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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

Also

IN THIS ISSUE

THE NOISE PRIMER

PART IX

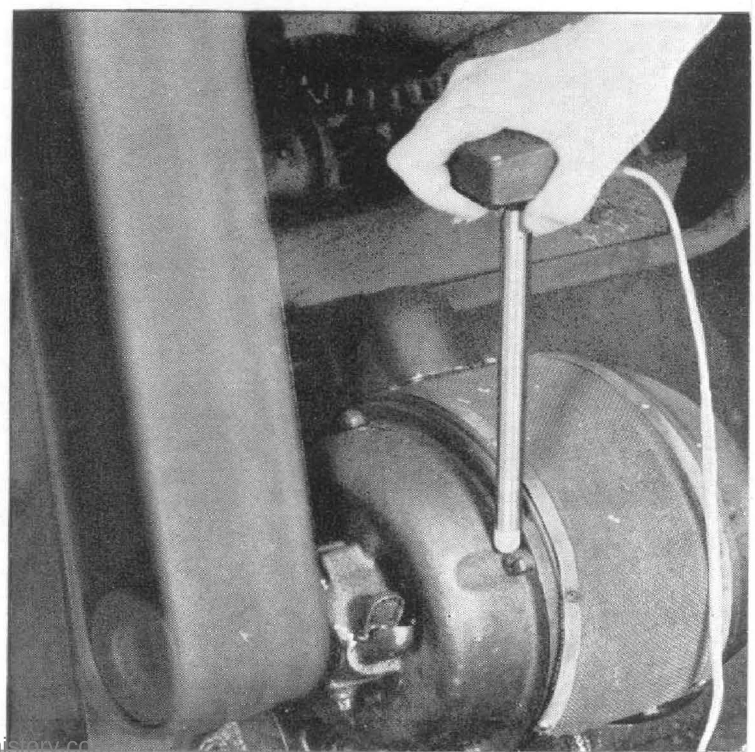
VIBRATION AND SOUND

● **THAT SOUND** and mechanical vibration are related is such a simple concept as to seem hardly worth mentioning. However, the relationships between these two phenomena and, in particular, their effects upon human beings are very complicated indeed.

Most of us use the terms sound and vibration in broad and overlapping senses, but for the purposes of this discussion it is best to keep to rather narrow meanings. Sound, therefore, will be considered as airborne vibrations of an audible frequency. The term vibration will be used to mean mechanical vibrations or vibrations occurring in solids. The frequency ranges for sound and vibration as thus defined are roughly the same, excepting that important vibrations may also be present considerably below the lower frequency limit of hearing, and satisfactory vibration-measuring equipment must operate at frequencies as low as 2 or 3 cycles per second.

The reasons for measuring or reducing vibration are generally two. In the first place, as is generally realized, audio-frequency vibrations of solids transmit sound vibrations to the air, thus creating noise. The process of quieting a machine or device generally

FIGURE 19. For most measurements the vibration pickup can be held in the hand.



includes, therefore, a study of the mechanical vibrations involved.

In the second place, serious vibration may cause actual failure which, in the cases of heavy machinery or airplanes, for instance, may have fatal consequences. Vibration, then, is not only a source of noise, but often a source of real danger. The present perfection of high-speed planes, ships, and automobiles could never have been achieved without a thorough study of vibration, its cause, measurement, and cure.

Displacement, Velocity, and Acceleration

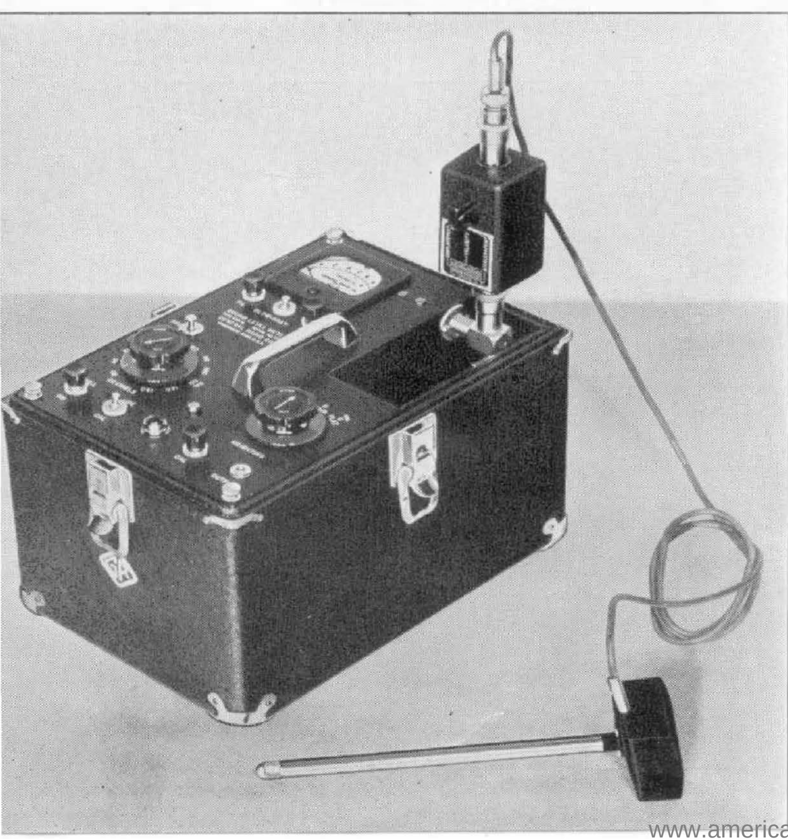
Vibration, like linear and angular motion, can be measured in terms of displacement, velocity, and acceleration. The easiest measurement to understand is that of displacement. In some cases where the displacement is large it can be measured directly with a ruler.

In its simplest case the displacement may be considered as simple harmonic motion, that is, a sinusoidal function having the form

$$x = A \sin \omega t \qquad (5)$$

where A is a constant, ω is 2π times the frequency, and t is the time. The maximum peak-to-peak displacement will be $2A$, and the r-m-s (sometimes loosely spoken of as the average) displacement will be $\frac{1}{\sqrt{2}}A$. The "average double amplitude" (r-m-s) will be $\sqrt{2}A$.

FIGURE 20. The control box and vibration pickup connect to the sound-level meter in place of the microphone.



Displacement, however, is not always the property of the vibration that is required in practical problems. A mechanical part radiating sound may be compared to a loudspeaker. In general the *velocities* of the radiating part (which corresponds to the cone of the speaker) and the air directly next to it will be the same, and, so long as the distance from the front of the part to the back is large compared to one-half of the wavelength of sound in air, the actual sound pressure generated in the air will be proportional to the velocity of the vibration. The sound energy radiated is the product of the velocity squared times the resistive component of the air load. Under these conditions, particularly where noise is important, it is the velocity of the vibrating part and not the displacement which is of greatest importance.

The velocity is the first derivative of the displacement, so that for the simple harmonic vibration in Equation (5) the velocity is

$$v = \frac{dx}{dt} = \omega A \cos \omega t \qquad (6)$$

Thus the velocity is proportional not only to the displacement but also to the frequency of the vibration.

In many cases of mechanical vibration, and particularly where mechanical failure is a consideration, the actual forces set up in the vibrating parts are important factors. Newton's laws of motion state that the acceleration of a given mass is proportional to the applied force, and that this force produces a resulting reacting force which is equal but opposite in direction. Any stresses and strains set up in a vibrating member, therefore, will be proportional to the acceleration of the vibration, which is the second derivative of the displacement. Acceleration measurements are important where vibrations are sufficiently severe to cause actual mechanical failure. Therefore,

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t \quad (7)$$

The acceleration, therefore, is proportional to the displacement and to the square of the frequency.

There is another use for acceleration measurements. The analogy cited above concerning the loudspeaker covers the usual case where the cone or baffle is large compared to the wavelength of the sound involved. In most machines this relationship does not hold, since relatively small parts are vibrating at relatively low frequencies. This may be compared to a small loudspeaker without a baffle. At low frequencies the air may be "pumped" back and forth from one side of the cone to the other with a very high velocity, but without building up much of a pressure or radiating much sound energy because of the very low air load, which has a reactive mechanical impedance. Under these conditions the acceleration measurement provides a better measure of the amount of noise radiated than does a velocity measurement.

To summarize, therefore, displacement measurements are used only in those few instances where the actual amplitude of motion of the parts is important. This would include, in particular, those cases where large amplitude of motion might actually cause parts to strike together, thus causing damage or serious rattle. Velocity measurements are generally used in noise problems where the radiating surfaces are comparatively large with respect to the wavelength of the sound. Acceleration measurements are the most practical where actual mechanical failure of the parts involved is of importance, and in most noise problems, particularly those involving small machinery. A vibration meter, therefore, should be able to measure all three vibration characteristics.

The above equations, (5), (6), and (7), represent only sinusoidal vibrations, but, as in the case of complex sound waves, complex periodic vibrations can also be represented as a Fourier series of sinusoidal vibrations. These simple equations may, therefore, be expanded to include as many terms as desirable in order to express any particular type of vibration.[25a] It will be noted that, since velocity is proportional to frequency, and acceleration is proportional to the square of the frequency, the higher frequency components in a vibration are progres-

[25a] General equations corresponding to (5), (6), and (7) are, respectively:

$$x = A_1 \sin(\omega_1 t + \alpha_1) + A_2 \sin(2\omega_1 t + \alpha_2) + A_3 \sin(3\omega_1 t + \alpha_3) + \cdots \quad (5a)$$

$$v = \frac{dx}{dt} = \omega_1 A_1 \cos(\omega_1 t + \alpha_1) + 2\omega_1 A_2 \cos(2\omega_1 t + \alpha_2) + 3\omega_1 A_3 \cos(3\omega_1 t + \alpha_3) + \cdots \quad (6a)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega_1^2 A_1 \sin(\omega_1 t + \alpha_1) - 4\omega_1^2 A_2 \sin(2\omega_1 t + \alpha_2) - 9\omega_1^2 A_3 \sin(3\omega_1 t + \alpha_3) - \cdots \quad (7a)$$

where α_1, α_2, α_3, etc., are the relative phase angles of various harmonics and $\omega_1 = 2\pi$ times the fundamental frequency.

sively more important in velocity and acceleration measurements than in displacement readings. The effective reading of the vibration meter on a complex wave is equal to the square root of the sum of the squares of the components which gives further emphasis to the higher amplitude components.[25b]

[25b]The vibration meter reading of displacement corresponding to (5a) would be

$$|x| = \frac{1}{\sqrt{2}}\sqrt{A_1^2 + A_2^2 + A_3^2 + \ldots} \quad (5b)$$

The velocity and acceleration readings would be, respectively

$$|v| = \frac{\omega_1}{\sqrt{2}}\sqrt{A_1^2 + 4A_2^2 + 9A_3^2 + \ldots} \quad (6b)$$

$$|a| = \frac{\omega_1^2}{\sqrt{2}}\sqrt{A_1^2 + 16A_2^2 + 81A_3^2 + \ldots} \quad (6c)$$

PART X—THE VIBRATION METER

Vibration Pickup with the Sound-Level Meter

For some years a vibration pickup has been available as an accessory for the General Radio Sound-Level Meter. The pickup, which responds to mechanical vibrations, is merely substituted for the microphone, and the sound-level meter used otherwise in a normal fashion. With the Type 759-B Sound-Level Meter the Type 759-P35 Vibration Pickup and the Type 759-P36 Control Box are used. The pickup itself is of the inertia-operated piezo-electric type, which responds to acceleration.[26] The control box, which connects between the meter and the pickup, provides electrical integrating circuits. The integrating circuits allow the conversion of this response for read-

[26]In this type of pickup the crystal is deflected by its own inertia when the pickup is subjected to vibration. The voltage generated is proportional to the actual force exerted on the crystal which is proportional to the acceleration.

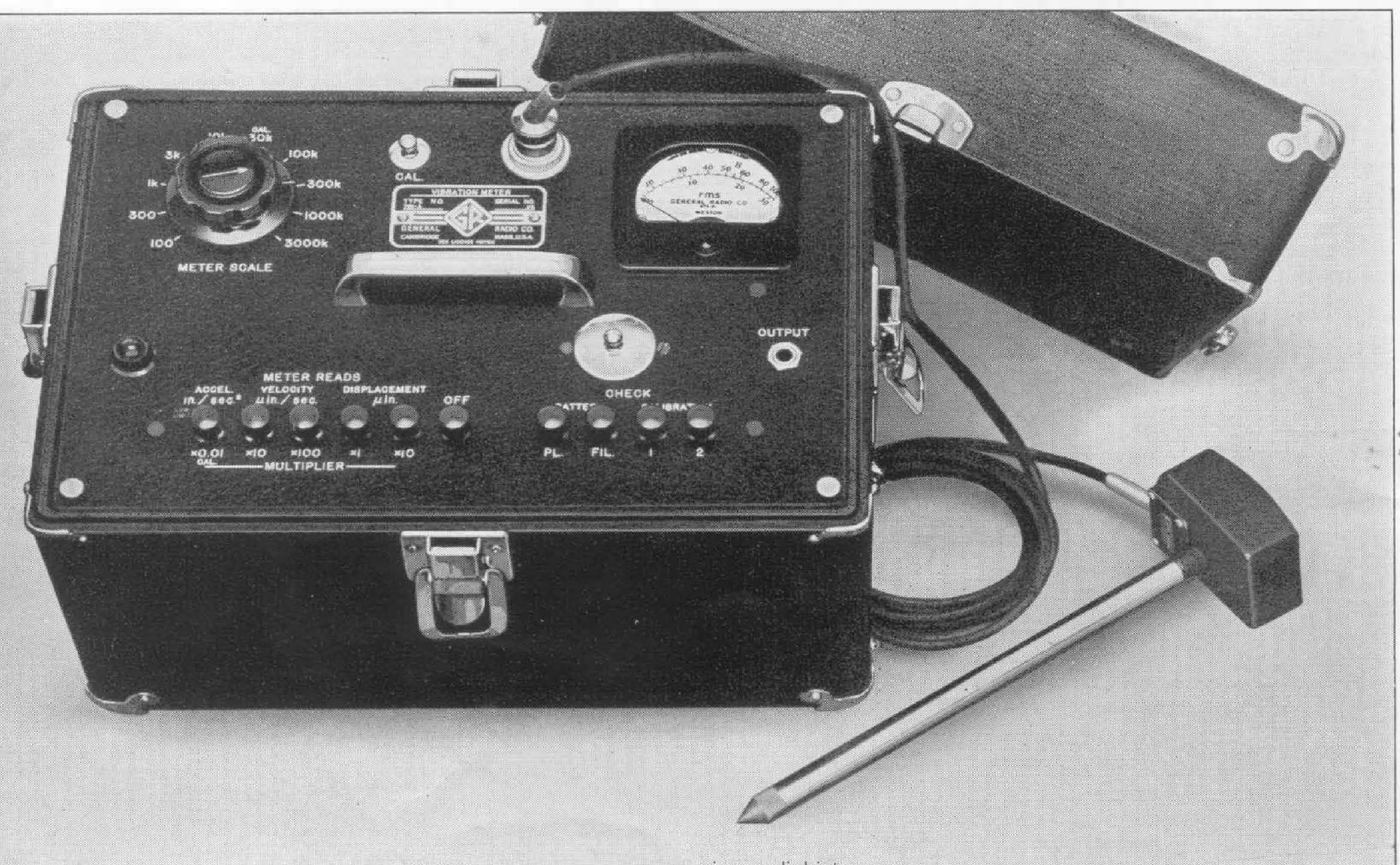


Figure 21. Type 761-A Vibration Meter, designed particularly for machinery vibration problems, covers the frequency range from 2 to 1000 cps (120-60,000 rpm).

ing velocity or displacement. This combination of pickup, control box, and sound-level meter provides a convenient and inexpensive way for owners of sound-level meters to make vibration measurements within the audio-frequency range. However, it should be remembered that the sound-level meter circuits were intended only to respond down to 25 cycles, and consequently this combination is not suitable for measuring lower-frequency vibrations.

Also, the sound-level meter reads in terms of decibels, which must be converted to other units if the readings are to mean anything in terms of vibration amplitude, velocity, or acceleration. A calibration chart is provided with each control box giving the proper correction figures for that pickup and control box when used with a particular sound-level meter. By means of these data plus a decibel table (supplied in the instruction book), the readings may be converted readily to the more logical units of micro-inches, micro-inches per second, or inches per second per second.

The TYPE 761-A Vibration Meter

For low-frequency vibrations, or where a large number of accurate observations must be made with a maximum degree of convenience, an instrument designed particularly for vibration measurements is desirable. The TYPE 761-A Vibration Meter is similar in many respects to the TYPE 759-B Sound-Level Meter. It is mounted in a case of nearly the same size, operates from the same size battery, and has a similar mechanical construction, including the free-floating tube shelf. However, the vibration meter was intended to take full advantage of the maximum frequency range of the piezo-electric type of pickup, which extends smoothly from 2 to 1000 cycles per second. Also, the meter is calibrated directly in terms of the r-m-s displacement, velocity, and acceleration and indicates these, respectively, in micro-inches, micro-inches per second, and inches per second per second.[27]

Since the vibration pickup used with this meter is of the acceleration type, two stages of electrical integration are necessary to provide the various types of response. Because the integrating cir-

[27]The TYPE 761-A Vibration Meter is completely described in "A General-Purpose Vibration Meter" by H. H. Scott, Journal of the Acoustical Society of America, Vol. XIII, No. 1, pp. 46–50, July, 1941. A brief description is also included in the General Radio *Experimenter*, Vol. XVI, No. 1, pp. 1–8, June, 1941.

FIGURE 22. Electrical frequency response of the TYPE 761-A Vibration Meter showing effects of integrating circuit.

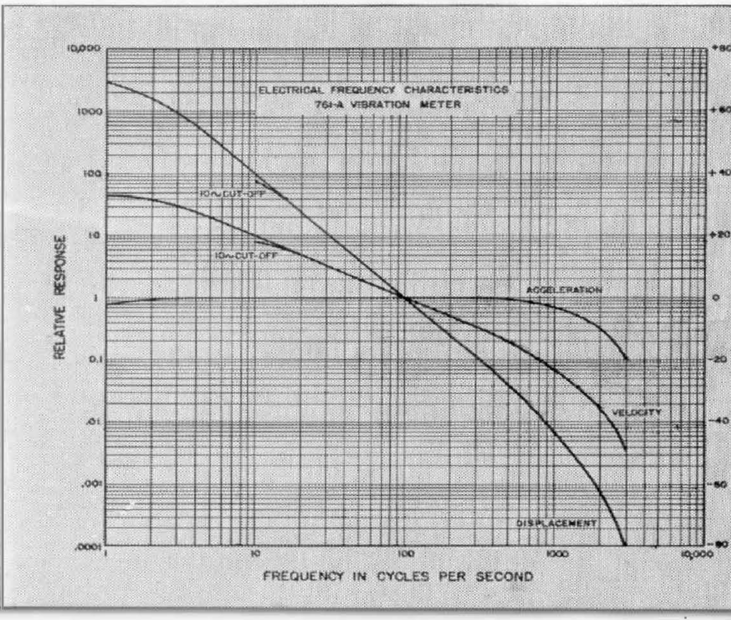
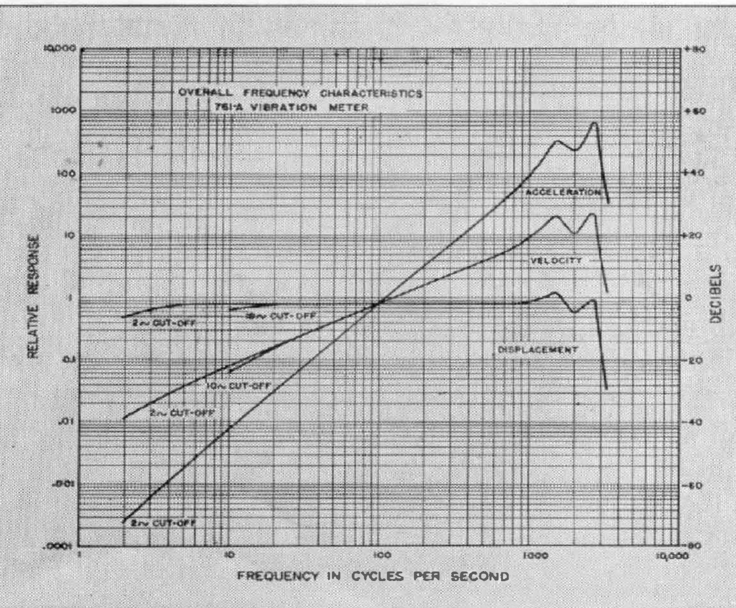


FIGURE 23. Over-all response of the TYPE 761-A Vibration Meter including the vibration pickup.



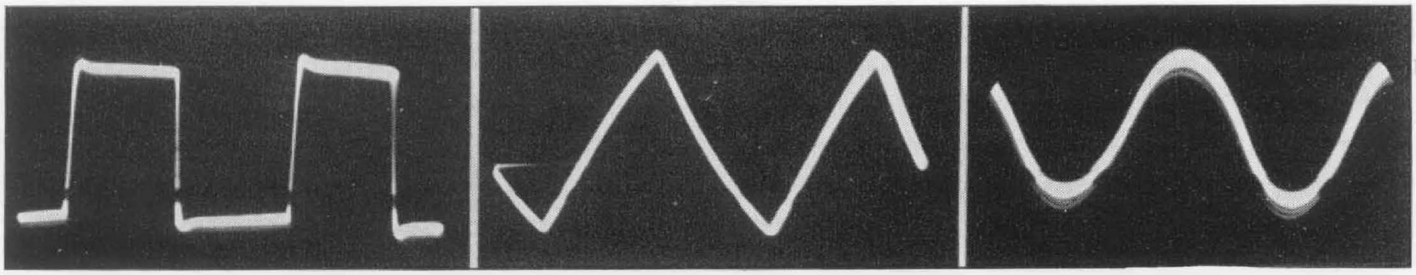


FIGURE 24. Illustrating the operation of the integrating circuits in the vibration meter (a) shows a square wave as transmitted by the amplifier when set for acceleration measurements, (b) shows the wave after one stage of electrical integration for velocity measurements, (c) shows the result of two stages of integration as used for displacement measurements.[28]

cuits are built in as part of the amplifier, better performance is possible than with a control box attachment. Figure 22 shows the electrical frequency characteristics of the vibration meter excluding the pickup. Figure 23 shows the over-all characteristic in terms of relative response for a constant-displacement vibration in terms of frequency. The irregularities above 1000 cycles are due to natural resonances in the pickup, but it will be noted that the average response is actually quite useful to 2000 cycles or higher. These figures show graphically how the integration process attenuates the higher frequencies with respect to the lower frequencies.

Figure 24 shows the actual effect of the electrical integration on a particular waveform. The square waveform of Figure 24a has strong harmonics. After two steps of integration the result in Figure 24c is substantially a sinusoidal waveform.[28]

PART XI — HOW TO USE THE VIBRATION METER

Operating Instructions

Like the sound-level meter, the vibration meter has an instruction sheet fastened in the cover which covers the actual operations involved in adjusting and reading the instrument. A knob marked METER SCALE provides in effect a multiplier for the indicating instrument. The red meter scale should be used with the red positions of the knob and the black scale with the uncolored positions. The reading of the control in all cases represents the full-scale deflection of the meter, so that it is merely necessary to add decimal places to the meter reading.

In addition, there is a row of five push buttons to select acceleration, velocity, and displacement response. For each of the latter two characteristics two but-

[28]The waveforms shown in Figure 24 may be represented by the following Fourier series:

$$\text{(a)}\quad a = -\omega_1{}^2 A \sin \omega_1 t - \frac{\omega_1{}^2 A}{3} \sin 3\omega_1 t - \frac{\omega_1{}^2 A}{5} \sin 5\omega_1 t - \cdots$$

$$\text{(b)}\quad v = \int a\,dt = \omega_1 A \cos \omega_1 t + \frac{\omega_1 A}{3^2} \cos 3\omega_1 t + \frac{\omega_1 A}{5^2} \cos 5\omega_1 t + \cdots$$

$$\text{(c)}\quad x = \int v\,dt = \int\!\!\int a\,dt = A \sin \omega_1 t + \frac{A}{3^3} \sin 3\omega_1 t + \frac{A}{5^3} \sin 5\omega_1 t + \cdots$$

These correspond, respectively, to Equations 7, 6, and 5, and to Equations 7a, 6a, and 5a in Note 25. Note that for this particular waveform, while the acceleration equation gives 33% third harmonic, the displacement one gives only 3.7%.

tons are provided. The normal buttons are those which provide a low-frequency limit of 2 cycles. The extra buttons, which are so marked, provide a low-frequency limit of 10 cycles, which, however, allows an increase in sensitivity of the meter by a factor of 10 : 1. This is a great advantage for measuring low-amplitude vibrations such as occur, for instance, in clocks, speedometers, and other small mechanisms. Such vibrations seldom have any important components below 10 cycles. Below each button is engraved a multiplying factor (always a multiple of 10) which should be applied to all readings when that particular button is used.

Push buttons are also provided for checking the battery and the calibration.

Sensitivity of Vibration Meter

The Type 761-A Vibration Meter will measure displacements as low as 16 micro-inches, velocities as low as 160 micro-inches per second, and accelerations as low as 0.160 inches per second per second.[29]

Pickup Placement

The pickup responds most strongly to vibrations perpendicular to its front surface (the surface with the name-plate). A threaded socket (¼″ — 28th) is provided on this surface so that the pickup may be bolted or clamped in any desired fashion. A conical and a rounded tip are also provided, and a long metal probe, all of which fit the threaded socket. The tips may be fastened directly to the pickup or to the end of the probe. By these means it is generally possible to hold the pickup against a vibrating surface or part so that it will pick up the vibration satisfactorily. Sufficient pressure should be used on the pickup so that it follows the vibration accurately without chattering, but care should be taken not to push so hard as to affect materially the vibration itself. Figure 19 shows how the pickup is normally used with the probe. For accurate reading the pickup should always be held or mounted so that its front surface is perpendicular to the direction of vibration.

Characteristics of Pickup

Like all piezo-electric microphones and pickups, the vibration pickup should not be allowed to reach temperatures above 130° F. or permanent damage may result. Measurements can be made on hotter machinery providing they are made quickly enough so that the pickup does not become heated.

At lower temperatures the temperature characteristics of the pickup are similar to the piezo-electric microphones (see Part V). The actual capacitance of the pickup is approximately 0.005 microfarad. The usual short cable supplied on the pickup does not require a temperature correction, but if a long cable is used the Equation (3) in Part V may be used.[30]

The pickup should not be subjected to accelerations greater than 10 times that of gravity (10g).[31]

Choice of Characteristic

The field of vibration measurement is not as well standardized as that of sound measurements. The choice among dis-

[29]The displacement and velocity figures are for a low frequency cut-off at 10 cycles. These limits are multiplied by 10 when the full range down to 2 cycles is used.

[30]For this purpose the capacity values in Figure 10(a) may be multiplied by two. Figure 10(b) will also apply if a cable of approximately 1350 $\mu\mu f$ is used. These figures are in Part V.

[31]On the calibration of the vibration meter this is equivalent to an acceleration of 3900 inches per second per second, a velocity of $\frac{6.3 \times 10^8}{f}$ micro-inches, or a displacement of $\frac{10^8}{f^2}$ micro-inches, where f is the frequency in cycles per second.

placement, velocity, and acceleration generally depends upon the use for which the data are needed and the considerations mentioned in Part IX. Typical examples of applications for the different types of measurements are as follows: Displacement measurements are widely used in measuring ship vibrations and vibrations in heavy machinery. Velocity measurements are used for measuring sound transmission through walls, the sound radiated by large surfaces such as power transformer shells, etc. Acceleration measurements are used in most machinery noise problems or where parts are liable to fail as a result of the vibration.

In all cases it should be remembered the acceleration measurements give the greatest emphasis to the high frequencies and displacement measurements to the low frequencies.

Applications of the Vibration Meter

All designers of airplanes, ships, and other expensive or elaborate structures, particularly where vibration may be dangerous, carefully calculate the vibratory conditions as a part of the design work.[16] Such calculations, however, generally involve assumptions which cannot always be rigidly justified, and measurements are necessary on the completed structure to check the calculations and make minor readjustments.

In the case of small machinery, it is sometimes more economical to build a sample and measure the vibration than to spend too much time on laborious calculations. The vibration meter, therefore, is not a substitute for thorough theoretical analyses, but should be used to supplement and check such analyses. In many cases its use will greatly simplify the calculations and reduce the number which are necessary.

The vibration meter is also an invaluable tool in checking finished equipment for vibration and, indirectly, for noise, as previously pointed out in Part V. This last application allows noise tests to be carried on under unfavorable conditions of ambient noise level, after correlating noise meter and vibration tests on a few sample machines.

— H. H. Scott

[16] Note 16 was included in Part VI.

INDEX TO VOLUME XVI AND VOLUME XVII

An index for Volumes XVI and XVII of the Experimenter has been prepared and will be mailed to any reader upon request.

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