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TECHNICAL PAPERS AND DISCUSSIONS

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It is with deep regret that the Institute of Radio Engineers announces the death of

Mr. N. H. C. Taylor

He was the radio operator on board the steamship "Marowijne," which foundered with all on board during a hurricane on the Caribbean Sea on August 14, 1915. Mr. Taylor, who was an Associate Member of the Institute, was in the employ of the Tropical Radio Telegraph Company at the time of his death.

The heroic steadfastness required by the radio operator in stress and storm is shown by this catastrophe.

THE TRAINING OF THE RADIO OPERATOR*

By

M. E. Packman

In contra-distinction to the advance that has been made by the scientific and commercial development of radio telegraphy, the operator problem stands much as it did in the earliest days. While radio equipment has undergone many improvements, traffic departments have accomplished much in the organization and collection of business, and the number of equipments has greatly increased, little has been done to increase the efficiency of the operating staff. In former days the only question asked of a man applying for a position as wireless operator was; "Are you an operator?" Since the advent of the Government License it has been changed to "Have you a license?" An affirmative answer was and is, in the majority of cases, all that is required to secure the applicant his position. In some cases, this was necessary; and it will undoubtedly continue to be so inasmuch as it is not always possible to obtain a competent man at the particular moment that an operator is needed. It seems, however, that if the question of the training and selection of operators for radio service were handled in a manner more in accord with that followed by railroads, the deplorable condition that exists in some land and ship stations would be greatly improved. As all persons acquainted with the commercial development of radio communication in this country well know, a very chaotic state of affairs existed prior to the time that the Department of Commerce placed certain restrictions on radio equipment, operators, and methods of operating. After the matter of wave length restrictions has been more nearly adjusted to meet present conditions, I think it will be possible to say that as a whole conditions have been much improved.

As far as the operators are concerned, the Government examinations have weeded out many undesirable men from the service and consequently have raised the standard a little higher, but it must be borne in mind that the Department of Com-

* Presented before The Institute of Radio Engineers, New York, October 6th, 1915.

merce looks at the efficiency of the operator from a different point of view than should the commercial company if it is seeking the most suitable men. The requirements of the department are only a part of what good commercial service demands, tho in many cases a first grade license is all that is required or asked for.

As one man of my acquaintance, who employs a great many operators every year has stated in referring to the operators on his ships, "Some of these men cannot send, some of them cannot receive, some of them cannot adjust a detector, and some of them cannot tune." Such men as he refers to, that do little else than ride back and forth on the ships, are common in the radio service everywhere; and the excuse is made that it is difficult to find men who are good telegraphers and at the same time are capable of handling a radio set to the greatest advantage. This condition is the natural result of attempting to get efficiency out of men who have had no telegraphic experience or who have had no training in the use of commercial radio apparatus. I have known men whose highest aim in life was to be so expert a telegrapher that they could sit down and work any telegraph circuit at which they might be placed. On the other hand, there are radio operators who are interested only in the handling of the instruments; and again there are men who have no particular interest in any phase of the business, other than its outside attractions. Obviously none of these classes of men completely fill the requirements of a "good" operator, but this list includes practically all the available men, with few exceptions, who have not been specially trained.

A question then arises: what are the qualifications of a "good" operator. In the first place, he should be capable of transmitting signals clearly, accurately, and rapidly in either American Morse or Continental Morse. He should understand that any communication not only has to be sent but also to be received; and realizing this, he will space his characters and words so that they will be easily understood by the receiving operator. He should know when to repeat and when it is unnecessary, when to send slowly and when he may send faster. In receiving, he should be able to read almost any kind of sending, and to make a neat copy with either pen or typewriter regardless of whether he has interference to contend with or not. If he is capable of getting the most out of his instruments he must thoroly understand the principles which underlie their design. This knowledge must be so perfect and of so practical a nature that he will know instantly what is needed in case of emergency or disaster. He must have a comprehensive knowledge of telegraphic tariffs, traffic, methods of routing, the location of various radio and telegraphic stations, etc., so that he can quickly determine in what way a message can be handled with the greatest dispatch and least expense. This will not only be an aid to him in procuring business but it will be of benefit to the general public with whom he deals. He must be ready and willing to perform the functions of his office at all times; and in every case he must be a gentleman.

Some of the qualifications enumerated involve inborn personal characteristics that are apart from any training that a school can give. Experience has shown that one phase of the work will appeal more strongly to one man than another and that it is seldom the case that a man of his own initiative becomes proficient in all the things essential to his work. I think it can be said, however, that given a man with average intelligence, willingness, and six months training in a well organized school; a proficient radio operator can be developed.

The object of this paper is briefly to outline the course in "Radio Telegraphy" as given at Dodge's Institute of Telegraphy at Valparaiso in Indiana, and to show some of the methods that are used in an effort to meet the requirements of a comprehensive training for radio operators or other persons interested in the art of radio communication. More than one-third of the students entering the school are enrolled in the Radio Department, and of this number about two-thirds also take the work in the Morse Department, thus familiarizing themselves with both codes. Aside from the special work in each of these departments all students are required to take a half hour's work in penmanship, under a competent instructor, each day. To anyone familiar with telegraphic work, the importance of this is apparent. The school is also equipped with a large number of typewriters so that the student may become proficient in copying directly from the circuits by this means. The greater portion of the students completing the course in Radio Telegraphy enter the service of the Marconi Wireless Telegraph Company with which company we have a working arrangement. Others of these students enter other commercial or government service.

As might be expected in an institution of this kind the student body is made up of all classes. The students are of all ages, and they come from all parts of the United States; many of them come even from foreign countries. Some of those who enroll in the Radio Department, especially, are college graduates; many are high school graduates, while some of them have very meagre education. Altho we have no regular entrance requirements, as far as education is concerned, it is frequently the case that we are compelled to refuse the applicant because of his lack of elementary education. The average student is between eighteen and twenty years of age and has had two years of high school training. This is, of course, a desirable qualification, and I have found that those students who have had amateur experience together with the high school work are the most apt in mastering the radio work. This is due, of course, to their great interest in the radio field. Another class of men that invariably develop into excellent radio operators is drawn from the commercial and railroad fields. There is also a great variance in the ulterior motives of the different students. The majority of them, of course, expect to prepare themselves for service as commercial radio operators. However, we have many special students with entirely different objects in view. Among these are men from the armies and navies of foreign countries as well as some from the United States government service. We have other students who are interested in the subject from a scientific standpoint only, some who expect to teach the subject in other schools and colleges, and so on.

In arranging a course that will meet these varied conditions, numerous points had to be considered. In the first place, it is impossible in most cases for the student to remain in school for a period exceeding six months. This means that all the work relating to the subject must be covered within this time at most. In the second place, owing to the lack of preparatory training in electrical science, it is necessary to begin with the very principles of electricity; and considering the extent of such electrical knowledge that an operator should have, and the complexity of some electrical ideas, it requires a very careful selection of the subject matter in order that the course be made as comprehensive as possible. Many of the theories which underlie the working of radio apparatus, involve the principles of alternating currents, a subject which is usually taken up in the third or fourth year in engineering courses; and altho simple to an electrical engineer is very complex and difficult of explanation to a student who has no foundation for such work. Nevertheless, these ideas have to be covered in order that the student is eventually able to reason out many of the problems and questions that may present themselves sooner or later. In our work the students are not shown lists of questions, the answers to which can be memorized, and their examinations are the result of actual knowledge of radio apparatus, its uses, merits, and failings. This, it appears, is the only possible manner in which a "good" operator can be trained so far as theory and manipulation of apparatus are concerned.*

In general, the theory, adjustment and operation of radio equipment is given in a series of lectures associated with laboratory work, which occupies three hours each forenoon, five days per week. The beginning class is held between 11 and 12, an intermediate class between 9 and 10, and the advanced class between 8 and 9. The hour between 10 and 11 is devoted to code practice. Other code classes are held from 1.30 to 3 P. M., 4 to 5 P. M., and 7 to 8 P. M. The penmanship class is held between 3.30 and 4 P. M. During the summer months, the evening session is discontinued. In describing the various parts of the course, I shall take up the code work first.

At the time the Department of Radio Telegraphy was established, the American Morse code was used almost exclusively in the radio service in this country, the South Wellfleet station being the only one that used Continental Morse extensively, so far as I am informed. With this condition existing, it was a simple matter to train a Morse operator to receive the same code thru telephone receivers. In the Morse Department of the school, there was and is every facility for training a student in Morse receiving where there are a great variety of speeds. A new student is started on a circuit where an instructor makes the characters of a letter and the student pronounces the letter; and as he becomes familiar with the combinations forming all the letters, he is advanced to a circuit on which short words are sent, which he calls off as he recognizes them. In this way a student is advanced from one circuit to another until he is receiving at a speed of twenty-five to thirty words per minute. On the more advanced circuits in the commercial and railway department, the instruction consists of commercial and railway messages or train orders. Examinations are held from time to

*One method of preparing a man for a government license, which I have been informed is used, is to furnish the student with a list of questions, such as are used by the inspectors in examining applicants for operators' licenses, together with the correct answer to each question. With such a list of questions and answers, it is a simple matter for a telegrapher to pass the examination as given by the Department of Commerce, provided he does not write down answer six for question five. Such a man is of course worthless in actual radio service altho he has nevertheless passed the requirements of the Department of Commerce.

time and the students making the fewest mistakes in their copies, which are marked with care, are advanced to higher circuits.

After a student who had enrolled in the radio course had progressed to the point where he could receive eighteen to twenty words per minute from a sounder, he was transferred to the radio code work. All such students were supplied with practice sets including a single slide tuning coil, detector, fixed condenser, head telephone receiver, and sufficient aluminum wire to construct an indoor antenna, as well as a key and buzzer mounted on a base board. An operator at the radio station in the school sent press matter at a speed of about fifteen words per minute for a period of an hour and a half or so, and at a faster speed for an equal period. Students, supplied with the small receiving sets which they had installed in their rooms, were supposed to spend the afternoon in copying these signals. This method was very good, in some respects, where the total number of students was small and where they were all able to receive ordinary Morse signals at a fair speed. They not only had the benefit of the code practice but they also had some practical experience in the adjustment of detectors and in tuning, which is, of course, a valuable part of an operator's training. On the other hand where students are depended upon to get their own practice in this way, they are very apt to waste a great deal of time and their progress is slow.

In the spring of 1913, the Department of Commerce regulations went into effect, calling for Continental code and examinations for licenses. It was evident that numerous changes would have to be made in the course of instruction in order that the students might receive the necessary and proper training. As a result a complete reorganization of the department was inaugurated. The Continental code is now taught exclusively in the Radio Department and the system used for many years in the Morse work is followed with the exception, of course, that all receiving is done with telephone receivers instead of by means of a sounder.

Ten code circuits, operated at various speeds, are used at the present time. On circuits 8, 9, and 10, which are the beginners' circuits open buzzers are used. An instructor sends letters singly on the lowest of these, and the student calls out the letter as he recognizes it. The student is not permitted to copy on paper at the start as it is found that he then invariably loses interest before he has accomplished anything. There is always a little rivalry among the members of a group of new students as to which one will be the first to call off the letters as they are sent; and this urges them to use increased energy in mastering the first few days' work. As they become familiar with all the letters, figures, and characters prescribed for radio signaling the students are advanced to a higher circuit where they pronounce words. Inside of a few weeks or less, they are able to receive words sent at a rate of four or five per minute. The length of time required, of course, depends upon the natural aptitude of the student, and his application.

Figure 1 shows the general arrangement of the code and lecture room; the photograph, being taken from the instructor's desk does not, however, include the switch board and instruments

FIGURE 1

used in controlling the circuits. On circuits 1 to 7, all receiving is done thru double head telephone receivers similar to those used in commercial working. The transmitter or signal producing device used on all of these circuits consists of a buzzer, controlled by a telegraph key, having line wires connected to the terminals of the interrupter, a condenser being interposed in one or both of these. On circuits 3 to 7, there is one transmitter on each circuit, with telephone terminals bridged across the line wires for seven students. Circuits 1 and 2 are arranged in long lines along the side of the code room, and each is divided into eight stations all of which are equipped with a buzzer transmitter, condenser and transfer switch for changing from sending to receiving. All of these circuits are connected with a Western Union switch board at the instructor's desk by means of which any one of the circuits may be connected with any other or any transmitter on the instructor's desk can be instantly connected, by means of a loop pin with any circuit. Or, in the same manner, a telephone transmitter may be connected with any circuit. The code speeds on circuits 7 to 1 are gradual variations from five or six words per minute on 7 to the highest speeds on circuits 1 and 2 which usually are connected together. Each of the buzzers on these circuits is separately adjusted so as to give out as good a note as possible, and it always happens that there are as many different tones as there are buzzers inasmuch as it is difficult to obtain a perfect tone from all. This I consider to be an extreme advantage, however, over the system used in some schools where one master vibrator or other audio frequency generating device is used as a source of power to supply all circuits. It is possible for a student to become so familiar with one spark frequency, especially if it is absolutely regular, that he will have great difficulty in copying commercial stations which have sparks varying in a more or less degree from the perfect tone. With the individual buzzer method, however, an experienced operator, when listening in on our faster circuits to the interchange of radio messages will not fail to recognize its true ring. Some of these student stations emit a high musical spark resembling the 500-cycle stations, while there are others of varying pitch down to the rough irregular spark such as is emitted by the old low frequency sets of the Shoemaker type. In fact nearly every condition of arcing or other irregularity in a commercial transmitter is automatically met.

To some this may appear to be a deplorable way in which to teach a student to copy telegraph code, but on further consideration it is evident that the student not having had a perfect spark to copy from at all times, has accustomed himself to just the conditions of regular commercial service. Altho the tendency is toward a universal use of musical sparks and apparatus with which such sparks can be readily and easily maintained, it will undoubtedly be a long time before the difficulties of tone adjustment will be done away with. Again, the moral effect of an irregular or rough spark is impressed upon the mind of the student. Some students of their own accord endeavor to get and maintain a clear tone, while others are more or less indifferent. In any case, the advantages of the good tone are evident and a certain pride is ordinarily taken in maintaining such tones. (I have been told by old operators that our students can be recognized by the adjustment of their test buzzers.)

Another advantage of the individual buzzer arrangement is that the loudness of the different stations can be readily adjusted to any fixed value, assuming that the frequency of interruption is constant. As previously mentioned, a condenser is interposed between either one or both sides of the line and the buzzer, and by constructing these condensers so that they have different capacities, their impedences are different and hence different stations can be made to send out signals of different strengths. Some of the stations have adjustable condensers and hence have a ready means of "varying their power." At the instructor's desk, variable condensers are used so that the signal strength can be varied from just audibility to any desired value. Other methods of varying the signal strength in such circuits readily present themselves, but the method outlined above has proven to be the most satisfactory of any that I have tested. In any case it is exceedingly important that the practice signals be not too loud and it is desirable to have them of different strengths. It has been my experience that a student may become so proficient in the code work on the circuits, that he can copy the most complicated code and cipher messages from the fastest senders, but when placed in the radio receiving room he will be able to get but little of what is being sent. This is partly due to the fact that he may be practicing with too loud signals. Ability to read very faint signals from distant stations is largely a matter of ear training and for this reason it is desirable to have the practice signals quite weak. On the other hand, an operator who is accustomed to receiving such signals may become so confused by very loud signals that he will be unable to copy them.

Each station on circuits 1 and 2 is provided with four calls; that is, it represents four different ship or land stations and the operator at each of these places is expected to answer any one of these calls. Inasmuch as the international call letters assigned to commercial stations in this country, into the service of which most of our students go, are combinations beginning with W or K, each of the student stations has one call beginning with each of these letters. The two remaining calls are selected from those used in the United States Naval service or foreign stations, thus giving a wide variety of combinations. Some of these calls are selected on account of the difficulty encountered in transmitting them, or again there may be two that are easily confused or wrongly interpreted by a receiving operator. Others admit of a rhythmical swing which experienced operators develop. Among the different stations and ships represented on these circuits are those at the principal ports on our sea coasts and on the Great Lakes and the vessels which are likely to be in communication with them. The instructor's desk answers to or uses any call not assigned to one of the student stations. Occasionally the entire system of calls is changed. It is thought that such a system as this trains the student to be quick to recognize as well as to send difficult or uncommon calls.

Practically all the code work on these faster circuits consists of message work, and such other communications as are actually carried between ships and radio stations. Everything is carried over these circuits in accordance with the provisions of the London Convention, the Marconi traffic regulations, and good judgment. Students are not permitted to converse over the circuit nor to carry on any conversation except where the exigency of the case demands it. Messages are sent from one station to another with proper prefixes and service instructions, being relayed where necessary and filed for future reference. As might be expected, a student is often tempted to give an O. K. on a message addressed to his station when a goodly portion of it is missing rather than to ask for a repetition or to say that he had not received it, and in order to circumvent this, nearly all the messages sent out from the instructor's desk are messages that call for answers or must be relayed. This proves to be a very satisfactory method of insuring that the communication has been actually received and soon the student ceases to commit such offenses. Sometimes a wrong check is purposely affixed to determine if the student will note this. The various ship stations send in their "T R" reports and positions which must conform to the practice of commercial stations; in fact, everything is handled as nearly in conformity with actual conditions as possible. Very little press is sent over these circuits as it not only gives the student a fictitious idea of his ability, but really furnishes little practice. In fact, I am convinced that it has a tendency to make him "guess" more than anything else, which is, of course, one of the worst habits that an operator can acquire. In case such material is sent over the circuits, it is generally an article having in it many uncommon words. In some instances subjects in French, German or Spanish, or long lists of code or cipher words are sent in an endeavor to train the student to write down just what he receives, by sending such material that he is unable to form any advanced ideas.

Another failing common to many operators, which we have endeavored to overcome in our men, is the inability to read signals thru interference. There always has been more or less of this to contend with, but now that all commercial work is ordinarily carried on at the same wave length, or at best at a few wave lengths the confusion has become greater; and when severe atmospheric disturbances exist in addition, it taxes the ability of the operator to the utmost. The acquiring of ability or skill in reading signals under these conditions is a matter of patience and concentration of the mind on one particular tone; a faculty which can be developed by training. In order, in a measure, to duplicate the condition of interference and to bring out this faculty of copying under adverse conditions, I make it a practice at times to have an interfering transmitter, or possibly several, working on the same circuit at the same time the regular code practice is being carried on over this circuit. This is done an hour or so each day and the results are exceptional. Ordinarily an omnigraph or other automatic transmitter is used to operate the interfering buzzer, the strength of the signals being variable. However, at times two groups of students will be carrying on communication simultaneously on the same circuit.

For the regular code practice, no automatic transmitters or sending machines of any kind are used, altho we have a number of varieties; except that a day or two before the government inspector visits the school, the omnigraph is used in order that the applicant shall not be confused by its mechanical accuracy. Hand sending by expert radio operators is depended upon entirely. All students receive such instruction either one and a half or two hours per day depending upon which circuit they happen to be working and the state of their advancement. During such periods of the day as there is no instruction on the circuit the students send among themselves, each being assigned a certain part of an hour, the schedules being rigidly maintained.

It is generally considered that not all men can become expert senders, this being a natural qualification with some; but it has been our experience that nearly any person can become a good sender if he is taught the full arm "pump handle" movement and rigorously practices it. Slow sending, if heavy, distinct, and properly spaced is always better than light rapid sending in the case of a new man. After he has once become a good sender, he easily acquires the speed which is essential in many cases. Taking operators as they are ordinarily found, better speed can generally be made and more business transacted in the same time, by sending good distinct characters at a speed of twenty or twenty-two words per minute than can be done by transmitting at a speed of twenty-eight or thirty words per minute. There is always a tendency for new men to endeavor to "burn up" some other operator by rapid sending and if he is a poor sender at best, a great deal of time is lost as a result. Our code circuits are fitted with good telegraph keys, some Western Union keys, and some heavier keys similar to those frequently found in radio stations; and students are urged to practice a correct method of sending on these during all their spare moments. In cases where a man is especially stiffened in the muscles of his forearm from manual labor, or in other cases, we insist on an hour or so of continuous sending every day. After a student has practiced a correct method of sending until he has become sufficiently competent to transmit signals clearly, accurately and without "breaking," he is allowed to operate the school station on field days. Students are always anxious to do this but good sending on the practice circuits is pre-requisite. In most cases a good sender is the result.

In the technical or theoretical phase of the work the first part of the course consists of a study of the elements of electricity and magnetism, emphasizing such points as relate directly to radio apparatus and abreviating such matters as are not of first-hand importance. Following these more elementary subjects, the course is extended into a study of dynamo electric machinery, going only briefly into the theory of such apparatus, but giving special attention to the actual principles of the generation of current, the factors influencing the output, the function of the various parts, as well as the use, and care of such machines and methods of making tests and repairs. In connection with the study of alternating current machines the student is familiarized with terms common to alternating current operation (frequency, wave form, power factor, etc.), so that when the study of capacity and inductance has been taken up, the elements of alternating current problems will be less difficult. To facilitate the study of dynamo-electric machinery, the laboratory is equipped with a number of types of motor-generator sets used in radio service as well as various other A. C. and D. C. motors and generators of different capacities, together with starters, rheostats and other auxiliary apparatus. Figure 2 is a view of the general laboratory which shows some of this equipment. In the foreground is a direct current generator, belted to a three-phase

FIGURE 2

induction motor, the set being used to supply direct current for operating the motor-generators and other direct current apparatus. The generator of this set is of very open construction and the terminals of all windings are brought to a connection block, making it a very convenient machine for demonstration work. Along the right hand side of the picture can be seen several radio transmitters, of different types, used for instruction and demonstration work. Along the wall to the left, not visible in the photograph, are cabinets containing various kinds of physical and electrical instruments, tuning devices, measuring instruments, and other apparatus useful in experimental work in radio telegraphy.

After completing that part of the work covering dynamo-electric machinery, a study of electro-magnetic induction is taken up, theoretically and experimentally. This is one subject upon which too much time cannot be spent and every effort is made to present the phenomena of inductance and self and mutual induction in such a manner that the student will get a clear conception of principles involved. The effects of self inductance are discussed in their relation to the primary circuits of induction coils and transformers. In the explanation of mutual induction I have found the use of audio frequency currents, generated by a buzzer, very helpful, using induction coils or coupling coils in which one of the coils can be rotated with respect to the other. Use is also made of coupling transformers of all types common to radio service, the result of which is that the student looks at the principle rather than any one form of construction. Many other schemes used by instructors in physics can be used to advantage. This part of the course is concluded with a study of the practical construction of commercial induction coils and transformers such as are used in radio installations, here as elsewhere, attention being drawn to methods of testing and making temporary or permanent repairs.

Following this work the next part of the radio set that is studied is the condenser, it being considered in its various forms and constructions. An effort is made to give the student a clear insight into the principles which are involved in certain important phenomena. Methods of calculating the approximate capacities of different types are shown, and then the means of obtaining any desired value of capacity, with definite dielectric strength, by the combination of standard units is demonstrated. Emphasis in this case is laid on the methods by wh ch the proper capacity in the condenser of a radio set can be obtained by re-arranging the separate units of the condenser which has been injured, thru breakdown or puncture, and the precautions which must be taken in thus using it. Methods of charging condensers as used in the closed circuit of transmitters and the necessity for and function of the spark gap are demonstrated; which leads to a study of oscillatory discharges.

It has been my experience that unless the theoretical work is varied or made attractive by the interposition of actual radio telegraphing, thus giving an actual demonstration of some of these theories, it often happens that the student will lose interest, with the result that he fails to grasp the very things which are most essential. At this point, then, a horizontal aerial is strung up a short distance from the ground thus constituting a very apparent air condenser, and an induction coil is connected thereto forming a plain aerial transmitter. With this arrangement signals or messages are sent to portable stations. Altho this type of transmitter is generally known to the student, the experiment proves an interesting diversion in which many of the practical difficulties encountered in the operation of such sets with large induction coils, and their remedies, can be easily demonstrated and in a forcible manner.

Just at this point, when the student has in mind the oscillations of the current in this plain aerial transmitter and the radiation of electric waves, I have found it to be a very opportune time to go ahead with the explanation of the terms period and frequency, and their relation to wave length. With an aerial 100 feet long, all of which is visible, the student can be made almost to see the oscillation running out to the end of that wire and returning in a given time, and if the wire is longer that it will take a greater length of time for the complete oscillation.

After concluding the study of plain aerial transmitters, with stress laid on the limiting quantity of charge that can be converted into radio frequency energy in consequence of the small capacity of the aerial, the work naturally leads into the study of coupled transmitters wherein much larger capacities can be used. With an understanding of oscillatory currents already acquired, the effects of the constants of the closed circuit on the wave length are quite apparent. I have found that a study of wave lengths in a circuit which does not radiate waves, leads to much confusion and lack of understanding hence the reason for a consideration of wave length in connection with the plain aerial transmitter first.

During that part of the work covering closed circuits of the transmitter, I assume a circuit having a condenser of a certain capacity about which is shunted an inductance of some twenty turns. It is stated that the wave length using say two turns is 300 meters. The inductance of the helix per turn is then calculated, and the results tabulated. The student then calculates the wave length with the movable clip on each of the twenty turns and enters this data in his tabulation after which attention is invited to the fact that the wave length varies as the square of the number of turns. From this it is apparent that in case he was working on a ship and for any reason was required to change his wave length say from 300 to 600 meters the position of the clips would be instantly known, with fair approximation, without the use of a wave meter or other device. In like manner, the effect of the condenser capacity on the wave length is demonstrated; and cases are assumed wherein a portion of the condenser is damaged and the use of half of the condenser with a definite increase in the inductance will give the wave length required, this to be obtained without the use of measuring instruments. It is a well known fact that the majority of operators, after having once located a 600-meter adjustment on their receiving tuners, actually do very little tuning, and in case of accident to a ship or its radio equipment, it is very necessary that the operator on such a ship should be able to maintain his apparatus in such a condition that he can send on a wave very nearly 600 meters in length. In cases where the distance is great, this may be of extreme importance. Close coupling will not answer in all cases and hence an endeavor is made to give the student a knowledge of the best and quickest way in which to meet such conditions (to say nothing of the value which such information is to him at all other times). In order to verify the calculations and to bring the facts more emphatically to mind, the wave length at each adjustment is measured in the laboratory by means of a wave meter and the results are tabulated along with the calculated values in the note book which every student keeps. The results of such measurement are also reproduced in curves.

Following this work, means for transferring the oscillating energy to the radiating circuit and the conditions under which the greatest current is produced in the antenna are taken up and explained. Various methods are demonstrated in the laboratory for indicating the maximum antenna current, so that an operator will have some way of determining if his antenna is radiating the maximum amount of energy whether he has an approved hot wire ammeter or not. In the study of resonance between an oscillating circuit and an oscillating E. M. F., no attempt is made to avoid the actual alternating current principles which determine the strength of current that will flow in a circuit containing resistance, inductance and capacity. Once this idea is formed in the mind of a student a great many questions such as resonance phenomena in the audio frequency circuits, the use and proper capacity of telephone condensers in receivers, etc., are readily understood. In any case of this kind, the general theory is explained and then demonstrated by experiment. After demonstrating the tuning of the open and closed circuits in different ways the effects of re-transference of energy between them and the production of two wave lengths are brought out. Here, as in many other cases, it is necessary to exaggerate the fact in order to make the desired impression, and for this purpose we have some special apparatus with which it is possible to produce two wave lengths differing from each other by several hundred meters, with the two circuits tuned to an intermediate value.

At this point we take up the study of spark gaps especially the quenched gap which, when placed in the primary circuit of the above coupled system, serves to demonstrate the quenching action in a forcible manner. I have constructed a small quenched gap having ten sections which quenches perfectly operating on 60-cycle current in connection with a one-fourth kilowatt leakage transformer. An attempt to measure the wave length of the circuit in which it is contained shows a very flat wave having a decrement that is difficult to determine. This is shown mounted on a small panel set (Figure 2) which was built for some experimental work in transmitting on low antennas.

The work in transmitting sets is concluded with a study of several standard sets that are in commercial use, showing the inter-relation of the various parts and auxiliary apparatus such as meters, circuit breakers, antenna switches, etc. One of the sets that we have permanently installed in the school is a Marconi 2-kilowatt, 240-cycle set which was loaned to us thru the courtesy of Mr. John Bottomley. This set is complete with storage battery-induction coil auxiliary set, and receiver. Several other complete sets of composite type are also installed. Figure 3 is a view of the radio station showing the Marconi

FIGURE 3

2-kilowatt, 240-cycle transmitter, the storage battery auxiliary transmitter, switch boards, and various types of receiving apparatus.

During the course, about one week's time is spent on storage battery work in which are set forth the details of types common in radio service, their care, methods of charging, etc. Some circuits applicable to emergency ship lighting are also shown.

In taking up the study of receiving circuits and receiving apparatus, we begin with a review of the principles of resonance, again emphasizing the factors which determine the impedence; wherein it is seen that the alternating E. M. F. produced in the antenna by the passage of a wave train can only produce a maximum current in the antenna to earth circuit when the inductance and capacity of that circuit bear a definite relation to each other. Therefore, in order that this circuit shall be adjusted so as to have a low impedence, its capacity and inductance must be made variable by the insertion of a variable condenser and a tuning coil at its earthed end. Before progressing farther into the theory of tuning, it is necessary to consider the action of some detector, such as a crystal rectifier, stating its function and the actual reason for its use. After this has been done, a detector can be included in our antenna circuit and we have the elements of the simplest form of receiving circuit. It is shown experimentally and theoretically how this circuit can be so adjusted that it will respond to waves of widely differing length; and then how it can be further adjusted so that it will respond only to frequencies which are very near to that to which it is tuned. Such a receiver is then compared with a standard receiver as to selectivity and strength of signals, which readily demonstrates its disadvantages. The next improvement on this simple outfit, the close coupled tuner is taken up in the same manner, theoretically and experimentally. In connection with this type of receiver, reference is made to commercial tuners embodying this principle, such as the Type "D" tuner of the United Wireless Telegraph Company, many of which are still in use. Every student, tho generally much to his displeasure at first, is required to use one of these tuners in the radio receiving room until he becomes familiar with its use and possibilities.

After the study of closely coupled receiving sets, and the various methods involving a direct coupling, their advantages and disadvantages, loose or inductively coupled receivers are taken up; first in an elementary way, and then in connection with regular receiving sets. Our laboratory is well equipped with tuning apparatus of various kinds so that quite an opportunity is offered for setting up any standard circuit, or most special circuits. Specific instructions in the use of commercial tuners, such as are used by the commercial companies, follow the theoretical circuits. A great many operators in commercial service are incapable of getting the most out of their receiving sets; and especially is this true in the case of some of the more complicated receivers involving intermediate circuits or special tuning apparatus. In order to train the student to make the most of the facilities at hand and to give him an actual knowledge of the use of such apparatus, I have used the following scheme with success. The tuner or receiving circuit under test is connected to an antenna in the usual manner or to a dummy antenna in which are induced sharply tuned oscillations from a wave meter excited by a buzzer operated by an omnigraph. With the wave meter in operation, the student adjusts the receiver as broadly as possible, thus picking up the signals; after which he tunes for selectivity, and readjusts for the optimum results. After he has become familiar with the various adjustments several wave meter transmitters differing more or less in wave length are simultaneously operated, all being inductively related to the antenna or dummy antenna. A student will send with one of these transmitters while the one manipulating the receiver will endeavor to separate his signals from the interfering signals. In a short time the student becomes quite adept in tuning, and is able to meet many of the difficulties encountered in practice.

In the study of detectors, many of the common types are included, tho the most emphasis is laid on those of the crystal rectifying type inasmuch as they are the ones most used in commercial service at the present time. A great deal of stress is laid on the use of carborundum, which is probably more used and more reliable than any other detector. In much the same manner that a student becomes apathetic toward the Type "D" tuner, he becomes averse to the use of carborundum for reasons which are well known, but if he is supplied with a suitable potentiometer and a large collection of these crystals, he can generally be convinced that this form of detector has some merits. Every student, during the course, spends several hours testing crystals. The laboratory is equipped with a large number of detector stands, potentiometers, and tuners fitted for the use of these crystals, and in using these he gains an idea of the correct method of using such detectors, and eventually has more confidence in them.

One forenoon each week, a special class is held at which all students in the radio work are in attendance. This period is devoted to the discussion and study of radio law, the international regulations, traffic rates, method of computing charges, and similar matters. On some occasions, classes in geography are held at this hour, and maps of the radio districts are studied, steamship lines and routes pointed out, location of radio stations noted, and so on. Each student is required to learn the name of every passenger steamship line on the Great Lakes, the names of their vessels, their runs, call letters, and stations with which they are likely to be in communication. This information is of great value to a new operator, and requires very little time to learn. Some students take a great interest in this work.

Altho our station license is an experimental license and calls for no specific hours of service, we have certain hours during which we always have one or more men on duty in the receiving room, where they get a great deal of practical experience. The requirements as to the matter of maintaining a continuous watch during the time that is assigned to a student are strict, the result of which is that the man acquires a sense of duty so that he is much more apt to realize the importance of his position after he is actually in the service. A complete log is kept of everything that transpires, and all messages are copied and filed. These later are sorted out and entered on report blanks such as are used by the commercial companies and which are furnished by them for this purpose. In fact, the business of the station, in every particular, is handled by the students in a manner as nearly in conformity with commercial practice as is possible. In rating students for positions their record in the receiving room, and number and completeness of the messages copied are taken into consideration.

The receiving room is well equipped with commercial tuners and some special receivers and other receiving apparatus. In the laboratory we have apparatus for receiving undamped, continuous wave stations as well as spark stations, and tuning apparatus for waves as long as 14,000 meters. Most of the equipment in the receiving room is commercial apparatus, while the experimental apparatus is used in the laboratory station. Figure 4 is a photograph in the laboratory receiving room, showing some special receiving apparatus used in research work. At the right is seen a long wave receiving coupling and on the left is a continuous wave generator which was built for testing receivers for undamped waves. It can also be used as a generator for heterodyne receiving. A radio-telephone, with which some experimental work has been done, can be seen in the foreground on the left. Any transmitter in the laboratory can be controlled from this room.

For the regular station work, we have a standard six wire aerial, supported on a 100-foot (31 meter) steel tower, brought down to a mast on the building. In connection with the labo-

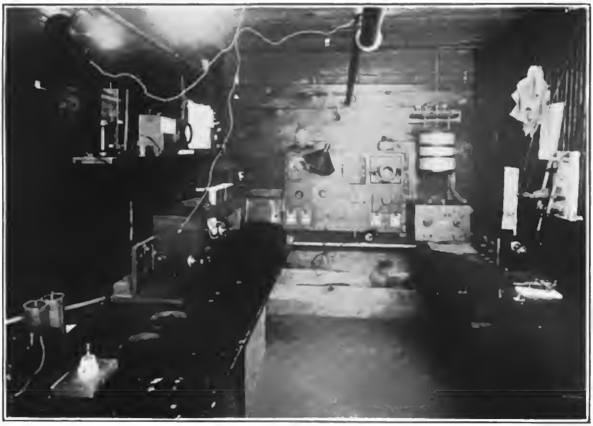

FIGURE 4

ratory apparatus, we have a smaller four wire aerial and a long single wire antenna, used principally for long wave reception. With the arrangement of these different aerials and certain apparatus, it is possible to have several groups of students receiving simultaneously without mutual interference. Ordinarily we have two well-advanced operators on duty in the station from 8 A. M. until midnight; however, in case of severe storms over the lake region, a continuous watch is maintained.

Another interesting feature of the work in the radio course is the so-called "field work." One afternoon each week, when the weather conditions permit, the students are divided up into parties of four to eight and supplied with portable receiving sets or complete field sets which are taken out into the surrounding country and set up. Aerials are erected on poles provided for the purpose or put up on high trees. Occasionally a kite will be used to elevate an aluminum wire or a small boat on one of the nearby lakes will be equipped with a small sending and receiving set. Figure 5 shows a field station, in the charge of a group of students, with which they are in communication with the station at the school. As will be seen this set includes both transmitter and receiver, and when the aerial is elevated to a suitable height,

Figure 5

it has a range of several miles. It can be used either as a "plain aerial" set or directly coupled, radiating about half an ampere either way when properly tuned.

Communication is established between these field stations and the school, where an operator is maintained. During the course of such work aerial construction, methods of quickly putting up an emergency aerial, and the importance of good earth connections are demonstrated in an interesting and forcible manner. I have also found this to be an excellent manner in which to combine practical detector adjustment, tuning and wiring up of apparatus. Many interesting experiments that can be performed in the open country readily present themselves, all of which are of advantage in an operator's training. The effects of broad and sharp waves, necessity of tuning, and the advantages of high spark frequencies and so on are readily set forth in an interesting manner.

For the benefit of special students or those who are particularly interested, we have a somewhat more advanced course in electrical and radio engineering subjects including radio telegraphic measurements and theories. The extent of this work, at the present time, is limited owing to a lack of necessary equipment, but at the same time it offers some advantages to those students who are ambitious and desirous of extending their knowledge of the art of radio communication.

During the coming fall it is our plan to erect a second steel tower 175 feet (54 meters) high at a distance of 400 feet (123 meters) from our present tower. It is also expected that we will add considerably to our electrical equipment at that time.

SUMMARY: The qualifications of a "good" operator are divided into inborn and acquired or teachable characteristics. A course of training for radio operators is then discussed in detail. The entrance requirements and objects of the students are considered, and the subject matter of the course is given.

1. OPERATING DIVISION. Students are taught to receive on buzzer-excited circuits, using head band telephone receivers. A number of circuits of gradually increasing speed and difficulty are provided. Different tones and intensities of signals are provided to accustom the student to actual conditions. All messages sent between student stations are in accordance with the radio laws and commercial practice. Messages must generally be checked and relayed by the student. Artificial interference is provided to teach reading of desired message thru such interference.

2. TECHNICAL DIVISION. The elementary principles of electricity and magnetism and the study of dynamo-electric machinery are given. Inductance, mutual inductance. capacity, wave length and frequency are studied, together with methods for their predetermination by calculation. Resonance phenomena are shown. Different types of commercial receivers and crystal detectors are tested. Field work is done with portable transmitting and receiving sets. Some facilities for research work are provided.

3. TRAFFIC DIVISION. The radio law, international regulations, geography and other material of value to operators are taught by lecture. Work in penmanship is obligatory.

DISCUSSION

Elmer E. Bucher: After careful consideration of Mr. Packman's contribution, I see that he recognizes the time-worn but desirable search for the "one hundred per cent perfect" employee. To a slight extent, I agree with him that in some respects the efficiency of the operating staff of commercial radio telegraph companies might be improved; but I must take complete exception to the allegation that the training of operators has suffered neglect, or that progress in this detail has not kept pace with general commercial radio development. The further reference to "deplorable conditions," assumed to exist in the operating staffs at certain commercial ship and shore stations, cannot carry weight without citations of specific instances of inability. It is useless to decry the service or personnel of an entire organization for the disability of a few, hence it may be of interest to give a brief outline of the method of instruction in vogue at the various radio schools maintained by the Marconi Wireless Telegraph Company, thereby disproving the assertion that radio telegraph operators have not been well trained.

It has been the practice of that Company since its inception to instruct its employees thoroly in the subject of radio telegraphy by the establishment of schools both here and abroad. In localities where the demand for operators has been insufficient to warrant the opening of a company-owned school, local telegraph schools have been subsidized or supplied with apparatus free of cost. In addition, these schools have had the free services in an advisory capacity of the Marconi officials and engineers, who have thereby assured themselves that the graduated students possess qualifications suitable to a proper standard. The foregoing policy has been adopted and rigidly adhered to thruout these years, and it is a fact that the courses given at privately owned institutions have been generally modelled after that given at the Marconi training schools.

In general, corporation-owned schools have the advantage over privately owned schools in that the former are in possession of a more complete radio equipment and are thus enabled to offer their pupils a more comprehensive course than is otherwise possible. Being in closer touch with commercial radio development and the demands of a well organized radio service, such companies are prepared to supply their students with the knowledge most necessary to their requirements, technically and commercially.

A particular problem which radio schools are compelled to meet is the varied degree of intelligence and ability manifested by the applicant for admission. In a university or college, before a student is enrolled on the roster, certain conditions must be met and complied with; consequently it is assured that the entrant is, in a large measure, fitted for the instruction he is about to receive. More clearly, such applicants have thru a number of successive years gradually fitted themselves for their more advanced work, and are therefore able to derive the fullest benefit of the instruction.

Obviously, in a radio telegraph school, such a long drawn out procedure is not possible: first, because the applicant has neither the inclination nor the financial means to support himself over an extended period of training; and second, because no commercial company would care to meet the financial drain imposed upon its treasury by carrying a student on the register for a great number of months. It may be of interest in this connection to remark that corporation-owned schools are not generally a financial success; yet companies are perfectly willing to stand the expense involved in order to maintain a high standard of service by the employment of a staff of well trained men, so important to its commercial success.

Therefore, in order that the student may receive a permanent assignment in the radio service with the least possible delay, it becomes the duty of the radio telegraph school to fill in the gaps in the student's knowledge of the art. In consequence, it is not always possible to inaugurate a definite course of procedure. In so far as possible, the mode of instruction must be varied to meet the individual needs of the pupils.

So far the best success has been achieved by first ascertaining the knowledge of the applicant in respect to the radio art in general and the fundamentals of elementary electricity and magnetism. This known, we are at once enabled to segregate the students into two classes. The missing links of the more advanced student's knowledge are then filled in by a number of general lectures on radio telegraphy, after which a series of experiments are made on the actual apparatus.

The student least informed on matters of electricity is placed in a separate class where he is given thoro instruction in the elements of electricity and magnetism. Slowly but surely, the supposed complexities of the art disappear, the pupil having formed a complete mental picture of the underlying action upon which the operation of radio telegraph apparatus is based.

In this work the instructor must exercise great patience, for it takes time to shape and mold the thought of a raw recruit in the right direction.

A similar procedure is adopted in respect to instruction in the telegraph code; *i. e.*, the student's ability is first ascertained, and then a division into classes made accordingly.

In the code classes artificial radio telegraph circuits are employed thruout, traffic being dispatched from individual to individual after the method employed at commercial ship and shore stations.

The foregoing instruction is followed up by a series of lectures on "Radio Traffic" in which the student is fully informed on the International, United States, and Navy regulations. Intricate problems which the student may encounter in dealing with various radio stations of foreign countries are discussed and solved, until it is certain that the pupil is thoroly familiarized with all possible future conditions which he may meet.

Contrary to the views expressed by the speaker of the evening, I am in favor of introducing a certain amount of automatic machine sending now and then in the code practice, for it has been observed to have a marked effect upon the student's sending. A good automatic Wheatstone sender, connected to a buzzer system, and operated at a speed suitable to the pupil's ability, will do wonders in impressing upon his mind the desirability and necessity of a uniform mode of sending. The ease of reception experienced impels the student to adopt a similar mode of formation more or less unconsciously, resulting in daily improvement.

I would lay down no hard and fast rule concerning the time required for a student to complete his tuition in radio telegraphy. I do not believe it possible to make an expert telegraphist from an absolute beginner in the space of six months, even tho I am aware that this condition has been approached in isolated instances. I do, however, maintain that by six months' study and close application a student is qualified to pass the U. S. Government examination and competent to take an assignment as junior operator at any ship or shore station.

It might be mentioned here that the Marconi Company uses every precaution in introducing a beginner into the commercial service. It is its custom to send a school graduate to sea as a junior operator, under the guidance of an experienced man. In this manner he is enabled to derive the full benefit of the senior operator's previous experience and all possibility of error thro lack of initiative on his own behalf is thereby eliminated.

In respect to training the student to read radio signals thru interference, a school located in a prominent seaboard city such as New York, does not require artificial "jamming" or interfering apparatus. A commercial receiver connected to a fair-sized aerial fulfills the requirements, the operator being enabled to separate interfering stations under actual commercial conditions. Obviously, no better method of training could be devised.

I note from Mr. Packman's contribution that certain pupils with whom he has come into contact possessed biased minds, even to the point perhaps of expressing their desires as to the type of apparatus they consider preferable! A student having pre-conceived notions in this respect is apt to possess proclivities along other lines not amenable to discipline. Hence I would lose no time, in extreme cases, for the good of the services, in eliminating his name from the records.

I contend that the profession of radio telegraphy requires young men of live and alert characteristics who are quickly capable of assimilating new ideas, progressive in their make-up and business-like in character. To secure a well-rounded employee, one equally proficient in several branches of a given art, is one of the problems of the hour; the natural result of this need has been an age of specialization which in many fields has been overdone.

A radio telegraphist cannot be a man of narrow vision. He must be broad enough to think in terms international for he comes in contact with peoples, business methods, and social customs, of all climes and races. Thru several years of experience I have not found it difficult to lay out a course of procedure that will fully fit the student in this respect; and I firmly believe that, in view of their previous training, the degree of proficiency attained by the average radio employee is remarkable, and that in no department of wire telegraphic or telephonic communication will there be found operatives of the attainments of the average telegraphist in charge of radio telegraph equipment to-day.

I think it will be found on investigation that as far as the Marconi Company is concerned, the training of radio operators has in nowise suffered neglect. Every possible available means has been brought to bear in the student's preliminary education so that he may be fully qualified to meet any emergency arising on his initial assignment to a ship or coast station.

David Sarnoff: I consider that the radio operator is one of the most important elements in radio communication. I agree, in some respects, with Mr. Bucher's refutation of the statement made by Mr. Packman regarding "the rather deplorable conditions which exist at present in the radio operating field." I believe there has been a marked tendency toward improvement in this direction during the past few years, and observation justifies the expectation that the improvement will continue.

The acquisition of the late United Wireless interests by the Marconi Company, thereby placing the large number of radio operators under the control of one organization, and the international requirements that a single code—continental—be universally employed, have helped matters considerably. By having all operators under the control of a single organization, antagonism and rivalry among operators otherwise employed by competing radio or steamship companies are removed and this is a very important factor. The advantages of a universal code are obvious in that it renders communication between operators of all nations more flexible.

Before operators are employed in the Marconi service, they are required to pass thru the Marconi School of Instruction where they are given thoro instruction in the principles and manipulation of the various types of radio equipment in general commercial use.

The procuring of a government radio license is not considered sufficient proof of the operator's ability and general fitness for the Marconi service. There are of course exceptional instances where deviation from this rule is imperative and under such conditions the choice of an operator must be governed by the exigencies of the moment.

I should like to say a word or two about the training of the radio operator, starting from the point where Messrs. Packman and Bucher leave the subject.

In my opinion the actual training of the operator commences after he leaves school and joins the operating staff. I have frequently thought that the present method of employing graduates from radio schools in the radio service is wrong; for the reason that at present their first positions are given them on shipboard whereas the better way would be to assign them first to coast stations, where they would obtain the benefit of the more skilled operators on shore, who are thoroly familiar with the proper methods of conducting radio traffic. Here also, the novices in the profession have a better opportunity of handling a larger amount of radio traffic, under the guidance and with the assistance of the more matured and trained coast station operators. The early Marconi operators, and those who now hold the more important positions in the organization, were thus trained.

Unfortunately, however, my theory is not possible of adoption by radio organizations at present, for the following reasons.

First. Because the number of coast stations now in operation, as compared with the number of ship stations, is proportionately very small.

Secondly. Because the majority of the coast stations are situated in out-of-the-way places where, by reason of existing circumstances, it is not practical to assign any but trained operators.

Thirdly. If a graduate from a school is sent to an important coast station and spends some time in becoming proficient, it is hardly to be expected that he would thereafter view with favor the assignment to a less important position on shipboard. Here, too, the difference in salaries paid at ship and shore stations would play an important part.

It is interesting to note from this evening's paper the different methods employed in training the student to become a proficient radio telegraphist, but experience has taught us that there is a marked difference shown by the young operator in transmitting or receiving messages in school, and in handling regular business at a commercial station. This is but natural, and a condition which must be expected. It is for this reason, however, that the disadvantages of placing a school graduate, even on an unimportant freight ship, are so apparent. One poor operator on shipboard, with even a weak radio equipment, can do more harm when in the vicinity of busy ship and shore stations, than can be undone by ten good operators, with an equal number of efficient sets.

In the paper on "Radio Traffic" which I delivered last year before the Institute, I dwelt at length on the importance of brevity in radio communication, and this all-important point cannot be impressed too strongly, especially on the young operator. Very often I have observed junior operators assigned to less important ship stations, transmitting a radiogram by the longest method possible, inserting unnecessary symbols and words, repeating where there is no need to do so, and thereby retarding the movement of traffic very seriously.

The young operator is often actuated by a desire to listen to himself sending. On ship stations where traffic is infrequent, the junior operator often indulges in quite a lot of unnecessary preliminaries and finishing touches, when transmitting or receiving a single message. While these matters may appear insignificant to some of those present, I submit that you need only consider the unfavorable conditions of static, or strays, interference, and frequently poor operating, to appreciate what it means to indulge in superfluities under such circumstances. On the other hand, the advantages of brevity under these conditions will likewise be apparent. Unfortunately, the government regulations pertaining to radio communication, are not adapted to the solution of these practical problems when they prescribe certain preambles and symbols in handling radio traffic. In my paper on the subject previously referred to, I gave examples of this condition.

I also reiterate my long-standing objection to the present wave length regulations enforced by international agreement. It avails us very little to produce transmitters of high radiating efficiency, and receivers capable of sharp tuning, when the majority of ship and shore stations employ 600 meters as their working wave length. I am aware, of course, that wave lengths below 600 meters may be used, and while this has been taken into consideration in the design of the more modern equipments, it fails entirely, nevertheless, to afford the measure of relief required.

In this connection I might say a word or two in admonition of the operators who fail to take full advantage of the opportunity afforded them by the latest Marconi equipments, which are provided with 300, 450 and 600 meter wave lengths, and with facilities for rapidly changing from one wave length to another, the change being effected by the throwing of a single switch. I have known operators who continue to struggle thru interference on 600 meters rather than change to 300 or 450 meters, and I have also observed others who do not even struggle. It is, however, not possible, under the present government regulations, to take full advantage of even the wave lengths mentioned; for the reason that by the rules of the London Convention it is required that when two stations communicate, both must employ the same wave length. Therefore, while it is feasible for a ship station equipped with the latest Marconi set to change quickly from 600 to 450 or 300 meters when communicating with another ship or coast station, it is not quite so feasible for the coast station to effect the same change. You will appreciate, therefore, the importance of reconsidering the whole subject of wave lengths and traffic regulations.

I would urge all of you who have opinions to express on this subject, to write the Institute.* It will be glad to accumulate and summarize all ideas so that a logical and comprehensive statement of facts and suggestions may be presented at the next International Convention, which is to be held at Washington, D. C. It may sound a trifle optimistic to talk of International Conventions in these days, but we are hopeful nevertheless.

In connection with the proper manipulation of radio equipments by operators; I have noticed during my experience that radio engineers are very often prone to criticize the operator for failure to obtain maximum efficiency, and I doubt not that the criticism is sometimes warranted; but on the other hand, something may be said about the radio engineer who, when designing radio equipment at the laboratory, fails to appreciate the operator's difficulty on shipboard. For instance, I have always felt that sufficient attention has not been paid by designing engineers to the subject of detectors. Sensitiveness seems to be the goal for which most engineers aim, but apparently stability is not given the same consideration. There is nothing more troublesome at radio stations than to handle a detector which is too frequently affected by vibration, induction from transmitting apparatus, or by the many other causes which disturb crystal detectors. Operators many times continue to call radio stations which promptly respond but are not heard because the detector at the calling station is temporarily out of adjustment. Every operator knows that this is a daily occurrence and the cause of unnecessary interference, repetition and consequent delay in the movement of traffic. I am of the opinion that some form of valve detector is probably most suitable for commercial operation at ship and shore stations, because the valve detector gives more promise of possessing the combination of the two important elements, namely, sensitiveness and stability.

* A paper dealing with "The Inadvisibility of Wave Length Regulation" will be delivered by Messrs. Goldsmith and Hogan later in 1916. All members having views on this subject are strongly urged to communicate them to either of the authors in writing.

As regards commercial radio schools versus radio telegraph company-owned schools: It is preferable, of course, where a student can do so, to take up his course of training in a school of a large radio organization, because such a school is conducted with the object of training the men for the company's service and not for the profit derived from tuition fees. But there are many cases where, for good reasons, it is not possible for a young man to attend the company's school, and for this reason the commercial schools are performing a very important mission in the radio art. Men must be trained, and they should be trained properly. Many a boy living at or near Valparaiso, Indiana—where the Dodge Institute of Radio Telegraphy is located—might have been unable to take a course in a Marconi School situated elsewhere, and for that reason the Marconi Company lends its support to, and assists in every possible way, this School, as well as all other schools which show a desire to train operators as they should be trained.

Alfred N. Goldsmith: There is no doubt whatever that the question of wave length regulation, which has been brought up by Mr. Sarnoff, is worthy of the most careful consideration. It is further desirable that it be carefully considered *at length*, in view of the possibility of an International Convention on this subject within the next two years. It is not at present obvious that wave length regulation is at all a necessity, and certainly the matter is one for considerable discussion.

As an illustration of an undesirable state of affairs, attention may be called to the restriction of the important range of wave lengths between 600 and 1,600 meters to the use of the Government. It will be noted that most of the stations using these wave lengths are Navy stations, primarily intended for use in times of war, but not for commercial service in times of peace. Whenever one realizes that in times of war the enemy would hardly refrain thru courtesy from using wave lengths within the restricted range, the valid objection to closing this range to all commerical ship and shore stations, becomes evident. The ability to tune skilfully and read thru interference is well worth cultivation.

I feel further that amateurs have been unduly hampered by the wave length restrictions which are now current. This, however, is of comparatively small importance when contrasted with the really serious crippling of commercial traffic by the enforced rules concerning 600 meter transmission and the equality of wave lengths between ships and their corresponding shore stations.

I expect that in the near future a paper will be written by Mr. Hogan and myself dealing with the question of wave length regulation and considering critically whether any wave length regulation should be adopted, and furthermore what rules of radio traffic are most desirable. I am very desirous that all members of the Institute or others interested, should correspond with Mr. Hogan or myself, on this subject in order that we may have the broadest expression of opinion on which to base our own judgments.

Referring further to a possible part of the training of the radio operator which has not been clearly brought out, it seems to me that it would be well to give the students in radio operating some courses in reading messages thru atmospheric disturbances. It would be possible to imitate their effect in the laboratory and thus train the student, at least to some extent, in receiving thru such strays.

John L. Hogan, Jr.: I have been much interested both by Mr. Packman's paper and by Dr. Goldsmith's statement as to the problem of wave length restriction which has been before us for some years. It is so nearly a self-evident fact that the present Federal regulations as to radio wave lengths are of an unjust and ineffectual nature that their adoption seems a most surprising thing. The restriction of wave lengths between 600 and 1600 meters, which is the range best adapted for low and moderate power ship communication, to Government use is an act which has caused and is causing unfortunate delay in the development of the commercial radio art. The requirement that inter-communicating stations both use the same wave length, and the insistence that ships communicate always with the nearest land station, are also regrettable features of the present Convention rulings.

With reference to Mr. Packman's paper, I must agree with Mr. Bucher in his indication that the operating situation of commercial radio telegraphy is not entirely "deplorable." Nevertheless, there are a number of points upon which the vast majority of radio operators could be better trained.

One of these, which was mentioned incidentally by Mr. Packman, receives far less attention than it really deserves. This is the matter of operators' handwriting. It has been my experience that the "copy" of radio operators is as a rule much poorer than that of wire telegraphers. One reason for this is, of course, that the average age of the radio men is considerably below that of the line men, and that the penmanship of the radio operator is therefore likely to be in a formative stage. Another reason is that the traffic in many radio stations, on account of its small volume, puts no especial premium upon and offers no especial opportunities for clear smooth handwriting. It is a fact, nevertheless, that a radio operator is not likely to advance rapidly to better operating positions, and thereafter to executive positions, if his handwriting is of an uncertain and illegible type. It is not probable that too much emphasis can be laid upon the desirability of careful drill in helping to form the habit of clear and characteristic handwriting.

A second point is that radio operators in a commercial telegraph school should be trained to copy signals thru static interference strays. I have known men to be graduated from telegraph courses with the ability to read good buzzer signals, at fairly high speeds, and after securing their Government licenses, to fail utterly in attempts to read incoming radio messages thru even moderate static disturbance. Until an operator has become accustomed to concentration upon signal notes in the midst of harsh irregular noises from strays, he is likely to become excited and useless if he encounters unusual atmospheric interference. If it were difficult to give such training during the usual telegraph course, it might be expected that the operators would have to wait until they entered commercial service for this part of their training. However, it is not at all impracticable to combine practice in receiving thru strays with the ordinary daily code practice which all students of radio operating must be given.

Figure 1 shows a device which is simple and easily set up, yet which I do not believe has been used for this purpose except by the National Electric Signaling Company. In this diagram, X represents a weighted pivoted contact which drags upon the heavily and irregularly knurled surface of a slowly revolving metal wheel. Connected in series with this imperfect contact is a battery B_1 and potentiometer R_1. By suitably choosing the speed of the wheel and the weight of the contact at X, the strength of battery B_1, and the position of sliding contact on R_1, irregular impulses corresponding to almost any sort of static may be applied to the line wires L_1, L_2, thru the telephone transformer T_1. These irregular current impulses, transmitted from the line wires, are reproduced in receiving

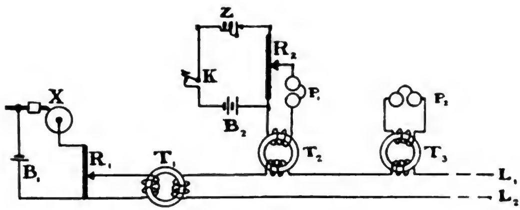

FIGURE 1

telephones P_2, thru the telephone transmitter T_3, as scratchy hissing sounds closely resembling those produced by atmospherics. A buzzer sender, consisting of a buzzer of any frequency Z, a key K and battery B_2, and a variable resistance R_2, may be associated with the line wires thru another telephone transformer T_2. By varying the potentiometers R_1 and R_2, the relative intensities of strays and signals can be made anything desired. It is, of course, obvious that additional transmitters of various frequencies and intensities can be associated with the same line wires, and that these wires may be used to conduct the signals to any reasonable number of student's receiving telephones, such as P_2. While many modifications of the device are obvious, the system as shown has proved very useful for such work as I suggest, and practice on it would form a desirable part of any radio operator's preliminary experience.

A third point upon which many radio operators are weak lies in the adjustment of their receiving tuners. Inductively coupled receiving apparatus, having variable primary and secondary inductances, and a tuning condenser directly connected across the secondary coil, represent the best practice of the commercial radio service to-day. This apparatus, simple as it is, is capable of giving widely different results in the hands of operators of different degrees of experience. Setting aside for the moment those men who are really able to handle an inductively coupled receiver properly, the remaining radio operators may perhaps be divided into two groups. The first of these, which we may call the "primary men," do all their adjusting by altering the inductance of the primary circuit and at the same time leave the secondary inductance and capacity at some average setting which gives fairly satisfactory results, so long as no interference is encountered, The second group, or "secondary men," have a great aversion to changing the settings of their primary coils and tune only with the secondary variable condenser. It is obvious that an operator who is in either of these classes will be certain to get only mediocre results from even the most carefully designed receiver. It is highly essential that all radio operators should appreciate that with an inductively coupled tuner they will secure maximum loudness of signals with maximum freedom from interference when their primary and secondary are both tuned to the wave length they desire to receive, and when the coupling between primary and secondary coils is properly adjusted. It requires a considerable amount of actual practice with inductively coupled tuners to learn just how the four variables (primary, secondary, inductive coupling, and secondary capacity) are inter-related and how compensating adjustments in each must be made as the others are changed.

In order that beginners may have training of this sort, they are usually given short periods of listening at an operating receiving radio station. It is manifestly impossible to handle a large class in this way, giving each one of them enough practice in tuning to be of much value to him. In order that radio schools may deal with this point in a way comparable with its importance, I suggest the circuit arrangements shown in Figure 2.

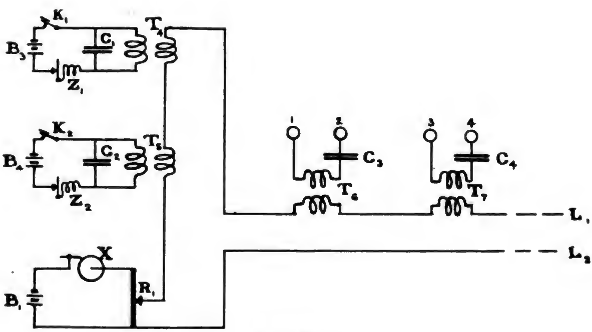

FIGURE 2

In this diagram, buzzer exciters producing radio frequency currents of any desired intensity and group frequency, and corresponding to waves of any length and decrement encountered in practice, are associated with the line wires L_1, L_2. The buzzer Z is connected with battery B, key K, capacity C, and the primary air core transformer T as shown. With the same line wires a static exciter such as that described in connection with Figure 1 may also be connected. At each student's desk, the line wire is connected thru a transformer, such as T_6, which has its terminals connected to binding posts such as 1 and 2, thru a condenser, as C_3. The radio frequency currents set up by the buzzer exciters impress forced radio frequency voltages upon the terminals 1, 2, and if the capacity C_3 and inductance of T_6 are chosen so as to represent properly an average antenna, any radio receiving set may be connected to the two binding posts exactly as it would be connected to antenna and ground in a radio station. By tuning the receiving sets so connected, signals from any of the buzzer exciters may be selected, as signals from outside stations may be selected in practice. The difficulties of eliminating highly damped disturbances, such as those of strays, may also be experienced in this way, since the impulse maker X will shock the primary circuit of each receiving tuner into oscillation of whatever period it has, exactly as static would in an ordinary receiving station. For tuning to damped waves, one or more of the buzzer exciters may have resistance inserted in their oscillation circuits, so as to increase the decrement of the current impulses there generated.

Simple modifications of Figure 2 which will permit students to inter-communicate under conditions very closely approximating those of actual radio practice may easily be devised by following the principles just outlined. It is certain that training of this sort would go far toward increasing the traffic handling ability of any operator who has not reached a point of high efficiency in the manipulation of his instruments.

By these comments, I do not wish to be understood as inplying that radio operators in general suffer from inability in these several directions. There are many men in the field of whom their respective service executives may well be proud. There are, nevertheless, many inexperienced telegraphers who would be greatly benefited by thoro drill in the three matters I have discussed. It is my hope that future courses of training in radio telegraphy will give beginners greater opportunities for thoro understanding of commercial radio conditions.

M. E. Packman: In reference to Mr. Hogan's scheme of using many commercial receiving sets, which, of course, is highly desirable, he apparently fails to appreciate the fact that it would require about 25 to 30 receiving sets, costing from two hundred and fifty dollars up. Such expense is not possible for commercial institutions.

The work with elaborate artificial antennas in receiving has been turned over to the more advanced students.

In reference to Mr. Bucher's and Mr. Sarnoff's remarks in connection with the deplorable conditions I referred to, I think that they are both considering the service on the Atlantic Coast which is indeed better than it is in other parts of the Marconi service. I am more or less familiar with Mr. Bucher's school and know that his training is very comprehensive but the point is that the demand is far greater than the school can supply. I have known men in service who have first grade licenses and who are actually unable to receive anything. I have known men in my own school who get thru the commercial examinations with no trouble who are practically worthless as far as commercial service is concerned. This condition has been the case for a good many years in the part of the work with which I am familiar, tho it has been improving from time to time. Many operators have been employed who have practically very little knowledge as compared with what they should have.

On the Great Lakes, it must be considered that the time of navigation does not exceed over nine months and that out of 60 or 80 ships, there are only 15 or 20 which run the whole year round. This means that there will be 40 or 50 operators required at the beginning of the season. Some of the old ones return, but only very few, and the first men that call at the office are the ones that secure the positions, regardless of their ability. The point to be noted is that they are employed without knowledge of the chief operator as to their ability. This condition does not exist in the East to this extent.

(Further material received from Mr. Packman too late for insertion in this issue of the PROCEEDINGS will appear in an early issue.—EDITOR.)

SUSTAINED RADIO FREQUENCY HIGH VOLTAGE DISCHARGES*

By

Harris J. Ryan and Roland G. Marx

INTRODUCTION

In high voltage work, discharges thru the air between conductors and over and thru insulators can be prevented only with the aid of ample knowledge of their characteristics. Discharges produced by low (audio) cycle voltages for given conditions are now fairly generally understood. In radio telegraphy, high (radio) frequency damped and sustained high voltage waves are employed. Accidents, including lightning, produce in high voltage power circuits, in the long run, almost every conceivable high voltage transient. Such transients may vary from a simple over-voltage at normal frequency thru all possible impulses and damped oscillations to perhaps a briefly sustained high frequency high voltage wave train. Little is known as yet of the relation between discharge distances and voltages of the various sorts just specified. The evidence so far accumulated indicates that for given values of maximum voltage, the discharge distances are almost *independent* of the characteristic variation of the voltage whenever the critical corona voltage is higher than the discharge voltage. It indicates, too, that the discharge distances are *dependent* upon the characteristic variation of the voltage whenever the critical corona voltage is below the discharge voltage. In regard to the latter condition, this evidence indicates further that the discharge distance will be longest when the voltage source or transient is most sustained, or when its frequency is the highest or when both of these characteristics are present. It follows that discharge distances should be found a minimum for low frequency high voltages and a maximum for sustained high (or radio) frequency high voltages. It thus appears that voltages which can be formed by accident may discharge thru

* A paper presented before a joint meeting of The Institute of Radio Engineers and The American Institute of Electrical Engineers, San Francisco, September 16th, 1915.

greater distances and do more damage than the same values of voltages as used in most commercial work. The following experiments were undertaken as a reconnaissance in this region of high voltage phenomena.

DISCHARGE INTO THE ATMOSPHERE FROM A SINGLE ELECTRODE

One terminal of a sustained high frequency high voltage source[1] was grounded, the other was a 1-inch (2.5 cm.) copper tube capped with a hollow copper sphere 2 inches (5 cm.) in diameter. This spherical end of the high voltage terminal was mounted properly remote from all grounded objects. When a voltage of 50,000 at 88,000 sustained cycles was applied, a dry redwood stick was brought near to the sphere and then removed. A spark passed from the sphere to the stick and immediately grew into a heavy brush discharge. See Figure 1. It consisted

FIGURE 1

essentially of an active mass of darting streamers. The character of this mass varied from that of a combustion flame at the base to the familiar static discharge at the extremities. We have been able to determine with a fair degree of approximation (by measurement correct to within ten per cent.) that the rate of energy supply in the discharge from the electrode, illustrated in Figure 1, was about one kilowatt. It is charac-

[1] Described in "*Sphere Gap Discharge Voltages at High Frequencies,*" by J. Cameron Clark and Harris J. Ryan, "Proceedings of the Am. Inst. Elec. Eng'rs," June, 1914, Vol. XXXIII, page 937.

teristic of this high voltage, radio frequency discharge, that it consumes a large amount of power; and if that power is not available, a discharge will not develop. It may start to develop and one may see some brush momentarily, but not the actual discharge. No "flashing-over" effect will be produced unless plenty of power is available. The discharge averaged about 10 inches (25 cm.) in length, was rather bright, produced a hissing, roaring sound, and was not accompanied by the familiar odor of ozone that is formed by the less violent audio frequency or intermittent radio frequency discharges. It is easily blown about by air currents. It may be blown by the breath from place to place on the ball. It can be fanned with a hat from the ball back along the 1-inch (2.5 cm.) conductor, and put out as it is driven into the region of lower capacity in the vicinity of the conductor, that is, where the fields are less intense and where the energy cannot be delivered at the rate that the flame or the discharge requires.

A modification of the above experiment was arranged to enhance the flame-like portion of this discharge, and to eliminate most of the "brush" part. A circular metal disk 16 inches (40 cm.) in diameter, provided with a 3-inch (7.5 cm.) hole at its center, and with ¼-inch (0.6 cm.) guard tubing facing all edges, was hung centrally over, and about 3 inches (7.5 cm.) above the 2-inch sphere (5 cm.) terminal by means of non-conducting supports. Figure 2 is a photograph of the steady flame-like discharge that occurred from the sphere to the plate. This photograph was naturally obtained by a legitimate artifice. In the laboratory, everything was dark when the first exposure was made and the flame photographed; and then by using some flash-light powder, all the apparatus was illuminated so that it could be photographed also. The flame, tho very strong, gives off no great amount of luminous radiation. The voltage and frequency were the same as before, viz., 50,000 and 88,000. The temperature of this flame was high. It melted quartz, rapidly disintegrated a tungsten lamp filament, and formed a bead on the end of a Nernst lamp filament. The metal of the electrodes was not greatly heated, and little or no metallic vapor appeared to enter the arc stream.

This flame discharge is not stable under all conditions. For example when the inductance and capacity of the disk were increased by placing in contact with it one end of an insulated 1-inch (2.5 cm.) copper tube 4 feet (1.2 meter) long, the flame discharge was no longer quiet and stable, but became noisy and snappy, tending to develop into an intermittent disruptive discharge. The flame became unstable also when the electrode gap length of the arc generator was too short in adjustment. It appeared to be identical with the flame-like portion of the

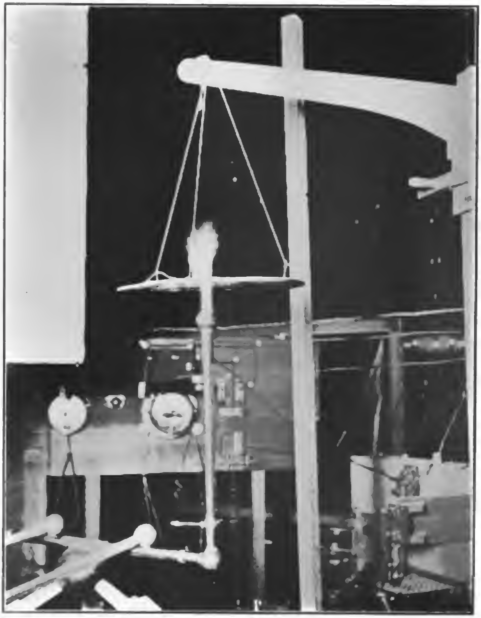

Figure 2

heavy brush discharge of Figure 1. Time did not permit a study of the extent to which the combustion of nitrogen was taking place in the flame. It seems as tho something of the sort is occurring for the reason that ozone is not in evidence when this discharge occurs.

The ability of the radio frequency brush to produce thermionic conduction thru glass, porcelain, quartz and all similar refractory insulations is perhaps its most remarkable property. This is illustrated by bringing any mass of high grade electrical porcelain near to or in contact with the sustained radio frequency electrode. In an actual case, the electrode was a ½-inch (1.2 cm.) aluminium tube laid in the top groove of a 33 kilovolt porcelain line insulator that was itself placed on an insulating support and mounted remote from all objects of opposite or ground potential. On the application of 35 kilovolts at 200,000 cycles, the air between the tube and insulator was overstressed, small flame discharges conducted the insulator charging currents to the porcelain surface where one or more brilliant hot spots would appear in about 30 seconds. Further study developed the fact that these hot spots were the heads of corresponding hot conducting cores that extended into the depth of the porcelain, thus establishing by conduction new routes for the delivery of the charging currents taken by the porcelain mass. No insulation that supports a conductor charged with high voltage at sustained high frequencies can endure, unless it is so designed that not a particle of air or other gas in contact with it is overstressed under actual working conditions. The Fortescue and Farnsworth principle can be employed in the design of such supporting insulators so as to suppress all overstress of air adjacent to the porcelain or other solid dielectric.[1]

SUSTAINED RADIO FREQUENCY CORONA ABOUT A WIRE

The general arrangement of the equipment employed for the sustained radio frequency corona study is shown in the diagram of Figure 3; and a photograph thereof in Figure 4. The corona was formed around a number 19, B. & S. gauge clean copper wire* held axially in a galvanized iron cylinder, 15 inches (38.1 cm.) in diameter and 35 inches (88.9 cm.) long. Twelve (12) inches (30 cm.) of the wire at the center of the cylinder were normally left clear, and the remainder was shielded by two brass tubes 7-16 inch (1.1 cm.) in diameter. A third tube ½ inch (1.2 cm.) in diameter was arranged to slip over the central portion of the wire, and shield that too when desired. In this manner the corona could be suppressed, or it could be allowed to develop by removing the copper tube from the wire, and thus greatly increasing the stress on the atmosphere adjacent to the wire (because of the smallness of the wire circumference). We could thus check up the accuracy of the cathode ray power measuring meter.

[1] "*Air as an Insulator when in the Presence of Insulating Bodies of Higher Specific Inductive Capacity,*" C. L. Fortescue and S. W. Farnsworth, "Trans. Am. Inst. Elec. Eng'rs," 1913, Vol. XXXII, page 893.

* Diameter of wire = 0.036 inch = 0.092 cm.

Various voltages up to about 30 kilovolts, (root-mean-square), were impressed on the wire at sustained radio frequencies of 88,000 and 188,000 cycles per second; also at 60 cycles per second for comparisons. The appearance of the coronas at radio and audio frequencies differed greatly, while those at the

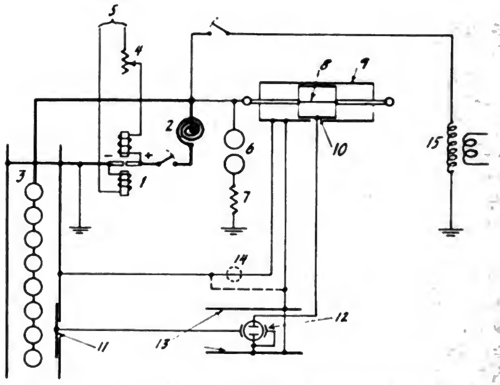

FIGURE 3—Diagram of Connections for Sustained Radio Frequency Corona Investigation

1—Arc Generator
2—Air Inductance
3—Air Condenser
4—Resistance
5—To 1200 Volt D. C. Supply
6—Sphere Gap Voltmeter
7—Carborundum Resistance
8—Corona Wire
9—Cylinder
10—Potential Tapping Cylinder
11—Potential Tapping Plate
12—Cyclograph Quadrants
13—Guard Plates
14—Carbon Lamp Resistance
15—60 Cycle High Voltage Transformer

two radio frequencies differed only slightly. That is to say, the enormous difference in corona at radio frequencies and at audio frequencies such as 60 cycles, is a difference that has come about perhaps gradually on the way up from 60 cycles to some such value as 50,000 cycles. At all events, to double, or a little more than double the frequencies when one is operating at a frequency of as high as 80,000 cycles, produces very little effect on the character of the phenomenon. The radio frequency corona appeared very active, it was quite brilliant and noisy and gave off an appreciable amount of heat. At 30 kilovolts the average diameter of the radio frequency corona was about 2 inches (5 cm.) whereas that at the audio frequency appeared to be less than 1-8 inch (0.3 cm.). A photograph of these coronas is reproduced in Figure 5. Two exposures were

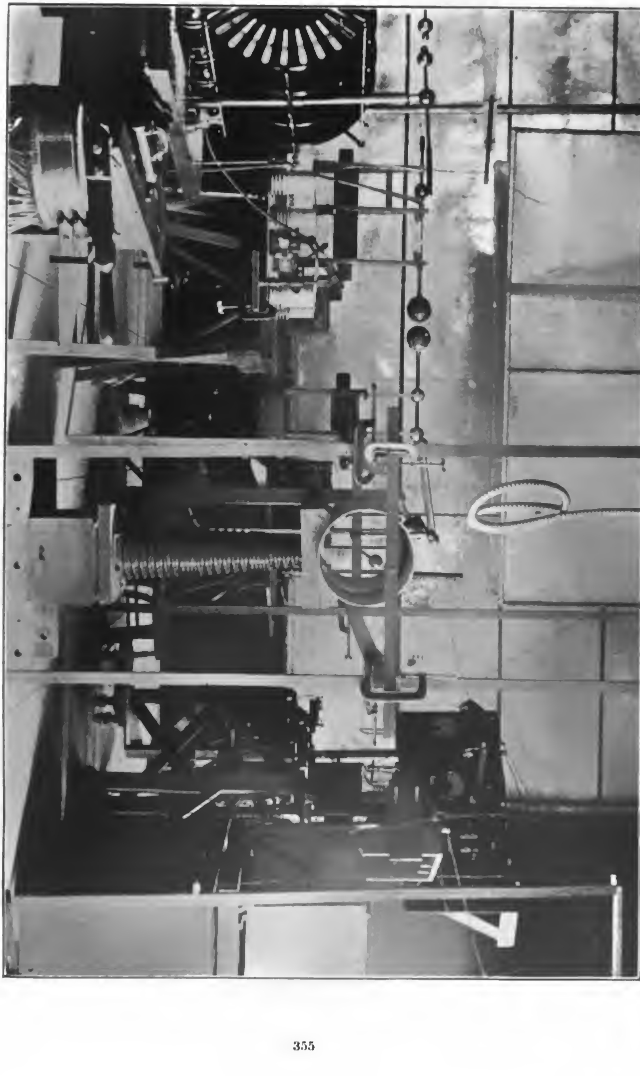

FIGURE 4

made on the same film; on the left is a 1-second exposure to the corona about the wire at 19.5 kilovolts and 188,000 cycles, while on the right is a 120-second exposure to the corresponding corona formed at the same voltage and 60 cycles. The camera was moved so as to separate the two images. In the original photograph the difference is very striking. Not only was the exposure 120 times as large, but the result was very nearly as many times less. The action is therefore a vastly more intense one.

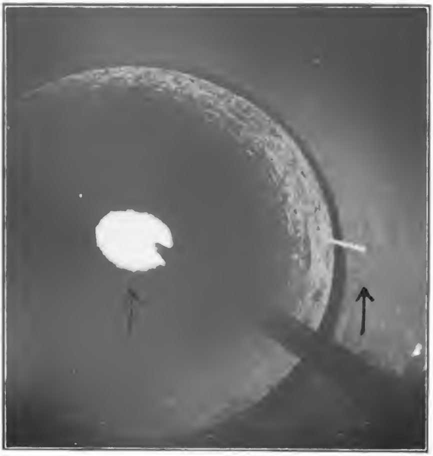

FIGURE 5

Some observations were made to determine the relative values of the voltages required to start corona about the wire at 188,000 cycles and at 60 cycles. All voltages were determined with the same 5-inch (12.7 cm.) spherical gap.* Attention is called to the lack of information that we have as yet in regard

* Little has been published as yet in regard to the standardization of the sphere-gap for the measurement of radio frequency voltage. It appears likely that not much more than a beginning has been made. Until such standardizations are available, the spherical gap will serve quite well as a radio frequency voltage gauge for purposes of record and comparison. The working scale for the 5-inch (12.7 cm.) spherical gap used herein was arbitrarily chosen as the one determined at radio frequencies for a 7-inch (17.8 cm.) spherical gap with the neutral of the voltage source grounded. *Loc. Cit.* No. 1; also "Dielectric Phenomena in High Voltage Engineering," by F. W. Peek, Jr., page 107.

to the standardization of these gaps. Our work has indicated very closely that there is little difference between the indications that a sphere electrode gap will give for given values of voltage at radio and at audio frequencies. However, for the exact interpretation of the result as given here, the footnote will be helpful. The density of the atmosphere was that due to ordinary temperatures near sea level. Twelve and seven-tenths (12.7) kilovolts were required to start the corona at 188,000 cycles and 13.2 kilovolts correspondingly at 60 cycles. The indications of the sphere gap were here assumed to be independent of changes in frequency.

Cyclograms were taken of the energy consumed per cycle in the corona about the wire at 60, 88,000 and 188,000 cycles and at voltages ranging from 15,000 to 20,000 to determine the relative power factors and the wave forms of the currents flowing from the wire. The cathode ray tube was used in taking these cyclograms. The details of the method used have been given in the "Transactions of the American Institute of Electrical Engineers."[1] The actual arrangement of the cyclograph with its voltage and current condensers as used in the present work is given in the diagram of Figure 3. Various trials were made to determine that the cyclograph gave true indications within its limits of action when high frequency high voltage was used. These trials were as follows. When the wire at number 8, Figure 3, was screened from corona formation by sliding the ½-inch (1.2 cm.) brass tube over it, the cyclogram would close up into a right line loop without area. Thus arranged, by inserting an ordinary incandescent lamp at number 14 the cyclogram would open so as to enclose a large elliptical area. Again using the radio frequency high voltage, the effect in the results due to the hysteresis or other loss in the glass of the cathode ray tube was found to be negligible by noting that a no-area cyclogram obtained with all four quadrants mounted on the exterior wall of the tube remained as such when all conditions continued the same except that one pair of quadrants was mounted within the tube.

In Figure 6 sample cyclograms are reproduced. With the aid of the lantern, enlarged images of these cyclograms were thrown upon a sketching board and tracings carefully made. Figure 7 was engraved from these tracings. The distortion noted is due to the fact that the only suitable tube available

[1] "*A Power Diagram Indicator,*" Harris J. Ryan, "Trans. Am. Inst. Elec. Engin'rs," 1911, Vol. XXX, pages 1089–1113.

for this sort of work was one of small size. To obtain sizeable cyclograms, it was necessary to permit some distortion in their lower portions. They are instructive, however, for they show that the radio frequency corona current wave suffers less dis-

FIGURE 6

tortion than the corresponding audio frequency corona current wave. They also show, under the conditions present, that the power factor of the radio frequency corona current was about *one-quarter* of the power factor of the corresponding audio frequency corona current. The present work, however, as stated in the introduction, is merely a reconnaissance of these interesting phenomena. It will be profitable to have them studied broadly and with great care, especially so with ample and suitable facilities.

DISCHARGE BETWEEN BLUNT POINT AND PLATE

A needle point is promptly melted and burned by radio frequency brush discharges. Only blunt points can be used, therefore, to determine the radio frequency high voltages required to discharge given distances when one electrode is or both electrodes are in corona. The scheme employed in this set of determinations is diagrammed in Figure 8 for the radio frequency or audio frequency discharges, and in Figure 10 for combined audio and radio frequency discharges. A photograph of the electrodes and the sustained radio frequency discharge between them is reproduced in Figure 9.

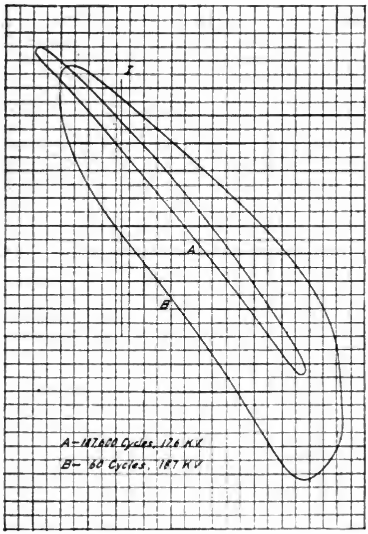

FIGURE 7

The blunt pointed electrode connected to the high frequency source was a square ended piece of number 12, B. & S. gauge copper wire,* projecting axially from the main radio frequency high voltage electrode, constituted as before of a 1-inch (2.5 cm.) copper tube ended with a 2-inch (5 cm.) copper sphere. A galvanized iron sheet, 3 feet (91.4 cm.) square, was used as the grounded electrode. Carborundum resistances (see number 5, Figure 8), were employed at strategic points to avoid short-circuiting the machines that supplied the arc generator with continuous current. The 5-inch (12.7 cm.) sphere gap at number 4, Figure 8, was used to measure all voltages. The sustained radio

* Diameter of number 12 wire = 0.081 inch = 0.21 cm.

frequency voltages that produced discharges between the point and plate also produced at slightly lower values heavy brushes

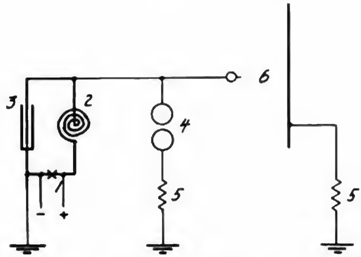

FIGURE 8—Diagram of Connections for Point to Plate Discharge

1—Arc Generator
2—Air Inductance
3—Air Condenser
4—Sphere Gap Voltmeter
5—Carborundum Resistance
6—Point to Plate Gap

that extended from the blunt point most of the distance to the plate. In fact, the discharges seemed to occur only when the brushes appeared to have fully bridged the space between the

FIGURE 9

electrodes. Facilities were lacking for the measurement of the large amounts of power that were evidently consumed in these brushes.

The 60-cycle voltage source was substituted for the arc generator in this sustained radio frequency point to plate discharge equipment diagrammed in Figure 8; and voltage discharge distance measurements were then made to compare with the corresponding sustained radio frequency discharge distance

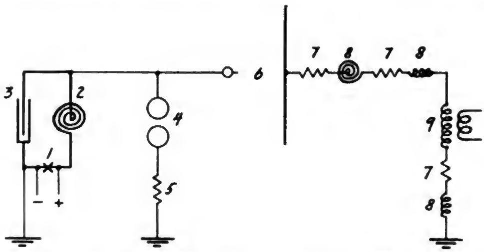

FIGURE 10—Diagram of Connections for Discharge with Combined Radio and Audio Frequency Voltage

1—Arc Generator
2—Air Inductance
3—Air Condenser
4—Sphere Gap Voltmeter
5—Carborundum Resistance
6—Point to Plate Gap
7—Carborundum Protective Resistance
8—Protective Air Inductance
9—60 Cycle High Voltage Transformer

measurements. Likewise for comparison a few determinations were made of the radio and audio cycle voltages required to discharge from the same blunt point to a similar blunt point in lieu of the galvanized iron plate.

The results obtained for the audio and radio frequency discharges are charted in Figure 11; and for the composite discharge values produced by the simultaneous application of sustained radio frequency voltage from earth to the blunt point and of 60-cycle voltage from earth to the plate are given in Table I. Two forms of discharge occurred and are designated "spark" and "arc" discharge. The former occurred at a somewhat lower voltage than the latter. The spark functioned to discharge the main condenser of the radio frequency generator and the arc to short circuit the 60-cycle and 1,200-volt direct current sources. The sums, equivalents and differences recorded also in Table I, and the values at corresponding differences charted in Figure 12 assist one to understand the parts that each voltage took in forming the composite discharges.

It is of interest to note (see Figure 11), that whereas 135 kilovolts at 60 cycles were required to discharge 16 inches (40.6 cm.) from the blunt point to the plate only 46.2 kilovolts were required correspondingly at 88,000 cycles. An increase of 7.5 kilovolts at 60 cycles was required by an increase of 1 inch (2.5 cm.) in

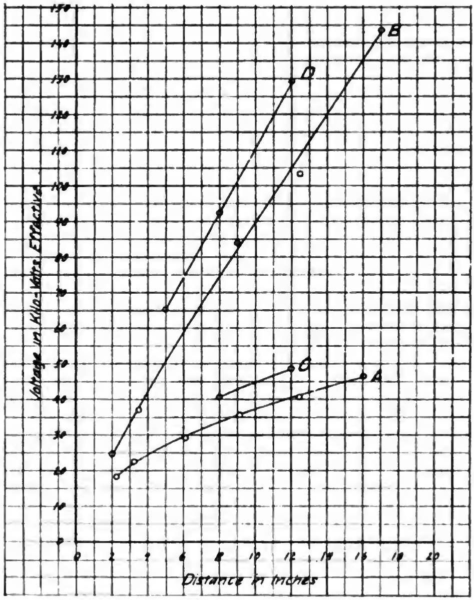

FIGURE 11—Point Discharge

A—Point to Plate, 88,000 Cycles
B—Point to Plate, 60 Cycles
C—Points, 88,000 Cycles
D—Points, 60 Cycles

NOTE—Radio Frequency Sphere Gap Voltmeter Calibration Used in all Cases.

the 15-inch (38.1 cm.) discharge gap while the corresponding increase at 88,000 cycles was only 1.5 kilovolts. In other words, as the length of a 15-inch (38.1 cm.) point to plate gap is increased the amount of increase of 88,000 cycle discharge voltage is *one-fifth* of that required at 60 cycles.

The composite discharge distances due to the combination of audio and radio frequency voltages are virtually the sum of the distances thru which the individual voltages discharge. In Table I, column 2, the radio frequency voltages alone would have discharged the distances given in column 6[1], which when subtracted from the actual discharge distances in column 1, give the distances in column 7 as the added discharge distances due to the audio frequency voltages in column 4. These audio frequency voltages and the added discharge distances they caused are charted in Figure 12. For comparison the A. I. E. E. standard

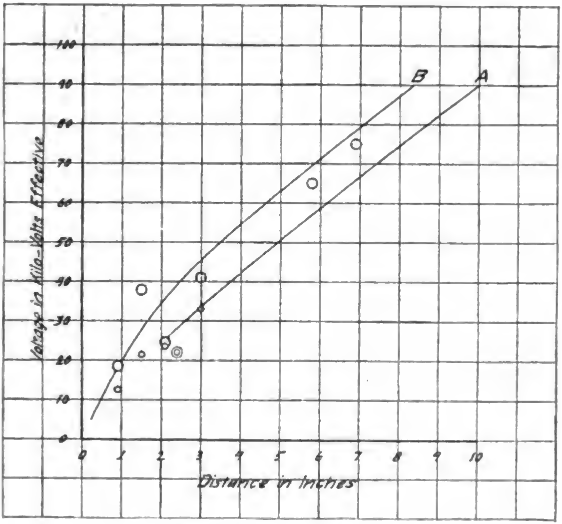

FIGURE 12—Plot to Accompany Table 1

For points marked thus—o Abscissas Represent Values in Column 7, Ordinates in Column 3
For points marked thus—O Abscissas Represent Values in Column 7, Ordinates in Column 4

A—Curve B of Figure 11
B—A. I. E. E. STANDARD Needle Gap Curve

needle gap voltage discharge curve and the 60-cycle point to plate discharge curve of Figure 11 are also charted as curves "B" and "A" in Figure 12. It is thus seen that the added dis-

[1] These distances were observed for the conditions shown in Figure 10, and are not identical with distances for corresponding voltages observed for the conditions in Figure 8 and charted in curve A, Figure 11.

charge distances due to the superimposed audio frequency voltage are practically the same as the corresponding discharge distances produced by the identical audio frequency voltages acting alone. In making this comparison, one must hold in mind the fact that the added discharge distance caused by the superposition of the 60-cycle voltage should naturally be somewhat greater than the discharge distance produced by such audio cycle voltage acting alone; because in the former case, no initial voltage is required to start corona at the blunt point; such corona is started by the sustained radio frequency voltage.

The authors desire to acknowledge herewith the valuable assistance rendered by their departmental co-worker Professor J. C. Clark.

SUMMARY: 1. Sustained radio frequency corona brushes or flames once started are maintained at much lower voltages than those required to start them by overstressing and ionizing the atmosphere. They quickly destroy even the most refractory insulations by their heating and ionizing properties.

2. The power factor of the charging current of a conductor in corona due to the application of sustained radio frequency high voltage is decidedly lower than the corresponding power factor at audio frequencies. Nevertheless, because of the high values of the currents that produce the radio frequency coronas, the losses they cause may be hundreds of times the corresponding audio frequency losses.

3. The sustained radio frequency voltage required to discharge between corona-forming electrodes may be as low as one-third of the corresponding audio frequency voltage. At higher voltages this ratio will probably be found to be less than one-third.

4. Sustained radio frequency and audio frequency voltages when combined, discharge thru distances between corona-forming electrodes that are substantially the sum of the distances thru which such voltages would discharge when acting alone, due account being taken of their mutual aid in starting the corona at one or both of the electrodes, as the case may be.

TABLE I

Combined Radio Frequency and Audio Frequency Voltages; Point to Plate Discharge

1 Gap Distance in Inches	2 Radio Frequency Voltage in Kilovolts	3 Audio Frequency Voltage for Spark Discharge	4 Audio Frequency Voltage for Arc Discharge	5 Sum of the R. F. and A. F. Voltages	6 Discharge Gap Equivalent to Radio Frequency Voltage	7 Difference, Column 6 Subtracted from Column 1
5	28.7	....	....	....	....	...
5	....	....	51	....	....	...
5	21.2	22	....	43	2.6	2.4
5	21.2	....	22	43	2.6	2.4
5	22.4	23.5	....	46	2.9	2.1
5	22.4	....	24.5	47	2.9	2.1
5	26.4	12.5	....	37	4.1	.9
5	26.4	....	18.5	45	4.1	.9
12	42.9	....	....	....	....	...
12	....	....	105	....	....	...
12	29	....	75	104	5.1	6.9
12	31.8	....	65	97	6.2	5.8
12	37.5	33	....	70.5	9.0	3.0
12	37.5	....	41	78.5	9.0	3.0
12	40.3	21.5	....	62	10.5	1.5
12	40.3	....	38	73	10.5	1.5

Radio Frequency Voltages at 88,000 cycles. All voltages in terms of five inch (12.7 cm.) gap; the calibration being taken as Kilovolts (effective) $= 2 + 45.5 \times$ (Gap Distance in inches) $= 2 + 17.9 \times$ (Gap Distance in cm.).

DISCUSSION

Robert B. Woolverton (Chairman): On behalf of The Institute of Radio Engineers, I wish to acknowledge the great courtesy of the American Institute of Electrical Engineers in the arrangements it has made for this joint session.

As the advantages of the use of long wave lengths in radio communication become more and more evident, it has become apparent to radio engineers that they are limited quite strikingly in the use of these long waves at high power by the formation of corona on the antenna. It is obvious, therefore, that any light that can be thrown on the subject of corona is of intense value to radio engineers.

Robert H. Marriott: As Mr. Woolverton has pointed out, a paper of this kind should enable us to anticipate what may be expected in the way of corona on high power station antennas, and in that way we can keep down costs. It will be remembered that the matter of antenna insulation has always been one of the important things in radio work.

Haraden Pratt: Does the resistance used in connection with direct current arc generator circuits vary with frequency? Another matter which arises in connection with this paper deals with harmonics produced in the working circuits. Taking a circuit of 100,000 cycles, I have been able to observe as many as 62 harmonics, some more or less strong than others. In the event that some parts of the apparatus subjected to the high potentials, such as the concentric brass tubes mentioned in this paper, should have a capacity that would reinforce one or more of these harmonics, might not the added steepness of the very high frequency wave affect the character of the corona ?

Harris J. Ryan: I have had no experience with the variation in the resistance of the carborundum rods with frequency. I understand that their resistance does vary with frequency. We were compelled to use these rods as a matter of strategy in preventing short circuit currents. Otherwise, it would have been disastrous for our apparatus. The values of the resistance, however, were so low that the results were not affected by the presence of these rods. We are confident of that. We made tests and assured ourselves of the fact that we were not using too much resistance.

Unavoidably harmonics are produced in the driving voltage of the Poulsen arc generator. However, in generating high voltage, the inductance of the oscillating circuit must be made relatively large and the capacitance relatively small. The harmonics in the arc voltage do not, as a consequence, drive corresponding currents in appreciable amounts thru the whole of such inductance. These currents penetrate only a few of the outer turns of the inductance whence they are shunted by the local capacitance of such turns; thus it comes about that the harmonic voltages are not impressed thru the entire inductance and do not reach the main electrode in appreciable amounts. This we have demonstrated conclusively by means of the cathode ray voltage oscillograph.

Ellery W. Stone: In the paper (on page 353), it is stated that "Further study developed the fact that these hot spots were the heads of corresponding hot conducting cores that extended into the depth of the porcelain." I should be interested in having Professor Ryan explain how the hot conducting cores in the porcelain were detected.

Harris J. Ryan: We have within the last two years again and again applied these sustained radio frequency high voltage discharges to porcelain insulators of many different patterns and sorts. We know that a molten conducting core is formed, because when a high voltage of radio frequency is applied in the manner indicated in the paper in the immediate neighborhood of the insulator, there is at first quite a corona display for a few moments due to the breakdown of the air near the electrode. This disappears immediately when a bright hot spot forms under or near the electrode, and this bright spot is of a yellowish white incandescence. As soon as such bright spot appears the charging current need no longer be furnished thru the outside conducting air (corona); but, since there is a conductor thru the porcelain, the charging current passes to it laterally thru the porcelain.

As regards the molten condition of this core, the discharge can be driven to the point that there is actual plasticity. In fact, if an opposing electrode is placed under the porcelain, so that directive forces are present, the conducting core is driven thru the porcelain from one electrode to the other. This experiment has been performed with porcelain one-half inch (1.27 cm.) thick, but there is no reason why it should not be performed with thicker porcelain. In these experiments, the hot spot has appeared at each side, and the corona has simultaneously disappeared. Upon stopping the application of the high voltage, the core promptly cools and solidifies. If the porcelain is broken apart thru the core it is found to be smooth grained, brittle and glass-like. Left mechanically undisturbed it is often, tho not always, found to have regained most of its original dielectric strength; i.e., it will endure the application of audio frequency voltage to the flash-over point. Renewed application of the radio frequency high voltage without change in the position of the electrodes will generally, tho not always, re-establish the hot conducting core in the former position.

An interesting variation in this experiment may be made to demonstrate the powerful mechanical drive that exists in the path of an electric spark. When a hot core thru the porcelain has been produced the main electrode is drawn away from the porcelain, say 3 to 5 inches (7.5 to 12.5 centimeters). This will stop the current flowing conductively thru the porcelain hot core and reactively thru the rest of the porcelain. Simultaneously the radio frequency voltage is raised to the value whereat the air between the main electrode and the hot core in the porcelain is ruptured. A spark is thus set up. It discharges the main condenser of the radio (high) frequency source thru the hot, plastic core in the porcelain. This spark stops the generation of the radio frequency high voltage. By the recovery of the generating action of the source in an obvious manner, such voltage is quickly renewed so that several sparks per second follow one another. When a few sparks have passed, the high voltage is turned off and the specimen is allowed to cool. It is then broken open whereupon one will often find that a clear hole of small calibre, diameter one-fiftieth of an inch (one-half millimeter), or thereabouts, has been made thru the porcelain core by the blast of the spark. There is here some evidence of the electro-physical manner in which a real open puncture is formed thru a refractory dielectric.

In a paper presented to the American Institute of Electrical Engineers before another section here to-day, Mr. F. W. Peek, Jr., demonstrates that it requires a much shorter time to build up and to produce under high voltage a discharge between spherical electrodes than between pointed, sharp or even blunt electrodes, as long as the "sharp" electrodes are not so blunt as to prevent corona from being formed in advance of the discharge. This is in contradistinction to an arrangement where spherical electrodes are employed and they are not widely separated, so that the corona is not formed in advance of the discharge. This is a matter of great practical importance in dealing with the question of arranging properly static arresters and reliefs. Incidentally, evidence related hereto was produced by the following experiment at sustained radio frequency high voltages. Near the main helix of the arc generator, a companion helix was mounted. Connected in series therewith was a high voltage adjustable condenser, so that one might easily, by turning the handle of that condenser, pass thru such a capacity value as to bring about resonance in the circuit thus formed. The detached helix was four or five feet away from the arc helix and the oscillating circuit of the generator, and was connected to rothing save the adjustable condenser. In order to ascertain when the circuit was in tune for the frequency of oscillation of the generator, there was connected across the terminals of the condenser a needle gap set at about an inch (2.5 cm.) length. As one passed thru the exact value for the capacity required to produce completely effective tuning, an arc would be set up between the needle points. They were promptly melted, because of the rather large amount of power present. Then it was noticed that unless one passed thru the correct capacity value slowly, the discharge did not have time to build up between the needle points. It was necessary to pass thru the resonance value very deliberately. To build up the discharge between the points required appreciable time because it required the absorption of considerable energy. Prolific ionization had to be produced to bring about the discharge.

Roy E. Thompson: Another explanation occurs to me, however. If two such circuits are coupled, in general (for electrical reasons) the second circuit will not follow the first one rapidly enough to admit of Morse signals in the first circuit being clearly indicated in the second. The "building up" of current takes too long in the second circuit, and detuning may occur thru reaction on the first circuit. Might this not be the explanation here also?

Harris J. Ryan: The point is well taken. If there is any effect in connection with these coupled circuits which throws one out of tune as the other is tuned to resonance when the action is performed rapidly but not when it is performed slowly, then this would be an explanation of the time required for the discharge to culminate. This is a matter which must be studied with great care to prevent arrival at erroneous conclusions.

Roy R. Thompson: If there were a means for controlling the energy of the primary circuit, it would be possible to note whether the discharge took place immediately after closing the primary circuit. The retardation due to variation of the secondary condenser could then be separately studied.

THE EFFECTIVENESS OF THE GROUND ANTENNA IN LONG DISTANCE RECEPTION*

By

R. B. Woolverton

The subject of this paper was suggested in October, 1914, when resonance curves were being taken by the writer at Eccles, Cal., on waves emitted by the various high powered commercial stations situated in the vicinity of San Francisco, at a distance of approximately 100 miles. The antenna used in taking these resonance curves consisted of the top wire of a 5-foot (1.6 meter) fence extending in a northwesterly direction for a distance of approximately 4,000 feet (1,300 meters). Altho the antenna so used was quite aperiodic, as might be expected, the received energy in the secondary circuit was remarkably large, signals being heard from stations in the Hawaiian Islands and Alaska. By using the ordinary crystal detector, full scale deflection was obtained on a Leeds & Northrup portable galvanometer when taking resonance curve data on the wave emitted by the high powered Marconi station at Bolinas, Cal.

In view of the results obtained at Eccles, the writer conducted on October 9th and 10th, 1915, experiments of a somewhat more quantitative character at the Palmer B. Hewlett ranch, situated 90 miles (140 kilometers) south by east of San Francisco. The receiving apparatus was of the de Forest "ultraudion" type (oscillating audion), using a second step amplifier audion bulb, and the audibilities were read on a "Wireless Specialty" audibility meter.† The connections are shown in detail in Figure 1.

It will be noted that two pairs of telephone receivers are connected in series, thus reducing the audibilities nearly 50 per cent., but it was found that the audion circuit would not oscillate when but one pair of receivers was used, with the audibility meter shunted about it.

Before beginning the experiments, it was thought that a com-

*Presented before The Institute of Radio Engineers, New York, November 3, 1915.

(†A variable multi-contact resistance, graduated directly in "times audibility" for use with a definite telephone receiver of the Pickard type.—Editor.)

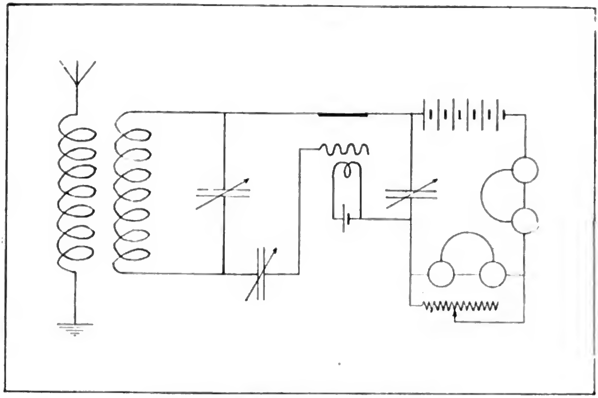

FIGURE 1—Diagram of Connections

paratively long single wire antenna would be so directional in effect that it was decided to confine the readings to one particular station; and Sayville, Long Island, was chosen, the antenna being made as nearly directional toward that station as possible. Buildings slightly interfered with this plan, however, and the antenna's true direction from the receiving apparatus was west-southwest, instead of more nearly west. As soon as readings were begun, it became apparent that this directional effect did not exist, as will readily be seen from the Honolulu audibilities in the "Audibility Table," Figure 2, and the "Direction and Range Chart," Figure 3. The two antennas consisted of 500-foot (160 meters) and 1,000-foot (320 meters) lengths respectively of a single Number 28 B. & S., cotton covered, magnet wire,* laid on dry earth without support at any point. The audibilities for the four transmitting stations are shown in Figure 2.

It will be noted that in the case of each station received from, the signal strength is more than sufficient for reliable communication, particularly when it is realized that the audibility of atmospherics was unity in each case. Atmospheric audibilities taken during the period of the tests, on a five-wire antenna, 45 feet high and 300 feet long, averaged 100.

Figure 3 shows the direction of the antenna with respect

* Diameter of Number 28 wire = 0.0126 inch = 0.0320 centimeter

ANTENNA	SAYVILLE	HONOLULU	ARLINGTON-ARC	ARLINGTON-SPARK
500 FEET	50	100	60	100
1000 FEET	80	160	80	160

FIGURE 2—Table of Audibilities

to the stations received from, with the distances of the stations plotted to scale; and it immediately suggests experiments to determine the most effective design of a ground antenna. These experiments will shortly be undertaken by the writer, using various lengths, heights from the earth, and high potential ends both open and earthed. In view of the comparatively high ohmic resistance of the antenna wire used in the above tests,

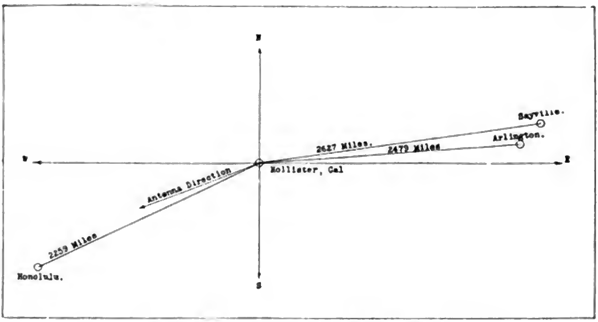

FIGURE 3—Direction and Range Chart

the use of a larger wire should give considerably greater audibilities. If such should be the case, and sufficiently high audibilities are obtained for daylight reception, it would seem that the ground antenna may be the solution of the serious problem of eliminating atmospheric interference, not to mention the difference in cost of the construction and maintenance of such an antenna, as compared with that of the present type.

In closing the writer wishes to express his appreciation for the courtesy and valuable assistance rendered by Mr. Palmer B. Hewlett, of Hollister, Cal.

SUMMARY: Using an antenna several hundred meters long stretched on the ground, signals of an audibility up to more than one hundred are received from sustained wave stations 4,000 kilometers away. Atmospheric disturbances are found by the experimenter to be less troublesome relatively than when using a normal antenna. A further series of development experiments are outlined and will be undertaken.

DISCUSSION

Alfred N. Goldsmith: In presenting this paper to the New York membership of the Institute, it is to be noted that the paper gives only preliminary experiments and that Mr. Woolverton is carrying on further experiments and will lay the results of these experiments before the Institute. The paper is merely an introduction. It is further to be noted that Mr. Woolverton is well aware of the previous work done in this field by Messrs. Marconi, Braun, Zenneck, Kiebitz, Taylor and others.

Lester L. Israel: So far as the ground antenna is concerned, it has been worked with very largely without much success in the past. In Cuba particularly, a ground wire 1000 feet (300 meters) long was used and signals were received with about the same intensity as on an antenna 100 feet (30 meters) high. So far as atmospheric disturbances were concerned, the results were anything but satisfactory. It must be mentioned that if any advantages were obtained by Mr. Woolverton in the use of the ground antenna, the ground conditions in the neighborhood where the experiments were tried would be largely responsible.

Alfred N. Goldsmith: In reading thru a number of papers on this topic by Kiebitz, it was found that this experimenter claimed that he found no change in the ratio of signals to atmospheric disturbances of reception by using the ground antenna, as compared with the ratio for an ordinary antenna.

Lester L. Israel: In experimenting with ground antenna, Mr. Hill found that an antenna grounded at one end could be tuned but in cases where it was entirely ungrounded, tuning was practically impossible.

Roy A. Weagant: From the statements which have been made so far, it is not very clear whether Mr. Woolverton was working in the immediate vicinity of the elevated aerial or not. I believe that Mr. Woolverton referred to an aerial about 45 feet (15 meters) high. The influence of such an aerial on reception by means of a wire stretched on the ground would be very great.

We are not able to judge completely as to the efficacy of the ground antenna in eliminating atmospheric disturbances, because Mr. Woolverton gives the strength of atmospheric disturbances and signals on his ground antenna and the strength of atmospheric disturbances on the elevated antenna, but he does not give the necessary data as to the strength of signals of the elevated antenna. Such information should be sent.

So far as my own experiments are concerned, I do not know if there is any advantage in using the ground antenna. It seems that the ratio between signal strength and disturbance strength is constant regardless of the type of antenna used. Sometimes advantages which are obtained with low aerials are due to the fact that the receiver used, for example the audion, has an upper limit of response. If it is struck by a stray impulse of more than a certain strength, it is simply temporarily paralyzed, and no further immediate response is obtained.

Alfred N. Goldsmith: In connection with the experiments which Mr. Woolverton is carrying out, any suggestions addressed to Mr. R. B. Woolverton, Custom House, San Francisco, Cal., will be welcomed by Mr. Woolverton, who is interested in obtaining the widest possible expressions of opinion relative to experiments of this type.

Robert B. Woolverton: Every effort was made to prevent the elevated antenna from affecting the results on the ground antenna. The elevated antenna circuit was kept wide open, and in addition its direction was exactly at right angles to the ground antenna. However, in future experiments, the elevated antenna will be taken down.

Every effort was made to keep the sensitiveness of the ultraudion constant. The Los Angeles station of the Federal Telegraph Company provided signals used as a reference constant before reading audibilities on other stations. Furthermore, the de Forest bulb used in all the tests was an especially goo l one, and practically no difficulty was experienced in keeping its sensitiveness constant.

THE DESIGN OF THE AUDIO FREQUENCY CIRCUIT OF QUENCHED SPARK TRANSMITTERS

By

Julius Weinberger

(Including a Supplementary Discussion of "Resonance Phenomena in the Low Frequency Circuit," by H. E. Hallborg, Proceedings of the Institute of Radio Engineers, Volume 3, Part 2, 1915, page 107.)

A large number of contributions to the literature of radiotelegraphy have been made upon the subject of the so-called "resonance transformer." These have been both experimental and theoretical. The experimental contributions, as a general rule, have been investigations of the resonance transformer under actual operating conditions (that is, with the secondary condenser discharging periodically thru a spark gap), while the theoretical contributions have generally assumed a steady state of affairs (the secondary condenser *not* being discharged); in this case the method of treatment has been that employed for two coupled circuits.

In actual practice, such as in the operation of quenched gap sets, the requirement of a clear note involves the discharge of the secondary condenser at the peak of the wave each half cycle. It would seem, therefore, that the *transient* phenomena in the circuit would be the determining factors of voltage and current, rather than those of the steady state of affairs; that is, conditions would never assume the steady state.

To investigate these conditions, we can reduce the whole resonance transformer circuit to that of a simple inductance, capacity and resistance in series (Figure 1), as has been shown by Mr. Hallborg. The inductance **L** includes all the inductances in the circuit—generator inductance, transformer leakage inductance, inductance of any series choke coils, and so on. The condenser **C** is the secondary condenser reduced to the primary circuit by multiplication by the square of the ratio of transformer voltages. The resistance **R** includes resistances in the primary circuit and resistances in the secondary circuit reduced to the primary by division by the square of the ratio of transformer voltages.

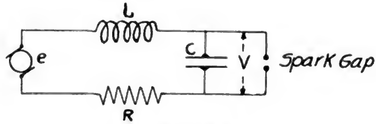

FIGURE 1

The differential equation for such a circuit is

$$e = E \cos(\theta - \theta_o) = R\,i + x\frac{d\,i}{d\,\theta} + x_c \int i\,d\,\theta$$

where

E = maximum generated * voltage
x_c = condenser reactance
x = inductive "
$\theta = \omega t$
θ_o = an angle to be subtracted from θ if e is not zero for $t = 0$.

The potential difference across the condenser terminals can be found from

$$V = x_c \int i\,d\,\theta$$

when equation (1) has been solved for **i**.

Since we are mainly concerned with this **V**, we will omit writing the solution for i, but give that for **V** immediately:

$$V = \frac{E\,x_c}{Z}\sin(\theta - \theta_o - \gamma) + \frac{E\,x_c}{Z}\varepsilon^{-\frac{R}{2x}\theta}\left\{\sin(\theta_o + \gamma)\cos\frac{q}{2x}\theta + \left[\frac{R}{q}\sin(\theta_o + \gamma) - \frac{2x}{q}\cos(\theta_o + \gamma)\right]\sin\frac{q}{2x}\theta\right\} + \varepsilon^{-\frac{R}{2x}\theta}\left\{e_o\cos\frac{q}{2x}\theta + \frac{2\,R\,e_o + 4\,x\,x_c\,i_o}{2\,q}\sin\frac{q}{2x}\theta\right\}$$

where

Z = impedance,
γ = phase difference between generated e. m. f and i
$q = \sqrt{4\,x\,x_c - R^2}$

e_o = value of potential difference across condenser terminals at the time $t = 0$

i_o = value of current thru the circuit at the time $t = 0$.

* This is *not* the voltage across the generator terminals. If the generator armature has appreciable inductance (in comparison with the rest of the circuit) there will be a drop in voltage inside of the armature and a very much higher voltage will actually be *generated* than that which is measured at the terminals.

Consider the conditions introduced in the circuit immediately after the condenser has sparked over, at one peak of a cycle, and the spark has ceased. This is the moment for which we take **t** = 0. The important thing to be determined is:—what will be the voltage across the condenser for $\theta = \pi$ (that is, at the next peak of the cycle)? Will it rise to a sufficient value to cause another discharge? Or, rather, will it rise to a value equal to that, at least, at which the previous discharge took place? If not, the requirements of a clear note, of twice the generator frequency, will not be fulfilled. Also, it is this discharge voltage which determines the energy absorbed by the condenser.

Taking the equation given for **V**, we can introduce the following simplifications:

(1) Since we will consider the circuit as being resonant, we have $x_c = x$, and shall substitute x for x_c accordingly, thruout.

(2) Since the circuit is resonant, the current and generated voltage are in phase, hence $\gamma = 0$.

(3) When the condenser discharges, the potential difference between its plates is reduced to zero. Hence. at the moment we are considering, $e_o = 0$

(4) The spark occurs when the generated voltage is zero. Since i_o is in phase with e_o, $i_o = 0$.

(5) Since the circuit is resonant, $Z = R$.

(6) R^2 can usually be neglected as compared with $4\,x\,x_c$.

Hence $$q = 2\sqrt{x\,x_c}$$

Or, since $$x = x_c$$

$$q = 2\,x$$

(7) In our case, $$\theta_o = \frac{\pi}{2}$$

Substituting these conditions, we obtain

$$V = \frac{E\,x}{R}\sin\left(\theta - \frac{\pi}{2}\right) + \frac{E\,x}{R}\varepsilon^{-\frac{R}{2x}\theta}\left\{\cos\theta + \frac{R}{2\,x}\sin\theta\right\}$$

$$= \frac{E\,x}{R}(-\cos\theta) + \frac{E\,x}{R}\varepsilon^{-\frac{R}{2x}\theta}\left\{\cos\theta + \frac{R}{2\,x}\sin\theta\right\}$$

To show the general shape of this curve, which gives the voltage across the condenser at any moment after the time t = 0, it has been calculated for a specific case (C = 20 microfarads and R = 1 ohm), and is shown in Figure 2. In the same figure the curve of generated voltage (a sine wave), is given for comparison.

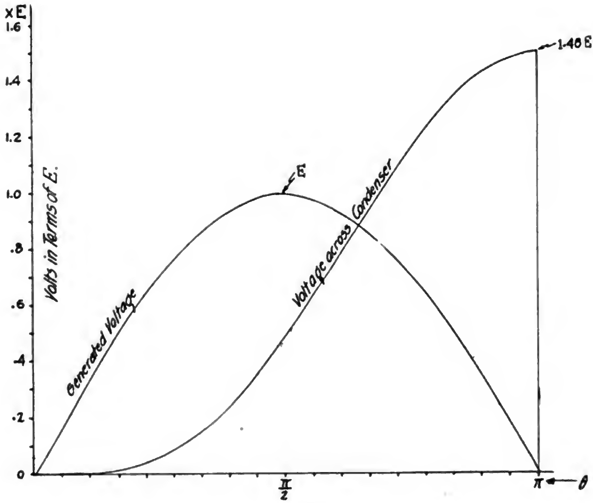

FIGURE 2

This condenser voltage, **V**, will reach its maximum for $\theta = \pi$. It will then be

$$V_{max} = \frac{E\,x}{R} - \frac{E\,x}{R}\varepsilon^{-\frac{\pi R}{2x}}$$

$$= \frac{E\,x}{R}\left(1 - \varepsilon^{-\frac{\pi R}{2x}}\right)$$

This, then, is the potential at which our "reduced" condenser will discharge. The actual condenser, across the transformer secondary, will, of course, discharge at a voltage which is simply this V_{max} multiplied by the transformer ratio. In Figure 3, curves are given for V_{max} in terms of **E** (the maximum generated voltage). It will be seen that for ordinary conditions of resistance (that is, **R** between zero and 1 ohm), $V = 1.5$ E is a good average value.

To find the R. M. S., or effective value of **V** is desirable, since this is the voltage that a voltmeter placed across the transformer primary will read, and this is also the voltage for which the transformer primary must be designed when the equation $V_{eff} = 4.44\,A\,B\,n\,f \cdot 10^{-8}$

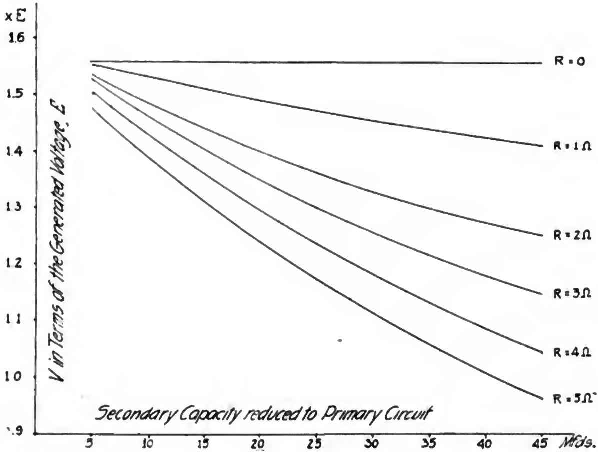

FIGURE 3

is used, where

V_{eff} = R. M. S. volts across transformer primary
A = cross sectional area of core in square cms.
B = flux density, in lines per square cm.
N = number of turns of primary winding.
f = supply frequency.

This effective value of **V** is

$$V_{eff} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(-\frac{E\,x}{R}\cos\theta + \frac{E\,x}{R}\varepsilon^{-\frac{R}{2x}\theta}\left\{\cos\theta + \frac{R}{2x}\sin\theta\right\}\right)^2}$$

It is found* that

$$V_{eff} = 0.504\ V_{max}.$$

The design of a quenched gap set to operate under resonance conditions becomes a relatively simple matter. Let us take a numerical example for a 500-cycle, 1 kilowatt set, operating with a 110-volt generator (154 volts maximum).

We shall first find the equivalent primary condenser (i. e. the secondary condenser reduced to the primary circuit), required to absorb 1,000 watts, from

$$W = n\,C\,V^2$$

*This value was determined graphically, the integration being done by measuring the area of the squared curve with a planimeter.

Since $V = 1.5\,E = (1.5)\,(110)\sqrt{2} = 233$ volts.

Hence $1000 = 500\,C\,(233)^2,$

$C = 37$ microfarads.

To tune to 500 cycles with this capacity, an inductance of 2.5 millihenrys is required. This can be made up partly from the generator armature inductance (usually this is between 1 and 5 millihenrys for a 1 kilowatt, 110-volt machine), and the rest obtained either by a transformer having this amount of leakage inductance, or else from a transformer with no appreciable leakage and series choke coils. I believe the latter method to be preferable as it admits of greater flexibility.

The value of the equivalent primary condenser (or rather, the "reduced" secondary condenser, as I have called it), being now fixed, the actual secondary condenser is determined by deciding on a suitable transformer ratio. The value of this secondary condenser is usually limited by conditions of wave length and also by the discharge current which the quenched gap in use will stand. A large condenser means heavy currents and considerable heating in the gap, while a high discharge voltage and a small condenser would require many gap sections and cause insulation difficulties. It is, I believe, common practice to employ about 0.006 microfarads as a secondary condenser for this type of set.

Having thus determined the ratio of primary to secondary capacities, the transformer ratio is of course fixed; and it is only necessary to design a transformer of the ratio desired—a simple matter with a closed core transformer of negligible leakage. Note should be taken of the fact previously mentioned that when the usual transformer formulas are used, the effective value of V (that is $0.504\ V_{max}$) should be used as the voltage across the primary.

Practically, the operation of quenched gap sets is at a point slightly "off" resonance. However, it is hardly necessary to operate with a condenser as much as 20 per cent larger than the resonance capacity. The foregoing results can, therefore, be applied as very good approximations to actual practice; and have been found to be quite satisfactory for this purpose.

[Since the above was written I have become aware of an article by L. B. Turner* upon the same subject. Following a somewhat different procedure, Turner reaches practically the

*L. B. Turner: "Electrician," Vol. 69, 1912, page 694; "Der Schwingungskreis niedriger Frequenz in der Funkentelegraphie," "Jahrb. d. Drahtl Tel.," Volume 9, Heft 2, page 141.

same results as given above, with the exception that he neglects the resistance of the circuit. As Figure 3 shows, however, this would lead to considerable inaccuracies, for large resistances, and is strictly correct only for $R = 0$. Turner obtains the result

$$V = \frac{\pi}{2} E.]$$

WASHINGTON, D. C., July 1, 1915.

SUMMARY: The paper gives the operating theory of the power transformer and alternator circuit of quenched spark gap transmitters. The secondary of the transformer and its loading capacity are reduced in the usual way to equivalent primary inductance and capacity. The theory of the transient phenomena occurring at the sudden discharge of the condenser is then developed. It is shown that under ordinary conditions the maximum condenser voltage (reduced to the primary circuit), is 1.5 times the maximum voltage generated in the alternator. The effective (or R. M. S.) condenser voltage, reduced to the primary, is found to be 0.504 times the maximum primary voltage. It is this R. M. S. voltage which is used in the usual transformer design. The theory is clearly illustrated for the case of a 500 cycle 1 K. W. set.

THE PUPIN THEORY OF ASYMMETRICAL ROTORS IN UNIDIRECTIONAL FIELDS

WITH SPECIAL REFERENCE TO THE GOLDSCHMIDT ALTERNATOR.*

By

Benjamin Liebowitz

Since its advent, the radio-frequency generator of Professor Rudolph Goldschmidt has been the subject of much discussion, and several theories of its action have been advanced. The theory of the Goldschmidt alternator, however, is but a special case of the general theory of asymmetrical rotors in unidirectional magnetic fields, which latter has been developed by Professor Pupin, and on which he has been lecturing during the past seven or eight years. The Pupin theory, therefore, antedates the Goldschmidt alternator by several years, but is little known except to those who have attended his lectures. The object of this paper is to give the theory its due publicity.

CIRCUIT HAVING VARIABLE INDUCTANCE AND NO RESISTANCE

It will be helpful, perhaps, before considering Pupin's problem, to take up a simple, hypothetical case first, viz., a circuit having a periodically varied self-induction and no resistance. Imagine a circuit made up of two coils connected in series, the one having inductance L_1, the other inductance L_2, and let M be the maximum value of the mutual inductance between the coils. When they make an angle θ with each other, the mutual inductance between the coils is $M \cos \theta$. (See Figure 1.) Let the circuit be supplied with a source of constant e. m. f., E (e. g., a battery), and let R be the resistance. If one of the coils is continuously rotated with angular velocity ω, the total self-induction of the circuit will vary periodically in accordance with the equation

$$L = L_1 + L_2 + 2 M \cos \omega t.$$

* Delivered before the Institute of Radio Engineers, New York, May 5, 1915.

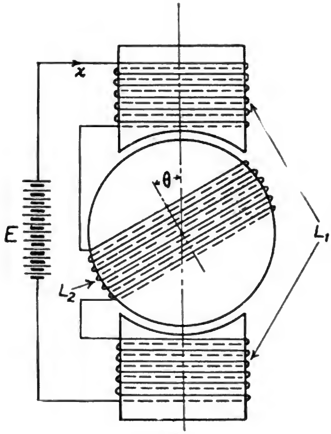

FIGURE 1

The inductance reaction will therefore be

$$\frac{d}{dt}\Big[(L_1 + L_2 + 2M\cos\omega t)\,x\Big],$$

where x is the current in the circuit at any instant. For brevity, put

$$L_1 + L_2 = \lambda,\ \ 2M = \mu,$$

and the inductance reaction becomes $\frac{d}{dt}\Big[(\lambda + \mu\cos\omega t)\,x\Big]$. The resistance reaction is Rx, hence the equation of reactions is:

$$\frac{d}{dt}\Big[(\lambda + \mu\cos\omega t)\,x\Big] + R\,x = E.$$

This equation, as it stands, comes under Pupin's problem; what we are interested in for the moment is the simplification which results when the resistance R is assumed to be vanishingly small. We must assume, of course, that E also becomes vanishingly small, altho the ratio $E/R = X$ is to be regarded as finite. With these assumptions the equation of reactions becomes

$$\frac{d}{dt}\Big[(\lambda + \mu\cos\omega t)\,x\Big] = 0,$$

the solution of which is

$$(\lambda + \mu\cos\omega t)\,x = K,$$

whence $$x = \frac{K}{\lambda + \mu \cos \omega t}.$$

K is a constant of integration, depending on the initial conditions. If we assume, for example, that

$$x = X \text{ when } t = 0,$$

then $$K = (\lambda + \mu)\, X = (\lambda + \mu) \frac{E}{R}.$$

Therefore $$x = \frac{(\lambda + \mu)\, X}{\lambda + \mu \cos \omega t}.$$

This equation shows how the current varies in a circuit having a periodically varied self-induction, an initial current, and no resistance. Its graph is given in Figure 2 for the case

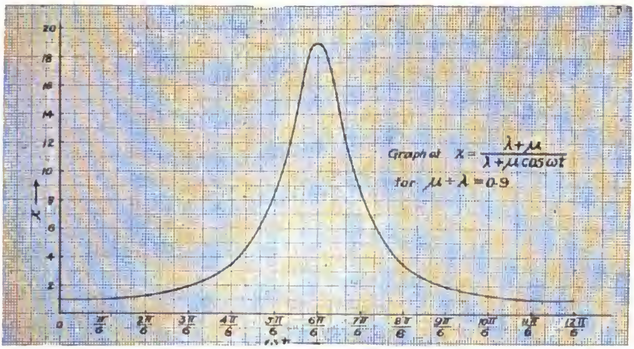

FIGURE 2

where $X = 1$, $\lambda = 1$, $\mu = 0.9$. Since it is an even function, it is developable into a series of cosines. Carrying out the development we obtain

$$x = \frac{(\lambda + \mu)\, X}{\lambda + \mu \cos \omega t} = 2X \sqrt{\frac{\lambda + \mu}{\lambda - \mu}} \left(\frac{1}{2} + B \cos \omega t + B^2 \cos 2 \omega t + B^3 \cos 3 \omega t + \ldots \ldots \right),$$

where $B = \sqrt{\left(\frac{\lambda}{\mu}\right)^2 - 1} - \frac{\lambda}{\mu} = \frac{\lambda}{\mu}\left(\sqrt{1 - \frac{\mu^2}{\lambda^2}} - 1\right).$

Now, the inductance of a circuit without capacity can never become negative, hence

$$L_1 + L_2 + 2\,M \cos \omega t > 0,$$

$$\therefore \qquad L_1 + L_2 > 2\,M \quad \text{and } \lambda > \mu.$$

Hence B is a negative quantity whose absolute value lies between 0 and 1. B is 0 when M is 0, and $B = -1$ when $L_1 + L_2 = 2\,M$; i. e., when $\lambda = \mu$. This can never happen, but if it did, we see that the amplitudes of all the harmonics of x would be equal but would alternate in sign, and the series would not be convergent. In all other cases we see that the amplitudes of the higher harmonics *decrease in geometric progression*, and that they alternate in sign as before. Obviously the series is convergent.

The case just considered is a purely hypothetical one, of course, but I have worked it out in some detail because of the light it throws on the difficult problem presented by the actual circuits.

For the benefit of those who are not familiar with the theory of transformation of equations, a few words on this topic may be said. Suppose we have an equation of any nature whatever, in any number of variables. To fix the ideas, let there be two variables, x and y, and let the equation be given by

$$f(x, y) = 0.$$

To aid in solving this equation we may substitute for x any legitimate function in any number of new variables, and likewise for y. Suppose these transformations involve 2n new variables; upon $(2n - 2)$ of them we may impose any conditions we please; this leaves two variables, the relation between which must be determined from the original equation, $f(x, y) = 0$.

CIRCUITS HAVING INDUCTANCE, RESISTANCE, AND VARIABLE MUTUAL INDUCTANCE

Turning now to Pupin's theory, we consider first the case of a circuit having resistance R, inductance L, and a constant impressed e. m. f., E.; in the field of this circuit is rotated another circuit having resistance S and inductance N. For any angle θ between the coils, the mutual inductance is given by $M \cos \theta$. (See Figure 3.) In the first circuit, the reactions are the inductance reaction $L \frac{dx}{dt}$, the resistance reaction Rx, and the e. m. f., $M \frac{d}{dt}(y \cos \omega t)$ due to the presence of the rotating circuit. In this latter the reactions are the inductance reaction

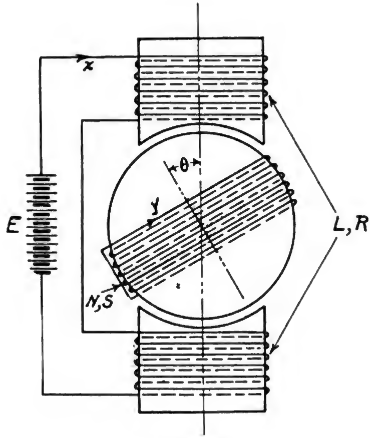

FIGURE 3

$N\frac{dy}{dt}$, the resistance reaction Sy, and the e. m. f. due to the presence of the stator circuit, $M\frac{d}{dt}(x\cos\omega t)$. (Thruout this paper, x shall denote the current in the stator, y the current in the rotor, and ω the angular velocity of rotation.) The equations of reactions therefore are:

$$(1)\quad \begin{cases} L\frac{dx}{dt} + Rx + M\frac{d}{dt}(y\cos\omega t) = E, \\ N\frac{dy}{dt} + Sy + M\frac{d}{dt}(x\cos\omega t) = 0. \end{cases}$$

Pupin's rigorous solution of these equations is the backbone of his theory. Having once obtained the solution of these equations, it is a relatively simple matter to extend the theory to more complicated cases, e. g., with condensers in one or both circuits, impressed e. m. f.'s varying periodically with the time, etc. We shall treat in some detail, therefore, the case now under consideration.

To equations (1) apply the transformations:

$$(2)\quad \begin{cases} x = x_0 + x_2 + x_4 + x_6 + \ldots\ldots\ldots\ldots, \\ y = y_1 + y_3 + y_5 + y_7 + \ldots\ldots\ldots\ldots, \end{cases}$$

getting:

$$
(3)\left\{
\begin{aligned}
& L\frac{d x_o}{d t} + R x_o \\
& + L\frac{d x_2}{d t} + R x_2 + M\frac{d}{d t}(y_1 \cos \omega t) \\
& + L\frac{d x_4}{d t} + R x_4 + M\frac{d}{d t}(y_3 \cos \omega t) \\
& + L\frac{d x_6}{d t} + R x_6 + M\frac{d}{d t}(y_5 \cos \omega t) \\
& + . \quad . \quad . \quad . \quad . \\
& + . \quad . \quad . \quad . \quad . = E, \\
& N\frac{d y_1}{d t} + S y_1 + M\frac{d}{d t}(x_o \cos \omega t) \\
& + N\frac{d y_3}{d t} + S y_3 + M\frac{d}{d t}(x_2 \cos \omega t) \\
& + N\frac{d y_5}{d t} + S y_5 + M\frac{d}{d t}(x_4 \cos \omega t) \\
& + . \quad . \quad . \quad . \quad . \\
& + . \quad . \quad . \quad . \quad . = 0.
\end{aligned}
\right.
$$

We may regard this transformation as one involving the $2n + 2$ variables:

$$x_o, x_2, x_4, \ldots . x_{2n}; \; y_1, y_3, y_5 \ldots . y_{2n+1};$$

where n is made to approach infinity. The transformation is therefore an infinite one, and we may impose an infinite number of arbitrary conditions; the only requirements to be fulfilled are that the sums $x_o + x_2 + x_4 + \ldots .$ and $y_1 + y_3 + y_5 + \ldots .$ shall satisfy their respective equations and that they shall be convergent.

Impose on $x_o, y_1, x_2, y_3, x_4 \ldots .$ the following conditions:

$$
(4)\left\{
\begin{aligned}
& (a)\quad L\frac{d x_o}{d t} + R x_o = E \\
& \qquad (b)\quad N\frac{d y_1}{d t} + S y_1 + M\frac{d}{d t}(x_o \cos \omega t) = 0 \\
& (c)\quad L\frac{d x_2}{d t} + R x_2 + M\frac{d}{d t}(y_1 \cos \omega t) = 0 \\
& \qquad (c)\quad N\frac{d y_3}{d t} + S y_3 + M\frac{d}{d t}(x_2 \cos \omega t) = 0 \\
& (d)\quad L\frac{d x_4}{d t} + R x_4 + M\frac{d}{d t}(y_3 \cos \omega t) = 0 \\
& \qquad (d)\quad N\frac{d y_5}{d t} + S y_5 + M\frac{d}{d t}(x_4 \cos \omega t) = .0 \\
& \quad . \quad . \quad . \quad . \quad . \\
& \qquad . \quad . \quad . \quad . \quad .
\end{aligned}
\right.
$$

These conditions obviously satisfy the transformed equations (3) for they make each part of the left-hand members of (3) vanish separately; hence they satisfy the original equations (1). Furthermore, these conditions lead to a convergent result, as will presently be shown.*

The result of the transformations (2) is to break up the original equations (1) into an infinite series of equations (4), each of which can be solved if those preceding it are solved first.

Disregarding transient states thruout, the solution of (4a) is:

$$x_0 = \frac{E}{R}. \tag{5a}$$

Substituting this in (4b) gives

$$N\frac{d\,y_1}{d\,t} + S\,y_1 = -M\,x_0\frac{d}{d\,t}(\cos\omega t) = \omega M\,x_0 \sin\omega t.$$

The solution of this is:

$$y_1 = \frac{\omega M\,x_0}{Z_1}\sin(\omega t - \theta_1)$$

$$\text{where } Z_1 = \sqrt{(\omega N)^2 + S^2} \text{ and } \theta_1 = \tan^{-1}\frac{\omega N}{S}. \tag{5b}$$

Substituting this in (4c) gives:

$$L\frac{d\,x_2}{d\,t} + R\,x_2 = -\frac{\omega M^2 x_0}{Z_1}\frac{d}{d\,t}\left[\frac{1}{2}\left(\sin(2\omega t - \theta_1) - \sin\theta_1\right)\right] = -\frac{(\omega M)^2 x_0}{Z_1}\cos(2\omega t - \theta_1).$$

The solution of this is:

$$\left.\begin{aligned} x_2 &= -\frac{(\omega M)^2 x_0}{Z_1 Z_2}\cos(2\omega t - \theta_1 - \theta_2), \\ &\text{where } Z_2 = \sqrt{(2\omega L)^2 + R^2} \text{ and } \theta_2 = \tan^{-1}\frac{2\omega L}{R}. \end{aligned}\right\} \tag{5c}$$

Substituting this in (4d) gives:

$$N\frac{d\,y_3}{d\,t} + S\,y_3 = \frac{\omega^2 M^3 x_0}{Z_1 Z_2}\frac{d}{d\,t}\left[\frac{1}{2}\cos(3\omega t - \theta_1 - \theta_2) + \frac{1}{2}\cos(\omega t - \theta_1 - \theta_2)\right]$$

$$= -\frac{(\omega M)^3 x_0}{Z_1 Z_2}\left[\frac{3}{2}\sin(3\omega t - \theta_1 - \theta_2) + \frac{1}{2}\sin(\omega t - \theta_1 - \theta_2)\right].$$

* Later we shall deal with a case where the series are divergent, but it will be shown that even in this case Pupin's transformation is justified by the physical phenomena.

The solution of this is:

$$(5d)\begin{cases} y_3 = -\dfrac{(\omega M)^3 x_0}{Z_1 Z_2}\left[\dfrac{3}{2 Z_3}\sin(3\omega t - \theta_1 - \theta_2 - \theta_3)\right. \\ \qquad\qquad \left. + \dfrac{1}{2 Z_1}\sin(\omega t - 2\theta_1 - \theta_2)\right], \\ \text{where } Z_3 = \sqrt{(3\omega N)^2 + S^2} \text{ and } \theta_3 = \tan^{-1}\dfrac{3\omega N}{S}. \end{cases}$$

We may continue in precisely the same manner to get $x_4, y_5, x_6, y_7, \ldots$ in turn. The complications multiply very rapidly, however, so I shall merely write down the values for a few more terms.

$$(5e)\begin{cases} x_4 = \dfrac{(\omega M)^4}{Z_1 Z_2} x_0\left[\dfrac{3}{Z_3 Z_4}\cos(4\omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4)\right. \\ \qquad + \dfrac{3}{2 Z_2 Z_3}\cos(2\omega t - \theta_1 - 2\theta_2 - \theta_3) \\ \qquad \left. + \dfrac{1}{2 Z_1 Z_2}\cos(2\omega t - 2\theta_1 - 2\theta_2)\right], \\ \text{where } Z_4 = \sqrt{(4\omega L)^2 + R^2} \text{ and } \theta_4 = \tan^{-1}\dfrac{4\omega L}{R}. \end{cases}$$

$$(5f)\begin{cases} y_5 = \dfrac{(\omega M)^5 x_0}{Z_1 Z_2}\left[\dfrac{15}{2 Z_3 Z_4 Z_5}\sin(5\omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4 - \theta_5)\right. \\ \qquad\qquad + \dfrac{9}{2 Z_3^2 Z_4}\sin(3\omega t - \theta_1 - \theta_2 - 2\theta_3 - \theta_4) \\ + \dfrac{9}{4 Z_2 Z_3^2}\sin(3\omega t - \theta_1 - 2\theta_2 - 2\theta_3) \\ \qquad\qquad + \dfrac{3}{4 Z_1 Z_2 Z_3}\sin(3\omega t - 2\theta_1 - 2\theta_2 - \theta_3) \\ + \dfrac{3}{4 Z_1 Z_2 Z_3}\sin(\omega t - 2\theta_1 - 2\theta_2 - \theta_3) \\ \qquad\qquad \left. + \dfrac{1}{4 Z_1^2 Z_2}\sin(\omega t - 3\theta_1 - 2\theta_2)\right], \\ \text{where } Z_5 = \sqrt{(5\omega N)^2 + S^2} \text{ and } \theta_5 = \tan^{-1}\dfrac{5\omega N}{S}. \end{cases}$$

(5g)
$$\left\{\begin{aligned}
x_6 = &-\frac{(\omega M)^6 x_o}{Z_1 Z_2}\left[\frac{45}{2 Z_3 Z_4 Z_5 Z_6}\cos(6\omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4\right.\\
&- \theta_5 - \theta_6) + \frac{15}{Z_3 Z_4^2 Z_5}\cos(4\omega t - \theta_1 - \theta_2 - \theta_3 - 2\theta_4 - \theta_5)\\
&+ \frac{9}{Z_3^2 Z_4^2}\cos(4\omega t - \theta_1 - \theta_2 - 2\theta_3 - 2\theta_4)\\
&\qquad + \frac{9}{2 Z_2 Z_3^2 Z_4}\cos(4\omega t - \theta_1 - 2\theta_2 - 2\theta_3 - \theta_4)\\
&+ \frac{3}{2 Z_1 Z_2 Z_3 Z_4}\cos(4\omega t - 2\theta_1 - 2\theta_2 - \theta_3 - \theta_4)\\
&\qquad + \frac{9}{2 Z_2 Z_3^2 Z_4}\cos(2\omega t - \theta_1 - 2\theta_2 - 2\theta_3 - \theta_4)\\
&+ \frac{9}{4 Z_2^2 Z_3^2}\cos(2\omega t - \theta_1 - 3\theta_2 - 2\theta_3)\\
&\qquad + \frac{3}{2 Z_1 Z_2^2 Z_3}\cos(2\omega t - 2\theta_1 - 3\theta_2 - \theta_3)\\
&\left.+ \frac{1}{4 Z_1^2 Z_2^2}\cos(2\omega t - 3\theta_1 - 3\theta_2)\right],\\
&\text{where } Z_6 = \sqrt{(6\omega L)^2 + R^2} \text{ and } \theta_6 = \tan^{-1}\frac{6\omega L}{R}.
\end{aligned}\right.$$

We see, therefore, that the current y_1 contains the frequency $\frac{\omega}{2\pi}$, the current x_2 the frequency $\frac{2\omega}{2\pi}$, the current y_3 the frequencies $\frac{3\omega}{2\pi}$ and $\frac{\omega}{2\pi}$, the current x_4 the frequencies $\frac{4\omega}{2\pi}$ and $\frac{2\omega}{2\pi}$, the current y_5 the frequencies $\frac{5\omega}{2\pi}$, $\frac{3\omega}{2\pi}$ and $\frac{\omega}{2\pi}$; etc. That is, the current y_{2n+1} contains all the odd frequencies from $(2n+1)\frac{\omega}{2\pi}$ down to $\frac{\omega}{2\pi}$, and the current x_{2n} all the even frequencies from $\frac{2n\omega}{2\pi}$ down to $\frac{2\omega}{2\pi}$. If we collect all the terms of frequency $\frac{\omega}{2\pi}$ and denote the result by η_1, those of frequency $\frac{2\omega}{2\pi}$ and denote the result by ξ_2, etc., we get:

(6a)
$$\begin{aligned}
\eta_1 = &\frac{\omega M x_o}{Z_1}\sin(\omega t - \theta_1) - \frac{(\omega M)^3 x_o}{2 Z_1^2 Z_2}\sin(\omega t - 2\theta_1 - \theta_2)\\
&+ \frac{3(\omega M)^5 x_o}{4 Z_1^2 Z_2^2 Z_3}\sin(\omega t - 2\theta_1 - 2\theta_2 - \theta_3)\\
&\qquad + \frac{(\omega M)^5 x_o}{4 Z_1^3 Z_2^2}\sin(\omega t - 3\theta_1 - 2\theta_2)
\end{aligned}$$

$$-\frac{9(\omega M)^7 x_o}{4 Z_1^2 Z_2^2 Z_3^2 Z_4} \sin(\omega t - 2\theta_1 - 2\theta_2 - 2\theta_3 - \theta_4)$$

$$-\frac{9(\omega M)^7 x_o}{8 Z_1^2 Z_2^3 Z_3^2} \sin(\omega t - 2\theta_1 - 3\theta_2 - 2\theta_3)$$

$$-\frac{3(\omega M)^7 x_o}{4 Z_1^3 Z_2^3 Z_3} \sin(\omega t - 3\theta_1 - 3\theta_2 - \theta_3)$$

$$-\frac{(\omega M)^7 x_o}{8 Z_1^4 Z_2^3} \sin(\omega t - 4\theta_1 - 3\theta_2)$$

$$+ \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$+ \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

(6b) $$\xi_2 = -\frac{(\omega M)^2 x_o}{Z_1 Z_2} \cos(2\omega t - \theta_1 - \theta_2)$$

$$+\frac{3(\omega M)^4 x_o}{2 Z_1 Z_2^2 Z_3} \cos(2\omega t - \theta_1 - 2\theta_2 - \theta_3)$$

$$+\frac{(\omega M)^4 x_o}{2 Z_1^2 Z_2^2} \cos(2\omega t - 2\theta_1 - 2\theta_2)$$

$$-\frac{9(\omega M)^6 x_o}{2 Z_1 Z_2^2 Z_3^2 Z_4} \cos(2\omega t - \theta_1 - 2\theta_2 - 2\theta_3 - \theta_4)$$

$$-\frac{9(\omega M)^6 x_o}{4 Z_1 Z_2^3 Z_3^2} \cos(2\omega t - \theta_1 - 3\theta_2 - 2\theta_3)$$

$$-\frac{3(\omega M)^6 x_o}{2 Z_1^2 Z_2^3 Z_3} \cos(2\omega t - 2\theta_1 - 3\theta_2 - \theta_3)$$

$$-\frac{(\omega M)^6 x_o}{4 Z_1^3 Z_2^3} \cos(2\omega t - 3\theta_1 - 3\theta_2) + \quad . \quad .$$

$$+ \quad . \quad . \quad . \quad . \quad . \quad .$$

(6c) $$\eta_3 = -\frac{3(\omega M)^3 x_o}{2 Z_1 Z_2 Z_3} \sin(3\omega t - \theta_1 - \theta_2 - \theta_3)$$

$$+\frac{9(\omega M)^5 x_o}{2 Z_1 Z_2 Z_3^2 Z_4} \sin(3\omega t - \theta_1 - \theta_2 - 2\theta_3 - \theta_4)$$

$$+\frac{9(\omega M)^5 x_o}{4 Z_1 Z_2^2 Z_3^2} \sin(3\omega t - \theta_1 - 2\theta_2 - 2\theta_3)$$

$$+\frac{3(\omega M)^5 x_o}{4 Z_1^2 Z_2^2 Z_3} \sin(3\omega t - 2\theta_1 - 2\theta_2 - \theta_3)$$

$$-\frac{45(\omega M)^7 x_o}{2 Z_1 Z_2 Z_3^2 Z_4^2 Z_5} \sin(3\omega t - \theta_1 - \theta_2 - 2\theta_3 - 2\theta_4 - \theta_5)$$

$$-\frac{27(\omega M)^7 x_o}{2 Z_1 Z_2 Z_3^3 Z_4^2} \sin(3\omega t - \theta_1 - \theta_2 - 3\theta_3 - 2\theta_4)$$

$$- \frac{27 (\omega M)^7 x_o}{2 Z_1 Z_2^2 Z_3^3 Z_4} \sin (3 \omega t - \theta_1 - 2 \theta_2 - 3 \theta_3 - \theta_4)$$

$$- \frac{9 (\omega M)^7 x_o}{4 Z_1^2 Z_2^2 Z_3^2 Z_4} \sin (3 \omega t - 2 \theta_1 - 2 \theta_2 - 2 \theta_3 - \theta_4)$$

$$- \frac{27 (\omega M)^7 x_o}{8 Z_1 Z_2^3 Z_3^3} \sin (3 \omega t - \theta_1 - 3 \theta_2 - 3 \theta_3)$$

$$- \frac{9 (\omega M)^7 x_o}{4 Z_1^2 Z_2^3 Z_3^2} \sin (3 \omega t - 2 \theta_1 - 3 \theta_2 - 2 \theta_3)$$

$$- \frac{3 (\omega M)^7 x_o}{8 Z_1^3 Z_2^3 Z_3} \sin (3 \omega t - 3 \theta_1 - 3 \theta_2 - \theta_3)$$

$$+ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$+ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

(6d) $$\xi_4 = \frac{3 (\omega M)^4 x_o}{Z_1 Z_2 Z_3 Z_4} \cos (4 \omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4)$$

$$- \frac{15 (\omega M)^6 x_o}{Z_1 Z_2 Z_3 Z_4^2 Z_5} \cos (4 \omega t - \theta_1 - \theta_2 - \theta_3 - 2 \theta_4 - \theta_5)$$

$$- \frac{9 (\omega M)^6 x_o}{Z_1 Z_2 Z_3^2 Z_4^2} \cos (4 \omega t - \theta_1 - \theta_2 - 2 \theta_3 - 2 \theta_4)$$

$$- \frac{9 (\omega M)^6 x_o}{2 Z_1 Z_2^2 Z_3^2 Z_4} \cos (4 \omega t - \theta_1 - 2 \theta_2 - 2 \theta_3 - \theta_4)$$

$$- \frac{3 (\omega M)^6 x_o}{2 Z_1^2 Z_2^2 Z_3 Z_4} \cos (4 \omega t - 2 \theta_1 - 2 \theta_2 - \theta_3 - \theta_4)$$

$$+ \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$+ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

(6e) $$\eta_5 = \frac{15 (\omega M)^5 x_o}{2 Z_1 Z_2 Z_3 Z_4 Z_5} \sin (5 \omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$- \frac{225 (\omega M)^7 x_o}{4 Z_1 Z_2 Z_3 Z_4 Z_5^2 Z_6} \sin (5 \omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4 - 2 \theta_5 - \theta_6)$$

$$- \frac{75 (\omega M)^7 x_o}{2 Z_1 Z_2 Z_3 Z_4^2 Z_5^2} \sin (5 \omega t - \theta_1 - \theta_2 - \theta_3 - 2 \theta_4 - 2 \theta_5)$$

$$- \frac{45 (\omega M)^7 x_o}{2 Z_1 Z_2 Z_3^2 Z_4^2 Z_5} \sin (5 \omega t - \theta_1 - \theta_2 - 2 \theta_3 - 2 \theta_4 - \theta_5)$$

$$- \frac{45 (\omega M)^7 x_o}{4 Z_1 Z_2^2 Z_3^2 Z_4 Z_5} \sin (5 \omega t - \theta_1 - 2 \theta_2 - 2 \theta_3 - \theta_4 - \theta_5)$$

$$- \frac{15 (\omega M)^7 x_o}{4 Z_1^2 Z_2^2 Z_3 Z_4 Z_5} \sin (5 \omega t - 2 \theta_1 - 2 \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$+ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

(6f) $\xi_6 = -\frac{45\,(\omega M)^6\,x_o}{2\,Z_1\,Z_2\,Z_3\,Z_4\,Z_5\,Z_6}\cos\left(6\,\omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4 - \theta_5 - \theta_6\right)$
$+ \ldots$

(6g) $\eta_7 = -\frac{7\times 45\,(\omega M)^7\,x_o}{4\,Z_1\,Z_2\,Z_3\,Z_4\,Z_5\,Z_6\,Z_7}\sin\left(7\,\omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4 - \theta_5 - \theta_6 - \theta_7\right) + \ldots$

. . . .

. . . .

Finally

(7)
$$x = x_o + \xi_2 + \xi_4 + \xi_6 + \xi_8 + \ldots$$
$$y = \eta_1 + \eta_3 + \eta_5 + \eta_7 + \ldots$$

These series are Pupin's solution of the fundamental differential equations (1). They are, in effect, Fourier's series, each amplitude of which is an infinite series. Their physical significance is most easily brought out after their convergence has been demonstrated.

PROOF OF CONVERGENCE WITH VANISHINGLY SMALL RESISTANCES

I have found a relatively simple proof of the convergence by first letting the resistances become vanishingly small, which leads us to series that are obviously convergent, and then showing that the re-introduction of finite resistances does not affect the convergence.

When $R = S = 0$, $\theta_1 = \theta_2 = \theta_3 = \ldots = \frac{\pi}{2}$,
and $Z_1 = \omega N$, $Z_2 = 2\,\omega L$, $Z_3 = 3\,\omega N$, $Z_4 = 4\,\omega L, \ldots$
Hence η_1 becomes

$$\eta_1 = x_o\frac{M}{N}\left\{\sin\left(\omega t - \frac{\pi}{2}\right) - \frac{M^2}{4\,L\,N}\sin\left(\omega t - \frac{3\pi}{2}\right)\right.$$
$$+ \frac{3}{4\times 4\times 3}\left(\frac{M^2}{L\,N}\right)^2\sin\left(\omega t - \frac{5\pi}{2}\right) + \frac{1}{4\times 4}\left(\frac{M^2}{L\,N}\right)^2\sin\left(\omega t - \frac{5\pi}{2}\right)$$
$$- \frac{9}{4\times 4\times 9\times 4}\left(\frac{M^2}{L\,N}\right)^3\sin\left(\omega t - \frac{7\pi}{2}\right)$$
$$- \frac{9}{8\times 8\times 9}\left(\frac{M^2}{L\,N}\right)^3\sin\left(\omega t - \frac{7\pi}{2}\right)$$
$$- \frac{3}{4\times 8\times 3}\left(\frac{M^2}{L\,N}\right)^3\sin\left(\omega t - \frac{7\pi}{2}\right) - \frac{1}{8\times 8}\left(\frac{M^2}{L\,N}\right)^3\sin\left(\omega t - \frac{7\pi}{2}\right)$$
$$\left. + \ldots \right\}$$

Hence:

$$
(8)\quad \begin{cases}
\eta_1 = -x_0 \frac{M}{N} \cos \omega t \left\{ 1 + \frac{1}{2^2}\left(\frac{M^2}{LN}\right) + \frac{2}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{5}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\text{Similarly} \\
\xi_2 = \frac{x_0}{2}\left(\frac{M^2}{LN}\right) \cos 2\omega t \left\{ 1 + \frac{2}{2^2}\left(\frac{M^2}{LN}\right) + \frac{5}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{14}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\eta_3 = -\frac{x_0}{2^2}\frac{M}{N}\left(\frac{M^2}{LN}\right) \cos 3\omega t \left\{ 1 + \frac{3}{2^2}\left(\frac{M^2}{LN}\right) + \frac{9}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{28}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\xi_4 = \frac{x_0}{2^3}\left(\frac{M^2}{LN}\right)^2 \cos 4\omega t \left\{ 1 + \frac{4}{2^2}\left(\frac{M^2}{LN}\right) + \frac{14}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{48}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\eta_5 = -\frac{x_0}{2^4}\frac{M}{N}\left(\frac{M^2}{LN}\right)^2 \cos 5\omega t \left\{ 1 + \frac{5}{2^2}\left(\frac{M^2}{LN}\right) + \frac{20}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{75}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\xi_6 = \frac{x_0}{2^5}\left(\frac{M^2}{LN}\right)^3 \cos 6\omega t \left\{ 1 + \frac{6}{2^2}\left(\frac{M^2}{LN}\right) + \cdots \right\} \\
\cdots\cdots\cdots
\end{cases}
$$

Now, by expansion in power series we find:

$$
(9)\quad \begin{cases}
\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right) = \frac{1}{2}\left(\frac{M^2}{LN}\right)\left\{ 1 + \frac{1}{2^2}\left(\frac{M^2}{LN}\right) + \frac{2}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{5}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right. \\
\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right)^2 = \frac{1}{2^2}\left(\frac{M^2}{LN}\right)^2\left\{ 1 + \frac{2}{2^2}\left(\frac{M^2}{LN}\right) + \frac{5}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{14}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\} \\
\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right)^3 = \frac{1}{2^3}\left(\frac{M^2}{LN}\right)^3\left\{ 1 + \frac{3}{2^2}\left(\frac{M^2}{LN}\right) + \frac{9}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{28}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots \right\}
\end{cases}
$$

$$\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^4 = \frac{1}{2^4}\left(\frac{M^2}{LN}\right)^4\left\{1+\frac{4}{2^2}\left(\frac{M^2}{LN}\right)+\frac{14}{2^4}\left(\frac{M^2}{LN}\right)^2 + \frac{48}{2^6}\left(\frac{M^2}{LN}\right)^3 + \cdots\right\}$$

. . . .

From (8) and (9) there results:

$$\eta_1 = -2x_o\frac{L}{M}\left(1-\sqrt{1-\frac{M^2}{LN}}\right)\cos\omega t$$

$$\xi_2 = 2x_o\frac{LN}{M^2}\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^2\cos 2\omega t$$

$$\eta_3 = -2x_o\frac{L}{M}\left(\frac{LN}{M^2}\right)\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^3\cos 3\omega t$$

$$\xi_4 = 2x_o\left(\frac{LN}{M^2}\right)^2\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^4\cos 4\omega t$$

$$\eta_5 = -2x_o\frac{L}{M}\left(\frac{LN}{M^2}\right)^2\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^5\cos 5\omega t$$

. . . .

. . . .

Hence, putting for brevity:

$$\left(\frac{LN}{M^2}\right)\left(1-\sqrt{1-\frac{M^2}{LN}}\right)^2 = \phi$$

we get

$$(10)\quad \begin{cases} x = x_o + 2x_o\left\{\phi\cos 2\omega t + \phi^2\cos 4\omega t + \phi^3\cos 6\omega t + \cdots\right\} \\ y = -2x_o\frac{L}{M}\left(1-\sqrt{1-\frac{M^2}{LN}}\right)\left\{\cos\omega t + \phi\cos 3\omega t + \phi^2\cos 5\omega t + \phi^3\cos 7\omega t + \cdots\right\}. \end{cases}$$

Hence Pupin's series reduce to Fourier's series, the amplitudes of which are proportional to integral powers of ϕ. This is what we should expect from the simple case treated above of a single circuit with no resistance and a periodically varied self-induction. In fact, if we had chosen in the case first treated, a pair of circuits with periodically varied mutual inductance and no resistance, instead of a single circuit with periodically varied self-inductance, we should have arrived immediately at equations (10).

The quantity ϕ takes the form $\infty \times 0$ when $M = 0$, and so does the quantity $\frac{L}{M}\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right)$. It may readily be shown, however, that

$$\operatorname*{Lim}_{M \doteq 0} \phi = 0$$

$$\text{and that } \operatorname*{Lim}_{M \doteq 0} \frac{L}{M}\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right) = 0$$

Hence, when $M = 0$, the only current which exists is $x_o = E/R$, which, of course, must be the case. Further, $\frac{M^2}{LN}$ is the coupling factor of the two circuits, and this must always be less than unity, and positive, of course. With these limitations on $\frac{M_2}{LN}$, it is clear that the quantities ϕ and $\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right)$ can never reach unity. Therefore

$$0 < \phi < 1 \text{ and } 0 < \left(1 - \sqrt{1 - \frac{M^2}{LN}}\right) < 1.$$

Hence the amplitudes of equations (10) are power series whose ratio is less than unity in absolute value, therefore the series are absolutely convergent.

PROOF OF CONVERGENCE WITH FINITE RESISTANCES

This result will now be extended to the practical case where the resistances are finite. In order to pass from equations (10) back to equations (7) and (6), we go through the following steps: (1), expand each expression $2\phi^n$ and $2\frac{L}{M}\left(1 - \sqrt{1 - \frac{M^2}{LN}}\right)\phi^n$ into power series in $\left(\frac{M^2}{LN}\right)$; (2), break up each term $h\left(\frac{M^2}{LN}\right)^k$ of the resulting series into a number of smaller terms

$$a_1\left(\frac{M^2}{LN}\right)^k + a_2\left(\frac{M^2}{LN}\right)^k + a_3\left(\frac{M^2}{LN}\right)^k + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

where a_1, a_2, a_3, . . . all have the same sign as the original term $h\left(\frac{M^2}{LN}\right)^k$; (it is important to note that in forming equations (8), the terms which were combined into single terms all had the same sign); (3), split up each term $a_i\left(\frac{M^2}{LN}\right)^k \cos m\,\omega t$ into two terms, viz.,

$$a_i\left(\frac{M^2}{LN}\right)^k \sin\Theta \cos m\,\omega t \quad\text{and}\quad a_i\left(\frac{M^2}{LN}\right)^k \cos\Theta \sin m\,\omega t,$$

each of which is smaller than the term from which it is derived. In this way each of the series in equations (10) is converted into two series, one in sines, the other in cosines, in each of which the amplitudes are infinite series. That is, we pass from the convergent series of (10) to Pupin's series (8) by a number of steps *which cannot alter the convergence.* Hence it is proved that Pupin's series are convergent, and therefore that the Pupin theory is entirely rigorous.

PHYSICAL INTERPRETATION OF THE SOLUTION

Turning now to the physical interpretation of equations (6) and (7), we see that whenever an asymetrical rotor is revolved in the field of a stator on which is impressed a constant e. m. f., there are generated an infinite number of harmonics in both the stator and the rotor. The harmonics in the rotor are all odd, in the stator they are all even. If the resistances are small in comparison with the inductances, the amplitudes of the harmonics decrease approximately according to integral powers of a quantity ϕ whose absolute value is less than unity. If the resistances are not small, it is obvious that amplitudes must decrease more rapidly. The smaller the coupling coefficient the smaller is the quantity ϕ, and hence the more rapidly do the amplitudes decrease. The wave distortion in single phase alternators is an example; if the air gap is small, the coupling $\frac{M^2}{LN}$ will not be very small, and the amplitudes of the odd harmonics in the rotor will not fall off very rapidly. The presence of these odd harmonics constitutes at least part of the distortion. If a large uncoupled inductance is connected in series with the field of a single phase alternator, the coupling $\frac{M^2}{LN}$ will be reduced, and the distortion consequently diminished. This might be of practical importance, for example, in enabling single phase alternators to be constructed with smaller air gaps and thereby reducing the amount of copper required in the field coils. The case of polyphase alternators with unbalanced load is precisely similar, of course.

In ordinary alternating current machinery, the harmonics are suppressed as far as possible; in the Goldschmidt alternator, on the other hand, the harmonics are encouraged by the use of condensers, the object being to get as much energy as possible into a single predetermined overtone. In his lectures, Professor Pupin indicated how the theory could bé extended to include condensers in the stator and rotor circuits. This extension will now be carried out in detail.

CIRCUITS HAVING RESISTANCE, INDUCTANCE, CAPACITY AND VARIABLE MUTUAL INDUCTANCE

Suppose that the rotor and stator circuits include any arbitrary arrangement of inductances and capacities. At a given frequency $\frac{2n\omega}{2\pi}$, the stator circuit will have a definite effective resistance, which we may denote by R_{2n}, and a definite effective inductance, which we may denote by L_{2n}. Similarly, at a given frequency $\frac{(2n+1)\omega}{2\pi}$, the rotor circuit will have an effective resistance S_{2n+1} and inductance N_{2n+1}. That is, the quantities R, S, L, N are no longer constants, but are functions of the frequency. R_{2n} and S_{2n+1} must always be positive, but L_{2n} and N_{2n+1} may be positive, negative or zero. If $L_{2n} = 0$, it means that the stator circuit is tuned to the frequency $\frac{2n\omega}{2\pi}$; similarly, if $N_{2n+1} = 0$, the rotor circuit is tuned to the frequency $\frac{(2n+1)\omega}{2\pi}$.

It is clear, therefore, that the fundamental differential equations (1) hold for the present case as well as for the previous case, provided that we consider steady states only, the only difference being that R, S, L and N are now functions of ω. Bearing this in mind we may proceed exactly in the same manner as before, arriving at equations (4). The solutions of these equations are of the same form as in the previous case, i. e., of the same form as equations (5); but now, Z_1, Z_2, Z_3, Z_4 . . . . and $\theta_1, \theta_2, \theta_3, \theta_4$, ... are given by:

$$Z_1^2 = (\omega N_1)^2 + S_1^2 \qquad \theta_1 = \tan^{-1}\frac{\omega N_1}{S_1}$$

$$Z_2^2 = (2\omega L_2)^2 + R_2^2 \qquad \theta_2 = \tan^{-1}\frac{2\omega L_2}{R_2}$$

$$Z_3^2 = (3\omega N_3)^2 + S_3^2 \qquad \theta_3 = \tan^{-1}\frac{3\omega N_3}{S_3}$$

$$Z_4^2 = (4\,\omega L_4)^2 + R_4^2 \qquad \theta_4 = \tan^{-1}\frac{4\,\omega L_4}{R_4}$$

. . . .

It is clear, therefore, that the solutions as given by equations (6) and (7) hold for all cases, provided that the proper meanings be attached to the Z's and the θ's.

We proceed now to investigate what happens to Pupin's series when the rotor circuit is tuned to a definite number of frequencies $\frac{\omega}{2\pi}, \frac{3\,\omega}{2\pi} \ldots$, and the stator circuit to the frequencies $\frac{2\,\omega}{2\pi}, \frac{4\,\omega}{2\pi}, \ldots$ To fix the ideas, let the rotor be tuned to the single frequency $\frac{\omega}{2\pi}$, and the stator to the single frequency $\frac{2\,\omega}{2\pi}$. Then Z_1 becomes simply S_1, and Z_2 becomes R_2. We assume, furthermore, that the effective resistances for these frequencies, i. e., S_1 and R_2, are small. Then the quantities $\frac{\omega M}{Z_1}$ and $\frac{\omega M}{Z_2}$, which now become $\frac{\omega M}{S_1}$ and $\frac{\omega M}{R_2}$, are very large.

It will be observed that the current η_1 contains the amplitudes:

$$\frac{\omega M x_o}{Z_1}, \quad \left(\frac{\omega M}{Z_1}\right)^2\left(\frac{\omega M}{2\,Z_2}\right)x_o, \quad \left(\frac{\omega M}{Z_1}\right)^3\left(\frac{\omega M}{2\,Z_2}\right)^2 x_o, \quad \left(\frac{\omega M}{Z_1}\right)^4\left(\frac{\omega M}{2\,Z_2}\right)^3 x_o$$

$$\left(\frac{\omega M}{Z_1}\right)^5\left(\frac{\omega M}{2\,Z_2}\right)^4 x_o, \ldots \quad . \quad . \quad .$$

Likewise, the current ξ_2 contains the amplitudes:

$$2\,x_o\left(\frac{\omega M}{Z_1}\right)\left(\frac{\omega M}{2\,Z_2}\right), \quad 2\,x_o\left(\frac{\omega M}{Z_1}\right)^2\left(\frac{\omega M}{2\,Z_2}\right)^2, \quad 2\,x_o\left(\frac{\omega M}{Z_1}\right)^3\left(\frac{\omega M}{2\,Z_2}\right)^3.$$

$$2\,x_o\left(\frac{\omega M}{Z_1}\right)^4\left(\frac{\omega M}{2\,Z_2}\right)^4, \ldots \quad . \quad . \quad . \quad .$$

and η_3 contains the amplitudes:

$$\frac{3\,\omega M x_o}{Z_3}\left(\frac{\omega M}{Z_1}\right)\left(\frac{\omega M}{2\,Z_2}\right), \quad \frac{3\,\omega M x_o}{Z_3}\left(\frac{\omega M}{Z_1}\right)^2\left(\frac{\omega M}{2\,Z_2}\right)^2,$$

$$\frac{3\,\omega M x_o}{Z_3}\left(\frac{\omega M}{Z_1}\right)^3\left(\frac{\omega M}{2\,Z_2}\right)^3, \ldots \quad . \quad . \quad . \quad .$$

and ξ_4 contains the amplitudes:

$$\frac{6\,(\omega M)^2 x_o}{Z_3 Z_4}\left(\frac{\omega M}{Z_1}\right)\left(\frac{\omega M}{2\,Z_2}\right), \quad \frac{6\,(\omega M)^2 x_o}{Z_3 Z_4}\left(\frac{\omega M}{Z_1}\right)^2\left(\frac{\omega M}{2\,Z_2}\right)^2$$

. . . .

We see, therefore, that every current contains amplitudes which are power series in $\left(\frac{\omega M}{Z_1}\right)\left(\frac{\omega M}{2 Z_2}\right)$; and if the circuits are tuned so as to reduce Z_1 to S_1 and Z_2 to R_2, and if these resistances are small, it follows that *all* the series of equations (6) *diverge*, and therefore that *all the amplitudes tend towards infinity.* A complete discussion of the convergence or divergence of these series is not very simple, but in the given case it is clear that if the resistances R_2 and S_1, are small, the higher powers of $\left(\frac{\omega M}{Z_1}\right)\left(\frac{\omega M}{2 Z_2}\right)$ which occur in all the amplitudes soon become so large as to make all the other terms in the amplitudes negligibly small, and the divergence of all the amplitudes is therefore assured.

EXPLANATION OF LIMITATION OF ROTOR AND STATOR CURRENTS IN PRACTICE

The question now arises, does the Pupin theory break down when tuned condenser circuits of low resistance are employed, or is the theory still justified by the physical phenomena? And if the theory is justified how can the operation of the Goldschmidt alternator be accounted for?

The answer to both of these questions is, I think, not far to seek. It does not require an elaborate theory to show that if the stator and rotor circuits are tuned, let us say to the frequencies $\frac{2\omega}{2\pi}$ and $\frac{\omega}{2\pi}$ respectively, the currents all tend toward infinity in an ideal machine of low resistance. For, suppose a current x_o is flowing in the stator; this will give rise to a current $\frac{\omega M x_o}{S} \sin \omega t$ in the tuned rotor circuit; this in turn will give rise to a current $-\frac{(\omega M)^2 x_o}{R S} \cos 2\omega t$ in the tuned stator circuit (leaving out of account the other currents generated): this stator current in turn will give rise to a current $-\frac{(\omega M)^3 x_o}{2 R S^2} \sin \omega t$ in the tuned rotor circuit, which is opposite in phase to the first current $\frac{\omega M x_o}{S} \sin \omega t$ but is very much larger than the same if the resistances are small. In this way, each new current of frequency $\frac{\omega}{2\pi}$ in the rotor will give rise to a much larger current of frequency $\frac{2\omega}{2\pi}$ in the stator, and this in turn to a still larger current of frequency $\frac{\omega}{2\pi}$ in the rotor, and so forth. Physical reasoning shows, therefore, that the currents to which the circuits are tuned tend toward infinity in ideal machines of low resistance. But obviously, if one of the currents becomes infinite, they all must become infinite, hence the Pupin theory as applied to the case of tuned condenser circuits is entirely in accord with the phenomena which would exist in an ideal machine. The correctness of the Pupin theory in all cases is therefore established.

As regards the practical operation of the Goldschmidt alternator, this is readily accounted for by the variable permeability of the iron. As the rotor and stator currents become larger and larger, the permeability of the iron becomes smaller and smaller, hence the circuits *automatically detune themselves* and thereby keep down the currents. At the same time, the losses in the iron increase rapidly as the currents become larger, hence the effective resistances also become larger, and this also tends to limit the values of the current. It is a physical impossibility, therefore, to keep the circuits in tune or to keep the resistances very low; the practical operation of the Goldschmidt alternator is thus accounted for.

The Pupin theory shows that by suitably controlling the impedances Z_1, Z_2, Z_3, ... it should be possible to make any given amplitude larger than the others, but it also shows that to make the given amplitude large, the neighboring amplitudes must also be large. Professor Pupin long ago pointed out the possibilities in this method of generating radio frequency currents, but in his opinion the difficulties and disadvantages outweighed the advantages to such an extent that he did not attempt to develop the method for practical purposes.

Thruout this paper, attention has been confined to the case of a constant e. m. f. impressed on the stator. It should be mentioned, however, that the Pupin theory includes the case where the impressed e. m. f. is any function of the time. In conclusion it should also be mentioned that Professor Pupin showed his solution of the fundamental differential equations to Professor Moulton of Chicago University, and that the latter has since applied the method to the general theory of linear differential equations with harmonic coefficients.

SUMMARY: The case of a simple circuit having periodically varying inductance is first examined. The solution shows that the current has a constant component and an infinite series of convergently diminishing higher harmonics. Circuits having inductance, resistance, and variable mutual inductance are next considered. To solve the equations obtained, an infinite transformation is carried out, each variable being replaced by the sum of an infinite series of new variables, thus enabling an infinite number of arbitrary conditions to be imposed. As a result, an infinite series of equations is obtained, each of which can be solved if those preceding it have been solved. The solutions are worked out to the fourth harmonic in one circuit and the third in the other. In one circuit, only odd frequencies appear; in the other, only even. The general solutions are in the form of a Fourier's series, each amplitude of which is an infinite series. The convergence of the solutions is completely established. The solutions are then extended to the case where rotor and stator circuits contain capacities.

It is shown that according to Pupin's theory, all currents in low resistance rotors and stators tend toward infinity if these circuits are appropriately tuned. This apparent discrepancy from practice is explained on the ground that the variable permeability of the iron in the Goldschmidt alternators automatically detunes the circuits and that the increasing losses of the iron tend further to limit all currents.

DISCUSSION

Louis Cohen (by letter): Aside from the interesting solution of the problem that the paper deals with relating to radio frequency alternators, the great importance of the paper consists in the general method that Professor Pupin has given us for solving differential equations having variable coefficients. I believe the method will prove of great value in the solution of many other problems in electrotechnics.

I recall that I have discussed this problem with Professor Pupin about six years ago, and he told me then that he had marked out the general solution of the problem, but reserved its publication for some future time. We ought to be grateful to Mr. Liebowitz for having put it in shape for publication and presenting it before the Institute.

As an illustration of the applicability of the method developed in the paper to the solution of other problems, it may be of interest to mark out the problem of the microphone circuit.

We have in this case an inductance, a variable resistance and a continuous e. m. f. in the circuit, and the circuit equation is,

$$L\frac{dI}{dt} + RI + rI\cos\omega t = E, \qquad (1)$$

where R+r is the total resistance of the circuit in stationary condition.*

As far as I know the complete solution of this problem has never been given. Following, however, the method developed by Professor Pupin, we can readily obtain the solution of the problem.

Put $$I = I_0 + I_1 + I_2 + I_3 + \ldots\ldots + I_n, \qquad (2)$$

and make the substitution in equation (1), we get

$$\left.\begin{aligned} &L\frac{dI_0}{dt} + RI_0 + rI_0\cos\omega t \\ &+ L\frac{dI_1}{dt} + RI_1 + rI_1\cos\omega t \\ &+ L\frac{dI_2}{dt} + RI_2 + rI_2\cos\omega t \\ &+ \ldots\ldots\ldots\ldots \\ &+ L\frac{dI_n}{dt} + RI_n + rI_n\cos\omega t = E \end{aligned}\right\} \qquad (3)$$

* (R is the constant resistance of the external circuit; the resistance of the microphone, which varies periodically under the influence of a sound of frequency $\frac{\omega}{2\pi}$, is $r\cos\omega t$. The term $I(r\cos\omega t)$ in equation (1) is therefore the drop of potential at time t across the microphone.—EDITOR.)

In accordance with the method given in the paper, we can break up equation (3) into a number of independent equations, as follows:

$$\left.\begin{array}{ll}
(a) & L\dfrac{dI_0}{dt} + RI_0 = E \\
(b) & L\dfrac{dI_1}{dt} + RI_1 + rI_0\cos\omega t = 0 \\
(c) & L\dfrac{dI_2}{dt} + RI_2 + rI_1\cos\omega t = 0 \\
(d) & L\dfrac{dI_3}{dt} + RI_3 + rI_2\cos\omega t = 0 \\
 & \cdots\cdots\cdots\cdots \\
 & \cdots\cdots\cdots\cdots
\end{array}\right\} \quad (4)$$

Disregarding the transients, we have for the solution of (4a),

$$I_0 = \frac{E}{R}. \qquad (5)$$

Substituting the value of I_0 from (5) into (4b), we get

$$L\frac{dI_1}{dt} + RI_1 = -\frac{Er}{R}\cos\omega t \qquad (6)$$

and

$$I_1 = -\frac{Er}{RZ_1}\cos(\omega t - \theta_1) \qquad (7)$$

$$Z = \sqrt{L^2\omega^2 + R^2}, \quad \theta_1 = \tan^{-1}\frac{L\omega}{R}.$$

Substituting the value of I_1 into (4c), we have

$$L\frac{dI_2}{dt} + RI_2 = \frac{Er^2}{RZ_1}\cos(\omega t - \theta_1)\cos\omega t$$

$$= \frac{Er^2}{2RZ_1}\left\{\cos(2\omega t - \theta_1) + \cos\theta_1\right\}$$

and

$$I_2 = \frac{Er^2}{2RZ_1Z_2}\cos(2\omega t - \theta_1 - \theta_2) + \frac{Er^2\cos\theta_1}{2R^2Z_1}. \qquad (8)$$

Repeating the operation, we find in a similar manner,

$$I_3 = -\frac{Er^3}{4RZ_1Z_2Z_3}\cos(3\omega t - \theta_1 - \theta_2 - \theta_3)$$

$$-\frac{Er^3}{4RZ_1^2Z_2}\cos(\omega t - 2\theta_1 - \theta_2)$$

$$-\frac{Er^3}{2R^2Z_1^2}\cos\theta_1\cos(\omega t - \theta_1) \qquad (9)$$

$$
\left.\begin{aligned}
I_4 = {} & \frac{E r^4}{8 R Z_1 Z_2 Z_3 Z_4} \cos (4 \omega t - \theta_1 - \theta_2 - \theta_3 - \theta_4) \\
& + \frac{E r^4}{8 R Z_1 Z_2^2 Z_3} \cos (2 \omega t - \theta_1 - 2 \theta_2 - \theta_3) \\
& + \frac{E r^4}{8 R Z_1^2 Z_2^2} \cos (2 \omega t - 2 \theta_1 - 2 \theta_2) \\
& + \frac{E r^4}{8 R^2 Z_1^2 Z_2} \cos (2 \theta_1 + \theta_2) \\
& + \frac{E r^4}{4 R^2 Z_1^2 Z_2} \cos (2 \omega t - \theta_1 - \theta_2) + \frac{E r^4 \cos^2 \theta_1}{4 R^3 Z_1^2}.
\end{aligned}\right\} \quad (10)
$$

and similarly for the other components.

If we collect separately the terms of the same frequency, and denote the results by γ_0, γ_1, γ_2, etc., respectively, we get

$$
\begin{aligned}
\gamma_0 = {} & \frac{E}{R} + \frac{E r^2 \cos \theta_1}{2 R^2 Z_1} + \frac{E r^4}{8 R^2 Z_1^2 Z_2} \cos (2 \theta_1 + \theta_2) \\
& + \frac{E r^4 \cos^2 \theta_1}{4 R^3 Z_1^2} + \cdots \\
= {} & \frac{E}{R} \left\{ 1 + \frac{r^2}{2 R Z_1} \cos \theta_1 + \frac{r^4}{8 R Z_1^2 Z_2} \cos (2 \theta_1 + \theta_2) \right. \\
& \left. + \frac{r^4 \cos^2 \theta_1}{4 R^2 Z_1^2} + \cdots \right\} \quad (11)
\end{aligned}
$$

$$
\begin{aligned}
-\gamma_1 = {} & \frac{E r}{R Z_1} \left\{ \cos (\omega t - \theta_1) + \frac{r^2}{4 Z_1 Z_2} \cos (\omega t - 2 \theta_1 - \theta_2) \right. \\
& \left. + \frac{r^2}{2 R Z_1} \cos \theta_1 \cos (\omega t - \theta_1) + \cdots \right\} \quad (12)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2 = {} & \frac{E r^2}{2 R Z_1 Z_2} \left\{ \cos (2 \omega t - \theta_1 - \theta_2) + \frac{r^2}{4 Z_2 Z_3} \cos (2 \omega t - \theta_1 - 2 \theta_2 - \theta_3) \right. \\
& + \frac{r^2}{4 Z_1 Z_2} \cos (2 \omega t - 2 \theta_1 - 2 \theta_2) \\
& \left. + \frac{r^2}{2 R Z_1} \cos (2 \omega t - \theta_1 - \theta_2) + \cdots \right\} \quad (13)
\end{aligned}
$$

. . . .

The total current in the circuit is

$$
I = \gamma_0 + \gamma_1 + \gamma_2 + \cdots \qquad (14)
$$

It is seen therefore that the current is of a complex character, having a continuous current component, and currents of frequencies $\frac{\omega}{2\pi}$, $\frac{2\omega}{2\pi}$, etc. It is also to be noted that the amplitudes of the different components decrease as the frequencies increase.

As a partial proof we may consider the case when there is no inductance in the circuit, $L = 0$, we have then

$$
\theta_1 = \theta_2 = \theta_3 = \cdots = 0
$$

$$
Z_1 = Z_2 = Z_3 = \cdots = R.
$$

Equations (11), (12), and (13) reduce to

$$\gamma_0 = \frac{E}{R}\left\{1 + \frac{r^2}{2R^2} + \frac{r^4}{8R_4} + \frac{r^4}{4R^4} + \cdots\cdots\right\}$$
$$-\gamma_1 = \frac{Er}{R^2}\cos\omega t\left\{1 + \frac{r^2}{4R^2} + \frac{r^2}{2R^2} + \cdots\cdots\right\} \tag{15}$$

. . . .

If we put L=0 in equation (1) we get

$$I = \frac{E}{R + r\cos\omega t} = \frac{E}{R}\left\{1 + \frac{r}{R}\cos\omega t\right\}^{-1}. \tag{16}$$

Expanding the above by the binomial theorem, we have

$$I = \frac{E}{R}\left\{1 - \frac{r}{R}\cos\omega t + \frac{r^2}{R^2}\cos^2\omega t - \frac{r^3}{R^3}\cos^3\omega t + \cdots\cdots\right\} \tag{17}$$

$$\cos^2\omega t = \frac{1}{2} + \frac{1}{2}\cos 2\omega t$$

$$\cos^3\omega t = \frac{1}{2}\cos\omega t + \frac{1}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t$$

$$\cos^4\omega t = \frac{1}{4} + \frac{1}{2}\cos 2\omega t + \frac{1}{8} + \frac{1}{8}\cos 4\omega t$$

. . . .

Making these substitutions, we get

$$\therefore\ I = \frac{E}{R}\left\{1 + \frac{r^2}{2R^2} + \frac{r^4}{4R^4} + \frac{r^4}{8R^4} + \cdots\cdots\right\}$$
$$- \frac{Er}{R^2}\cos\omega t\left\{1 + \frac{1}{2}\frac{r^2}{R^2} + \frac{1}{4}\frac{r^2}{R^2} + \cdots\cdots\right\} \tag{18}$$
$$+ \cdots\cdots\cdots\cdots$$

The results by the two methods are in exact agreement.

Benjamin Liebowitz (by letter): Owing to the fact that Pupin's series diverge when tuned condenser circuits of low resistance are employed, great care must be exercised in interpreting the theory as applied to the Goldschmidt alternator. The theory shows that if a current of a given frequency is large, the currents of neighboring frequencies must also be large; but it also shows that by properly controlling the impedances (detuning some of the circuits, if necessary) *the series for a given frequency can be made to diverge more rapidly than any other.* There is nothing in the theory, therefore, which says that a high efficiency is impossible. On the other hand, a high efficiency would hardly be expected, owing to the inevitable large losses in the iron, and in practice the efficiency is not more than fifty-four per cent., according to Mr. Mayer.

It has been remarked that the currents of frequency $\frac{2\omega}{2\pi}$, for example, generated in the stator by successive "reflection" from the rotor, being of opposite signs, tend to neutralize each other. It must be borne in mind, however, that any power series whose ratio is greater than unity is divergent, even if the signs alternate. Therefore, all the currents tend toward infinity in an ideal machine, in spite of the differences in sign of successive amplitudes. The series will begin to converge only when the ratios $\frac{\omega M}{Z}$ become sufficiently small, and in tuned condenser circuits this cannot happen until the currents attain sufficiently large values to produce detuning, a decrease in M, and increases in the effective resistances, by the approach of saturation.

Lester L. Israel (by letter): From the theory developed in this paper it appears that currents of lower frequency due to reactions of the higher harmonics become increasingly large.

Since in practice the Goldschmidt alternator is quite efficient, this can hardly be so. Perhaps the apparent discrepancy may be accounted for by the fact that these induced lower harmonics are opposed in phase, together with a limitation or modification of the series representing them arising from the high energy absorption at one of the higher harmonics.

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All references to any individual, company, or radio station are fully listed in this index. Answers to points brought out in the Discussions will be listed under the name of the questioner. With the exception of the names of companies, all topics will be listed in general under the *noun* referred to. The numbers correspond to pages in the text. The following abbreviations are used: r. f.—radio frequency; a. f.—audio frequency; l. v.—low voltage; h. v.—high voltage.