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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

IMPEDANCE BRIDGES ASSEMBLED FROM LABORATORY PARTS

PART VI—INDUCTANCE MEASUREMENTS

Also IN THIS ISSUE

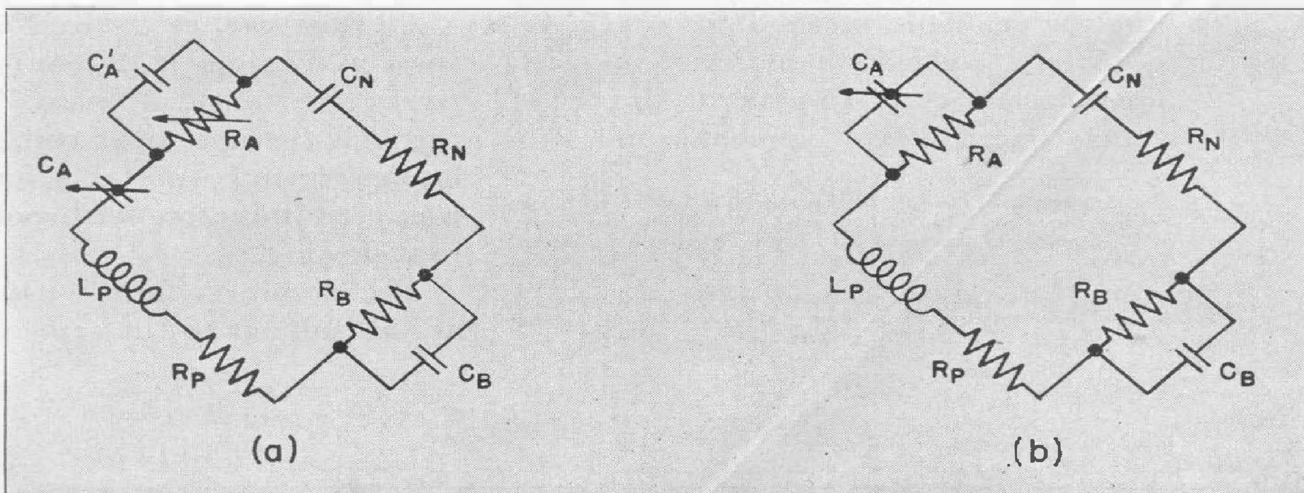
INDEPENDENCE OF BALANCE

● IN ANY ALTERNATING-CURRENT BRIDGE there are two conditions that must be simultaneously satisfied to obtain a true null balance. For maximum convenience in the use of the bridge it is desirable that the two adjustments for balance be independent of each other, so that the element that is varied to secure one balance shall not affect the other balance. Otherwise, the condition commonly known as a "sliding zero" occurs. It is characterized by the fact that balance must be approached by comparing a number of successive adjustments for minimum. The degree of dependency of the two components of balance

FIGURE 1. Schematic diagrams of (a) the Owen bridge, and (b) the "Schering," or parallel, form of the Owen bridge. The conventional form of the Owen bridge balances the losses in the P-arm by a condenser in series with R_A, while the circuit shown in (b) utilizes a parallel condenser across the A-arm. Circuit residuals are represented by R_N, C'_A and C_B in both diagrams. The approximate equations of balance are

$$\text{Owen:}\quad L_P = R_A R_B C_N;\ Q_P = \frac{D_A}{1 + D_A(Q_B + D_N + Q'_A)}$$

$$\text{"Schering":}\ L_P = \frac{R_A R_B C_N}{1 + Q_A{}^2};\ Q_P = \frac{1}{Q_A + Q_B + D_N}$$



on each other (i.e., the amount of "sliding") is dependent only on the storage factor, Q, of the unknown impedance.[1] The higher the storage factor of the unknown impedance, the less pronounced is the sliding effect.

It can be shown that truly independent balances are obtained only when the two adjustments for balance *are made in the same arm*, or when one adjustment is made in each complex arm. An example of the first method is the Owen bridge, while a bridge devised by Sinclair[2] illustrates the second.

The two elements that provide independent balances can be made direct reading for the resistive and reactive (or conductive and susceptive) components of the unknown impedance, independent of each other.

As an example, consider the Hay and Maxwell bridges, already discussed. Both bridges are subject to a sliding balance, since the elements that are varied to secure balance do not satisfy the conditions stated above. The Hay circuit is commonly used for inductors of high Q, and the effect is not pronounced. The Maxwell bridge, however, is frequently used for measurements of low storage factors, and in this case the sliding zero becomes very noticeable. If C_N and R_N (where N is the arm opposite the unknown inductance) are chosen as variables, the two components of balance are completely independent, as pointed out above. This method, however, requires a standard condenser of the variable, decade type. In general, the use of such a condenser will mean some sacrifice in accuracy of inductance measurements, as compared to the accuracy attainable with a fixed standard condenser. Furthermore, the bridge no longer can be made direct reading for Q. For these reasons, this arrangement is not commonly used, and the method suggested earlier (varying R_A and R_N) is generally preferred in spite of the concomitant sliding zero.

THE OWEN BRIDGE

Another well-known circuit for the measurement of inductance is the Owen bridge shown in Figure 1(a). In this circuit the resistive component of the unknown impedance is balanced by a capacitance in series with one of the resistive arms. If the reactive balance is obtained by varying this resistive arm, the two balances are independent, as shown by the equations of Figure 1. We have here a situation analogous to that pointed out above for the Maxwell bridge, wherein independent balances are secured if the resistance and capacitance of the standard arm are both varied.

For the Owen circuit the variable condenser determines the resistance balance, whereas for the Maxwell bridge the variable condenser determines the inductance balance. A decade condenser, with its relatively poor accuracy (typically 1%), is generally satisfactory for the resistance measurement, however, as larger errors from other sources generally determine the accuracy of measurement of this component.

Another well recognized advantage of the Owen bridge is that it is a comparatively easy matter to pass direct current through the unknown coil. This bridge is consequently suitable for measuring iron-core inductors with polarizing current flowing.

The circuits so far discussed, however, are all subject to the same limitation in

[1]This statement refers to the four-arm bridge with two complex arms. If three or more arms are complex, the degree of dependency is expressed in a somewhat more complicated fashion.

[2]D. B. Sinclair, "A Radio-Frequency Bridge for Impedance Measurements from 400 Kilocycles to 60 Megacycles," *Proc. I.R.E.*, November, 1940, pp. 497–503.

measuring coils of high Q, namely that the error in the determination of Q (or of resistance) is directly proportional to Q, and an error of 25%, 50%, or even greater is not uncommon when Q is of the order of 100 or greater.

THE RESONANCE BRIDGE

The resonance bridge, shown in series form in Figure 2, is undoubtedly the most accurate method available for measuring coil resistance at audio frequencies. Since the reactance of the P arm is reduced to approximately zero, the resistance balance becomes the dominant one, and the circuit residuals have only a second-order effect on it, while having a comparatively large effect on the reactance balance. This method, then, although quite accurate for resistance measurements, is not very accurate for inductance measurements.

To obtain the coil resistance, when the total P arm resistance is known, obviously requires a knowledge of the effective resistance of the tuning condenser. Additional measurements are thus required, unless condensers of known dissipation factor are used, or unless R_C is negligible with respect to R_L.

THE "SCHERING" CIRCUIT

The analogue of the Schering (capacitance) bridge discussed in a previous article is the circuit shown in Figure 1(b).[3] Here the inductance is balanced by a precision air condenser in the opposite arm, while the resistance of the unknown is balanced by a capacitance across one of the fixed resistance arms. As is the case for the Hay bridge, the two components of balance are dependent on each other, but, for storage factors greater than 10, the annoyance from the sliding balance is not serious.

For capacitance measurements, one of the advantages of the Schering circuit lies in the fact that it is a relatively simple matter to establish an initial balance by adjusting the difference $Q_A - Q_B$, using trimmer condensers across the ratio arms. For inductance measurements, however, the Q's of the resistance arms and the condenser are directly additive, as pointed out in a previous article. Thus, in this case, it is not possible to use parallel capacitance across the resistance arms. The possibility immediately suggests itself, however, of using *series inductance* in one of the resistance arms[4] to compensate for the residual Q's of the circuit. The establishment of the initial balance requires an inductance of high Q, the value of which is known to a moderate accuracy, or, alternatively, a moderate value of Q, accurately known. This, in turn, re-

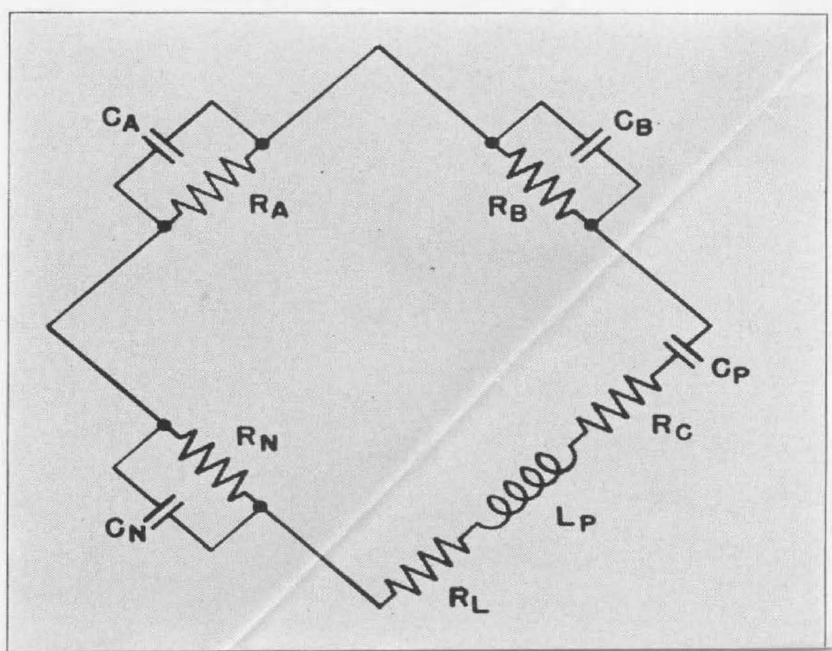
[3]This circuit may equally well be considered as a parallel form of the Owen bridge.

[4]This is comparable to Grover's arrangement for using series inductance in a ratio arm to measure the D of a condenser.

FIGURE 2. The series-resonance bridge. The capacitances C_A, C_B, and C_N are circuit residuals. The equations of balance are

$$R_P = \frac{R_B R_N}{R_A} \frac{1 - Q_A{}^2}{1 + Q_N(Q_A + Q_B) + Q_A Q_B}$$

$$L_P = \frac{1}{\omega^2 C_P} \text{ (approximately)}$$

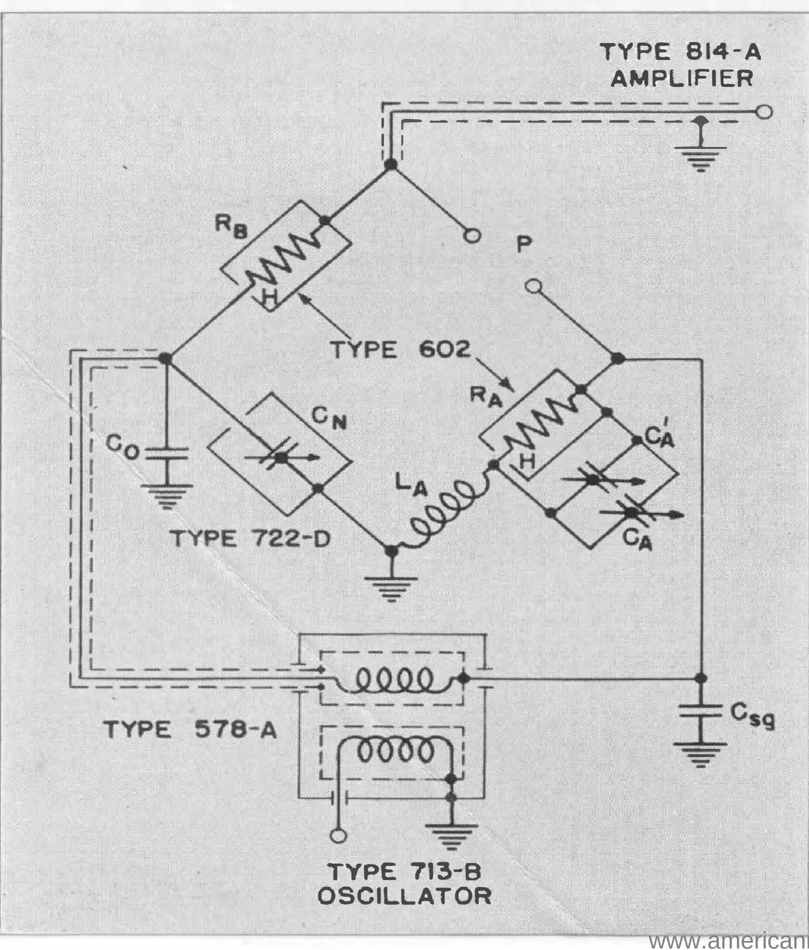


quires an independent method of measuring resistance accurately, but fortunately such a method is available in the series-resonance bridge.

A compensated circuit of the type outlined, using the same components as the Schering capacitance bridge already described,[5] is shown schematically in Figure 3. The secondary shield-to-ground capacitance of the transformer is placed across the resistance arm A, with the smaller (10 $\mu\mu$f) terminal capacitance across the capacitive arm N (TYPE 722-D Precision Condenser). A fixed inductance, L_A, is used, with an additional trimmer capacitor C'_A, to make the final adjustment in establishing the initial balance. For an A arm resistance of 20,000 ohms, a 50 or 100 mh choke[6] may conveniently be used for L_A, together with a 100 $\mu\mu$f condenser used for C_A.

[5]General Radio *Experimenter*, August, 1941, p. 2.

FIGURE 3. Connections for a bench set-up of a parallel form of Owen bridge.



The stray capacitance placed across the standard condenser can be determined by a method similar to that used with the capacitance bridges. If two balances are made for a given L_P, one with the standard condenser disconnected at its high terminal, the other in the usual manner, with C_N set at about 100 $\mu\mu$f, the stray capacitance C_O will be given by

$$C_O = C_{N_1} \frac{1}{\frac{B_2}{B_1} - 1}.$$

Here B_1 is the reading of the B arm resistance box with C_N disconnected, B_2 its reading with the standard set at a value of C_N.

With this arrangement an initial balance was established against a Q of 20 (known to about $\pm 1\%$, from resonance bridge measurements). The Q's of several other coils were then measured, the results checking the known values within essentially the accuracy of reading of the condenser C_A. Additional correction terms are introduced into the inductance equation by the transformer capacitance shunting the series inductance L_A. For the case cited, this correction is negligible, but it can become significant if large inductances are used in the A arm.

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[6]The resistance of the choke must, of course, be added to R_A. The choke resistance is small compared to R_A, however, and the d-c value may be used without introducing any appreciable error.

THERMOCOUPLES

We regret that, owing to circumstances beyond our control, we can no longer supply vacuum thermocouples. Consequently, all models of TYPE 493 Thermocouples are discontinued, effective December 1.

USING THE CATHODE-RAY OSCILLOGRAPH IN FREQUENCY COMPARISONS

● IN FREQUENCY MEASUREMENT AND CALIBRATION, where an unknown frequency is to be compared with, or adjusted to, a standard frequency, the cathode-ray oscillograph offers a convenient and precise means of making the necessary comparison. The following summary of the various ways in which a cathode-ray oscillograph can be used advantageously in frequency measurement presents no new methods. The methods discussed are presented particularly for those who may have available a cathode-ray oscillograph, but who may not appreciate its potentialities in this field.

I LISSAJOUS FIGURES

Starting with the simplest comparison, if a voltage from a frequency standard is applied to one pair of the deflecting plates (say, the horizontal) of the cathode-ray oscillograph and a voltage from a source whose frequency is to be adjusted in terms of the standard to the other pair, patterns of the type illustrated in Figure 1 will be obtained.

These are the well-known Lissajous figures. For simple frequency ratios, expressible by small whole numbers, the patterns are not too complicated, and identification of the frequency ratio is possible, even when the pattern is rotating slowly.

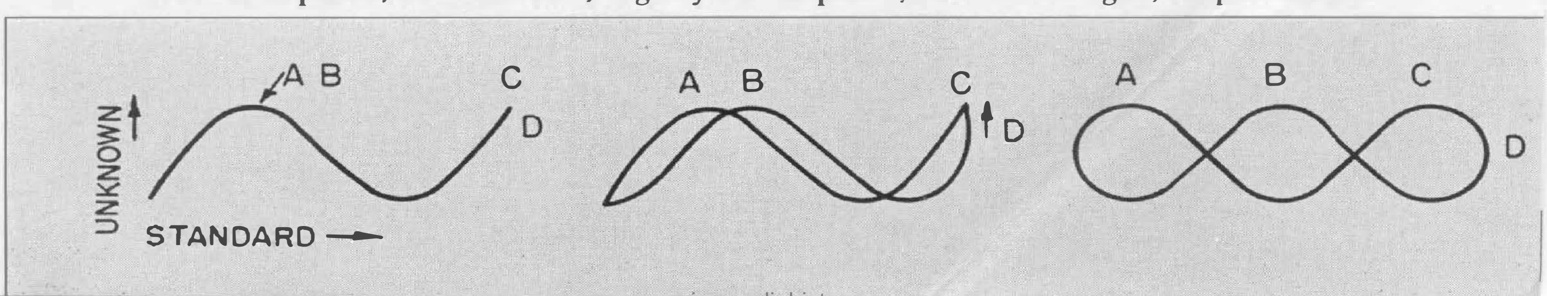
If the pattern can be made to be nearly stationary, by adjustment of the frequency to be checked, then the frequency ratio is found as follows: Count the horizontal tangent points (such as *A*, *B*, *C*, Figure 1); count the vertical tangent points (such as *D*). The frequency ratio is the ratio of the number of horizontal points to the number of vertical points, which, for the example of Figure 1, is 3 : 1. If the unknown frequency is on the vertical plates, then the unknown is three times (as illustrated in Figure 1) the standard frequency.

As indicated by the successive parts of Figure 1, the appearance of the pattern changes progressively if there is a slight difference in frequency between the unknown and standard frequencies. Under such conditions the tangent points can be counted only for simple frequency ratios. If a frequency ratio of 7 : 5 is obtained, for example, the pattern appears almost as a network covering the area and, unless the pattern is steady, it is very difficult to count the tangent points.

II MODULATED WAVE PATTERNS

A variation of this procedure occurs in the problem of matching a low beat frequency, obtained as a result of comparing an unknown radio frequency with a standard frequency. If this beat is obtained in an oscillating receiver, the oscillating frequency being offset appreciably (by a kilocycle or so from both of the radio frequencies), then the output of the receiver is an audible tone, the

FIGURE 1. Lissajous figures for a frequency ratio of 3 : 1. At the left, the two frequencies are shown in phase; at the center, slightly out of phase; and at the right, in quadrature.



amplitude of which waxes and wanes at a rate equal to the low beat frequency difference of the two radio frequencies.

If one of the two radio frequencies is of somewhat greater amplitude than the other, and if the receiver output is connected to the vertical plates while a calibrated audio oscillator is connected to the horizontal plates, then patterns of the type shown in Figure 2 are obtained.

The receiver output is equivalent to an audio-frequency carrier modulated by the beat-frequency difference. The pattern of Figure 2 is familiar as a means of checking the percentage modulation and indicating roughly the quality of modulation. The advantage of this method in frequency comparisons is that the audio-frequency system of the receiver is not called upon to transmit a frequency of only a few cycles; it transmits the audible carrier, which may be placed anywhere in the audible range. Also, the matching oscillator can be operated at a multiple of the beat frequency, in a range where the accuracy of its calibration is usually much improved.

In Figure 2, the illustration is made for a case in which one radio frequency is roughly twice the amplitude of the other, resulting in a modulation of the audible carrier of $A/B = 0.5$ approximately, or 50%. If the matching oscillator frequency is not exactly equal to the beat frequency, the pattern will slowly progress through the sequence indicated. The illusion of a three-dimensional figure is strong; the pattern appears like a tube, with the ends cut at an angle to the axis.

If the matching oscillator frequency is adjusted to a multiple of, or in simple ratios to, the beat frequency, the pattern developed at the ends of the figure corresponds to the Lissajous figures for the same ratios. If the matching oscillator is adjusted to twice the beat frequency, a pattern of the type shown in Figure 3 results. It will be seen that a "two-to-one" pattern is developed at each end of the figure or that, in illusion, two tubes are developed, side by side. If the matching frequency is not exactly twice the beat frequency, the pattern changes progressively through the sequence indicated.

III CIRCULAR SWEEP PATTERN

Very useful patterns are obtained if a circular sweep is used. These patterns are of a form in which the frequency ratio is easily identified, even when the ratio is expressed by numbers which are not small integers.

To produce a circular sweep, it is necessary to obtain two equal voltages having a phase difference of 90 degrees from the standard frequency source. One method is illustrated in Figure 4.

The standard frequency is supplied through transformer T-1, to match the total load. Resistances, R, and reactances, C, are made equal in magnitude,

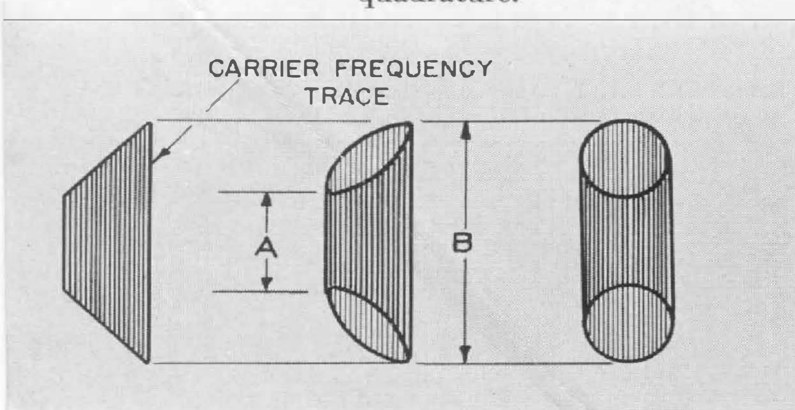


FIGURE 2. Modulated wave pattern. Left, in phase; center, slightly out of phase; right, in quadrature.

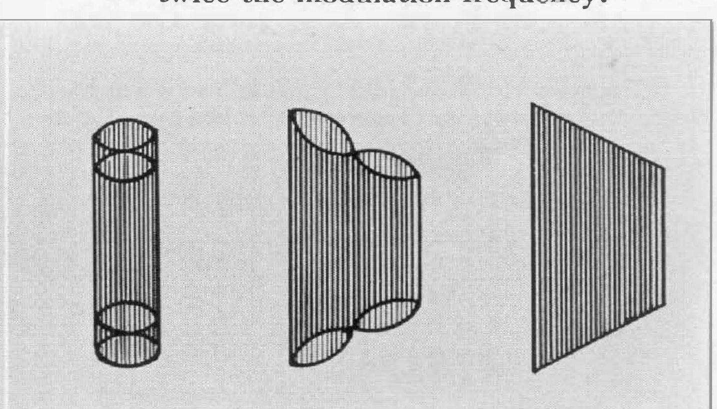


FIGURE 3. Successive phases of a modulated wave pattern when the matching frequency is twice the modulation frequency.

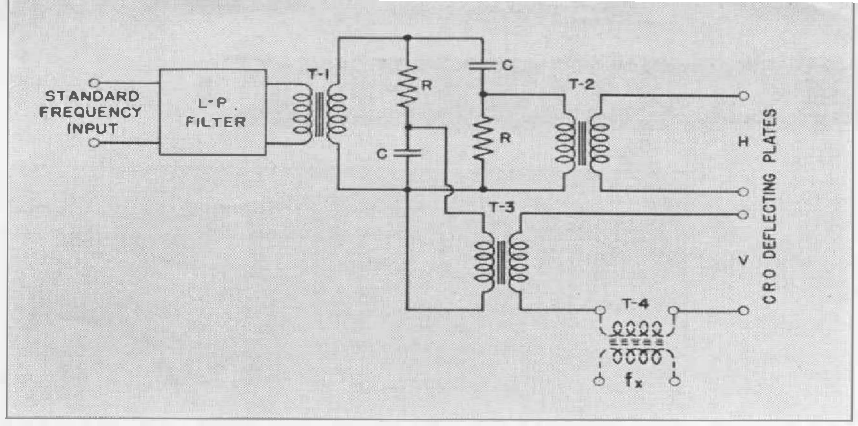


FIGURE 4. Connections for obtaining a circular sweep.

for convenience. A value of 10,000 ohms at the standard frequency is handy. The phases of the voltages taken to the primaries of *T*-2 and *T*-3 are 45 degrees ahead and behind the voltage supplied from *T*-1, or have a phase difference of 90 degrees. If the impedances of *T*-2 and *T*-3 are high (interstage coupling transformers with a step-up ratio of 6 : 1 are suitable), then connecting them across the elements *R* and *C* will not materially affect the phase of the voltages. The combined effects of the transformer loading and of the impedances across the secondaries generally make it necessary to readjust the elements of the phase shifter somewhat.

If the standard frequency supply is distorted in waveform, a circular sweep cannot be obtained. A low-pass filter is then necessary, as indicated in the diagram.

If a cathode-ray oscillograph having a radial deflector is used, the unknown frequency can be placed on that electrode. In the more general case, it is convenient to introduce the unknown on the vertical plates as shown at *T*-4.

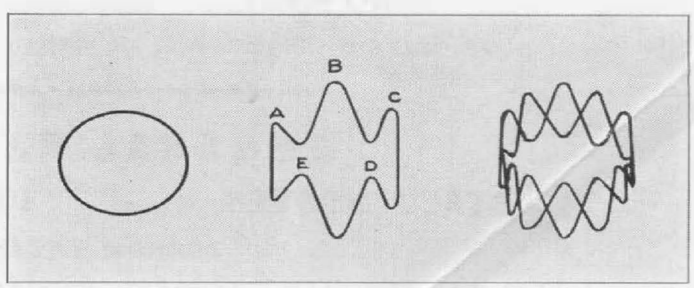
The type of patterns obtained with a circular sweep are illustrated in Figure 5. With no unknown frequency introduced, the pattern is a circle, with the spot traveling once around the circle for each cycle of the standard frequency. If a frequency equal to five times the standard frequency is introduced on the vertical plates, a pattern as illustrated in the second part of the figure will result. The frequency ratio can be determined by counting the tops of the waves, as at *A*, *B*, *C*, *D*, *E*. (If radial deflection is used, the pattern is not distorted, and the frequency ratio is found by counting the outer tips of the waves.)

Note that, even when the unknown frequency is not set exactly to five times the standard frequency, the pattern does not change form, but appears to *rotate* slowly. The successive transitions from one line to two lines (as in Figure 1) do not occur. The pattern remains a "one-line" pattern as long as the frequency ratio is not far from any integral multiple of the standard frequency.

If the unknown were adjusted to one-fifth the standard frequency, a set of five nearly concentric circles would be seen. (Not illustrated.)

The "one-line" pattern is extremely convenient when using equipment whose calibration is known to be nearly correct. For example, an audio-frequency oscillator may be known to have no error greater than a very few cycles at any point in its range. If it is desired to set this oscillator to, say, 1700 cycles in terms of the standard (say 100 cycles), then simply set the oscillator to 1700 cycles by its calibration. The pattern will then be a "one-line" pattern (having 17 "tops," but it is not necessary to

FIGURE 5. Circular sweep patterns. Left, standard-frequency circular sweep; center, circular pattern with superimposed frequency equal to five times the standard; right, with superimposed frequency equal to 9/2 the standard.



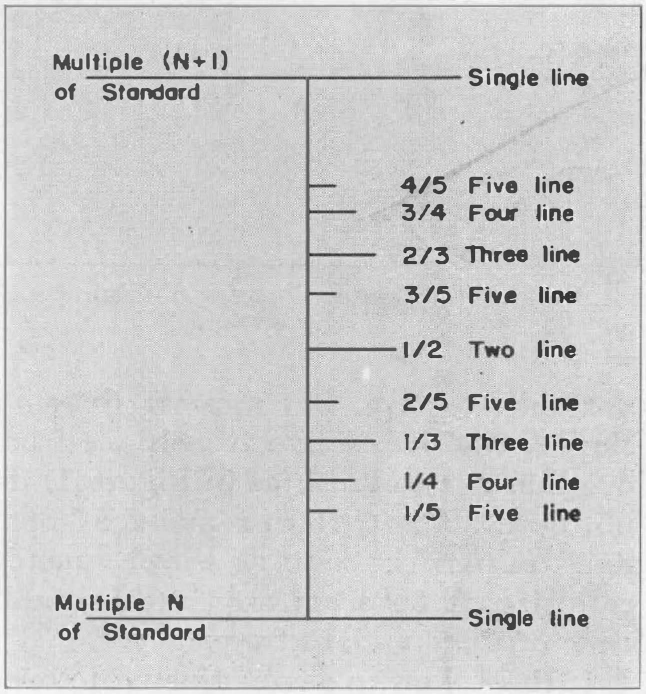


FIGURE 6. Table showing the sequence of patterns in each standard-frequency interval.

count them), rotating at a rate depending on the error of the oscillator at the setting of 1700 cycles. Readjust the oscillator slightly until the pattern stands still, when the frequency will be 1700 cycles in terms of the standard.

It will be seen from the above that an audio-frequency oscillator can be calibrated using "single-line" patterns only, at every 100 cycles (from a 100-cycle standard) up to the highest frequency at which the successive waves on the pattern can still be distinguished.

If the oscillator is set to an odd multiple of one-half the standard frequency, a "two-line" pattern, illustrated in the third part of Figure 5, is obtained. Using "one-" and "two-line" patterns, the oscillator can be calibrated at every 50 cycles up to a limit determined by the ability of the observer to distinguish the pattern.

In a similar manner, "three-line" patterns are obtained when the oscillator is set at one-third and two-thirds of the way between successive 100-cycle points; "four-line" patterns when set at one-quarter and three-quarters, etc.

The important feature of this method is that the sequence of patterns repeats in *each* 100-cycle interval, as illustrated in Figure 6. Since the 100-cycle multiples are readily identified, it becomes a simple matter to calibrate an oscillator within 20 cycles of any desired audio frequency.

For purposes of illustration, reference has been made throughout the above to a standard frequency of 100 cycles, which is a convenient value for use in measurements up to a few thousand cycles. If the standard frequency be multiplied by 10, the patterns obtained will be identical at frequencies ten times higher than before, giving a useful range up to the low radio frequencies.

The General Radio TYPE 699 Comparison Oscilloscope has been designed particularly for use with the CLASS C-21-HLD Primary Frequency Standard and Frequency Measuring Equipment. Provision is made for permanent shielded wiring to all necessary components of the standard and measuring equipment. By means of key switches, the desired sources and method of comparison can be selected. One hundred-cycle and one thousand-cycle filters and phase shifters are provided for obtaining circular sweeps at either frequency.

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