Lowering the antenna not only increases the loss in the static field but it also increases the loss in the magnetic field. Experience with high power stations has shown that the loss in poor conductors cut by intense magnetic fields rises to enormous amounts. In one high power station the loss in concrete walls cut by the magnetic field of the loading coil was high enough to make the wall burning hot. Undoubtedly the ground in the region of the center of the low antenna is cut by strong magnetic fields with consequent high eddy current loss. It would be interesting to have a resistance figure computed for this eddy loss in the ground. I suspect that it would considerably increase the totals given by Dr. Bennett.

I would be glad to offer an explanation for an observation mentioned; namely that altering the length of the counterpoise alters antenna ammeter reading at constant output, as I once had the same trouble in similar experiments.

When the counterpoise length is changed the position of the potential node changes too, so that even tho the antenna input and resistance are the same the current indication varies. By connecting the ammeter each time at the potential node, the antenna current changes are small or at least more in conformity with what one would expect from the other conditions of the experiment.

There is another interesting point in this paper worthy of mention, namely, the variation of radiation resistance with the height of the antenna. Dr. Bennett's research is based on the assumption that the radiation resistance varies as the square of the height of the antenna. This may be true for antennas which have a small flat top area in comparison with their height, but I doubt whether it is true for antennas which have a very large flat top area. If we apply the square law of radiation resistance to ground antennas, we are hard put to it to see why the ground antennas work at all. I feel that a paper supplementing this one and taking up this aspect of the matter would be very helpful. There are a great many engineers in the field who become confused by the new facts which are being discovered in the use of ground antennas.

At one time we thought that a ground antenna received because of sloping wave fronts, but the efficient working of ground antenna in sea water leaves us "high and dry." And how can we explain antennas working thousands of feet below the earth's surface?

These low antennas must change the energy distribution or form of the wave so that more of the wave energy is absorbed. Such variations of radiation resistance from the square law contribute greatly to the difficulty of the low antenna problem. They tend to increase the radiation resistance of the low antenna, minimizing the importance of the losses, and bringing the whole problem into the class that can only be satisfactorily solved by physical trial. Dr. Bennett's deductions will certainly form a most valuable guide thereto.

Alexander E. Reoch: About one year ago, I read extracts from a "Bulletin" by Dr. Bennett, entitled "High Versus Low Antennas for Radio Telegraphy," and having considerable interest in the design of large antennas, I obtained a copy of the bulletin. The first part of the bulletin arrives at conclusions which are generally accepted; but, with respect to the second part, I came to the conclusion that the author had not given much consideration to the wasteful resistance in antennas of present-day type. A complete understanding of the subject would be much easier for the practical engineer if the energy in the antenna was given in terms of the current rather than the voltage. The antenna current in antennas of different capacity, maintained at the same voltage, varies considerably; and if the total resistance of the antenna remains about the same the power required to maintain the same voltage will likewise vary. The total resistance of the antenna consists of useful radiation resistance and wasteful resistance. The radiation resistance varies with the height but the wasteful resistance does not necessarily so vary.

In this bulletin is cited a case of an antenna 10 meters high with a radiation resistance of 0.011 ohm and it is suggested that the total resistance of this antenna would be 0.12 ohm. In which case the radiation efficiency would be 9 per cent.

It will be generally acknowledged, however, that in an antenna of these dimensions, with a natural wave length of the order of 4,000 meters, the total resistance cannot be reduced at the best to less than 1 ohm. In this case then, the radiation efficiency would become 1.1 per cent. and 98.8 per cent. of the energy supplied to the antenna would be utilized in overcoming wasteful resistance.

If the wasteful resistance, instead of being the controlling factor, was small compared with the radiation resistance, then the argument of this first paper would be well based and antennas of this type would call for practical consideration.

It is evident that the assumption that the total resistance of a large antenna can be reduced to an exceedingly small figure has been carried into the present paper.

That this is not the case is fully borne out by published information concerning the Marconi stations in Ireland, at Glace Bay, and New Brunswick, the arc stations at Arlington, San Francisco, and Darien, and the Sayville and Tuckerton stations. From this information, while no exact measurements are given, it can be deduced that the minimum total resistance is about 2 ohms.

Assuming the total resistance of the Darien antenna to be 2 ohms, and the radiation resistance to be 0.3 ohm, there is a wasteful resistance of 1.7 ohms to be accounted for.

Lieutenant Crenshaw in a description of the Darien station described how an antenna inductance situated near a reinforced concrete wall caused the wall to become exceedingly hot (as mentioned by Mr. Israel). Large stations have been built with umbrella antennas with a central tower of lattice work steel and the main lead to the antenna has been run parallel to this tower at a distance of thirty feet from it to a height of 500 ft. (150 m.).

Eliminating such evidently fruitful sources of antenna wasteful resistance and taking every care to reduce the wasteful resistance to the lowest possible figure, as no doubt has been done in the stations mentioned above, the wasteful resistances still remain exceedingly high in comparison to the radiation resistances of the antennas of these stations.

In the instance mentioned above in the Darien antenna there is a resistance of 1.7 ohms which remains to be eliminated or reduced. The sum of the resistances of all the elements which Dr. Bennett has considered is negligibly small in comparison with this figure, and these elements could, therefore, be passed over for the time being and other elements which have not as yet been attacked could be profitably investigated.

As the matter stands at present, however, in stations of the type considered, the higher the antenna the greater its radiation efficiency, and a compromise must be made between efficiency and cost, the price of steel being the deciding factor, the cost of copper for the construction of the antenna proper being of minor amount.

There are two particularly remarkable points raised in dealing with the details of the wasteful resistance. (1) The use of a counterpoise ground instead of buried wires does not materially reduce the ground resistance as suggested, because currents corresponding to those which flow in the counterpoise network must flow in the ground beneath the counterpoise. No great reduction in antenna resistance has been obtained in practice by the use of a counterpoise net work. (2) That the section of the ground resistance due to choking effect is increased when the ground material is of low resistance. That this has not been noticeable in practice may also be due to the fact that the maximum value of this section of the ground resistance is negligibly small.

In present-day high power stations, the efficiency of the antenna and ground systems combined is the weakest link in the chain between prime mover and radiation. Dr. Bennett's work as published in his original bulletin and the present paper, is undoubtedly of the greatest value; but in my opinion, this value lies in the unflinching attack which has been made on the inefficiency of the radiator rather than in the suggestion of the feasibility of the low antenna. An explanation of the methods which Dr. Bennett has used in his calculations would be of great value to engineers familiar with antenna design. Many engineers who are at present unable to approach intelligently these problems would be greatly assisted thereby, and useful results would surely ensue.

Edward Bennett: I agree with Professor Hazeltine that it is largely a matter of judgment as to the distance from the periphery at which to stop in computing the extra-peripheral resistance. The question is raised as to why the zone for which the extra-peripheral loss has been computed should be arbitrarily taken as a quarter wave length in width. The practical justification for not taking the zone any wider than one-quarter wave length is that any reasonable increase in the width of the zone—for example, an increase to a width of one wave length—would not materially increase the computed resistance. The reason for this is that the surface density of charge decreases rapidly as the zones are taken farther and farther from the periphery, and at the same time the cross sectional area of the earth thru which the current flows increases, so that the loss in the outer zones drops off very rapidly indeed. The computations indicate, for example, that the current flowing beyond the grounded periphery is only six per cent. of the current to be measured in the tail of the antenna, while the current crossing a circle at a distance of one-eighth wave length from the periphery is only one-half of one per cent. of the current in the tail. The principal justification for taking the zone as wide as a quarter wave length is that by so doing an upper limit is established for a resistance of hitherto unknown magnitude. Since the computation indicates that the extra-peripheral resistance is small in comparison with the radiation resistance, the lower limit is not of much interest.

Dr. Goldsmith points out that it is not satisfactory to work large and high power stations for trans-oceanic transmission at twenty words per minute, and states that the trend is toward speeds of 100 to 1,000 words per minute. From this he concludes that the trend is toward the continued use of the high antenna, since the long time-constant of the low-antenna would preclude its use at the higher speeds. High speed in the sense of a high number of words per station means high power, expensive stations, because high speed stations cannot be made very selective and the strays must be swamped by the use of high power. It does not follow, however, that high speed in the sense of a high number of words per station is synonymous with high speed in the sense of a high number of words per dollar of expenditure. In my estimation the high number of words per dollar may possibly mean the use of several slow speed, low power, inexpensive, highly selective stations, rather than one large high power station.

Mr. Ballantine has questioned the propriety of the statement "Since the radiation resistance of an antenna of given form and dimensions is inherent in the form and dimensions, it follows that the efficiency of a given antenna as a receiver is 100 per cent. if the equivalent resistance of the detector is made equal to the radiation resistance, and if the wasteful resistance is made negligibly small in comparison with the radiation resistance." I welcome this opportunity to take issue with the practice which assigns to an antenna under the conditions described above, an efficiency of only 50 per cent. The statement that the efficiency is only 50 per cent. signifies to ninety-nine engineers out of a hundred that there may be a possibility of doubling the amount of power which the 50 per cent. antenna is abstracting from the impinging waves. The statement is, therefore, grossly misleading, because by no conceivable method can the antenna be made to abstract and deliver more power to the detector.

Mr. Israel has referred to the large eddy current losses experienced in a concrete wall in the neighborhood of the loading coil of the Darien station, and has expressed the fear that the larger current of the low antenna may lead to increased difficulty in reducing such losses. In this connection it should be recalled that the magnitude of the eddy current losses depends upon the intensity of the magnetic field. Now the intensity of the magnetic field in the vicinity of the loading coil is proportional to the product of current times the number of turns of wire. Since the low antenna has a large capacity and requires a small loading inductance or a small number of turns in the loading coil, it follows that notwithstanding the larger current, the intensity of the nagnetic field in the vicinity of the loading coil of the low antenna may not be greatly different from the intensity in the vicinity of the high loading coil.

To several of the speakers two things have seemed questionable or incredible. The first is that the wasteful resistance can be of the order of only 0.01 ohm for the low antenna as contrasted with 0.06 ohm for the high antenna of the Darien type. The second questionable point is that a low antenna in which the voltage generated by the impinging waves is only one-tenth or one-twentieth as great as in the high antenna can abstract the same power from the waves as the high antenna. As these views have been privately expressed again and again it may be well to consider them at length.

In considering the wasteful resistances we ought first to disabuse our minds of the notion that the wasteful resistances are resistances which may be measured with a Wheatstone bridge. If with this impression, we look at two antennas, a high and a low, no reason can be seen for such a great difference in the magnitude of the two resistances. The wasteful resistances are computed resistances found by dividing the power losses by the square of the current to be measured at the base of the antenna tail. If then, for the sake of argument, we assume that the power loss is the same in the ten-meter antenna as in the Darien antenna when operated at the same voltage and at 30,000 cycles, it follows that since the low antenna current is 14.6 times as great as the high antenna current the computed wasteful resistance of the high antenna will be 14.6^2 or 215 times as great as that of the low. It may assist to refer again to the extra-peripheral resistance, the estimated value of which has been set at 0.0006 ohm. Contrast this with the resistance which would be obtained by an ammeter and a voltmeter for a surface layer of earth of depth equal to the skin thickness and extending from the grounded periphery of the lower net work to a second circle of pipes surrounding the net work at a distance of a quarter wave length. This measured resistance would be of the order of 1 ohm, but in reducing this resistance to its equivalent in the tail of the antenna, consideration must be taken of the fact that only 6 per cent. of the antenna current flows beyond the periphery of the antenna and only one-half of one per cent. flows beyond the one eighth wave length point.

To consider the effect of the height and dimensions of a flat top antenna upon the power which the antenna can abstract from sustained waves and deliver to a resistor detector, we assume that the equivalent detector resistance R_d, is equal to the sum of the radiation resistance R_r plus the wasteful resistance R_w and that the total resistance R_t is

$$R_t\,(=R_d+R_w+R_r)=k\,R_r,$$

k may be expected to lie between 3 and 8.

If an antenna has an extended capacity area at a height h and is resonant to the sustained impinging waves, the average power P, delivered to the detector after the current attains the steady-state value is

$$P=\tfrac{1}{2}\,\frac{E^2_{r.m.s.}}{k\,R_r}=\tfrac{1}{2}\,\frac{(hF_{r.m.s.})^2}{k\,R_r}=\frac{2\,(hF_m)^2}{\pi^2 k\,R_r}$$

in which F_m represents the peak value of the electric intensity of the impinging waves.

Substituting in the above equation, the expression for the radiation resistance of an extended flat top antenna namely,

$$R_r = \frac{160\,\pi^2 h^2}{\lambda^2}$$

the expression for the power becomes

$$P = \frac{\lambda^2 F_m{}^2}{80\,k\,\pi^4}$$

In other words, the power abstracted from the impinging waves and delivered to the detector is independent of the dimensions of the antenna, except in so far as these dimensions may thru the wasteful resistance affect the value of k.

Thruout this discussion, it is assumed that the greatest dimension of the antenna is a small fractional part of the wave length.

It may be of interest to put the above expression for the power in a form fraught with greater physical significance

$$P = \frac{\lambda^2 F_m{}^2}{80\,k\,\pi^4} = \frac{\frac{1}{2}\,p F_m{}^2 \lambda^2}{\frac{1}{2}\,p\,80\,k\,\pi^4} = \frac{3}{k\,\pi^3}\,s\,\lambda^2\,(\tfrac{1}{2}\,p F_m{}^2)$$

in which p represents the permittivity of air, namely $\frac{1}{36\,\pi\,10^{11}}$ coulombs per sq. cm. per volt per cm.

Now $\frac{1}{2} p F_m{}^2$ represents the energy in the dielectric per cubic cm. when the peak of the electro-magnetic wave impinges upon the antenna, s represents the velocity of propagation, and λ^2 represents the area of a square the side of which is equal to the wave length—"a wave length square." Since the electric intensity F is a sine function of time, the average value of $\frac{1}{2}\,pF^2$ taken over the wave as it streams past the receiving station is $\frac{1}{2}$ of $(\frac{1}{2}\,pF^2)$ and since the electro-kinetic energy $\frac{1}{2}\,m H^2$, is equal to the electro-potential energy $\frac{1}{2}\,pF^2$, it follows that $s\,\lambda^2\,(\frac{1}{2}\,pF_m{}^2)$ represents the energy which streams past the receiving station per second across an area equal to the wavelength square. That is to say, the greatest average power which can be delivered to a detector by any antenna from impinging sustained waves equals $\frac{3}{k\,\pi^3}$ times the power flowing across a wave length square at the receiving station.

THE AMPLIFICATION OBTAINABLE BY THE HETERODYNE METHOD OF RECEPTION*

By

G. W. O. Howe, D.Sc.

(Professor at the City and Guilds (Engineering) College, London)

A certain difference of opinion appears to exist as to the maximum amplification obtainable by the simple heterodyne method of receiving undamped signals. In a paper read before this Institute in 1915, Dr. B. Liebowitz[1] maintained that the amplification of audio power could not exceed four times, except in so far as indirect influences come into play, and cause an increase in efficiency of the receiving apparatus. This point of view was strongly opposed by Mr. Louis Cohen; both arguments were supported by mathematical proofs and Mr. Cohen maintained that his was further supported by experimentally observed facts.

In a recent paper by Mr. Armstrong[2], it is stated that the results obtained with the auto-heterodyne appear to support the view put forward by Dr. Liebowitz.

The present communication is intended to throw some light on this question.

Before considering the heterodyne, it is important that one should be quite clear with regard to the assumptions made as to the rectifying action of the detector employed. Two very simple assumptions are possible; one may assume that the detector acts as a thermo-junction, heated by the radio-frequency current passing thru it, and, owing to a combination of electro-thermal phenomena, generating an electromotive force proportional to the square of the radio-frequency current; on the other hand, one may assume that the rectifier has a constant resistance in one direction, and a higher, preferably infinite, resistance in the other direction.

The former assumption is in agreement with the experimental

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* Received by the Editor, March 25, 1918.

[1] "Proceedings of The Institute of Radio Engineers," Volume 3, page 185.

[2] "Proceedings of The Institute of Radio Engineers," Volume 5, page 145, 1917.

observation that, when using a crystal detector, the audibility factor, and consequently the audio current, is proportional to the square of the received radio frequency current. On the latter assumption, the audibility factor would be directly proportional to the radio frequency current. Since the former assumption implies that the power supplied by the detector to the telephone receiver increases as the fourth power of the received radio current, i. e., that the output of the detector is proportional to the square of its input, it is evidently not of unlimited application. In view, however, of its known agreement with experimental observation over a very wide range, it will be assumed that the ordinary contact detector may be considered from this point of view.

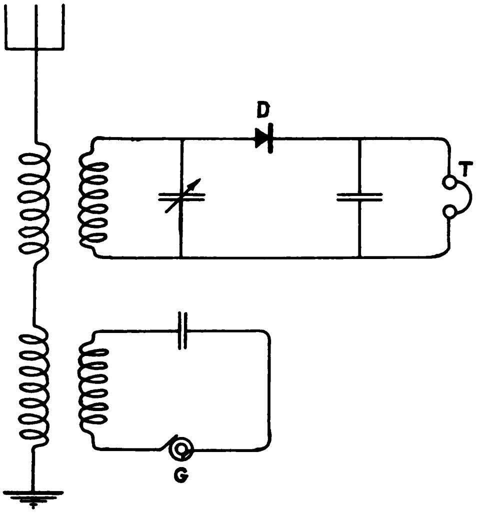

FIGURE 1

With the circuit arrangement shown in Figure 1, let the radio frequency current thru the detector *D* due to the received wave be $i_1 = I_1 \sin \omega_1 t$, and let that due to the local high frequency generator *G* be $i_2 = I_2 \sin \omega_2 t$, then the resultant current passing thru the detector is $i = i_1 + i_2 = I_1 \sin \omega_1 t + I_2 \sin \omega_2 t$. This resultant current is represented in Figure 2 (a) on the assumption that $I_2 = 2 I_1$. When the two currents come into phase, the resultant amplitude is $I_1 + I_2$, whereas, when they come into opposition it is I_2-I_1. Now the rate (averaged over a radio period) at which heat is developed at the detector contact is equal to the effective resistance of the latter multiplied by a half of the square of the amplitude of the radio current. Hence the telephone current will vary between $0.5\,k\,(I_2+I_1)^2$ and $0.5\,k(I_2-I_1)^2$ where k is a constant depending on the detector and on the impedance of the telephone receiver. This telephone current is shown in Figure 2 (b) in which the maximum and minimum ordinates are as 9 to 1. This is equivalent to a steady current of $0.5\,k\,(I_1^2+I_2^2)$ upon which is superposed the real sine-wave audio-current with an amplitude of

$$0.25\,k\left[(I_2+I_1)^2-(I_2-I_1)^2\right]=k\,I_1\,I_2.$$

Hence the audio current is directly proportional both to I_1 *and* I_2.

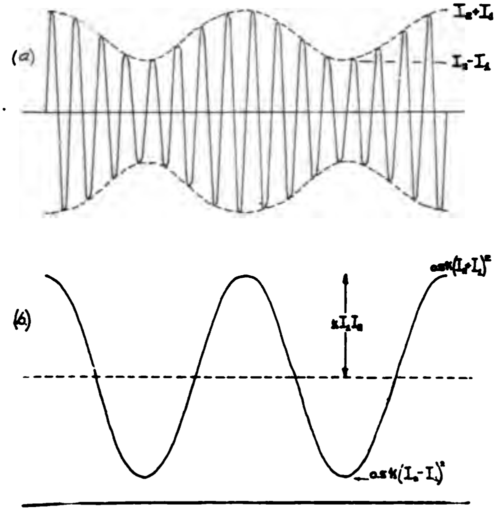

FIGURE 2

The problem may be looked at in a slightly different way, more suggestive of the method of Dr. Liebowitz. The rate at which energy is supplied to the detector is proportional to i^2, that is, to

$$(I_1 \sin \omega_1 t+I_2 \sin \omega_2 t)^2=I_1^2 \sin^2 \omega_1 t+I_2^2 \sin^2 \omega_2 t+ 2\,I_1\,I_2 \sin \omega_1 t \sin \omega_2 t.$$

Putting aside any question of the thermal capacity of the detector preventing the temperature changes following the variations of radio frequency, the electromotive force will be proportional to this expression at every moment. The effects of these electromotive forces on the telephone receiver have now to be determined. That the first two terms produce no audio current, can be shown in several ways. In Figures 3 (a) and (b), for example, the two terms are shown separately, on the left when in phase and on the right when out of phase. The resultant in each case is shown in Figure 3 (c), and it is seen that the mean value is constant whilst the amplitude of the radio oscillation varies. These two terms, therefore, represent a steady current thru the telephone receiver, the radio oscillations passing thru the shunt condenser.

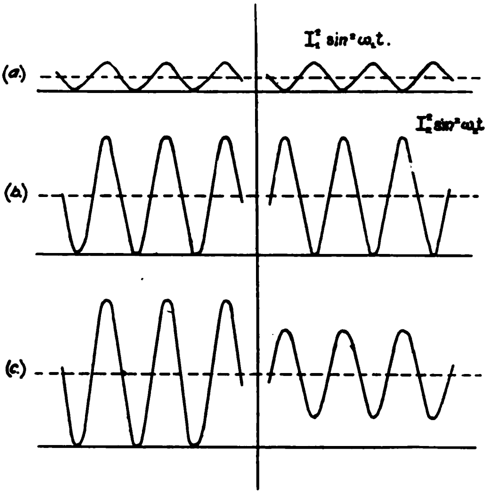

Figure 3

The last term is best split up into two cosine terms, thus:—

$$2 I_1 I_2 \sin \omega_1 t \sin \omega_2 t = I_1 I_2 \left[\cos\left(\omega_1 - \omega_2\right) t - \cos\left(\omega_1 + \omega_2\right) t\right]$$

The latter of these two terms represents an electromotive force with a frequency equal to the sum of the two component radio frequencies, and is, therefore, without any effect on the telephone. *The remaining term* $I_1 I_2 \cos(\omega_1 - \omega_2) t$ *represents the electromotive force which produces the audio frequency current of amplitude* $k I_1 I_2$.

If the heterodyne is not used, and the received waves are made audible by being interrupted at an audible frequency, then, assuming equal duration for the open and closed periods, the radio frequency current thru the detector will be as shown in Figure 4 (a), where, to make the consideration more general, it is assumed that the detector carries a steady polarizing current I_o.

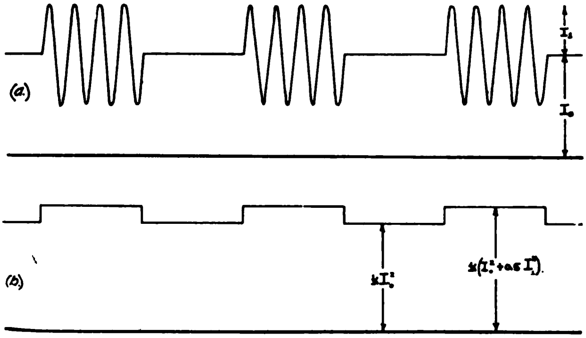

FIGURE 4

The heat production and consequent electromotive force will consist of rectangular pulses as in Figure 4 (b) and the telephone current will vary between $k\left(I_o^2+\frac{I_1^2}{2}\right)$ and $k\,I_o^2$, which is equivalent to a steady current of $k\left(I_o^2+\frac{I_1^2}{4}\right)$ upon which is superposed a rectangular alternating current with an amplitude of $\frac{k\,I_1^2}{4}$. This rectangular wave may be analysed into a number of sine waves, the fundamental of which determines the pitch of the note, whilst the higher harmonics determine its character. Altho it is difficult to determine upon what basis two sounds of different wave-forms should be compared, the simplest method and the one least open to objection is to compare the amplitudes of their fundamentals, assuming, of course, that these are of equal pitch. On this basis the amplitude of the audio current should be taken as $\frac{4}{\pi}\cdot\frac{k\,I_1^2}{4}$, and the amplification of audio current obtained by the use of the local heterodyne generator is

$$\frac{k\,I_1\,I_2}{\frac{4}{\pi}\cdot\frac{k\,I_1^2}{4}}=\pi\frac{I_2}{I_1}$$

Turning now to the other simple assumption which may be made with regard to the operation of the detector, viz.: that it has a constant resistance to current in one direction and an infinite resistance to current in the reverse direction, it may be assumed as a close approximation that the current passing thru the detector will be simply $I_1 \sin \omega_1 t + I_2 \sin \omega_2 t$, except that all current in one direction is suppressed, as shown in Figure 5 (a).

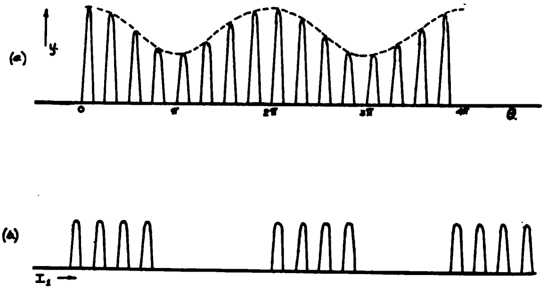

FIGURE 5

Assuming that I_2 is greater than I_1, the average value of the current over a radio period varies between $\frac{(I_1+I_2)}{\pi}$ and $\frac{(I_2-I_1)}{\pi}$; the equivalent steady current thru the telephone, and the amplitude of the fundamental audio-frequency current can be found as follows:—

Let the ordinate of the envelope shown dotted in Figure 5 (a) be y, then

$$y = \sqrt{(I_1^2 + I_2^2 + 2 I_1 I_2 \cos \theta)}$$

where θ is the instantaneous value of the phase difference between the two component currents. This may be written

$$y = \sqrt{\left[I_1^2 + I_2^2 + 2 I_1 I_2 \left(1 - 2 \sin^2 \frac{\theta}{2}\right)\right]}$$

$$= (I_1 + I_2) \sqrt{\left(1 - \frac{4 I_1 I_2}{(I_1+I_2)^2} \sin^2 \frac{\theta}{2}\right)}$$

Putting $I_2 = n I_1$

$$y = (n+1) I_1 \sqrt{\left(1 - \frac{4n}{(n+1)^2} \sin^2 \frac{\theta}{2}\right)}$$

The mean value of y over a complete period is

$$\frac{1}{\pi}\int_o^\pi y\,d\theta=\frac{2\,(n+1)}{\pi}\,I_1\int_o^{\pi/2}\sqrt{\left(1-\frac{4\,n}{(n+1)^2}\,sin^2\frac{\theta}{2}\right)}\,d\frac{\theta}{2}$$

$$=\frac{2(n+1)}{\pi}\,I_1E\,(k\cdot\pi/2)$$

where E is the complete elliptic integral of the second type, and the modulus $k=\frac{2\sqrt{n}}{n+1}$. The values of E can be obtained from any table of elliptic integrals; they are given in the following table for $n=1$, 2, 3, and 4.

$n=\frac{I_2}{I_1}$	k	$\alpha=sin^{-1}k$	E	$\frac{y_{\text{mean}}}{I_1}$	$\frac{I_{\text{mean}}}{I_1}$	$\frac{I_{\text{mean}}}{I_2}$	Amplitude of Fundamental of y	Amplitude of Fundamental Audio Current
1	1.0000	90°	1.000	1.274	0.4055	0.4055	0.849 I_1	0.27 I_1
2	0.9425	70°24′	1.118	2.137	0.68	0.34	0.968 I_1	0.308 I_1
3	0.8667	60°	1.211	3.085	0.982	0.327	0.986 I_1	0.314 I_1
4	0.8000	53°8′	1.27	4.042	1.287	0.322	0.991 I_1	0.316 I_1

Since the radio current passes for only half the total time, $I_{\text{mean}}=\frac{y_{\text{mean}}}{\pi}$. To find the amplitude of the fundamental audio current, the curve of y must be analysed into its Fourier components. This the writer has done and the results are given in the last column but one; the last column is obtained by dividing this by π.

If the heterodyne is not used, but the received waves made audible by being interrupted to give the same audio frequency, as shown in Figure 5 (b), the average current varies between $\frac{I_1}{\pi}$ and 0. The steady component and the amplitude of the rectangular alternating component are both equal to $\frac{I_1}{2\pi}$. The amplitude of the sine-wave fundamental is $\frac{I_1}{2\pi}\times\frac{4}{\pi}=\frac{2\,I_1}{\pi^2}$. The current amplification is obtained by dividing the figures given in the last column of the above table by this figure. Its values for $n=1$, 2, 3, and 4 are 1.33, 1.52, 1.55, and 1.56 respectively, with a maximum value of $\frac{\pi}{2}$ as n is still further increased.

It is seen therefore that the amplification of the audio current depends on the type of detector employed. With a detector which gives an audibility factor proportional to the radio current, there is no doubt of the correctness of Dr. Liebowitz's contention that the amplification is not increased indefinitely with the current I_2. On the above assumptions the maximum amplification of audio power would be $(1.56)^2=2.43$. With the other type of detector, however, there are some grounds for a difference of opinion. On page 194 of Volume 3 of the "PROCEEDINGS," Dr. Liebowitz states that the maximum *true* amplification of audio power obtainable in the most efficient form of heterodyne receiver is four, and it is probable therefore that he would maintain that the amplification as here calculated is not a *true* amplification. It will be interesting then to determine to what extent the amplification as here calculated is due to increased efficiency of the detector. The efficiency will be taken to be the ratio of the audio power, which is the useful output, to the total input power. The ratio of the output with the heterodyne to that without it is $\left(\frac{\pi I_2}{I_1}\right)^2$, whilst the ratio of the corresponding inputs is $\frac{I_1^2+I_2^2}{0.5 I_1^2}$. Since the supply of power to the detector is interrupted during half the time when the heterodyne is not used, the power actually supplied to the detector has been put in the denominator; this is a point upon which some difference of opinion may exist.

The efficiency of the detector is increased in the ratio

$$\frac{\pi^2 I_2^2}{I_1^2} \cdot \frac{0.5 I_1^2}{I_1^2+I_2^2} = \frac{\pi^2}{2} \cdot \frac{1}{1+\left(\frac{I_1}{I_2}\right)^2}$$

If $I_2=I_1$, this equals $\frac{\pi^2}{4}$; the total ratio in which the audio power is increased is π^2, leaving a ratio of 4 to 1 for what Dr. Liebowitz calls the true amplification of the power. This is evident, moreover, from the fact that, with $I_2=I_1$, the power supplied by the heterodyne generator is equal to that supplied from the antenna, and since they are supplied continuously, whereas without the heterodyne the current I_1 is interrupted half the total time, four times the power is supplied and the audio power given out would be increased in the same ratio if there were no change in the efficiency.

If $I_2=2I_1$, the input is ten times as great as with the interrupted continuous waves; the efficiency of the detector, as here defined, is increased in the ratio $\frac{\pi^2}{2.5}$; the total amplification of audio power is $4\pi^2$, and the amplification apart from the increased efficiency is 10, which is, of course, the ratio in which the input has been increased.

This subdivision of the amplification, which depends on the method of defining the efficiency of the detector, is carried out here merely to show that such a procedure does not necessarily support the view put forward by Dr. Liebowitz. In the opinion of the writer, such a distinction is not warranted. Dr. Liebowitz differentiates between amplification obtained (1) by infusing new energy into the oscillations and (2) by increasing the efficiency of the receiving apparatus. In the latter example considered above, the newly infused energy was four times that already there, and if I_2 had been equal to $3\,I_1$ it would have been nine times as great. The amplification, however, depends not upon the amount of infused energy but upon its effect on the detector. By simply increasing the amplitude of I_2 from I_1 to $2\,I_1$ the audio power output is quadrupled. If Dr. Liebowitz's contention is correct, that there is no increased true amplification apart from increased efficiency, the latter must have been quadrupled, that is, the efficiency must have increased in the same ratio as the output. In the writer's opinion there is no basis for such an assumption. If such an increased efficiency could be obtained by means of a continuous polarising current, thus allowing the same amplification to be obtained with a smaller value of I_2, there would be more reason for denying the heterodyne the full credit for the total amplification, but it can be shown very simply that, on the assumptions made, the calculated amplification is independent of such a continuous polarising current.

It appears, therefore, that the large amplifications claimed by Mr. Cohen may be quite possible without tuning the audio frequency to resonance with the telephone diafram. Dr. Liebowitz on page 201 ascribes a part of the amplification to "adjusting the amplitude of the local current so as to work the crystal on the best part of its characteristic;" it should be noted, however, that the calculations of this paper are based on the assumption of an ideal detector with no "best part of its characteristic," and that, moreover, as already pointed out, the same increase of efficiency cannot be obtained by means of a continuous polarising voltage; it is essentially a part of the heterodyne amplification.

SUMMARY: The author contrasts the Cohen theory that heterodyne amplification can be increased indefinitely by increasing the local current (using an ideal detector of unlimited current-carrying capacity) with the Liebowitz theory that the maximum "true heterodyne amplification" is four.

It is then shown by several different methods of considering detector and heterodyne action as compared with chopper detection of received energy, that if the detector gives an audibility current proportional to the received current, the maximum amplification of audio power is 2.43, and does not increase indefinitely with the local current.

With detectors giving an audibility current proportional to the square of the received current (e. g., ordinary contact detectors thru a considerable range), the amplification may greatly exceed four, and its excess over four cannot be accounted for on the basis of "increased detector efficiency," since a steady polarizing current will *not* produce the same increase.

FURTHER DISCUSSION ON "ON THE INTERPRETATION OF EARLY TRANSMISSION EXPERIMENTS BY COMMANDANT TISSOT AND THEIR APPLICATION TO THE VERIFICATION OF A FUNDAMENTAL FORMULA IN RADIO TRANSMISSION" BY LEON BOUTHILLON

By

Oscar C. Roos

Equation (1) on page 226 should be, if j be substituted for $i=\sqrt{-1}$, to conform to equation (4) on page 228:

$$C_o=\frac{ej}{Ln}\left[1-\left\{\frac{R_o-jLv}{R_o}\cdot\cos\frac{nx}{v}+\sin\frac{nx}{v}\right\}\right]\quad\cdot\;\cdot\;\cdot\;\cdot\qquad(1)$$

Two lines below the above equation the expression $\frac{evi}{R_o n}$ becomes $\frac{ev}{R_{on}}\cdot$ In equation (1) $\frac{nx}{v}$ is the electrical angle from the base of the antenna to the point x on the antenna, thru which the current stationary wave passes, as the current changes in value along the antenna.

The last line in my discussion is obviously to be read as

$$\frac{na}{v}\left(\underline{\text{not}}\quad\cos\frac{na}{v}\right)=\pi,\ 3\pi,\ 5\pi\ \ldots\ \text{radians,}$$

which is to be interpreted as the current-wave phase change over the whole antenna. Note that equation (1) is deduced from equation (4) by remembering that

$$\sinh\frac{pa}{v}=j\sin\frac{na}{v}+\cosh\frac{pa}{v}=\cos\frac{na}{v}$$

also that

$$\frac{na}{v}=\frac{\pi}{2}+Z_o=R_o$$

in the case under discussion.