The AEROVOX Research Worker

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The Aerovox Research Worker is a monthly house organ of the Aerovox Corporation. It is published to bring to the Radio Experimenter and Engineer authoritative, first hand information on condensers and resistances for radio work.

VOL. 7, NO. 11 — NOVEMBER, 1935 — 50c per year in U.S.A. 60c per year in Canada

Laws of Alternating Currents

PART 1

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It can probably be taken for granted that readers of this article are familiar with the laws of direct current and could compute the voltage and current in any branch of an involved d.c. circuit. Although many are acquainted with the laws of the simpler a.c. circuits, a knowledge of the solution of networks with many branches is not general. This is a distinct handicap for the lack of ability to figure things out forces an acceptance of someone else's statements. This article is an attempt to clarify the subject to readers with limited mathematical accomplishments.

Perhaps the best way to begin is by re-stating the laws of direct current.

These are:

Ohm's Law: $E=IR$

Kirchoff's Laws: I. The sum of the currents flowing to a junction is zero.

II. The sum of the voltages taken around a circuit is zero.

Although Kirchoff's laws are not often quoted they are quite necessary for the solution of d.c. circuits. They are quite obvious statements which would immediately follow when applying common sense to the problem. The sum of all currents flowing to a junction must be equal to the sum of the currents flowing away from it. Similarly, the voltage of a generator must be equal to the sum of the voltages across the elements which are connected in series across the generator.

ALTERNATING CURRENT CIRCUITS

Alternating currents are continously varying but at any instant the laws of Kirchoff hold although the measuring instruments might lead one to believe otherwise. Ohm's law is now written

$$E=IZ$$

where Z is called the "impedance," the combined effect of inductance, capacitance and resistance on the e.m.f. in the circuit. In order to combine inductive and capacitive reactance with resistance when they are in series and in parallel it is necessary to pay due attention to phase differences. This brings us to two new notions; vectors and phases.

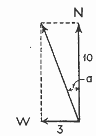

Fig. 1

Although it was hoped that it were unnecessary to introduce these two concepts here, the use of vectors and phases is quite necessary for the understanding of the example to follow.

VECTORS

A *vector* is a quantity which defines magnitude as well as direction, while ordinary quantities which indicate only magnitude are called scalar quantities. So, for instance when one says that the length of a road is 10 miles, the length is expressed as a scalar quantity. But when one says: a ship is sailing at a speed of 10 knots in a south-easterly direction, one is using a vector quantity. These vectors can be added in a way familiar to all. Suppose a ship is sailing north at a speed of 10 knots and there is a cross current from east to west which has a speed of 3 knots. In which direction and at what speed is the ship actually moving? Figure 1 illustrates the problem; it is well known that the resultant vector can be found by completing the parallellogram and drawing the diagonal. When the drawing has been made to scale, the length of the diagonal represents the resultant speed and the angle *a* shows the direction. Calculating the magnitude and the direction of the vector, we notice that the original vectors were at right angles, hence the diagonal can be found by Pythagoras' theorem:

$$S=\sqrt{10^2+3^2}=\sqrt{109} \quad 10.44 \text{ KNOTS}$$

while the direction, indicated by the angle *a* is found from

$$a=\tan^{-1}\frac{3}{10}=16°42'$$

or the resultant velocity of the ship is 10.44 knots in a direction: "North, 16 degrees 42 minutes West." In the case of the addition of the two vectors which are not at right angles to each other, the algebraic way might sometimes be made easier by first resolving each one into two vector-components which lie along two axes perpendicular to each other. The addition then becomes similar to the example above. It is always possible to add two vectors or to resolve them into two or more others as long as the vectors represent the same quantity; i.e. they must both be speeds, or both displacements, both forces, etc. You cannot add a velocity to an acceleration.

PHASE DIFFERENCES

Alternating current of a single frequency, a *pure sine wave*, has its voltage and current varying continuously according to the law of simple harmonic motion. This motion can be

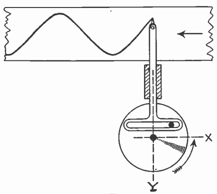

Fig. 2

reproduced mechanically from a crankshaft and a piston rod as shown in Figure 2. Assume that the crankshaft revolves at a constant speed, then the rod will move up and down in accordance with the law of simple harmonic motion. This could be recorded by attaching a pen to the upper end of the rod and letting it rest upon a strip of paper which were being moved towards the left at a uniform speed. The pen would then draw what is known as a *sine wave*. This name is used because the displacement of the rod from its center (average) position is at all times equal to the projection of the radius r on the Y axis and this in turn is proportional to the line of the angle the crank is making with the X axis. Consequently, when thinking of the magnitude of an alternating e.m.f. one thinks of it as a vector which is rotating at the rate of one revolution per cycle and having a length in proportion to the peak value of the alternating e.m.f. From this imaginary rotating vector we can find the instantaneous value of the voltage by projecting it upon the Y axis for the particular angle in question.

Now suppose there is a second vector rotating at the same speed, representing the voltage in another circuit, either the two vectors keep exact step or one is ahead of the other by a constant angle. This difference is called the *phase difference*. When two such voltages are in series across two impedance elements in a circuit, the sum of the voltages is found again in a similar way as illustrated in Figure 1. It is customary to draw the vectors "standing still" just showing the angle between them. When adding them as described before we find the magnitude of the voltage across the series combination and the phase difference between it and either of these voltages. Vectors can be added or resolved into components again as long as they represent the same things, you cannot add a voltage to a current.

We are now ready for an illustration. As an example of a rather involved circuit which will necessitate the combination of impedances in parallel and in series, a power supply filter was chosen.

Figure 3 illustrates a two section filter with choke input and showing the series and parallel resistances (leakage and power factor) of the electrolytic condensers as well as the resistance of the chokes. For simplicity's sake the rectifier has been omitted. The question is now when a given voltage, say 100 volts a.c. at 120 cycles appears across the input terminals, how much of it appears across the chokes, across the condensers and how much across the load. In other words, why is it a filter? We are taking the 120 cycles as an example because it is the fundamental component of the ripple in most power supply filters but the reader will see that the same question can be answered for any other frequency.

In order to solve this problem it will be necessary to reduce the circuit to a simpler one, having the same impedance and causing the same phase difference between current and voltage. Since the available formulas will allow the addition of impedances in series only or in parallel only, it will sometimes be necessary to replace a series circuit consisting of a reactance and a resistance by a parallel circuit—again a reactance and a resistance—in such a way that the parallel circuit lets exactly the same current flow as the series circuit did and with the

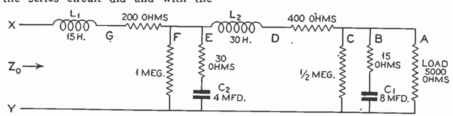

Fig. 3

same phase difference between voltage and current. The reverse may also be done.

Now returning to Figure 3, the first thing to do is to find a single series circuit which is equivalent to the branches A, B and C together. For the reason that branches F and E are each exactly twice the branches C and B it is best to first combine C and B. But B is a series circuit and C is in parallel. This means that B has to be transformed into an equivalent parallel combination of a resistance and a capacitance. Figure 4 shows the voltage vectors across the two parts of the circuit B with their relation to the current vector. The sum of the voltages across condenser and resistor is apparently

$$IZ_B = \sqrt{(IR)^2 + (IX_C)^2}$$

or, dividing by I, we find the total impedance Z is

$$Z_B = \sqrt{15^2 + 166^2} = \sqrt{27{,}781} = 166.7 \text{ OHMS}$$

The resultant voltage, according to Figure 4 is out of phase with the cur-

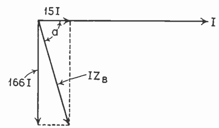

Fig. 4

rent through B by an angle a. The magnitude of this angle is

$$a = \tan^{-1} \frac{166}{15} = 84°51'$$

The next problem is to find a parallel circuit which has the same impedance and causes the same phase difference. In parallel circuits the *currents* have to be added vectorially since they obviously have the same voltage across the branches. Figure 5 shows the vector diagram of the parallel circuit and the current vectors in their proper relation to the voltage across them. Beginning with the single current vector of $\frac{E}{166.7}$ amperes, 84° 51' ahead of the voltage, we resolve it into two components, as follows

$$\frac{E}{R} = \frac{E}{166.7} \cos 84°51' \quad \frac{E}{X_C} = \frac{E}{166.7} \sin 84°51'$$

or, solving for R and X_c,

$$R = \frac{166.7}{\cos 84°51'} = 1860 \text{ OHMS}$$

$$X_C = \frac{166.7}{\sin 84°51'} = 167.35 \text{ OHMS}$$

The obtained resistive branch should now be combined with the branch C

$$R_{BC} = \frac{500{,}000 \times 1860}{501{,}860} = 1859 \text{ OHMS}$$

Thus, the circuit of the condenser, (branches B and C) can be replaced by two branches (H and I), having 167.35 ohms reactance and 1859 ohms resistance. Similarly, the other condenser, (branches E and F) can be replaced by two new branches J and K of twice these values. The new circuit now appears in Figure 6.

The next step is the vectorial addition of branches I, H and A. First adding the resistive branches

$$R_{AI} = \frac{5000 \times 1859}{6859} = 1355 \text{ OHMS}$$

In order to find the resultant of this with branch H the reciprocals must be added vectorially, because the currents are added vectorially and the

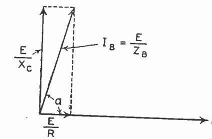

Fig. 5

current is inversely proportional to the impedance.

$$\frac{1}{Z_{AHI}} = \frac{1}{Z_{ABC}} = \sqrt{\left(\frac{1}{R_{AI}}\right)^2 + \left(\frac{1}{X_H}\right)^2}$$

$$Z_{ABC} = \frac{R_{AI} X_H}{\sqrt{R_{AI}^2 + X_H^2}}$$

substituting values

$$Z_{ABC} = \frac{167.35 \times 1355}{\sqrt{167.35^2 + 1355^2}} = 166.1 \text{ OHMS}$$

and the phase angle

$$b = \tan^{-1} \frac{1355}{167.35} = 82°58'$$

This newly found impedance has to be replaced by an equivalent series circuit consisting of a single resistance and reactance in series, so that it can be added to circuit D. This operation is the reverse of that accompanying Figure 3. It will be seen easily that the required values are

$$R = Z_{ABC} \cos b$$
$$X = Z_{ABC} \sin b$$
$$R = 166.1 \cos b = 20.8 \text{ OHMS}$$
$$X_C = 166.1 \sin b = 164.8 \text{ "}$$

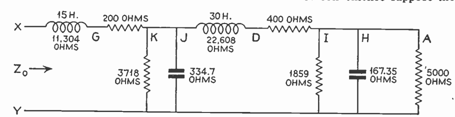

Fig. 6

Vectorially adding the above to the circuit D, the following well known equation is used:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

in this case R is 20.8 ohms+400 ohms = 420.8 ohms, X_c is 164.8 ohms and XL = $2\pi fL$ = 6.28x120x30 = 22608 ohms.

$$Z_{A\text{-}D} = \sqrt{420.8^2 + (22{,}608 - 164.8)^2} = 22{,}447 \text{ OHMS}$$

and the phase angle

$$C = \tan^{-1} \frac{22{,}608 - 164.8}{420.8} = 88°39'$$

This impedance has to be replaced again by an equivalent parallel circuit, so that it can be added vectorially to branches J and K. Note that the newly found impedance has its current lagging the voltage and that this time the equivalent circuit will consist of a resistance and an inductance. Incidentally when impedances are replaced by equivalent circuits as has been shown here, the newly found circuit is equivalent to the original one only at the *one frequency* under consideration, at any other frequency the values of the reactances will be different and the equivalent circuit will not be the same.

Resolving the impedance Z (A-D), into two parallel branches, the values of the required branches are

$$R = \frac{22{,}447}{\cos C} = 951{,}400 \text{ OHMS}$$

$$X_L = \frac{22{,}447}{\sin C} = 22{,}452 \text{ OHMS}$$

The network has now been reduced to that of Figure 7. Finding the resultant of branches K and L

$$R_{KL} = \frac{951{,}400 \times 3718}{951{,}400 + 3718} = 3694.4 \text{ OHMS}$$

$R_{K\text{-}L}$ must now be added vectorially to the branches J and M and be replaced by an equivalent series circuit. All three can be added in one operation. The current in M is lagging the voltage by 90 degrees and in J it is leading by 90 degrees; consequently, the currents in the two branches are in opposite direction and they partly cancel each other. The difference between the two must be added—vectorially—to the current in the resistive branch. For convenience suppose the applied voltage is 1 volt, then

$$\frac{1}{Z_{J\text{-}M}} = \sqrt{\left(\frac{1}{R_{KL}}\right)^2 + \left(\frac{1}{X_J} - \frac{1}{X_M}\right)^2}$$

$$= \sqrt{\left(\frac{1}{3694.4}\right)^2 + \left(\frac{1}{334.7} - \frac{1}{22{,}452}\right)^2}$$

$$Z_{J\text{-}M} = 338.3 \text{ OHMS}$$

and

$$d = \tan^{-1} \frac{\frac{1}{X_J} - \frac{1}{X_M}}{\frac{1}{R_{KL}}} = 0.0029432 \times 3694.4 = 84°44'$$

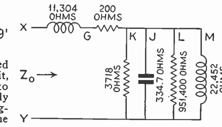

Fig. 7

The equivalent series circuit consists of

$$R = 338.3 \cos d = 31 \text{ OHMS}$$
$$X_C = 338.3 \sin d = 336.9 \text{ OHMS}$$

Finally adding the above to the remaining impedance G, the result is

$$Z_0 = \sqrt{31^2 + (11{,}304 - 336.9)^2} = 10{,}530 \text{ OHMS}$$

This is the total impedance at 120 cycles between the points X and Y. It is a well known law that in a series circuit, the voltage across each element is proportional to the impedance of that element (this follows from Ohm's law). For convenience we assume 100 volts applied to the input of the filter. Then the part across the first condenser is

$$E_{C_2} = \frac{Z_{J\text{-}M}}{Z_0} \; 100 \text{ VOLTS}$$

or, substituting values,

$$E_{C_2} = \frac{338.3}{10{,}530} = 3.21 \text{ VOLTS}$$

This 3.21 volts is the input to the second filter section, consisting of impedance D in series with the resultant of A, B and C. So the voltage across the last condenser is

$$E_{C_1} = \frac{Z_{ABC}}{Z_{A\text{-}D}} \times 3.21 \text{ VOLTS}$$

or, substituting values,

$$E_{C_1} = \frac{166.1}{22{,}447} \times 3.21 = 0.024 \text{ VOLT}$$

Where did all the voltage go? It is all across the chokes; in fact, the voltage across L_1 is more than 100 volts and across L_2 is more than 3.21 volts which the reader can easily compute for himself.

There is a much simpler method of solving complicated networks like these. This method is universally used in engineering texts; it does not necessitate finding equivalent circuits each time. Next month the same problem will be solved by means of this simpler method.