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EXPERIMENTER

ELECTRICAL MEASUREMENTS
TECHNIQUE AND ITS INDUSTRIAL APPLICATIONS

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LOSSES IN AUDIO-FREQUENCY COILS

● **IN DESIGNING** electrical communication equipment, it is frequently necessary to select low-loss inductance coils for use in tuned circuits and filters. Experimental data accumulated over a period of several years at the General Radio Company have shown that a set of formulae derived by means of approximations of limited validity hold very well in practice. These expressions are so simple that they have been of considerable assistance in selecting the best type of coil for a particular use, and they are presented here because of their wide application.

The discussion will be limited to measurements in the frequency range between 10 and 100,000 cycles per second and to frequencies far removed from resonance. For iron-core coils it will be assumed that the voltages are very low in order that hysteresis may be neglected in comparison to eddy-current loss. Most of the measurements have been made on iron-core coils (with and without air gap) and multi-layer audio-frequency coils of the type usually met in filter practice.

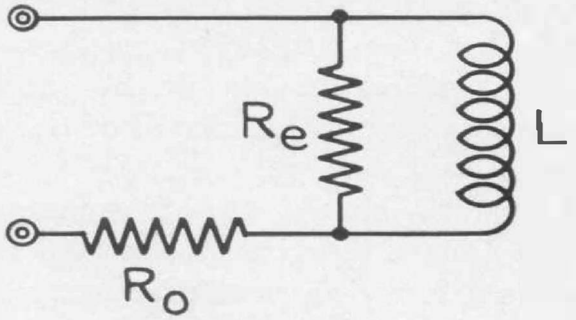

FIGURE 1. Equivalent circuit of a low-frequency coil

With these restrictions it has been found that all coils may be represented by the equivalent circuit shown in Figure 1 where R_o is the d-c resistance and R_e is a resistance due to eddy-currents and is *independent of frequency*.

This is based on the following considerations: A conductor placed in the magnetic field of a coil may be considered a terminated secondary winding. As long as the circulating currents are mainly determined by the *resistance* of the path rather than by its inductance the termination impedance is resistive, and the secondary circuit will reflect a resistance in parallel with the primary

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winding. In particular, the circulatory currents may exist in an iron core and may include induced currents set up in the wire itself, usually spoken of as skin effect.

In many applications the reactance to resistance ratio, known as "Q", serves as a convenient factor of merit. From the simplicity of the equivalent circuit shown in Figure 1, it might be expected that the curve of Q versus frequency for any coil would be given by a fairly simple expression. The application of ordinary circuit theory to Figure 1 gives the approximate results:

$$X = 2\pi fL$$

$$R = R_o + \frac{(2\pi fL)^2}{R_e}$$

which gives for Q,

$$Q = \frac{2\pi fL}{R} = \frac{1}{\dfrac{R_o}{2\pi fL} + \dfrac{2\pi fL}{R_e}}$$

which has a maximum value, Q_m, of

$$Q_m = \frac{\pi f_m L}{R_o} \qquad (1)$$

or

$$Q_m = \frac{1}{2}\sqrt{\frac{R_e}{R_o}} \qquad (2)$$

at a frequency $f_m = \dfrac{\sqrt{R_o R_e}}{2\pi L}$ (3)

where,

f_m is the frequency of maximum Q. At any other frequency, f, the corresponding value of Q (denoted by Q_f) is

$$Q_f = \frac{2Q_m}{\dfrac{f}{f_m} + \dfrac{f_m}{f}} \qquad (4).$$

It will be noticed that this expression is symmetrical with respect to $\frac{f}{f_m}$ and $\frac{f_m}{f}$ and will give a curve shape which is invariant on logarithmic paper.

To tabulate available Q data it is sufficient to plot the point of maximum Q and use a standard template for drawing the curve (this template may be obtained by replotting a curve of Figure 2 on standard logarithmic paper).

In iron-core coils, the eddy-current losses in the copper are usually negligible in comparison to those in the core so that, for a given core and volume of copper, wire size has little influence on Q. One interesting point to be noted for iron-core coils is that the maximum Q for a given structure but with various air gaps is very nearly constant. This can be explained by the fact that the core may be regarded as a single turn secondary of constant termination. This reflects a given shunt resistance across the winding regardless of its inductance. Since the copper resistance is constant, it follows that the ratio of shunt eddy-current to series copper resistance must be constant and hence from expression (2) Q_m must stay the same. Experimental results check fairly closely with the relationship given by (3) which shows that the frequency of maximum Q varies inversely with the inductance as the air gap is varied.

Figure 2 shows the characteristics of an audio transformer core as the air gap in the magnetic circuit is varied. It will be noticed that the maxima are practically independent of the magnitude of the gap. With no gap, the maximum Q is about 50 and occurs at a frequency of 90 cycles. With a 0.125-inch air gap the maximum Q is 45. With a gap of 0.95 inches the maximum Q stays about the same, at 42, even though the gap corresponds to a total removal of the center leg.

Figure 3 shows the frequency at which Q is a maximum for the same core as a function of the inductance in accordance with equation (3).

Figure 4 gives the loci of points of maximum Q for several types of coils. Curve B corresponds to the data of Figure 2 and Curve A shows the characteristics of another type of iron core.

Curve C gives the locus of Q_m points for a set of multi-layer air-core coils of given mechanical form but varying wire size. It will be noticed that the larger wire sizes have lower maxima as might be expected from the larger dimensions of the individual eddy-current circuits. The straight line drawn through the three experimental points was drawn in accordance with equation (1), with a unit slope corresponding to what would be expected of the variation of Q_m with the frequency at which the maximum occurs when the ratio of $\frac{L}{R_o}$ is held constant, as it is with a given geometry and weight of copper.

Tests made on high-frequency dust cores show that at audio frequencies the effective series resistance of a coil is virtually unchanged as the core is brought into the field, but the inductance is increased, and Q increases directly as the inductance. The particular case is more complicated than the previous ones because essentially the coil may be thought of as consisting of two parts, first an air-core coil of the type previously considered and, sec-

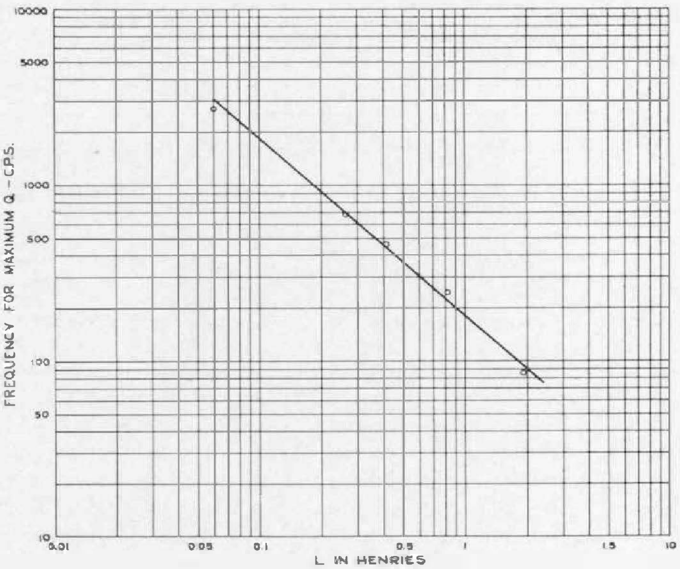


FIGURE 3. Frequency of maximum Q of the iron-core coil of Figure 2 as a function of inductance. The inductance variation is obtained by varying air gap. The curve is drawn as a straight line of slope (-1) in accordance with equation (3)

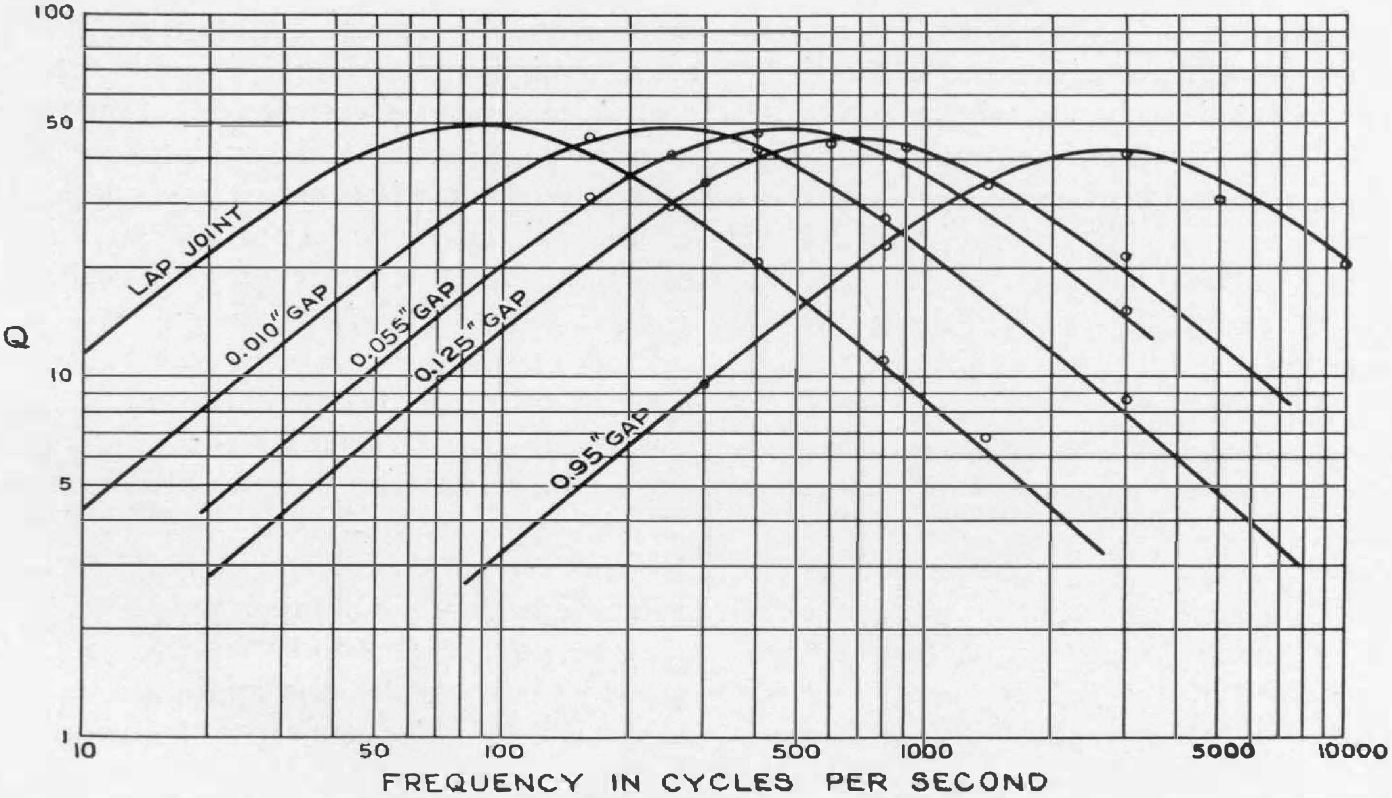


FIGURE 2. Frequency characteristic of Q for an iron-core coil with various air gaps. The curves are drawn with a template made in accordance with the equation

$$y = \frac{1}{x + \frac{1}{x}}$$

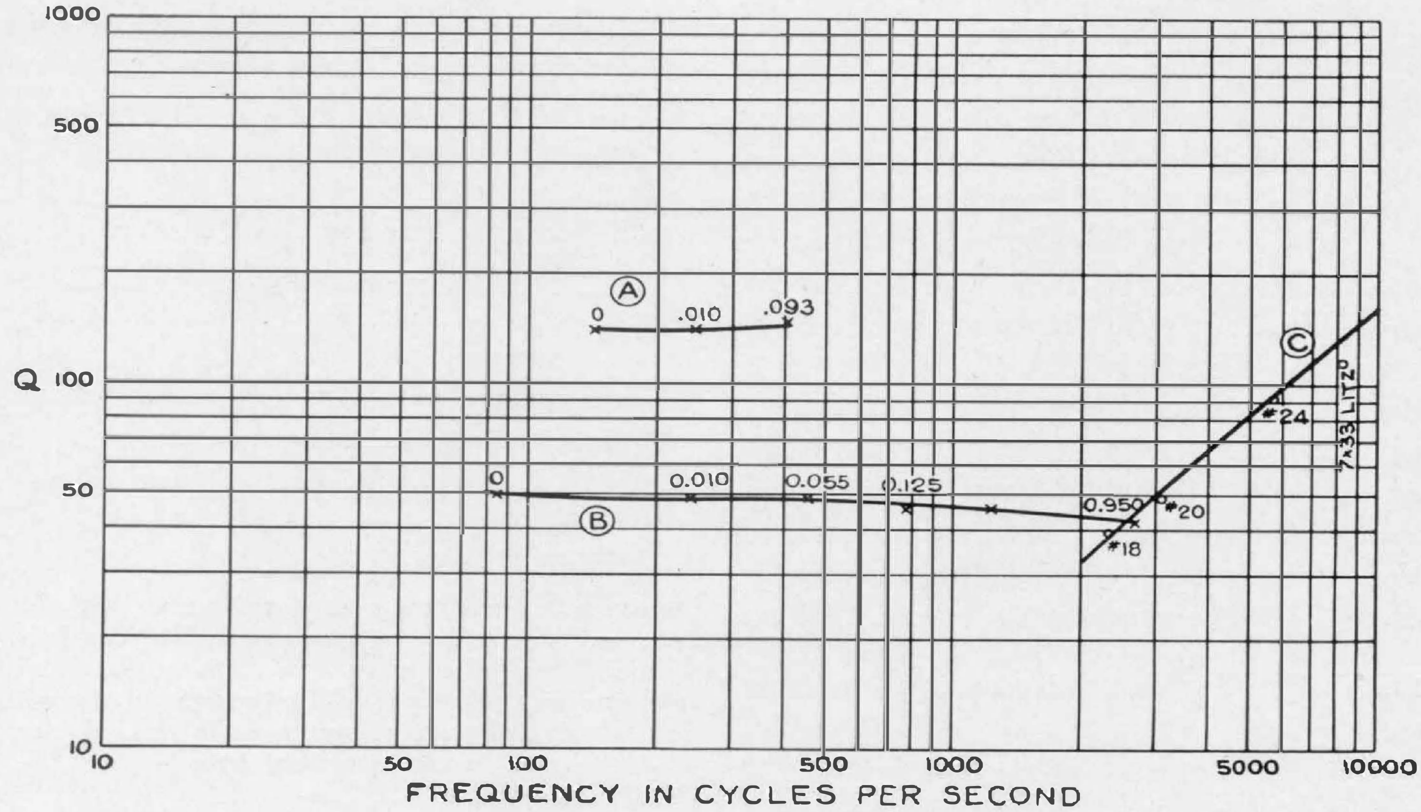


Figure 4. Loci of maximum *Q* points for various coil types

Curve A is for a large transformer with 0.007-inch silicon-steel laminations. Curve B is for the coil of Figure 2, which is an audio transformer with 0.018-inch silicon-steel laminations. In both curves, the numbers refer to air-gap distance in thousandths of an inch

Curve C is for a group of multi-layer air-core coils with the same cross section but using different wire sizes

ond, a loss-free inductance. As the amount of iron in the field is varied, such a structure will have a vertical locus, that is, the maximum *Q* will vary, but the frequency at which it occurs will be unchanged.

One application of these ideas about *Q* may be noted. In constant-K low-pass filters it is frequently desirable to keep the attenuation within the transmitted band very nearly constant. It is often stated that, in order to achieve this, *Q* should be very high. Actually the change of attenuation is due almost entirely to the shunt component of the loss. To assure constant transmission, a coil should be operated at frequencies well below that which makes *Q* a maximum; in this region the effective series resistance is constant, and the loss, which is almost entirely governed by this effective series resistance, is also constant.

Another application is in oscillator coil design. To assure constant output as a tuning condenser is varied, the equivalent parallel resistance should be constant, because this resistance is equal to the anti-resonant impedance of the coil and represents the loading on the tube. Constant parallel resistance is achieved when the coil is operated at frequencies well above the frequency of maximum *Q* since in this range the effect of series resistance is negligible and the remainder must clearly be constant.

The value of this analysis, after a few measurements have been made on a sample coil, lies largely in the viewpoint which it gives the engineer, enabling him to make more intelligent estimates of coil behavior without actually calculating the constants of successive models. — L. B. Arguimbau

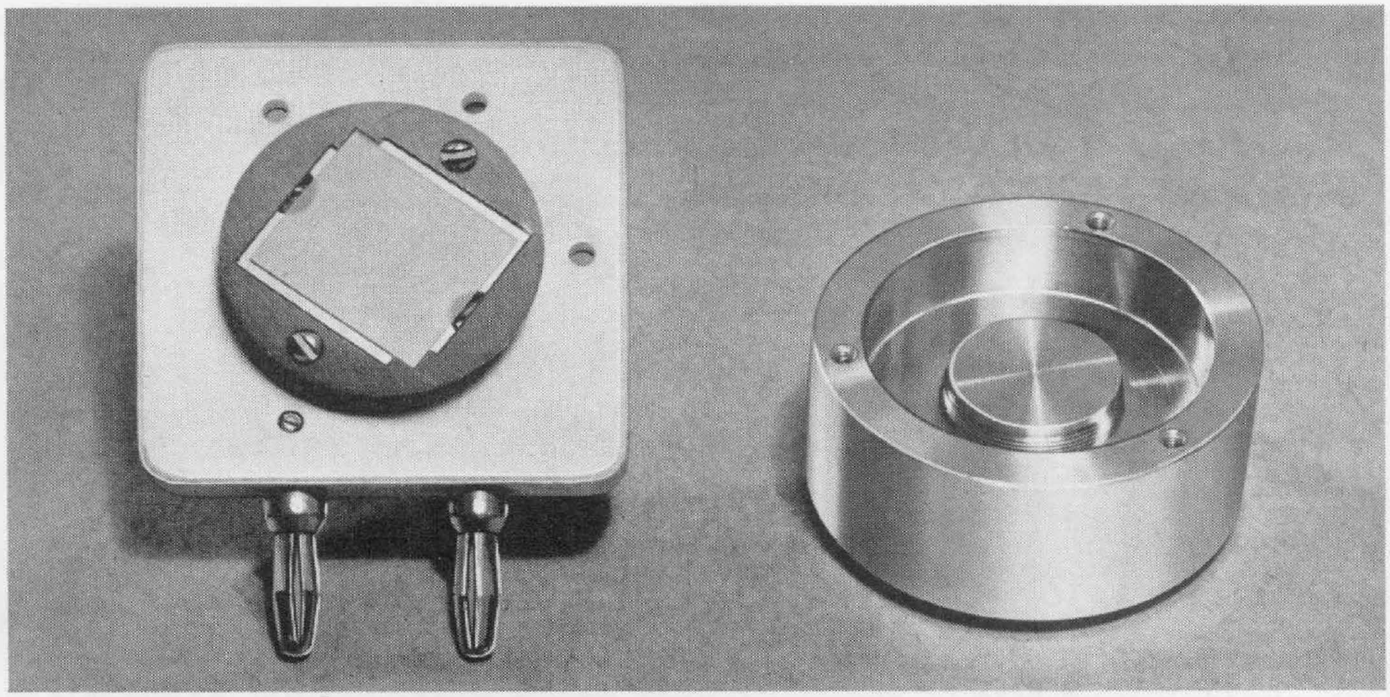
LOW-TEMPERATURE-COEFFICIENT QUARTZ PLATES

● **THE WIDE USE** of piezo-electric quartz plates as standards for frequency measurement and monitoring has stimulated research directed toward more stable piezo-electric elements and circuits. For the past few years considerable work has been done on reducing the temperature coefficient of frequency, which is the largest single source of frequency variation. An investigation of this field, together with a study of general methods of cutting and grinding quartz, has been carried out by the General Radio Company over a period of more than a year. Studies have been made not only on the thin plates used in monitoring, but also on the quartz bars used in primary standards.

This work has produced a plate which not only has a low temperature coefficient but is more active piezo-electrically and is free from both spurious frequencies and abrupt jumps in frequency when the temperature is varied.

FIGURE 1 (above). External view of holder used for TYPE 376-L Quartz Plates

FIGURE 2 (below). TYPE 376-L Quartz Plate with cover removed. The retaining ring is so shaped that little random motion of the plate is possible. The heavy aluminum cover with its threaded plug provides stability of air-gap adjustment and uniform heat distribution

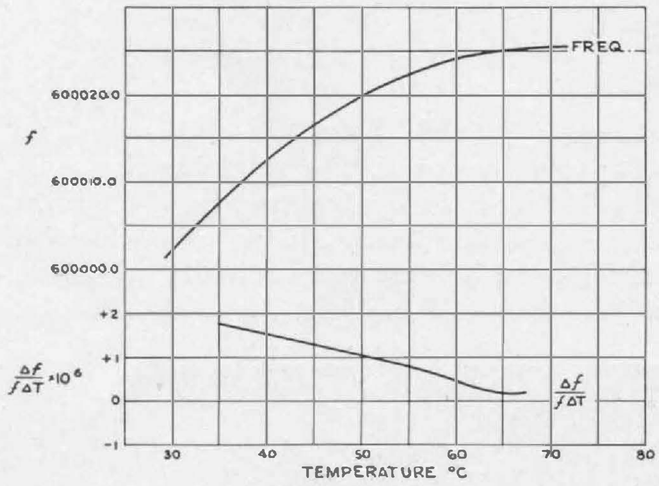


While the temperature coefficient is, in general, very low — less than one part per million per degree C. — it is evident that *zero* temperature coefficient cannot always be obtained in a desired temperature range. If a zero coefficient is reached, it is only for a single temperature.

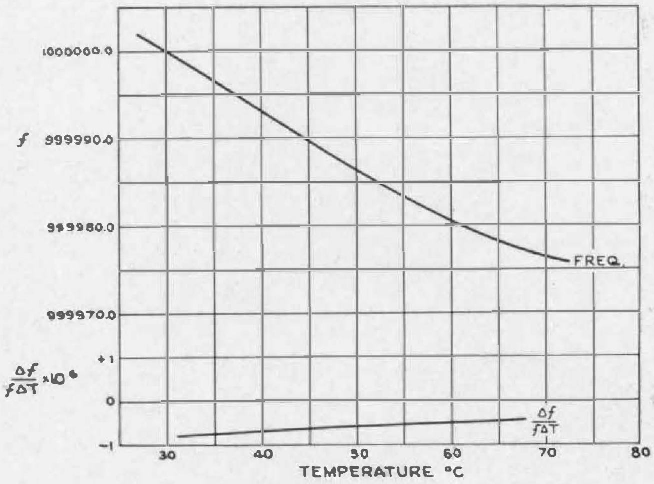
The temperature-frequency characteristics of these plates are shown in Figure 3, in which the upper curves show the actual variation in frequency with temperature, while the lower curves show the temperature coefficient (measured at 5-degree intervals) in parts per million per degree Centigrade.

A method of mounting has also been developed which restricts random motion of the crystal in the holder without materially affecting its freedom to vibrate. In the top of the holder is a fine-threaded plug, the bottom of which serves as the upper electrode. The quartz plate and the retaining ring rest on the lower electrode which is in the form of a plate secured to the isolantite base.

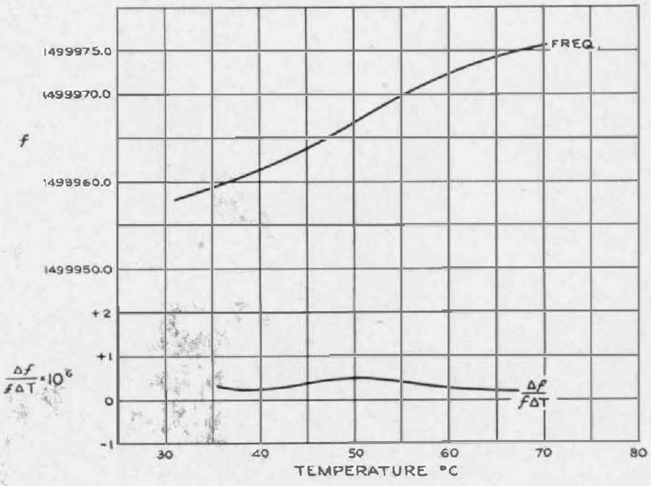
TYPE 376-L Quartz Plates now replace the TYPE 376-J and TYPE 376-K Quartz Plates formerly used in monitoring instruments. Each TYPE 376-L Quartz Plate is adjusted at its normal operating temperature until its frequency differs from that specified in the order by less than one cycle per second or 0.0001% (1 part in 10^6) whichever is the larger. When used in General Radio TYPE 475-A Frequency Monitor or TYPE 675-H Piezo-Electric Oscillator, the frequency is guaranteed to 0.002% (20 parts in 10^6) for a period of one year, provided the plate is operated under the conditions specified in the certificate of calibration. The temperature coefficient of frequency is guaranteed to be less than 3 parts per million per degree Centigrade between 20° and 70° C.



(A) 600 kc



(B) 1000 kc



(C) 1500 kc

FIGURE 3. Curves showing the variation of both frequency and temperature coefficient of frequency as the temperature is varied for three plates in the standard broadcast band. The upper curves show the change of frequency in cycles per second; the lower curves, the temperature coefficient in parts per million per degree Centigrade

FIGURE 4. One of the grinding operations in the production of General Radio quartz plates. The edge of the plate is held against a rotating disk, and the calibrated head shows the amount ground from the edge

Although General Radio frequency monitors are provided with a precise temperature-control system, adequate to maintain the above-mentioned accuracy with older types of plates, the use of the new low-temperature-coefficient plates provides a better margin of safety. In addition, if, for any reason, temporary failure of the temperature control occurs, the frequency error is greatly reduced. The price of TYPE 376-L Quartz Plate is $85.00, but for those users of General Radio frequency monitors who wish to replace their present crystals with the new type, a liberal credit allowance for the returned holder will be made as explained below.

TYPE 376-L Quartz Plates have been approved by the Federal Communications Commission for use in General Radio frequency monitors.

Studies on low-frequency quartz bars have reduced the temperature coefficients of TYPE 676-A Quartz Bars, used in the Class C-21-H Standard-Frequency Assemblies and in Class C-10 Secondary Frequency Standards to between one and two parts per million per degree C. In quartz bars, the temperature coefficients are practically constant over the range of temperatures of test, in contrast to the quartz plates in which variations in the temperature coefficients are observable.

— J. K. CLAPP

General Radio quartz plates are licensed under all patents and patent applications of Dr. G. W. Pierce pertaining to piezoelectric crystals and their associated circuits.

REPLACEMENT QUARTZ PLATES FOR FREQUENCY MONITORS

● **WHEN** a TYPE 376-J or TYPE 376-K Quartz Plate is returned to be replaced by a TYPE 376-L Quartz Plate, the new plate will be mounted in the returned holder, regardless of whether or not this holder is of the type shown on page 5. Under these conditions a credit of $20.00 will be allowed for the returned holder, making the price of the replacement plate $65.00. Those who wish to make the exchange should write directly to the Service Department for shipping instructions in order that each exchange may be handled as promptly as possible. Under no conditions should plates be returned without first communicating with our Service Department.

MISCELLANY

● **AMONG THE RECENT** distinguished visitors to our laboratories and factory were His Excellency, Professor Ing. Giancarlo Vallauri and his Assistant, Professor Gori.

Professor Vallauri is Vice-President of the Accademia d'Italia, President of the Societa' Idroelettrica Piemontese, President of the Ente Italiano Audizioni Radiofoniche, President of the Elettrica Piemonte Centrale, Dean of the Politecnico of Turin, Italy, and one of the leading scientists invited to participate at the Tercentenary Celebration at Harvard University.

● **AN INDEX** has been compiled for the General Radio *Experimenter* covering the period from June, 1931, to May, 1935, inclusive. This is printed in two parts, one for Volumes VI and VII (1931-1933) and the other for Volumes VIII and IX (1933-1935). Copies will be furnished to all who request them.

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