through Capacitor $C_3$ and Switch S.

One signal voltage (No. 1) is presented to the grid circuit of the left triode, and the other signal voltage (No. 2) to the grid of the right triode. When Switch S is at Position 1, the first signal voltage ($E_a$) is indicated by the vtvm. When S is at 2, the second voltage ($E_b$) is indicated. When S is at 3, the output voltage ($E_c$) of the amplifiers is read. The amplitude of $E_c$ will depend upon the phase relations of $E_a$ and $E_b$. When $E_a = E_b$ and these two voltages are in phase, $E_c$ is maximum. When they are equal and 180° out of phase, $E_c = O$.

If $E_a$ and $E_b$ are of the **same frequency** and **waveform** and are measured separately (Switch S successively at 1 and 2), and $E_c$ also is measured (Switch S at 3), the cosine of the phase angle may be calculated:

$$(1)\quad \cos \ominus = \frac{E_c^2}{E_a^2 + E_b^2 + 2E_aE_b}$$

and the phase angle from:

$$(2)\quad \ominus = \cos^{-1} X,$$ where X is the right side of Equation (1).

To eliminate the necessity for calculations, the scale of the v-t voltmeter may be graduated directly in degrees.

Like the oscilloscopes described earlier in this article, the electronic phase meter may be used to measure the phase relations between two voltages, two currents, or a voltage and a current.

### Phase Relations in Circuit Components

Reactive circuit components (capacitors and inductors), if perfect in their actions, would introduce a 90° phase shift between current and voltage — leading current in capacitors: lagging current in inductors. Because a certain amount of resistance in unavoidable in every reactor, however, the phase shift is somewhat less than 90 degrees. The quality of the component (or of a dialectric) thus can be expressed in terms of the difference between the ideal 90° angle and the actual phase shift. This **phase difference** angle is designated $\varnothing$ and is equal to $90° - \ominus$, where $\ominus$ is the actual phase angle.

Bridge measurements and Q-meter measurements appraise the quality of a reactive component by determining the ratio of resistive to reactive factors. If $\varnothing$ is taken as the phase angle $(90° - \ominus)$ of a capacitor, for example, sin $\varnothing$ is the **power factor** (pf) indicated by bridge measurements. Phase angle may be determined from the bridge-determined power factor value thus:

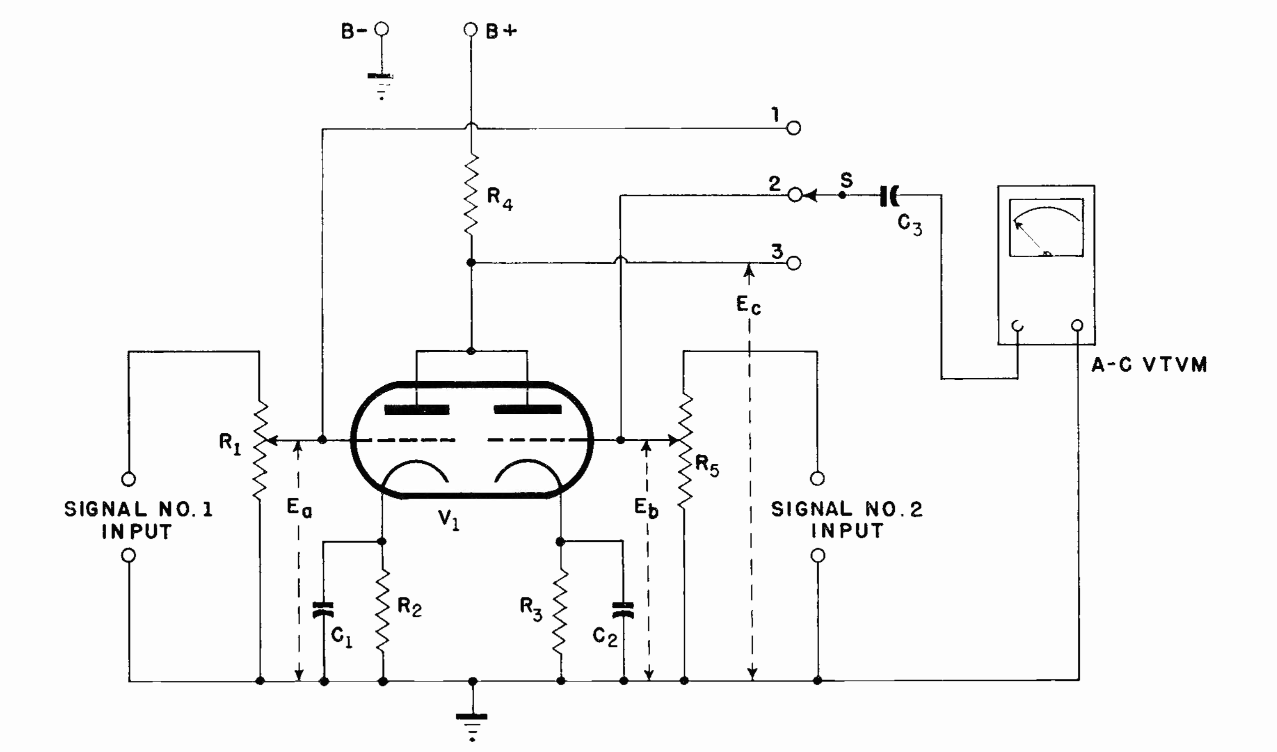
$$(3)\quad \varnothing = \sin^{-1} pf$$

Some bridges give indications of **dissipation factor** (D), rather than power factor. $D = \tan \varnothing$. From which:

$$(4)\quad \varnothing = \tan^{-1} D$$

Some bridges and all Q-meters give indications of Q, which is the ratio of reactance to resistance, X/R. Q is the reciprocal of dissipation factor ($Q = 1/D = 1/\tan \varnothing$). Therefore:

$$(5)\quad \varnothing = \tan^{-1} 1/Q.$$




ELECTRONIC PHASE METER

FIG. 7



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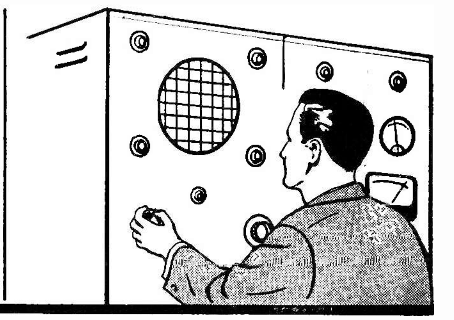



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# Phase Measurements

By the Engineering Department, Aerovox Corporation

AMONG experimenters and some students, phase measurements are considerably less common than the a-c measurements of current, voltage, capacitance, frequency, inductance, resistance, and power. The importance of phase and phase angle measurements must not be minimized, however. Actual phase measurements, rather than estimates or calculations, should be made wherever practicable. These measurements do a great deal to establish circuit operation and are an invaluable aid in checking a design and in trouble-shooting. Phase relations are particularly important in modern electronic instrumentation.

Some of the instances in which phase relationships are significant and their measurement serves to pinpoint operating characteristics include: phase shift introduced by a complete amplifier or amplifier stage, phase difference between two voltages, a current and a voltage, or two currents, and phase shift introduced by various passive networks.

Several methods are available for the practical measurement of phase relations. The method used in a particular instance depends upon available instruments and required accuracy. Some of the schemes require calculations; others provide direct readings of phase angle. This article describes representative measurement techniques.

## OSCILLOSCOPE METHODS

The oscilloscope is widely used for phase measurements both in the laboratory and field. Oscilloscope methods are fairly simple and provide good accuracy when employed by a careful technician.

**Conventional Oscilloscope.** Phase measurement is somewhat similar to frequency identification with a conventional oscilloscope. The process is reliable, provided the vertical and horizontal amplifiers of the instrument have identical phase shift characteristics and that both vertical and horizontal linearity are excellent. In this method, one signal voltage is applied to the vertical amplifier input and the other signal voltage to the horizontal amplifier input, and their phase difference determined from observation of the displayed pattern. The internal sweep and sync functions are disabled during the test.

Figure 1(A) shows the simple arrangement. With zero signal input, the cathode ray spot is centered exactly on the scope screen. Signal voltage $E_1$ then is applied to the vertical input; signal voltage $E_2$ to the horizontal input. Both voltages must be of the same frequency and sinusoidal. These voltages may be derived from different parts of a circuit; e. g., the grid-signal and plate-signal voltages of an amplifier, voltage drops across separate legs of an RCL circuit, etc.



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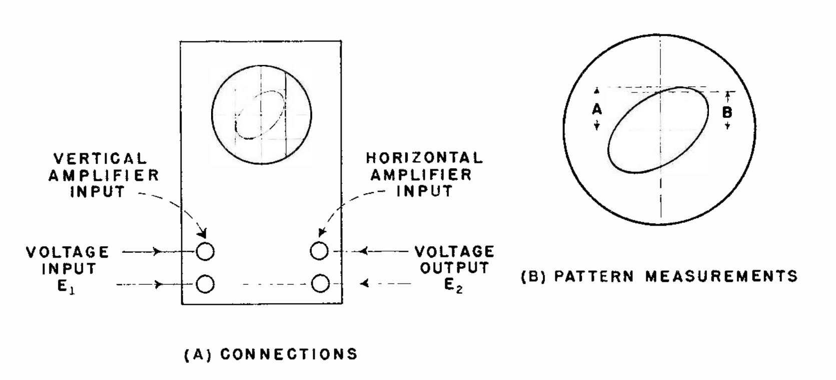
The phase difference (e) between the two voltages is determined from the displays, as shown in Figure 1(B). The vertical distance from center-screen to the point at which the pattern intersects the vertical axis is designated B. The maximum vertical height of the pattern is designated A. These dimensions may be measured in scale divisions, inches, centimeters, or any other convenient units. The ratio B/A gives the cosine of the phase angle: thus $\cos \Theta = B/A$, and $\Theta = \cos^{-1} B/A$.

The pattern will vary from a right-tilted 45° single-line trace (Figure 2A) when the phase shift between $E_1$ and $E_2$ is zero, to a left-tilted 45° single-line trace (Figure 2E) when $\Theta = 180°$. Halfway between these limits, a circular trace (Figure 2C) is produced at 90°. At 45°, a right-tilted ellipse (Figure 2B) is obtained, and at 135° a left-tilted ellipse appears. Note from Figure 2 that identical patterns are obtained for 45° and 315°, 90° and 270°, and 135° and 225°. The difference is that the pattern rotates clockwise (as shown by the solid arrowhead) for the lower phase angle, and counterclockwise (as shown by the dotted arrowhead) for the higher angle.

For best accuracy with the conventional oscilloscope method, the horizontal and vertical amplifiers must have identical phase shift characteristics, and the oscilloscope must have excellent linearity, sharp focus, and complete freedom from interaction between deflection and beam centering.

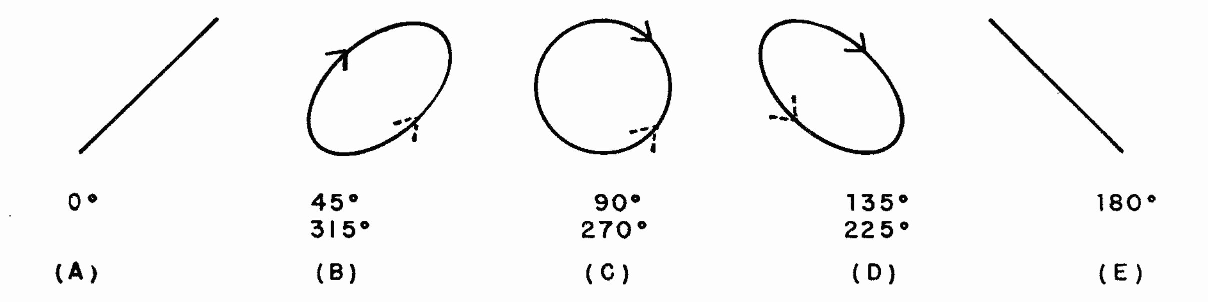
In addition to checking the phase difference between two voltages per se, current-voltage phase relationships also may be checked by means of a conventional oscilloscope. The current of interest is passed through a non-inductive resistor and the resulting voltage drop across the resistor is applied to the oscilloscope as one of the signal voltages. Figure 3 illustrates this scheme. Here, the purpose is to measure the phase between the voltage applied to an inductor and the current flowing through it. For this test, a non-inductive resistor, R, is connected in series with the inductor, L. The resistance of R must be very small, with respect to the resistance and reactance of the inductor, otherwise it will severely limit the inductor current. Current (I) flowing through the resistor sets up a voltage drop, $E_2$, which is proportional to I, and this voltage is applied to the horizontal channel of the oscilloscope. The voltage, $E_1$, which is applied to L and R in series is presented to the vertical channel. While an inductor is shown in this example, the same method may be employed to check current-voltage phase relations in a capacitor, a resistive device, or a combination of L, C, and R components. In either case, the resistance of R must be negligible with respect to the impedance of the device under test.

This same scheme may be employed to check the phase relations between two currents. Each current is passed through a separate resistor. The voltage drops will be proportional to the currents and are applied to the oscilloscope as the vertical and horizontal signal voltages.



USE OF CONVENTIONAL OSCILLOSCOPE FOR SINE-WAVE VOLTAGES

FIG. 1



TYPICAL SINE-WAVE PHASE PATTERNS

FIG. 2



**Dual-Beam Oscilloscope.** The **dual-beam** or **double-trace** oscilloscope is somewhat more convenient than the conventional scope for phase measurements, since the former permits simultaneous observation of the two signals. Figure 4 illustrates use of the dual-beam instrument. Here, one sine-wave signal is presented to one vertical amplifier, and a second sine-wave signal to the second vertical amplifier. The internal sweep and sync are adjusted for a desired number of stationary cycles on the screen.

The dual-beam oscilloscope has a common sweep and sync circuit for the two signal channels, so both signals may be "stopped" simultaneously. The phase relation between the two signals is determined merely by linear measurement along the horizontal axis (time base). Thus, if the sweep frequency and horizontal gain are adjusted so that each signal cycle occupies 20 scale divisions, the horizontal (phase) scale will be 180/20 = 9 degrees per division. The phase difference between the two signals is determined by measuring the lead or lag between the two along the horizontal axis. Thus, in Figure 4(A), Signal 2 is 180° out of phase with Signal 1, and vice versa.

It is convenient that the gain is adjustable separately in each vertical channel. This enables one signal to be "blown up" on the screen, with respect to the other, for easier comparison. Further advantages of the dual-beam method are that the two signals need not be of the same waveform nor of the same frequency. (However, for sweep and sync purposes, the signals should bear a harmonic relation). Thus Figure 4(B) shows sine-wave and square-wave signals displayed simultaneously for phase comparison. Pulse signals may be examined if the pass-band of the oscilloscope amplifiers will transmit these signals faithfully.

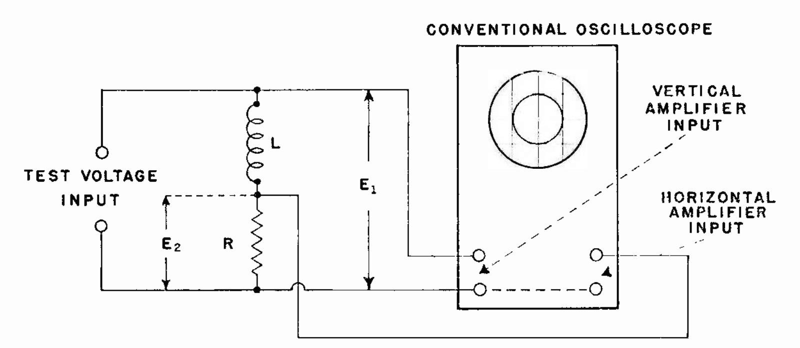
Since the dual-beam oscilloscope has separate beam-centering controls, the signal patterns may be moved vertically on the screen, by adjustment of the vertical beam-centering controls, for easier comparison. Once adjusted, however, the **horizontal** beam-centering controls must not be disturbed, since horizontal displacement of the pattern would give a false picture of phase relations.

The dual-beam method is not restricted to voltages only. Phase relations between current and voltage also may be determined by taking as one signal the voltage drop produced by the current across a small resistance. (See Figure 3). Phase relations between two currents likewise may be checked by utilizing as signals the voltage drops produced by these currents across separate small resistances.

The requirements, listed previously for the conventional oscilloscope, apply also to the dual-beam instrument used for phase measurement: identical phase shift in the amplifiers, excellent linearity and focus, and stability of beam centering.

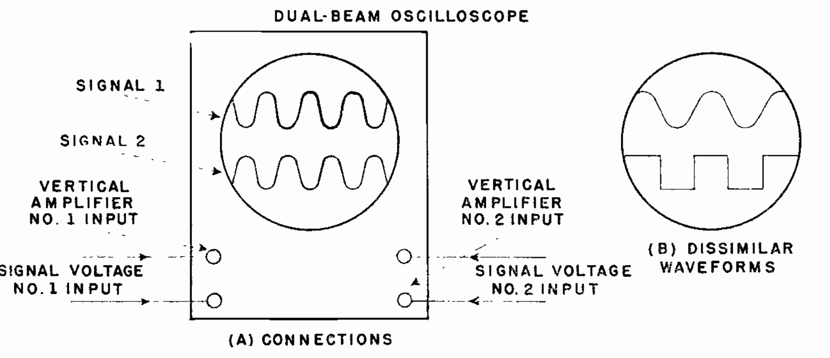
**Electronic Switch with Conventional Oscilloscope.** When a dual-beam oscilloscope is not available, the advantages of this instrument may be obtained to some extent by operating an **electric switch** ahead of a conventional oscilloscope. The electronic switch feeds the vertical channel of the oscilloscope, as shown in Figure 5(A), and the two separate signal inputs of the switch accommodate the two signal voltages to be compared for phase. The rapid switching action causes the two signal voltages to be sampled at different instants of time, and this action causes the two signal patterns to be displayed separately on the oscilloscope screen, as shown in Figure 5(A).

Considerable flexibility is provided by a good electronic switch. For example, the signal amplitudes are separately adjustable by means of independent gain controls, the signals need not be of the same waveform (although, some electronic switches are limited in their ability to handle other than sine waves), and the signals need not be of the same frequency. (However, for sweep and sync purposes, the signal frequencies must bear a harmonic relationship to each other). A further advantage is the **position control** of the electronic switch which permits one pattern to be moved with respect to the other on the screen. Thus, in addition to the display of one signal above the other, as in Figure 5(A), the signals may be superimposed (Figure 5B) for direct comparison of phase.



ARRANGEMENT FOR CURRENT-VS-VOLTAGE PHASE RELATIONSHIPS

FIG. 3



USE OF DUAL-BEAM OSCILLOSCOPE

FIG. 4



The conventional oscilloscope to be used with an electronic switch must have excellent focus and linearity; stable sweep, sync, and beam centering; and good hum suppression. The electronic switch itself must have identical phase-shift characteristics in its two signal channels. The operator should investigate beforehand the frequency response and switching frequency of the switch, since these factors will determine the maximum signal frequency which can be handled and to what extent non-sinusoidal waveforms can be accommodated.

## METER METHODS

**Non-Electronic Meter.** Meters of various types have been used in the determination of phase values. One such instrument is the electrodynamometer-type **power factor meter** illustrated by Figure 6. This instrument indicates power factor (pf) directly, as a decimal, with 1.00 as the higrest scale graduation. Since $pf = \cos \Theta$, the scale also reads directly in cosine of the phase angle, and the phase angle may be determined from $\Theta = \cos^{-1} pf$. As illustrated in Figure 6, the meter will indicate the phase between voltage and current of a load.

The rotating member of this meter consists of two coils, $L_2$ and $L_3$, fastened together at right angles and attached to a shaft which rotates in jewelled bearings. Unlike familiar d'Arsonval moving-coil meters, there is no spring to return the coil assembly to zero. The pointer is attached to the movable coil assembly which rotates inside a fixed coil, $L_1$.

Coil $L_1$ is connected in series with one side of the power line, while $L_2$ and $L_3$ are connected in parallel with the line — through Resistor R in series with $L_2$, and Inductor $L_4$ in series with $L_3$. Since there is only a resistor in series with $L_2$, current $I_1$ through this coil is in phase with the line voltage. But the inductor in series with $L_3$ causes current $I_2$ through this latter coil to lag behind the line voltage by 90 degrees.

When the power factor of the circuit (line-load) is unit ($pf = 1$, $\Theta = 0°$), $I_1$ and $I_2$ are in phase. The $L_2$-$L_3$ assembly then rotates to align the flux between $L_2$ and $L_1$. The meter accordingly reads power factor = 1, which represents a phase angle of zero. This is the position in which the meter is shown in Figure 6.

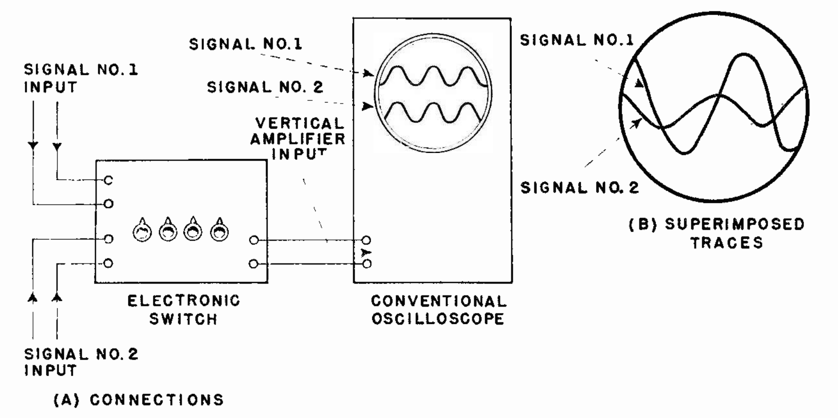
When the line current is 90° out of phase with the line voltage, the fluxes of $L_1$ and $L_3$ are in phase, and Coil $L_3$ is caused to turn the assembly to the right (driving the pointer upscale) when the line current leads the line voltage, or to the left (pointer downscale) when the line current lags.

From this action, it is seen that the meter scale may be graduated directly in power factor, or in phase angle the cosine of which if given by the numerical power factor.

Because of the presence of Inductor $L_4$, this type of meter is limited to use at or near the line frequency for which it is designed and calibrated.

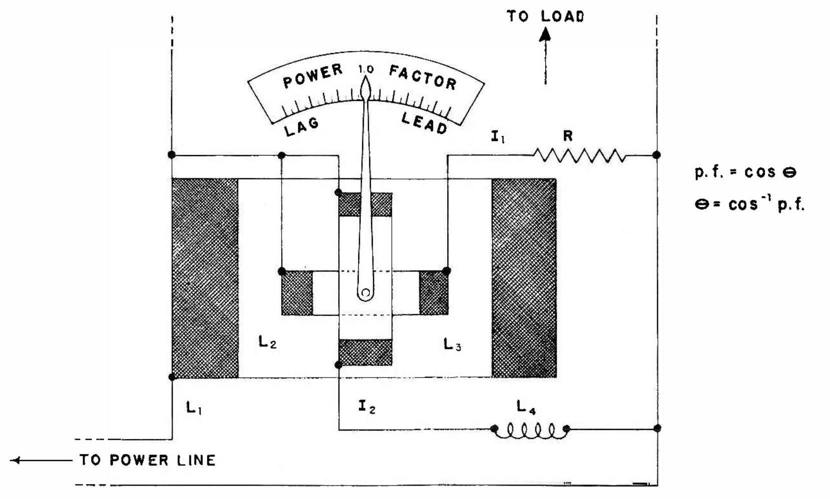
**Electronic Meter.** Electronic phase angle meters are presently available in several types and ranges for use over a wide frequency range. In addition to their desirable frequency characteristics, these instruments (like v-t voltmeters and oscilloscopes) have high input impedance which makes circuit loading negligible.

Figure 7 shows the basic arrangement of an electronic phase meter circuit. In this arrangement, two identical amplifier stages are provided by the sections of the twin-triode tube, $V_1$. These amplifiers are designed for minimum distortion and identical phase shift throughout the specified frequency range. The gain of each amplifier is adjustable separately by means of gain controls $R_1$ and $R_5$. The amplifiers have a common plate resistor, $R_4$. The indicator is an a-c vacuum-tube voltmeter coupled to the amplifier plates



DUAL-TRACE PRESENTATION WITH CONVENTIONAL OSCILLOSCOPE

FIG. 5



ELECTRODYNAMOMETER-TYPE POWER FACTOR METER

FIG. 6