George R. Town

HAZELTINE SERVICE CORPORATION

(A SUBSIDIARY OF HAZELTINE CORPORATION)

NEW YORK LABORATORY:
333 West 52nd Street
COlumbus 5-0793

CHICAGO LABORATORY:
325 West Huron Street
Superior 0790

BAYSIDE LABORATORY:
Bayside, Long Island.
FLushing 7-5300

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Title. WIDE-BAND AMPLIFIERS FOR TELEVISION

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(Jobs 94 and 155)

Wide-Band Amplifiers for Television*

by
Harold A. Wheeler
Hazeltine Service Corporation
Bayside, N. Y.

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Summary

The maximum uniform amplification that can be secured over a wide frequency band by means of a single vacuum tube is much greater than that of the usual simple circuits. It can be secured by either of two arrangements, one using an individual filter coupling each tube to the next, and the other using degenerative feedback in each stage to make the stage behave as a section of a confluent filter. In either case the shunt capacitance on each side of each tube is included in an individual full-shunt arm of a band-pass or low-pass filter. One end of each interstage filter, or of each filter including one or more feedback stages, is extended to a dead-end termination with resistance approximately matching the image impedance. The other end is terminated at one of the tubes in a full-shunt arm, where the filter presents the maximum uniform impedance that can be built up across the tube capacitance. These concepts in terms of wave filters lead to practical wide-band circuits adapted to meet any given requirements.

The following general formula is shown to express the maximum uniform amplification that can be secured in one tube:

$$A = \frac{g_m}{\pi f_w \sqrt{C_g C_p}}$$

in which

- A is the voltage ratio between input and output circuits of equal impedance,
- g_m is the transconductance of the tube,
- C_g and C_p are the grid and plate capacitance of the tube, and
- f_w is the width of the frequency band.

I. Introduction

For more than twenty years, the design of amplifiers to cover a wide band of frequencies has been a major problem. The severe requirements of television have forced the solution of this problem. Even now, there has not been published either a treatment of the fundamental limitations or an outline of the basic methods of design. The purpose of this treatment is to give both as concisely as possible.

The problem dates from the "untuned radio-frequency amplifiers" of the World War. It was discovered that the amplification in a wide-band amplifier was limited not only by the amplifying ability of the vacuum tube at low-frequencies, but also by its shunt capacitance. Later, we tried to design amplifiers to cover the broadcast band without tuning. We met with failure until the advent of screen-grid tubes, and by that time the need for selectivity in radio receivers had quenched the demand for wide-band amplifiers.

As soon as radio receivers no longer wanted wide-band amplifiers, television appeared on the horizon. Television came to demand wide-band amplifiers such as never before had been conceived. They must not only amplify over megacycles of band width, but they must do that with unusual fidelity. They must amplify megacycles more faithfully than the sound receivers amplify kilocycles.

For fifteen or twenty years, the development of wide-band amplifiers was casual and sluggish. Only during the past few years have the fundamental limitations been appreciated. The British publications of W. S. Percival, which appeared last year, do show this appreciation. Only recently have we learned of independent work in this country, but this has not been published.

Our problem is to secure the maximum product of the band width and the amplification ratio of one stage. The product is the logical criterion, because either can be increased at the expense of the other. The product is limited by the quotient of the transconductance over the shunt capacitance. The shunt capacitance is involved because it limits the wide-band coupling impedance that can be built up across the input and output circuits of a vacuum tube. The real problem is merely building up the impedance across a shunt condenser, effective over a wide band of frequencies.

There are many forms of networks which can be employed to maintain nearly uniform impedance across a shunt condenser. The condenser is regarded as one element of the network. These networks all have in common some filter properties, the total width of the frequency band being limited by the shunt capacitance. Therefore it is logical to take the wave filter as a basis for a study of this subject.

Fig. 1 shows three ways in which a low-pass filter may be connected as a coupling impedance between successive tubes of an amplifier. In each case, there is included shunt capacitance across each tube. There is no obvious reason for choosing a configuration such as that shown. There is no obvious way of assigning circuit values to obtain the maximum product of coupling impedance and band width.

The configuration is chosen and the elements are evaluated by an unusual application of the theory of wave filters. The coupling impedance Z is the input impedance of a low-pass filter. The filter is a dead-end filter, employed only to secure the desired impedance. It is possible to design the filter to secure nearly uniform impedance, to any degree of approximation, by the simple methods to be described.

In Fig. 1(a), the impedance Z is employed as a two-terminal self-impedance coupling the two tubes. The network includes C_o, the total shunt capacitance of both tubes. The product of the impedance and the band width is limited by the total shunt capacitance. The amplification is proportional to the impedance, so greater impedance is desirable.

Greater impedance can be secured by separating the capacitance of the preceding tube from that of the succeeding tube, so only one of these is lumped across the impedance terminals. This separation is shown in Fig. 1(b) and (c).

In Fig. 1(b), the uniform impedance Z is developed in the output circuit of the first tube, across its shunt capacitance C_o. Therefore the amplification (voltage ratio) from grid to plate is uniform. The filter offers no attenuation from the first plate to the second grid, in the pass band. Therefore the amplification is uniform from the first tube to the second tube.

In Fig. 1(a) the two-terminal self-impedance Z is the coupling impedance. In Fig. 1(b), the coupling of the four-terminal network is measured by its transfer impedance. The transfer impedance is the quotient of the output voltage over the input current. In this case, it is the quotient of the second grid voltage over the first plate current. In the absence of filter attenuation, the transfer impedance has the same magnitude as the self-impedance. In the pass band, they differ only by the phase angle of the intervening filter.

Since the transfer impedance is the same in both directions, the four-terminal network of Fig. 1(b) may be reversed end for end, as in Fig. 1(c), without changing its coupling properties. The dead-end filter is backward instead of forward. The advantage is the location of the dead-end resistor on the plate side for carrying the plate current.

With Fig. 1 as an introduction, the theory of uniform coupling impedance and the filter method of design are to be described.

II. Uniform coupling impedance obtained by means of a dead-end filter.

In the usual applications of wave filters, uniform impedance is found only in the form of image impedance which is approximately uniform resistance over the pass band. Relatively small shunt capacitance is found to be associated with such image impedance, as compared with other arms of the same filter. Therefore a more favorable arrangement has been found. This is to be derived with reference to Fig. 2.

A low-pass filter is shown in Fig. 2(a). It contains several constant-k sections with mid-shunt termination. The far end is concluded with an m-derived half-section to secure image impedance nearly matching the terminal resistor R_o over the pass band.

The image impedance Z_1 at the near end is of the constant-k mid-shunt form, as shown in the impedance diagram. The image impedance is the actual impedance of an infinitely long filter. The actual impedance of the filter shown is nearly the same, because image impedance matching is followed through the filter to the terminal resistor. It can be made the same, to any degree of approximation, by multiple m-derivations at the far end.

The image impedance is purely resistive over the pass band, which is desirable, but it is not uniform. It can be made uniform by adding more shunt capacitance across the impedance terminals, as shown in Fig. 2(b). The added capacitance C_n should be equal to the constant-k mid-shunt arm C_m within the filter, across the impedance terminals. These together comprise a full-shunt arm C_o. The resulting impedance Z is uniform over the pass band, as shown in the impedance diagram.

The curves of Fig. 2(c) show the variety of impedance characteristics available from this combination.*

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* Relative impedance as a ratio is expressed in napiers, one napier being equal to 8.7 decibels. A small fraction of one napier represents an equal departure from unity ratio; for example, ±0.01 napier represents a ratio of 1.01 or 0.99. The corresponding unit of angular measure is the radian, equal to 57.3 degrees. This comparison enables angles to be expressed in decibels if desired, one radian being 8.7 decibels, or one decibel being 6.6 degrees.

The parameter n is the number of mid-shunt arms added in parallel with the image impedance Z_1. That is, n is the ratio of the added capacitance C_n to the constant-k mid-shunt capacitance C_m.

The image impedance Z_1 in Fig. 2(a) has the form,

$$Z_1 = \frac{R_o}{\sqrt{1 - \omega^2/\omega_c^2}} \qquad (1)$$

It is modified by adding more capacitance C_n in parallel:

$$C_n = \frac{n}{R_o \omega_c} \qquad (2)$$

The resultant impedance is

$$Z = \frac{1}{\frac{1}{Z_1} + j\omega C_n} = \frac{R_o}{\sqrt{1 - \omega^2/\omega_c^2} + nj\omega/\omega_c} \qquad (3)$$

In the pass band, $\omega < \omega_c$, the magnitude of this impedance is

$$|Z| = \frac{R_o}{\sqrt{1 - (1-n^2)\omega^2/\omega_c^2}} \qquad (4)$$

If $n = 1$, the variable term disappears so the impedance is uniform over the pass band.

The corresponding phase curves are shown in Fig. 2(d). The phase angle of the impedance, in the pass band, has the form

$$b = \text{anti-tan}\, \frac{n\omega/\omega_c}{\sqrt{1 - \omega^2/\omega_c^2}} \qquad (5)$$

This is the same as the phase angle of an m-derived half-section of a low-pass filter, but with n in place of m in the formula.

In the special case of $n = 1$, the impedance Z is developed across a full-shunt arm of the filter. The amplitude and phase characteristics of this self-impedance are identical with the transfer characteristics of a constant-k half-section filter.

The phase characteristic corresponding to uniform impedance is curved, causing some phase distortion. Uniform (zero) phase slope is secured by $n = 0$, but with an abrupt change at the cutoff frequency. Nearly uniform phase slope is secured with n slightly greater than one. There is no value of n corresponding to uniform impedance and uniform phase slope, both at once. This result can be approximated by combining in different stages, self-impedance coupling with different values of n .

If this network is used as a four-terminal coupling impedance, as in Fig. 1(b) or (c), the attenuation and phase are merely increased by the amount of the intervening sections of the filter. Therefore any desired characteristics can be obtained by proper choice of the intervening filter sections. Great attenuation outside the pass band, and phase correction in the band, are the properties most likely to be desired. Either or both can be secured at will.

This derivation is equally applicable to band-pass filters having any set of cutoff frequencies. The essential requirements are merely that Z_1 is a mid-shunt image impedance of the constant-k form and that the added shunt arm is a corresponding constant-k mid-shunt arm. The impedance is then uniform over all pass bands.

Uniform impedance Z_o equal to R_o is developed in Fig. 2(b) across the total shunt capacitance equal to a full-shunt arm of a constant-k filter:

$$C_o = \frac{2}{R_o \omega_c} \tag{6}$$

This is the greatest shunt capacitance across which this uniform impedance can be developed over a frequency band of this width. The level impedance Z_o is double the reactance of the shunt capacitance C_o at the cutoff frequency ω_c.

This relation leads to the ultimate theoretical limitation on the wide-band performance of this coupling impedance:

$$Z_o C_o \omega_w = 2 \tag{7}$$

in which ω_w is the band width in terms of angular frequency. This formula is valid not only for low-pass filters, but also for band-pass filters of the same total band width. This product is the figure of merit by which any wide-band uniform impedance should be judged. Its theoretical upper limit is two. Its practical value depends not only on the theoretical factors but also on the tolerance of departure from uniformity.

The theoretical limit is based on an infinite number of circuit elements, and cannot be exceeded in any passive network. The very simple practical circuits in common use are makeshifts and fall far short of the limit. The addition of a few circuit elements in a preferred arrangement is sufficient to obtain a very close approximation to the theoretical performance.

The design of a wide-band coupling impedance by this method is best accomplished by working from the simple to the complex, until a sufficiently good figure of merit is realized. Fig. 3 is an example of this procedure. The low-pass filter components are shown in the first column. The first example (a) includes no filter sections, only the shunt capacitance with a resistor in parallel. The second example (b) includes a constant-k half-section. This turns out to be the ordinary video-frequency coupling impedance with series inductance and resistance in the parallel path. The third example (c) has instead an m-derived half-section. The fourth example (d) has first a constant-k half-section and then an m-derived half-section. The constant-k half-section provides for maximum capacitance directly across the impedance terminals on the left-hand side. The m-derived half-section provides for matching the image impedance with the resistor at the dead end on the right-hand side.

The percentage notations give an approximate indication of the figure of merit of these networks, relative to the ideal. They are based on a tolerance of ± 0.03 radian or $\pm 1/4$ decibel over the useful band (with reference to the curves of Fig. 5). There is an improvement from 20% to 95% by the addition of three more circuit elements in the low-pass filter.

The second column of Fig. 3 shows the practical low-pass circuits. The resistor and reactors are rearranged in ladder networks, in the most convenient order. One to three circuit elements are added to the shunt capacitance and the essential resistor.

The third column shows the band-pass networks exactly analogous to the low-pass networks of the second column. The band-pass dead-end filters are reduced to a chain of coupled circuits, each resonant within the pass band. Each reactance element of the low-pass filter becomes a tuned circuit in the band-pass analogue.

The practical circuits, especially the band-pass examples, are arranged to include shunt capacitance to ground wherever the circuit permits. Such capacitance may be a disturbing factor if neglected. This is the reason for deriving the band-pass analogues in terms of parallel-tuned circuits instead of the obvious combination of parallel-tuned and series-tuned circuits to replace shunt capacitors and series inductors.

The performance of the simpler low-pass networks can be checked by computation. Also they can be designed by different methods, without reference to filter theory. The method commonly used involves expanding the impedance formula into a power series in terms of frequency. The added circuit elements are so evaluated as to cancel the same number of frequency terms in the power series. With reference to the familiar Taylor series, this means cancellation of a number of the lower-order derivatives. This method involves too much labor when the number of circuit elements becomes large. Experience with this method and the filter method serves to demonstrate the advantages of the latter, which is equally useful for any number of circuit elements.

Fig. 4 shows the impedance curves of the low-pass networks of Fig. 3, the circuit elements having their critical values determined by the series method. All the peaks are merged into a single peak. The ideal curve (e) is that of Fig. 2, for n = 1.

This and the following figure are plotted in such a manner as to show directly the figure of merit in terms of the useful band width. There are three quantities involved, the impedance, the shunt capacitance, and the frequency band width. Two of these have definite values while the third is indefinite in practical cases. The mid-band or zero-frequency impedance has a definite value, and the impedance varies but little over the useful band. The shunt capacitance has a fixed value. The useful band width, however, depends on the tolerance of departure from uniform impedance. The curves are plotted for unit impedance and unit capacitance, so the figure of merit is equal to the useful band width on the frequency scale. The figure of merit of the ideal curve is two, the maximum theoretically possible.

If the same low-pass impedance networks have their circuit elements evaluated by the filter method instead of the series method, the resulting impedance curves are those of Fig. 5. The computations are simple and direct. The m-derived half-sections are based on the usual value, m = 0.6. Curve (d), for three added elements, approaches the ideal so closely that the difference has no practical significance.

The question arises why curve (c), based on Fig. 3(c), should fall so far short of curve (d), based on Fig. 3(d). Both have an m-derived image impedance facing the resistor, with equal approximation of matching. In (c), however, the total capacitance across the impedance terminals is a constant-k mid-shunt arm plus an m-derived mid-shunt arm, the total being only (1 + m) constant-k mid-shunt arms. In (d), the total shunt capacitance has the maximum value, two constant-k mid-shunt arms. When these networks are reduced to unit impedance and unit shunt capacitance, the filter cutoff frequency of (c) is only 1.6 while that of (d) has the maximum value two. It is noted that (d) includes no shunt capacitance directly across the coils and the resistor. If such incidental capacitance is appreciable, it may be taken into account, but it always reduces the cutoff frequency. The method of plotting used in Fig. 5 shows clearly the result of these factors.

Referring back to Figs. 1(b) and 2(b), greater coupling impedance between two tubes may be obtained by separating their shunt capacitance in a four-terminal network. The self-impedance then has to be developed across the shunt capacitance of only one of the tubes. Double the impedance is possible over the same frequency band. In Fig. 2(b), the self-impedance is developed across the shunt capacitance of one tube, C_o. That of the other tube, C_o', is displaced along the filter, so it does not limit the self-impedance. The transfer impedance from one tube to the other, over the pass-band of the filter, has the same magnitude as the self-impedance Z . Therefore the four-terminal transfer-impedance network has a higher standard of performance that the two-terminal self-impedance.

Several examples of four-terminal networks are shown in Fig. 6. The simple examples of (a) and (b) are simple filters without the dead-end extension of the filter. They are makeshifts in view of the present theory of design. Symmetrical damping by resistors on both sides is shown in the first row (a). The more effective unsymmetrical damping by a resistor on only one side is shown in the second row (b). The latter is the first step toward the dead-end filter, which is further unsymmetrical. Good practical embodiments of the dead-end filter are shown in the third and fourth rows (c) and (d). They have the same dead-end termination as (c) and (d) of Fig. 3. The low-pass examples have the dead-end filter reversed as in Fig. 1(c).

In the band-pass examples of Fig. 6, the part of the filter between the two pairs of terminals is not an exact analogue of the corresponding low-pass example. The low-pass section requires three reactors, that is, two capacitors and an inductor, whereas the band-pass section requires only two tuned circuits.

The result is less attenuation and phase shift than would be found in the exact band-pass analogue.

It is interesting to compare, for the various examples, the phase shift within the pass band. That is the change of phase angle over the band. It is one right angle for the two-terminal low-pass case and two right angles for the band-pass. For the four-terminal cases, it increases to three and four right angles, respectively. The attenuation outside the band compares in the same ratios.

The phase shift may be detrimental in two ways. The phase slope represents delay of the signal, which may be undesirable, but usually does no harm if uniform over the pass band. The phase angle also includes departure from uniform phase slope. This is phase distortion and is always detrimental. In comparing the two-terminal and four-terminal cases, the relative phase distortion affects the choice of which yields the optimum compromise between maximum coupling impedance and minimum phase distortion. The impedance is about twice as great in the four-terminal cases. The low-pass phase shift is three times as great, while the band-pass is twice as great. Therefore the four-terminal coupling has less advantage in a low-pass case than in a band-pass case.

Without the aid of filter theory, the study of phase characteristics is even more difficult than the study of amplitude characteristics illustrated in Figs. 4 and 5. Neither is practical for the more complicated networks.

Phase correction, with nearly uniform amplitude, may be obtained among two-terminal networks by designing the different ones of a group to be complementary. In a four-terminal dead-end filter, phase correction may be obtained without affecting the amplitude characteristics. All that is needed is the insertion of a phase-correcting filter between the two pairs of terminals. Such a filter is the m-derived section with m greater than one, obtained by negative mutual inductance. The availability of systematic phase correction is a great advantage of this method of design.

Before summarizing the theoretical limitations on wide-band amplifiers, the use of feedback deserves attention. It is interesting, not only because it is a useful extension of the method of dead-end filters, but also because it proves to be subject to the same theoretical limitations.

III. A filter section including a feedback amplifier.

Feedback of a degenerative or stabilizing nature has the effect of decreasing the amplification in an amplifier. It also increases the width of the frequency band over which the amplification is uniform. Therefore it is useful in a wide-band amplifier.

The feedback amplifier has usually been treated in terms of its forward amplification and backward attenuation. The amplitude and phase characteristics of both have been required, and are usually difficult to handle, especially if there are several stages involved.

A much simpler method of design can be used if the feedback is associated individually with each stage. This is one of the best arrangements for a wide-band amplifier, as well as one of the simplest.

Each stage of the amplifier, with its feedback, is designed as a section of a wave filter. It can then be combined with other stages and other filter sections, in accordance with filter theory. This gives greater insight into the behavior of the feedback amplifier, and facilitates the design of circuits to obtain any characteristics available in wave filters.

The essential elements of such a filter section are a pair of reactance arms with forward and backward coupling. The forward coupling is the transconductance of the amplifier tube.

A low-pass filter section including a feedback amplifier stage is shown in Fig. 7(a). It comprises shunt capacitance across input and output terminals, C_1 and C_2, coupled by forward and backward transconductance, g_{12} and g_{21}. This combination is termed bidirective transconductance. Each transconductance may be positive or negative. Each may be obtained in a screen-grid tube having negligible input or output self-conductance. The usual transconductance of a vacuum tube is called negative, because the signal polarity is reversed by coupling through the tube. Two such tubes resistance-coupled in cascade, or some special types of tube, can be used to secure the opposite polarity of transconductance, called positive. The absence of self-conductance is necessary if dissipation in the filter is to be avoided. This condition is met in screen-grid tubes. The transconductance property by itself does not involve dissipation.

In order to derive the filter properties of this network, the usual method is followed, based on short-circuit and open-circuit impedance. Looking into one end of the network, the impedance with the other end on short-circuit is simply that of the shunt arm:

$$Z_{sc}' = \frac{1}{j\omega C_1} \tag{8}$$

The feedback comes into play when the other end is on open-circuit:

$$Z_{oc}' = \frac{1}{j\omega C_1 - \dfrac{g_{12}g_{21}}{j\omega C_2}} = \frac{j\omega C_2}{-g_{12}g_{21}\left(1 - \dfrac{\omega^2 C_1 C_2}{-g_{12}g_{21}}\right)} \tag{9}$$

The image impedance is the geometric mean of these two:

$$Z_i' = \sqrt{Z_{sc}'Z_{oc}'} = \frac{1}{\sqrt{-g_{12}g_{21}}} \cdot \frac{1}{\sqrt{1 - \dfrac{\omega^2 C_1 C_2}{-g_{12}g_{21}}}} \cdot \sqrt{\frac{C_2}{C_1}} \tag{10}$$

This has the same form as the mid-shunt image impedance of a constant-k low-pass filter:

$$Z_i' = \frac{R'}{\sqrt{1 - \omega^2/\omega_c^2}} \tag{11}$$

The nominal image impedance, or that at zero frequency, is

$$R' = \frac{1}{\sqrt{-g_{12}g_{21}}}\sqrt{\frac{C_2}{C_1}} \tag{12}$$

The cutoff frequency is

$$\omega_c = \sqrt{\frac{-g_{12}g_{21}}{C_1 C_2}} = \frac{1}{C_1 R'} \tag{13}$$

It is noted that similar expressions are obtained looking into the input end or the output end. The only difference is the interchange of the subscripts. This has no effect on the cutoff frequency. The image impedance is the same at both ends if the shunt capacitance is the same across both ends.

The nominal image impedance and the cutoff frequency are real only if the forward and backward values of transconductance are of opposite polarity. This is the condition for degenerative feedback at zero frequency. The total phase angle of input and output capacitance causes the feedback to become regenerative to the point of oscillation at the cutoff frequency. This corresponds to the free oscillation which would occur in a non-dissipative filter section at its cutoff frequency. The oscillation is damped by the resistance termination of the filter, in either case.

The graphs of Fig. 7 summarize the filter properties of the low-pass feedback amplifier. The attenuation a and the phase angle b of a wave filter are given by the relation,

$$\tanh (a + jb) = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \frac{\sqrt{\omega^2/\omega_c^{\,2} - 1}}{\omega/\omega_c} \tag{14}$$

Simplifying this expression,

$$\cosh (a + jb) = \pm\omega/\omega_c \tag{15}$$

In the attenuation band, $\omega > \omega_c$,

$$a = \text{anti-cosh } \omega/\omega_c \quad ; \quad b = 0 \text{ or } \pi \tag{16}$$

In the pass band, $\omega < \omega_c$,

$$a = 0,\ b = \text{anti-cos } \pm\omega/\omega_c = \text{anti-sin } \omega/\omega_c \pm \pi/2 \tag{17}$$

The choice between the two values of b is not determined by the filter characteristics but rather by those properties yet to be discussed, which are not found in passive wave filters.

The forward amplification and the backward attenuation through the feedback amplifier filter can be described separately from the filter characteristics. They arise from the inequality of the forward and backward transconductance. The filter properties alone would be secured if the transconductance were the same in both directions. But the product of the forward and backward values must be negative, so each value would have to be imaginary. Also their values are assumed constant in this treatment. This set of conditions cannot be realized, so the filter characteristics have to be supplemented by the effect of unequal transconductance. This would have to take care of the phase angle of $\pm\pi/2$ at zero frequency, given by the filter analysis but not possible in a physical network.

Just as the transconductance product $g_{12}g_{21}$ determines the filter characteristics, the transconductance ratio g_{12}/g_{21} determines independently the amplification. This is uniform over the entire frequency range, because the ratio is constant.

The uniform amplification is regarded as superimposed on the filter properties. The amplification must be expressed in terms of power, because the zero attenuation of a filter in the pass-band is based on equal power input and output. The power ratio of amplification can be shown to have the value,

$$A^2 = \left|\frac{g_{12}}{g_{21}}\right| = \frac{-g_{12}}{g_{21}} = \frac{g_{12}{}^2}{C_1 C_2 \omega_c{}^2} = g_{12}{}^2 R' R'' \tag{18}$$

If the filter is terminated by its image impedance, the actual voltage ratio at zero frequency is

$$A\sqrt{\frac{R''}{R'}} = \sqrt{\frac{-g_{12}C_1}{g_{21}C_2}} = g_{12}R'' \tag{19}$$

This result is inevitable because the output impedance is R" under these conditions. The voltage ratio and power ratio are uniform in the pass band, and are subject to the filter attenuation at higher frequencies.

The transconductance ratio simply causes the current or voltage ratio of the filter to be multiplied by the "directive factor",

$$q = \sqrt{\frac{g_{12}}{g_{21}}} \tag{20}$$

which is imaginary so it has a phase angle of $\pm \pi/2$. Taken with the filter phase angle b , this gives 0 or π as the net phase angle at zero frequency. Either is physically possible. The dotted phase curves in Fig. 7 show these conditions. The usual vacuum tube as the forward transconductance gives a negative value, representing a reversal of polarity, that is, a phase angle of π .

The backward directive factor is the reciprocal of the forward, while the filter properties are the same in both directions. These together determine the backward attenuation. This distinguishes the feedback amplifier from the unidirective amplifier having no coupling in the backward direction.

The attenuation and phase characteristics of the filter are those of a half-section constant-k low-pass filter. The image impedance, however, has the mid-shunt form at both ends of the filter. Such a low-pass filter has not previously existed. It is the low-pass analogue of the band-pass filter comprising a symmetrical coupled pair of tuned circuits. The analogy is complete as far as the filter characteristics, but not the amplification, are concerned.

This filter can be designed for any set of cutoff frequencies. A constant-k mid-shunt arm, designed for these cutoff frequencies, is included at each end of the section. It is arranged to include directly in parallel, the maximum capacitance consistent with the band width and the nominal image impedance. Then this capacitance is embodied in the incident capacitance of the vacuum tube and associated circuit elements.

Separate forward and backward tubes would be required to meet the conditions in the theoretical feedback amplifier filter of Fig. 7(a). This is neither desirable nor essential for practical purposes. Large amplification requires that the forward transconductance g_{12} be much greater than the backward g_{21}. Therefore the latter can be replaced by a resistor without departing too much from the ideal conditions of zero self-conductance.

This expedient is shown in Fig. 7(b). The negative forward transconductance is furnished by the vacuum tube g_m, with slight opposition from the smaller self-conductance G_3 of the feedback resistor. The latter furnishes the backward transconductance.

$$-g_{12} = g_m - G_3 \quad ; \quad g_{21} = G_3 \tag{21}$$

The self-conductance G_3 has the undesired effect of introducing dissipation just as if an equal conductance were in parallel with C_1 and another with C_2. This effect is small if the amplification is large. The resulting cutoff frequency, power ratio and voltage ratio are:

$$\omega_c = \sqrt{\frac{G_3(g_m - G_3)}{C_1 C_2}} \tag{22}$$

$$A^2 = \frac{g_m - G_3}{G_3} = \frac{(g_m - G_3)^2}{C_1 C_2 \omega_c^2} = (g_m - G_3) \quad R'R'' \tag{23}$$

$$A\sqrt{\frac{R''}{R'}} = (g_m - G_3)R'' \tag{24}$$

The treatment of a feedback amplifier as a filter involves an undue amount of work for a single simple example such as that of Fig. 7. (The same would have been true of a single-section low-pass filter in the evolution of wave filters.) Its benefits are found in its general application. It introduces a new concept in the cooperation of amplifiers and filters. Feedback amplifiers and filter sections can now be designed individually and joined in succession with image impedance matching at the junctions. They behave like an ordinary filter plus an amplifier, with the added advantage of distributing the filter attenuation among the amplifier stages. It is now possible to design a feedback amplifier with complex circuits which could not be computed directly. Its performance can be predicted with very close approximation.

In designing a wide-band amplifier with feedback, the feedback filter stage is inserted as a section of a dead-end filter. All of its properties described above are retained, while adding the amplification of the feedback stage. Several examples are shown in Fig. 8. The first two, (a) and (b), have two-terminal interstage coupling networks. The second two examples have four-terminal networks to secure the advantage of separating the shunt capacitance of one tube from that of the noxt.

A simple low-pass example is shown in Fig. 8(a). The middle tube is in the feedback stage. This section is included in a filter which is extended on the output end to a dead-end termination with uniform image impedance matching a resistor. The termination is like that of Figs. 3(c) and 6(c). Uniform impedance is developed at the input end, in the output circuit of the preceding tube. The amplification is uniform over the pass band. The percentage notations have the same meaning as those of Fig. 3.

The principal advantage of using feedback is the reduction of the number of circuit elements. Only one dead-end termination is required for several interstage circuits, instead of one for each. An incidental advantage is the greater percentage of the ideal figure of merit, for a small number of circuit elements in the dead-end filter. Another is the reduction of distortion from non-linear amplification, such as overloading. A disadvantage is the interdependence of the filter properties with the amplifying properties of the tube. The cutoff frequency depends on the transconductance, so this must remain constant. Some of the interstage circuits include no resistance, relying entirely on the feedback for their damping. The cutoff frequency must be the same for all stages in a single filter.

The band-pass analogue is shown in Fig. 8(b). Here the feedback resistor is tapped down on the input and output coils to permit the use of the most convenient value of resistance. This is a value small enough to minimize the disturbance of parallel capacitance but large enough to minimize that of series inductance.

The examples of Fig. 8(c) and (d) are the corresponding examples with four-terminal instead of two-terminal interstage coupling impedance. The percentage notations are relative to the higher standard of performance of the four-terminal networks. The comparison is about the same as between the dead-end filters of Fig. 3 and those of Fig. 6. The dead-end termination is shown toward the input. This places the resistor ahead of the amplification, so it has to dissipate less power and a smaller unit can be used. Only the feedback is required to damp the higher-power coupling networks, following the amplification. This minimizes the power output required from the tube in each feedback stage, especially from the last stage in the filter.

The maximum amplification obtainable by the use of a given tube in a feedback amplifier filter is subject to the same limitations as if an individual dead-end filter were used in each interstage network. The two alternatives are comparable in most respects.

IV. The theoretical limitations on the wide-band performance of an amplifier.

There is a maximum uniform amplification that can be obtained over a wide band of frequencies from a single tube in the systems described. It depends on the grid and plate capacitance of the tube, C_g and C_p, as well as its transconductance g_m. If it is measured between input and output circuits of equal impedance, it has the value

$$A = \frac{2g_m}{\omega_w\sqrt{C_gC_p}} = \frac{g_m}{\pi f_w\sqrt{C_gC_p}} = \frac{f_o}{f_w} \qquad (25)$$

The total frequency band width is f_w, or ω_w in terms of angular frequency.

This formula is based on a band-pass dead-end filter of ideal properties, that is, freedom from dissipation and exact matching of image impedance with the terminal resistor over the pass band. It is based on a band-pass rather than a low-pass case, to permit the use of transformers to match capacitance in different shunt arms of the same filter. Also this permits of measuring the amplification between input and output circuits of equal impedance. This is the only fair measure of amplification.

The formula may be derived in any of several circuit arrangements with four-terminal coupling networks, with or without feedback. The simplest is the band-pass circuit of Fig. 6(d).

Like tubes are assumed and the voltage ratio from one grid to the next is computed. The uniform impedance developed in the plate circuit of the first tube, across C_p, is

$$Z_o = \frac{2}{C_p \omega_w} = \frac{1}{\pi f_w C_p} \tag{26}$$

The gain from grid to plate of the first tube is this impedance multiplied by the transconductance g_m. This is multiplied by the voltage ratio from plate to grid, which is the transformer ratio needed to match the plate and grid capacitance, C_p and C_g. The amplification is then found to be

$$A = g_m Z_o \sqrt{\frac{C_p}{C_g}} = \frac{g_m}{\pi f_w \sqrt{C_g C_p}} \tag{27}$$

This formula contains the grid and plate capacitance, only in terms of the geometric mean value. In the case of a low-pass coupling filter, a transformer is not available, so not quite as much amplification can be secured between unequal values. The same theoretical limit is still valid, however, and can be approximated in low-pass networks designed to operate between unequal values of shunt capacitance. They are designed to secure the effect of a transformer over all of the pass-band, except near zero frequency where there is no difficulty in building up the amplification to its level value.

There is a certain band width over which the theoret_cal maximum voltage ratio is unity, corresponding to neither gain nor loss. This band width is called the "band-width index" (a term suggested by my associate, Mr. L. F. Curtis):

$$f_o = \frac{g_m}{\pi \sqrt{C_g C_p}} \quad ; \quad A = \frac{f_o}{f_w} \tag{28}$$

The amplification is easily computed as the ratio of the band-width index over the required band width. The band-width index is an interesting property of a vacuum tube, to express its relative merit as a wide-band amplifier. The practical value of the band-width index is about half as great, being reduced by the added circuit capacitance and by the failure to realize exactly the ideal dead-end filter.

It is recommended that the band-width index be included in the specifications of vacuum tubes, especially of those for use in wide-band amplifiers. The following table includes its value for various types of pentode amplifier tubes.

Type	g_m (μmhos)	C_g (μμf)	C_p	f_o (Mc)
6K7 (metal)	1600	8	12	52
6D6 (glass)	1600	6	6	85
954 (acorn)	1400	3	3	150
1851 (metal)	9000	15	5	330

(29)

The formula given for the band-width index is valid only for tubes having the grid-plate coupling shielded to such a degree that neutralization is not required. If push-pull neutralization is to be used, the grid and plate total capacitance should each be increased by the amount of the neutralizing capacitance, which is equal to the grid-plate capacitance.

Another band-width index, useful with reference to power tubes, would be the maximum band-width over which the optimum load impedance could be developed across the plate capacitance by the use of a dead-end filter.

Conclusion

We have derived the theoretical limitations on the performance of a wide-band amplifier. We have shown how circuits can be devised to approximate this performance as closely as required. The methods of design are simple and direct. They involve no laborious computations because they draw from the wealth of information available in the art of wave filters. Examples have been given to show how the methods lead to practical designs which meet any reasonable demands. These methods are the connecting link between amplifiers and wave filters.

References

The references are classified as follows, in terms of the subject matter to which they relate.

(1, 2, 4, 6) The general theory of filters and methods of design.

(3, 16) Wide-band amplifiers using the dead-end filter.

(5, 7, 8, 9, 10, 14, 18, 20) Wide-band amplifiers using series inductance and resistance across the shunt capacitance.

(3, 4, 5, 15, 17, 18) Wide-band amplifiers using other forms of interstage coupling networks.

(11, 12, 13, 19, 31) Degenerative feedback associated with a pair of tuned circuits in an amplifier.

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(2) O. J. Zobel, "Theory and design of uniform and composite electric wave filters", B.S.T.J., vol. 2, pp. 1-46, January, 1923.

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HAWheeler
Engineer

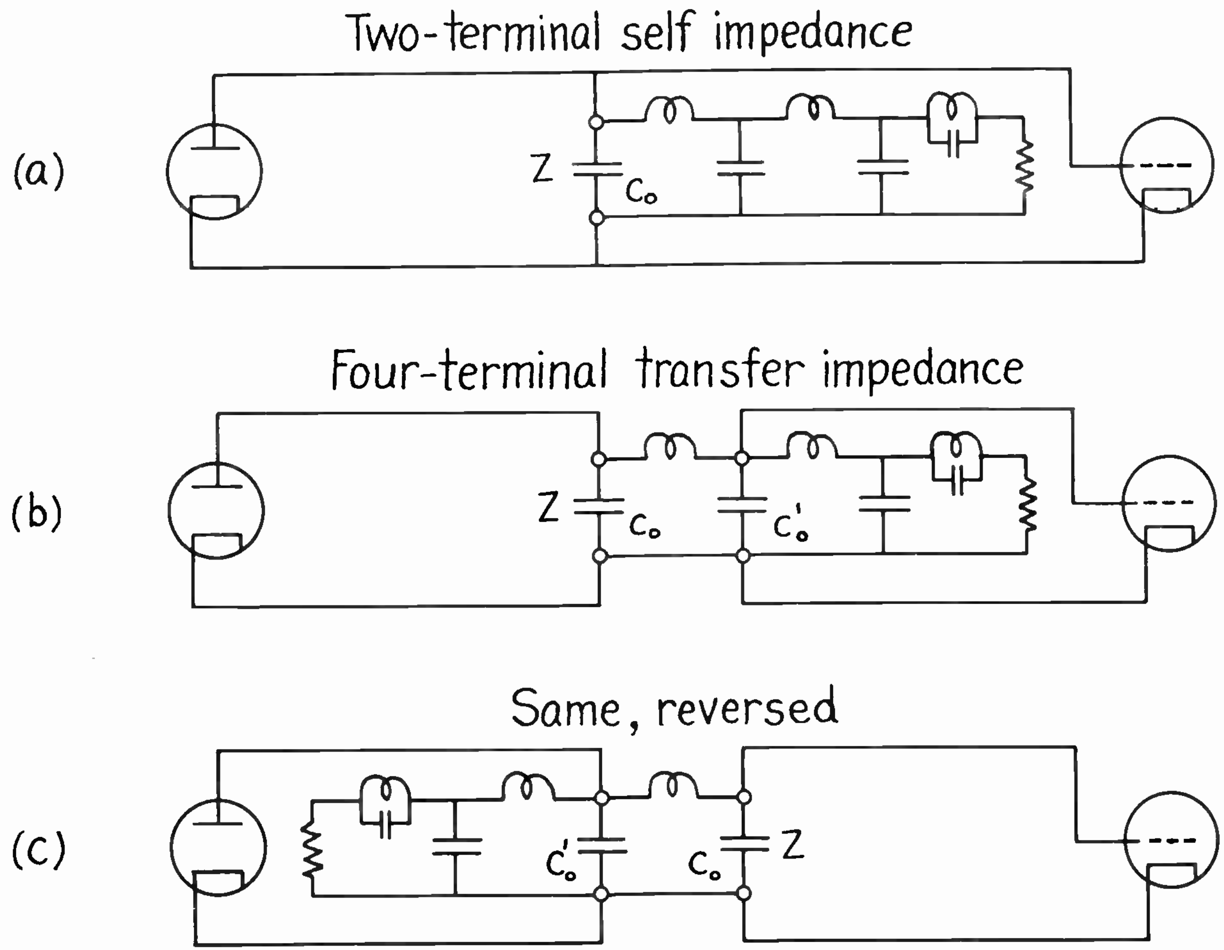


Fig.1 - A low-pass coupling impedance obtained by means of a dead-end filter.

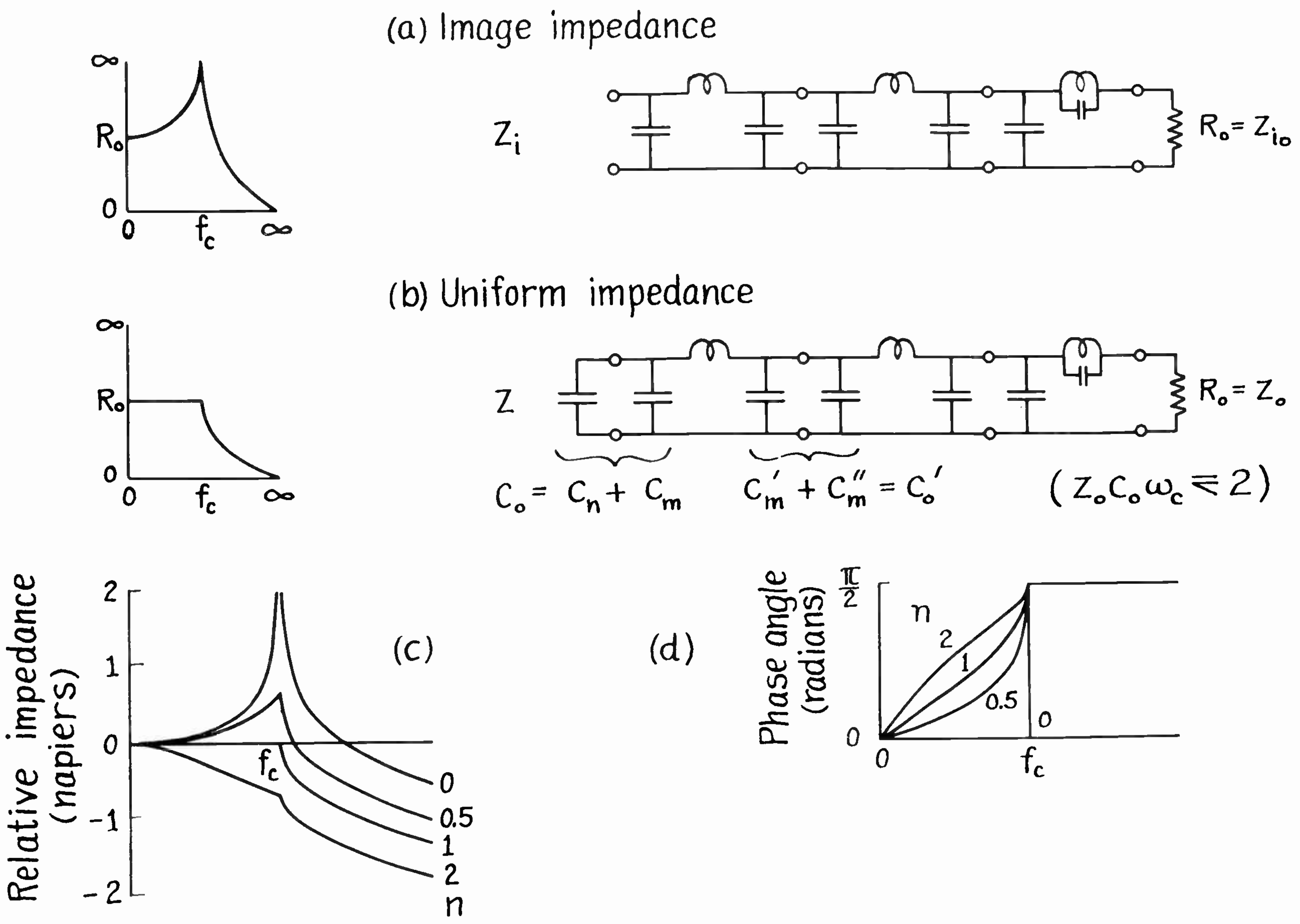


Fig. 2 - The derivation of a low-pass coupling impedance from filter theory.

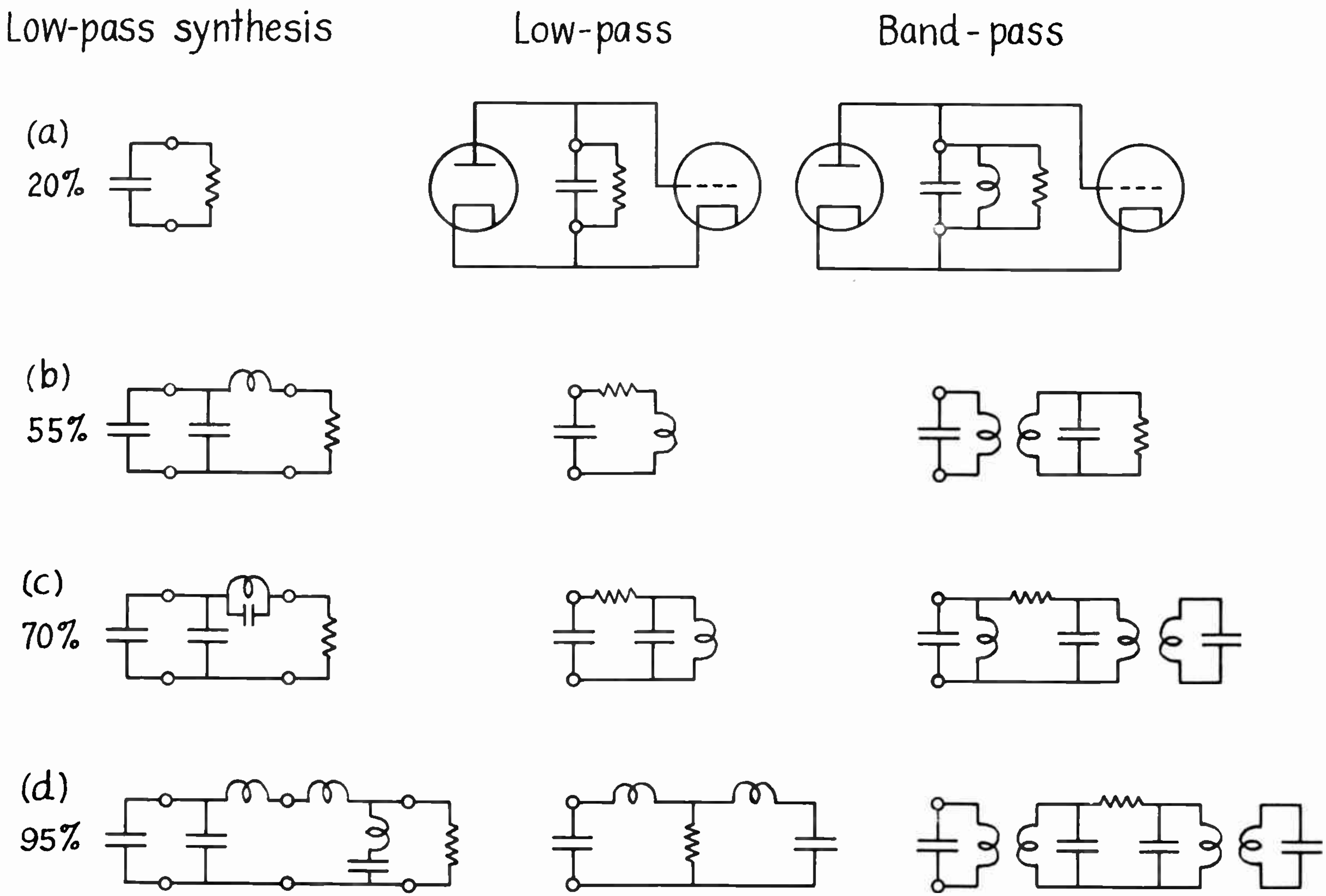


Fig. 3 - Two-terminal self-impedance coupling networks, low-pass and band-pass.

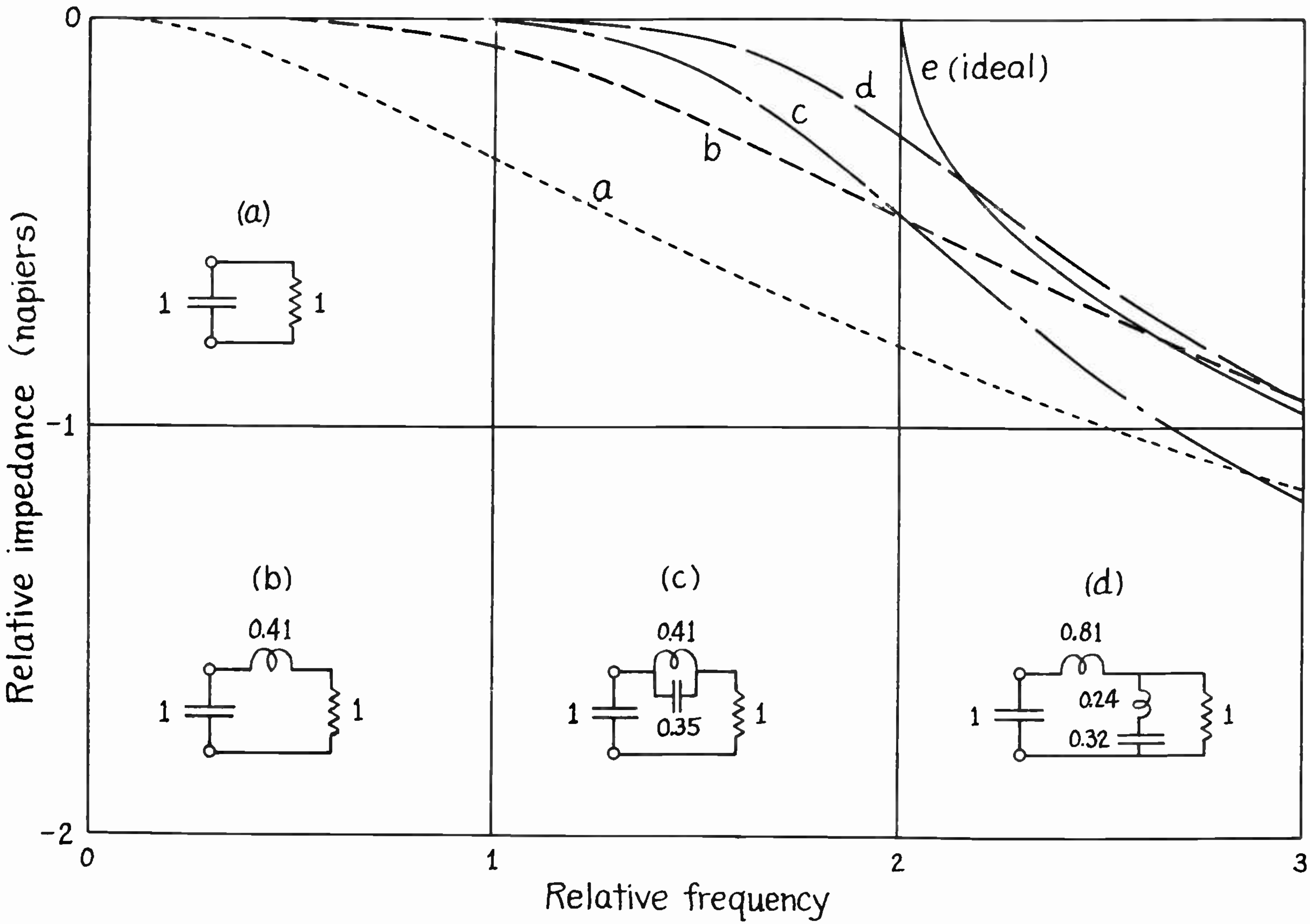


Fig. 4 - Low-pass self-impedance characteristics for the critical values of the added circuit elements.

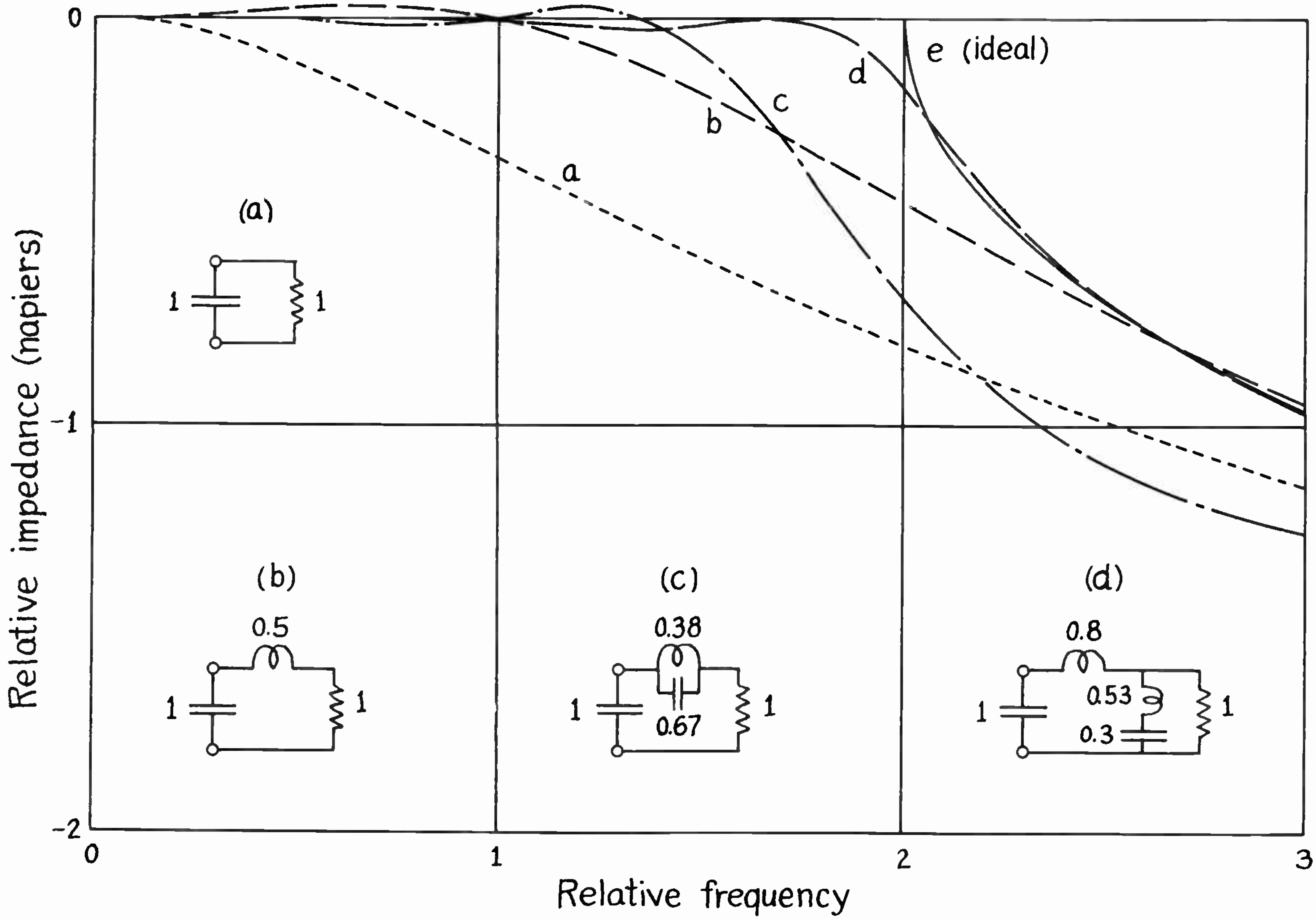


Fig.5 - Low-pass self-impedance characteristics for the filter values of the added circuit elements.

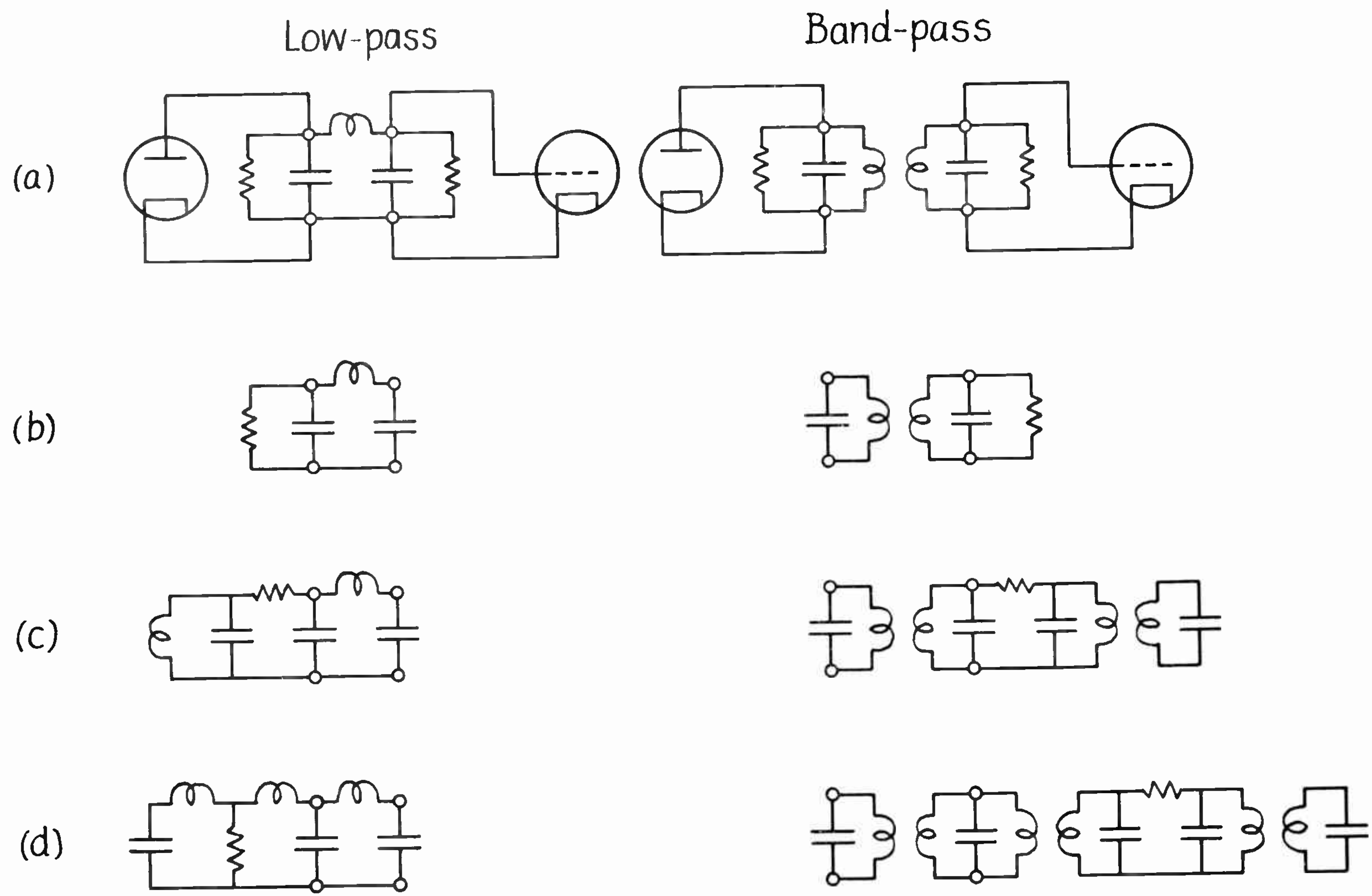
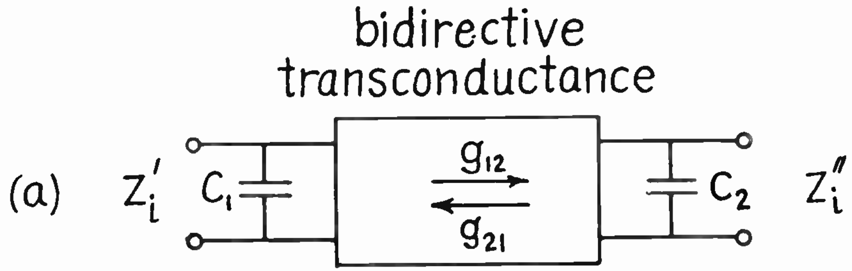


Fig.6-Four-terminal transfer-impedance coupling networks, low-pass and band-pass.

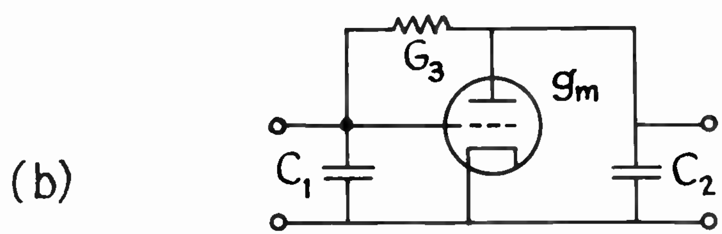


Cutoff frequency: $\omega_c = \sqrt{\dfrac{-g_{12}\,g_{21}}{C_1 C_2}}$

Power ratio: $A^2 = \left|\dfrac{g_{12}}{g_{21}}\right| = \dfrac{g_{12}^2}{C_1 C_2 \omega_c^2}$

$$\left.\begin{aligned} -g_{12} &= g_m - G_3 \\ g_{21} &= G_3 \end{aligned}\right\} \qquad \omega_c = \sqrt{\frac{G_3(g_m - G_3)}{C_1 C_2}}$$

$$A^2 = \frac{g_m - G_3}{G_3} = \frac{(g_m - G_3)^2}{C_1 C_2\, \omega_c^2}$$



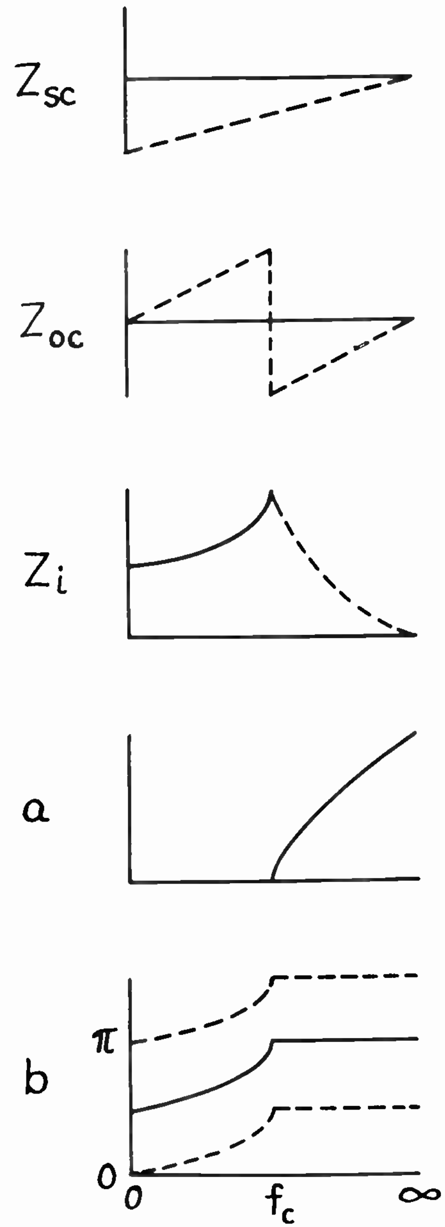


Fig.7- The derivation of a feedback amplifier operating as a low-pass filter section.

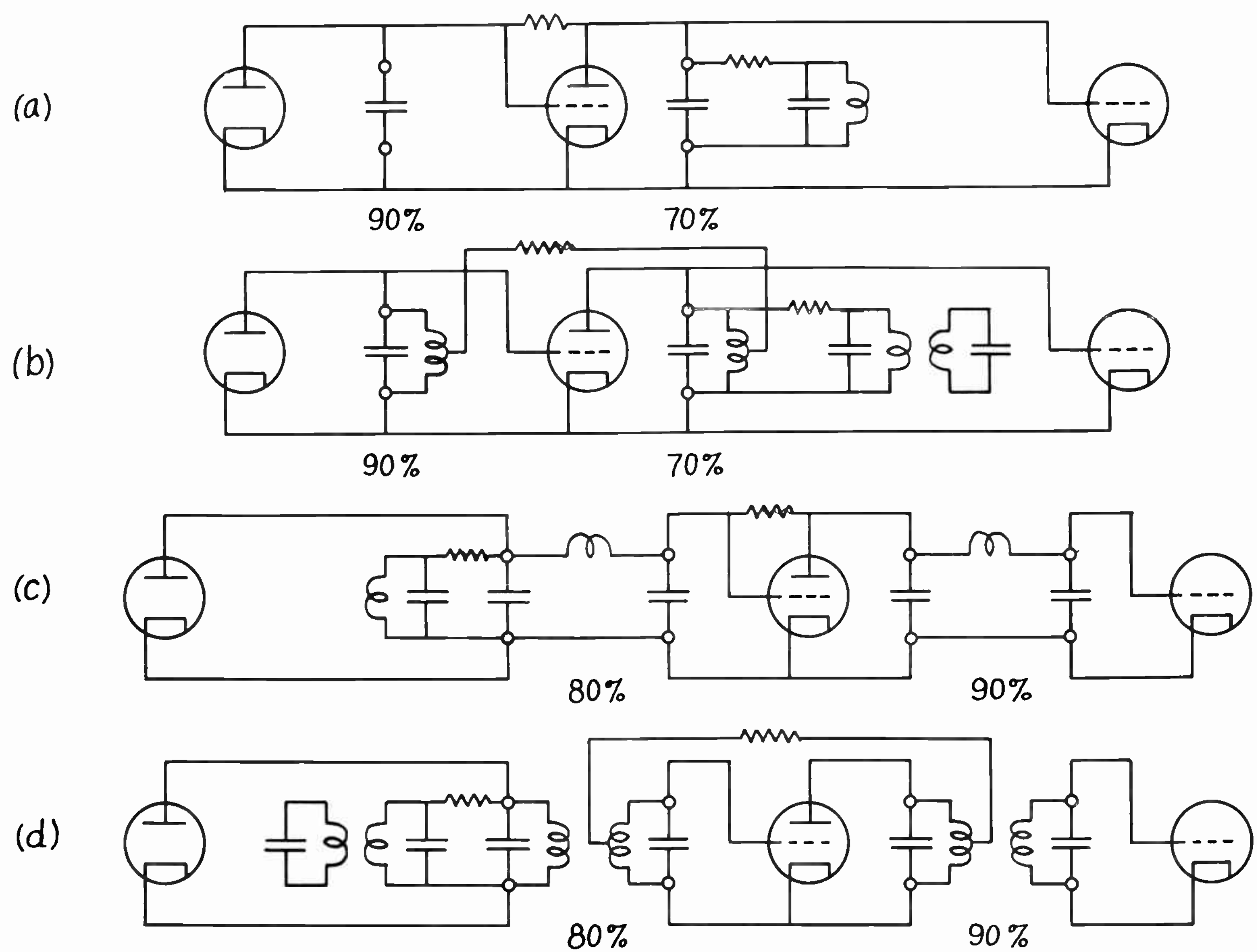


Fig. 8 - The insertion of a feedback amplifier as a section of a dead-end filter, low-pass and band-pass, two-terminal and four-terminal interstage coupling impedance.