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Industrial Mathematics*

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The report consists of three major sections. The first discusses mathematical specialists in industry, calls attention to the essentially consultative character of their work, and makes some observations regarding the education, employment and supervision of this type of personnel.

The second section deals, not with the work of these specialists, but with the uses to which mathematics is put at the hands of industrial workers in general, the various ways in which it contributes to the economy and effectiveness of research, and the kinds of mathematics that are most used. A number of illustrations are given, together with brief surveys of the utilization of mathematics in four important industries: communications, electrical manufacturing, petroleum and aircraft.

The third section is devoted to statistics, which touches industrial life at rather different points, and hence could not conveniently be included in the general discussion.

Introduction

MATHEMATICAL technique is used in some form in most research and development activities, but the men who use these techniques would not usually be called mathematicians.

Mathematicians also play an important role in industrial research, but their services are of a special character and do not touch the development program at nearly so many points.

Because of this contrast between the ubiquity of mathematics and the fewness of the mathematicians, this report is divided into sharply differentiated parts. Under "Mathematicians in Industry" an attempt is made to explain what sort of service may be expected of industrial mathematicians, and to develop some principles of primary importance in employing and managing them. An attempt is also made to appraise future demand for men of this type, and to discuss the sources from which they can be drawn. Under "Mathematics in Industry" appear brief surveys of the extent and character of the utilization of mathematics in a few special industries, and examples of specific problems in the solution of which mathematical methods have been necessary or advantageous.

* This discussion of the part which Mathematics might play and to a certain extent is playing in industry was prepared for the National Research Council *Survey of Industrial Research*, a survey undertaken at the request of the National Resources Planning Board. The document which the survey produced has been published as "Research—A National Resource, Part II, Industrial Research" and is available through the Government Printing Office.

In these two sections mathematics is interpreted broadly to include not only the fundamental subjects, algebra, geometry, analysis, etc., but also their manifestations in applied form as mechanics, elasticity, electromagnetic theory, hydrodynamics, etc. Statistics, however, touches industrial activity in a rather different way, and is therefore discussed separately under a third heading, "Industrial Statistics and Statisticians."

One observation which will be made in more detail later is worthy of mention here, because of the present and prospective scarcity of suitably trained industrial mathematicians. Though the United States holds a position of outstanding leadership in pure mathematics, there is no school which provides an adequate mathematical training for the student who wishes to use the subject in the field of industrial applications rather than to cultivate it as an end in itself. Both science generally, and its industrial applications in particular, would be advanced if a group of suitable teachers were brought together in an institution where there was also a strong interest in the basic sciences and in engineering.

Mathematicians in Industry

What is a Mathematician?

If every man who now and then computes the average of a set of instrumental readings or solves a differential equation is a mathematician, there are few research workers who are not. If, on the other hand, only those who are primarily engaged in making additions to mathematical knowledge are mathematicians, there are almost none in industry. Neither definition is sound. The first is absurd; the second not closely related to the essential nature of mathematical thought. This report adopts a definition based upon the character of the man's thinking rather than the ultimate use to which his thinking is put.

Some men would be called mathematicians in any man's language; others physicists or engineers. These *typical* men are differentiated in certain essential respects:

The typical mathematician feels great confidence in a conclusion reached by careful reasoning. He is not convinced to the same degree by experimental evidence. For the typical engineer these statements may be reversed. Confronted by a carefully thought-out theory which predicts a certain result, and a carefully performed experiment which fails to produce it, the typical mathematician asks first, "What is wrong with the experiment?" and the typical engineer, "What is wrong with the argument?" Because of this confidence in thought processes the mathematician turns naturally to paper and pencil in many situations in which the engineer or physicist would resort to the laboratory. For the same reason the mathematician in his "pure" form delights in building logical structures, such as topology or abstract algebra, which have no apparent connection with the world of physical reality and which would not interest the typical engineer; while conversely the engineer or physicist in his "pure" form takes great interest in such useful information as a table of hardness data which may, so far as he is aware, be totally unrelated to any theory, and which the typical mathematician would find quite boring.

A second characteristic of the typical mathematician is his highly critical attitude toward the details of a demonstration. For almost any other class of men an argument may be good enough, even though some minor question remains open. For the mathematician an argument is either perfect in every detail, in form as well as in substance, or else it is wrong. There are no intermediate classes. He calls this "rigorous thinking," and says it is necessary if his conclusions are to be of permanent value. The typical engineer calls it "hair splitting," and says that if he indulged in it he would never get anything done.

The mathematician also tends to idealize any situation with which he is confronted. His gases are "ideal," his conductors "perfect," his surfaces "smooth." He admires this process and calls it "getting down to essentials"; the engineer or physicist is likely to dub it somewhat contemptuously "ignoring the facts."

A fourth and closely related characteristic is the desire for generality. Confronted with the problem of solving the simple equation $x^3 - 1 = 0$, he solves $x^n - 1 = 0$ instead. Or asked about the torsional vibration of a galvanometer suspension, he studies a fiber loaded with any number of mirrors at arbitrary points along its length. He calls this "conserving his energy"; he is solving a whole class of problems at once instead of dealing with them piecemeal. The engineer calls it "wasting his time"; of what use is a galvanometer with more than one mirror?

In the vast army of scientific workers who cannot be tagged so easily with the badge of some one profession, those may properly be called "mathematicians" whose work is dominated by these four characteristics of greater confidence in logical than experimental proof, severe criticism of details, idealization, and generalization. The boundaries of the profession are perhaps not made sharper by this definition, but it has the merit of being based upon type of mind, which is an attribute of the man himself, and not upon such superficial and frequently accidental matters as the courses he took in college or the sort of job he holds.

It is, moreover, a more fundamental distinction than can be drawn between, say, physicist, chemist and astronomer. That is why the mathematician holds toward industry a different relationship than other scientists, a relationship which must be clearly understood by management if his services are to be successfully exploited.

The Place of the Mathematician in Industrial Research

The typical mathematician described above is not the sort of man to carry on an industrial project. He is a dreamer, not much interested in things or the dollars they can be sold for. He is a perfectionist, unwilling to compromise; idealizes to the point of impracticality; is so concerned with the broad horizon that he cannot keep his eye on the ball. These traits are not weaknesses; they are, on the contrary, of the highest importance in the job of finding a system of thought which will harmonize the complex phenomena of the physical world, that is in reducing nature to a science. The job of industry, however, is not the advancement of natural science, but the development, production and sale of marketable goods. The physicist, the chemist, and especially the engineer, with their interest in facts, things and money are obviously better adapted to contribute directly to these ends. To the extent that the mathematician takes on project responsibility, he is forced to compromise; he must specialize instead of generalize; he must deal with concrete detail instead of abstract principles. Some mathematicians cannot do these things at all; some by diligence and self-restraint can do them very well. To the extent, however, that they succeed along these lines they are functioning not as mathematicians but as engineers. As mathematicians their place in industry is not to supply the infinite attention to practical detail by which good products, convenient services, and efficient processes are devised; their function is to give counsel and assistance to those who do supply these things, to appraise their everyday problems in the light of scientific thought, and conversely to translate the abstract language of science into terms more suitable for concrete exploitation.

In other words, the mathematician in industry, to the extent to which he functions as a mathematician, is a consultant, not a project man.

Qualifications Necessary for Success as an Industrial Mathematician

The successful industrial mathematician must not only be competent as a mathematician; he must also have the other qualities which a consultant requires:

First, though his major interests will necessarily be abstract, he must have sufficient interest in practical affairs to provide stimuli for useful work and to reconcile him to the compromises and approximations which are necessary even in the theoretical treatment of practical problems. This usually means that the type of mathematician who could not do a good engineering job if he turned his hand to it will not get on very well in an industrial career.

Second, he must be gregarious and sympathetic. If he shuts himself off from his associates, much of his thinking will have no bearing on their needs and that which does will exert less influence than it might. If he does not translate his thoughts into their language, they will miss the significance of much of his work and he will have but a limited clientele.

Third, he must be cooperative and unselfish. A man cannot be at once consultant and competitor to his associates. Self-seeking attempts to gain credit for his contributions to the industry will inevitably alienate his clientele. There are two reasons for this: In the first place a mathematician's appraisal of mathematical work, even if made from a detached point of view, is heavily weighted on the side of its fundamental scientific significance, whereas its industrial value should be judged on very different grounds and can best be appraised by the engineer. In the second place, the engineer in charge of a project can give credit without embarrassment for help received; it is to his credit to have known where help was to be had. The same story told by another, and particularly by the consultant himself, has an entirely different flavor.

Fourth, he must be versatile. Jobs change, and even the same job may give rise to questions which require very different mathematical techniques.

Fifth, he must be a man of outstanding ability. No one wants the advice of mediocrity. Among industrial mathematicians there is no place for the average man.

Employment and Supervision

Perhaps the greatest hazard in hiring mathematicians for industry arises from the fact that the employment officer is not often a judge of mathematical ability. Paradoxically, however, his mistakes are not usually made in judging mathematical aptitude, since general scholastic rating is an unusually trustworthy index of mathematical ability. But because of a feeling of incompetence bred by his lack of mathematical lore, he spreads the mantle of charity over other characteristics with regard to which he should trust his own judgment. If, for example, the applicant gives an incoherent account of the problems on which he has been working, the interviewer excuses it on the ground of his own lack of mathematical training, an excuse which would be quite adequate if the circumstances demanded that he meet the applicant on the applicant's ground. What he overlooks is that the applicant has failed to meet him on his own ground; has failed, in other words, to display the essential ability to translate his thoughts into the language of his hearer. Or perhaps a personality defect is excused on the ground that "after all, he will be working by himself and won't have to meet people," whereas in fact the real value of a consultant comes not in what he does at his desk, but in how much of it gets through to his associates. The applicant who is boastful or pushing or querulous should not be hired on the general theory that "all mathematicians are queer."

High standards in all such matters, and an interest in practical things as well, are as important as technical mathematical ability. These are stiff specifications, and the men to fill them are not to be found in every market place. They are, however, the requirements implicit in the nature of the job and no good can come from failing to recognize them.

After the right man is hired, he is not a difficult person to supervise if his function as a consultant to the rest of the staff is kept clearly in mind. The broad objectives must be to avoid barriers which would tend to deter his associates from seeking his services, and to assure that his work is justly appraised and fairly compensated.

The three barriers most likely to arise between him and his associates are jealousy, red tape and unavailability.

Jealousy is unavoidable if the man himself is self-seeking; once such a man is hired trouble is inevitable. But the man is not always to blame. A generous and cooperative recruit will be spoiled by an atmosphere too highly charged with progress reports, or by a salary policy which bases revisions upon the dollar value of the last year's work. Actually the "progress" which is significant to management will be far more accurately appraised by his colleagues than by himself, hence his reports have little value except as they give him an opportunity to review and criticize his own activities. If too much emphasis is placed upon them, even this value will be lost and they will be written in the spirit of making a case for himself, which is exactly the spirit most certain to breed jealousy. Similarly, a salary policy based on dollar returns is essentially unjust, for the money value of various bits of theoretical work has almost no correlation with the scientific acumen which they require. This does not mean that a mathematician's pay should, in the long run, be independent of the dollar value of his services. It means only that whether he gets a raise this year, and how big it shall be, should properly be based on the size, character and satisfaction of his clientele, and not upon the commercial importance of the questions they saw fit to bring him last year.

Red tape is easily avoided by avoiding it. No engineer, whatever his rank in the organization, ought ever need permission to consult a mathematician in the company's employ, and the mathematician in turn ought not need a specific work order or expense allowance before giving his advice. In this respect he should be on the same basis as the free-lance investigators who are to be found in most large research laboratories, and who are generally known as staff engineers.

Unavailability is a more serious matter. It is well recognized that in industrial research the urgent job always tends to take precedence over the important one. Left to themselves, fundamental studies give way to the detailed development "which ought to go into production next month." Mathematical studies are no more susceptible than other fundamental research to such interruptions, but the effect upon the career of the mathematician may be more far-reaching, for as soon as he is assigned an urgent project of special character his availability as a consultant ceases or at best is temporarily impaired. If his value to the industry is greater as a project man than as a consultant this need not be a cause for regret; but to turn a good mathematician into a poor engineer, or an irreplaceable mathematician into a replaceable engineer, is unfortunate for both employer and employee.

The Mathematical Research Department of the Bell Telephone Laboratories

In the Bell Telephone Laboratories men of this type have been grouped together as a separate organization unit. They have no more specific function than to be helpful to their associates in other parts of the Laboratories. No engineer is obliged to consult them about any phase of his work; no particular jobs come to them by reason of prerogative; conversely, there is no sort of help which an engineer or physicist may not seek from them if he so desires. No routine need be complied with in advance in order to secure their services, and no report is required afterwards, though written reports are frequently prepared when needed for scientific record. The expense of the group is distributed broadly over the activities of the Laboratories, not charged to specific jobs. Every effort is made to maintain a spirit of service among the members of this group, and though responsibility for engineering projects occasionally descends upon them, it is regarded as an undesirable necessity to be avoided whenever possible and liquidated at the earliest opportunity.

The group has functioned successfully for a number of years. Its members are respected by their engineering associates, and like their jobs. Information regarding their activities reaches management almost entirely through spontaneous acknowledgments made by the engineers they assist. These expressions of appreciation are generous, but rather erratic in that they concentrate attention first on one man, then on another, as the genius and training of the individual happen to click with the important job of the moment. This has not affected the morale of the group adversely, probably because a serious effort is made to avoid erratic salary revisions in which the man who is at the moment in the limelight benefits at the expense of others who are doing equally good but less conspicuous work.

From the standpoint of the men, the principal advantages of being associated together instead of distributed through the engineering departments, is the stimulus of contact with men of like interests. From the standpoint of management, the advantages are wider availability, greater flexibility in matching the talents of the man with the requirements of the job, and a more uniform appraisal of ability because of supervision by a man of adequate mathematical background.

So far as is known, mathematicians have not been organized into separate administrative groups in other industries. In most laboratories their numbers have been thought too small to make such an arrangement feasible, and they have been treated as staff engineers distributed throughout the various general departments. It is believed, however, that there are a few industries in which this arrangement could be introduced with profit at this time, and that it has sufficient merit to justify its adoption wherever possible.

The Mathematician in the Small Laboratory

What has been said above relates primarily to conditions in large industries. The qualifications for success in the small industry are not dissimilar, though the relative emphasis to be placed upon them is somewhat different. Matters of personality (gregariousness, unselfishness, etc.) are not quite so important, because they are offset to some extent by the friendly coherence of the small group. On the other hand, a strong interest in things as well as ideas, and the ability to translate from the language of concrete experience to that of abstract thought and conversely, take on even greater importance. As Dr. H. M. Evjen, himself a worker in a small laboratory, says:

"In order to be of optimum value, the mathematician must keep in close touch with realities. In a sufficiently large organization, employing both theoretical and experimental men, the best results, therefore, can be obtained only by the closest cooperation between the two groups. In smaller organizations, employing —for instance—only one scientifically qualified man, it is difficult to say whether this man should be of the theoretical or the experimental type. If he is a theoretical man, no success can be expected unless he is willing to roll up his sleeves and get his feet firmly planted on the ground. In fact, even if he has highly qualified experimental assistants, he should not feel averse to 'getting down in the dirt.' Secondhand information is always of inferior quality. . . .

The mathematician not only is useful as an auxiliary to whom the practical man can turn with special problems. A properly trained mathematician, with a sufficiently broad vision, can be very much more useful as an active participant in the industrial problems. Due to his training in exact thinking he should be better able to see through the maze of intricate details and discover the fundamental problems involved."

Number Employed

The number of mathematicians employed in communications, electrical manufacturing, petroleum and aircraft is estimated at about 100. The number employed in other places is no doubt somewhat less, but it is probably not an insignificant part of the whole, since mathematicians are found here and there in some very small industries. For example, the Brush Development Company with a total engineering force of only 17, has found it desirable to supplement this group with a man hired specifically as a consultant in mathematics.

It is perhaps not too wide of the mark to estimate the total number at 150, not including actuaries and statisticians.

This number can be checked in another way. The membership list of the American Mathematical Society lists 202 men with industrial addresses. Of these, 102 are in financial and insurance firms and are presumably statisticians. The remaining 100 names are those of industrial employees with mathematical interests strong enough to belong to an organization devoted exclusively to the promotion of mathematical research. Some of these are not mathematicians by the definition adopted in this report. On the other hand, there are also 158 names for which only street addresses are given, some of whom are known to be industrial mathematicians. Balancing these uncertainties against one another, and remembering that many industrial mathematicians find little profit in belonging to an association devoted primarily to pure mathematics, the estimate given above does not appear unreasonable.

Future Demand

The appraisal of future demand is even more speculative than the estimation of present personnel. Two statements, however, seem warranted: (1) The demand for mathematicians will never be comparable to that for physicists, chemists or engineers. (2) It will certainly increase beyond the number at present employed.

The first statement is justified by the fact that physicists, chemists, and other experimental workers deal directly with the natural laws and natural resources which it is the business of industry to exploit, whereas mathematicians touch these things only in a secondary way.

The second statement would perhaps be granted on the general ground that throughout the whole of industry, research is becoming more complex and theoretical, and hence the value of consultants in general, and of mathematical consultants in particular, must increase. It is not necessary, however, to rely solely on such general considerations. Direct evidence exists in certain industries, notably aircraft,[1] where many of the major research problems are generally recognized to be more readily accessible to theoretical than experimental study, and in certain others, such as industrial chemistry,[2] where one may reasonably assume that modern molecular physics will soon begin to play an important part in determining speeds of reaction. There is also the general alertness of executives to the dollar value of a theoretical framework in planning expensive experiments, and the gradually changing attitude toward mathematics that stems from it. As Dr. W. R. Burwell, Chairman of the Brush Development Company, writes:

> "There is a definite trend toward a greater use of mathematics in industry which is somewhat commensurate with the trend toward the acceptance of research and development departments as necessary adjuncts to successful businesses. It is becoming more and more generally recognized that mathematics is not only a necessary tool for all engineers, physicists and chemists who make any pretense of going beyond strictly observational methods and experimental solutions to their problems but that it is also performing an important function as the recording medium for those generalizations which lay the foundation for the advances of scientific knowledge. . . .
>
> Even in an organization as small as ours, the use as a consultant is really important and we are constantly having instances where the mathematician because of his training is serving as an interpreter of mathematical and physical theories, sometimes influencing the direction of experimental work and sometimes eliminating the need for it."

If, therefore, the estimate of 150 mathematicians in industry at present is realistic, it may not be too wide of the mark to forecast several times that number a decade or so hence.

Source of Supply

Based on these estimates, a demand for new personnel of the order of 10 a year may be predicted. This number sounds small; but if we reiterate that mediocrity has no place in the consulting field, and that

[1] See pages 31–34.
[2] See pages 30–31.

these 10 must be *exceptional* men, it does not seem unreasonable to ask where they may be found.

Most mathematicians now in industry were trained as physicists or as electrical or mechanical engineers, and gravitated into their present work because of a strong interest in mathematics. Few came from the mathematical departments of universities. As scientists they are university trained, but as mathematicians they are self-educated.

Their training has not been ideal. Industrial mathematics is being carried on by graduates of engineering or physics not so much because of the value of that training as because of the weakness of mathematical education in America. The properly trained industrial mathematician should have, beyond the usual courses of college grade, a good working background of algebra (matrices, tensor theory, etc.), some geometry, particularly the analytic sort, and as much analysis as he can absorb (function theory, theory of differential and integral equations, orthogonal functions, calculus of variations, etc.). These should have been taught with an attitude sympathetic to their applications, and reinforced by theoretical courses in sound, heat, light and electricity, and by heavy emphasis upon mechanics, elasticity, hydrodynamics, thermodynamics and electromagnetic field theory. He should understand what rigor is so that he will not unwittingly indulge in unsound argument, but he should also gain experience in such useful but sometimes treacherous practices as the use of divergent series or the modification of terms in differential equations. He should have enough basic physics and chemistry of the experimental sort to give him a realistic outlook on the power as well as the perils of experimental technique. By the time he has acquired this training he will usually also have acquired a Ph.D. degree, but the degree itself is not now, and is not likely to become, the almost indispensable prerequisite to employment that it is in university life.

There is nowhere in America a school where this training can be acquired. No school has attempted to build a faculty of mathematics with such training in mind. Hence industry has had to make such shift as might be with *ersatz* mathematicians culled from departments of physics and engineering. To make matters worse, a student with strong theoretical interests who enrolls in physics these days is almost certain to spend most of his time on modern mathematical physics, which insists almost as little upon fidelity to experience and experiment as does "pure" mathematics, from which it differs more essentially in matters of language and rigor than of general philosophic attitude. At the moment, therefore, engineering schools must be looked upon as the most hopeful sources of industrial mathematicians.

Historically it is easy to explain how this situation came about. Fifty years ago America was so backward in the field of mathematics that there was not even a national association of mathematicians. A quarter of a century later it was just coming of age in mathematics and was properly, if not indeed necessarily, devoting its entire attention to improving the quality of instruction in the "pure" field. The first faint indications that industrial mathematics might someday become a career had indeed begun to appear, but they were not impressive enough to attract the attention of university executives.

Today we lead the world in pure mathematics, and perhaps also in that other field of mathematics which has somehow come to be known as modern physics. We have strong centers of actuarial and statistical training. But in the field of applied mathematics which is the particular subject of this report, we stand no further forward than at the turn of the century, and far behind most European countries.

A quarter of a century ago it would have been difficult to find suitable teachers. Just now it could be done, primarily because a number of European scholars of the right type have been forced to come here, and a few others have developed spontaneously within our own borders. There are perhaps half a dozen of them, but they are so scattered, sometimes in such unpropitious places, as to have little influence on the development of industrial personnel.

It is unfortunate that no university with strong engineering and science departments has seen fit to bring this group together and establish a center of training in industrial mathematics. We have estimated a demand of about 10 *exceptional* graduates per year. If that estimate is even remotely related to the facts, such a department would have a most important job to do.

Mathematics in Industry

Subjects Used

As Dr. H. M. Evjen, Research Physicist of the Geophysical Section of the Shell Oil Company, remarks:

> "Higher mathematics, of course, means simply those branches of the science which have not as yet found a wide field of application and hence have not as yet, so to speak, emerged from obscurity. It is, therefore, a temporal and subjective term."

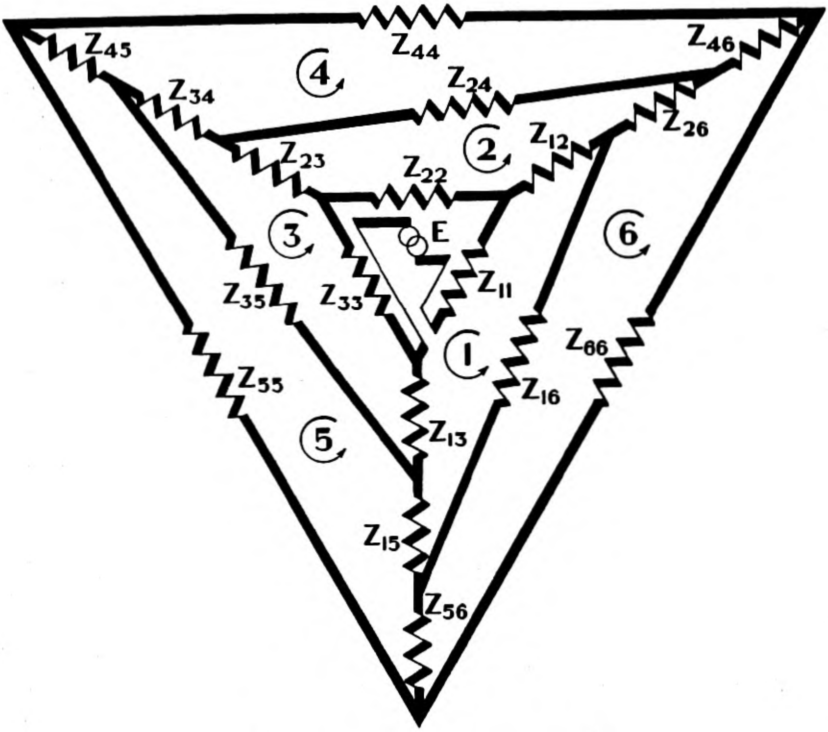
If this is accepted as a definition of higher mathematics—and it is a valid one for the pure science as well as for its applications—it follows automatically that industry relies principally upon the lower branches. What it uses much, ceases by the very muchness of its use to be high. The theory of linear differential equations, for example, is a subject by which the average well-trained engineer of 1890 would have been completely baffled. The well-trained engineer of 1940 takes it in his stride and regards it as almost commonplace. The well-trained engineer of 1990 will certainly regard as equally commonplace the theory of analytic functions, matrices and the characteristic numbers (Eigenwerte) of differential equations, which today are thought of as quite advanced.

With this as a background, there need be no apology associated with the statement that such simple processes as algebra, trigonometry and the elements of calculus are the most common and the most productive in modern industrial research. They frequently lead to results of the greatest practical importance. The single sideband system of carrier transmission, for example, was a mathematical invention. It virtually doubled the number of long distance calls that could be handled simultaneously over a given line. Yet the only mathematics involved in its development was a single trigonometric equation, the formula for the sine of the sum of two angles.

Next in order of usefulness come such subjects as linear differential equations (e.g., in studying the reaction of mechanical and electrical systems to applied forces, the strains in elastic bodies, heat flow, stability of electric circuits and of coupled mechanical systems, etc.); the theory of functions of a complex variable (particularly in dealing with potential theory and wave transmission, propagation of radio waves and of currents in wires, gravitational and electric fields as used in prospecting for oil, design of filters and equalizers for communication systems, etc.); Fourier, Bessel, and other orthogonal series (in problems of heat flow, flow of currents in transmission lines, deformation and vibration of gases, liquids and elastic solids, etc.); the theory of determinants (particularly in solving complicated linear differential equations, especially in the study of coupled dynamical systems); and the like.

Less frequently we meet such subjects as integral equations, which has been made the basis of one version of the Heaviside operational calculus, and which has also been used in studying the seismic and electric methods of prospecting for oil; matrix algebra, which has been applied to the study of rotating electric machinery, to the vibration of aircraft wings, and in the equivalence problem in electric circuit theory; the calculus of variations, in improving the efficiency of relays; and even such abstract subjects as Boolean algebra, in designing relay circuits; the theory of numbers, in the

DETERMINANTS



$$D = \begin{vmatrix} Z_1 & -Z_{12} & -Z_{13} & 0 & -Z_{15} & -Z_{16} \\ -Z_{12} & Z_2 & -Z_{23} & -Z_{24} & 0 & -Z_{26} \\ -Z_{13} & -Z_{23} & Z_3 & -Z_{34} & -Z_{35} & 0 \\ 0 & -Z_{24} & -Z_{34} & Z_4 & -Z_{45} & -Z_{46} \\ -Z_{15} & 0 & -Z_{35} & -Z_{45} & Z_5 & -Z_{56} \\ -Z_{16} & -Z_{26} & 0 & -Z_{46} & -Z_{56} & Z_6 \end{vmatrix} ; Z_j = \sum_{k=1}^{6} Z_{jk}$$

Driving point impedance in mesh j = $Z_{(jj)} = \frac{D}{D_{jj}}$

Transfer impedance between mesh j and mesh k = $Z_{(jk)} = \frac{D}{D_{jk}}$

(D_{jk} = the first minor of the element Z_{jk} in D)

Many properties of the complicated networks studied at Bell Telephone Laboratories are most conveniently expressed by means of determinants. Above are shown a six-mesh network; its "circuit discriminant", D; and some formulae which illustrate how simply the properties of the system can be found from D. Note that, since $Z_{jk} = Z_{kj}$, D is symmetrical.

design of reduction gears, and in developing a systematic method for splicing telephone cables; and analysis situs, in the classification of electric networks.

Least frequently of all, but by no means never, the industrial mathematician is forced to invent techniques which the pure mathematician has overlooked. The method of symmetric coordinates for the study of polyphase power systems; the Heaviside[3] calculus for the study of transients in linear dynamical systems; the method of matrix iteration in aerodynamic theory;[4] much of the technique used in the design of electric filters and equalizers—these may stand as illustrative examples.

The student of modern mathematics will be impressed at once by two aspects of this review: first, by the heavy emphasis on algebra and analysis, and the almost complete absence of geometry beyond the elementary grade; second, the complete absence of the specific techniques which play such a large role in modern physics and astrophysics. It is not easy to say just why advanced geometry plays no larger part in industrial research; however, the fact remains that it does not.[5] As regards modern physics, one may perhaps extrapolate from past history and infer that what is now being found useful in interatomic physics will soon be needed in industrial chemistry. In making this extrapolation, however, it is well to bear in mind that the physics in question is for the most part a mental discipline, its connection with the world of reality still ill-defined and incompletely understood. Therefore it may not prove to be as quickly assimilable into technology as have other disciplines whose symbols could be more immediately identified with experience.[6]

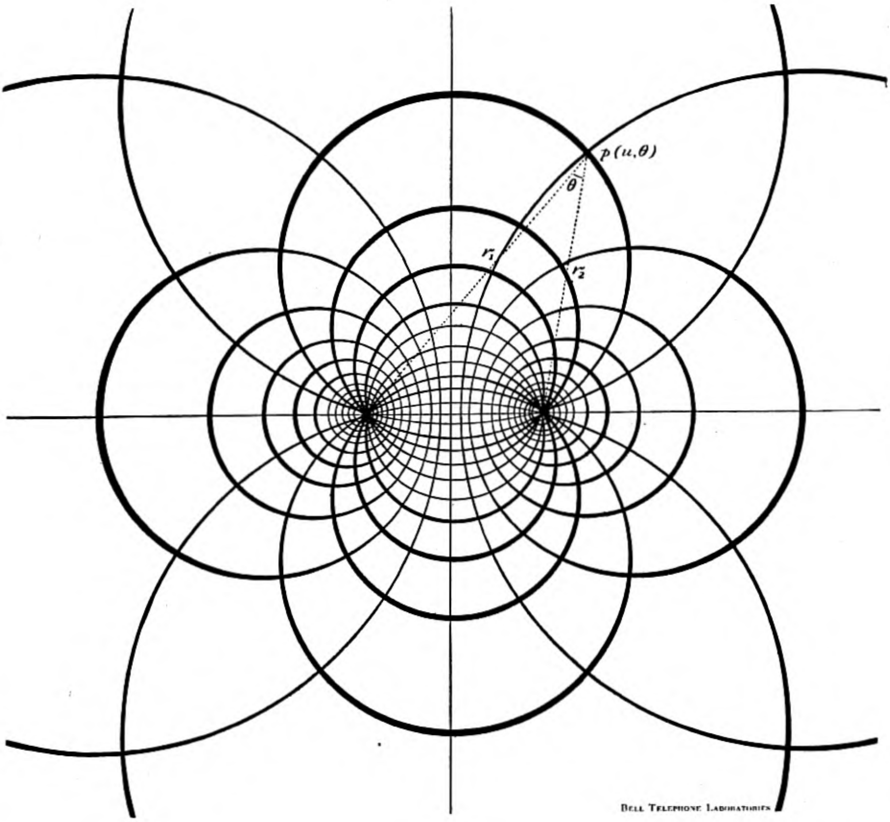
[3] Heaviside was not himself an industrial employee, but the reformulation of his work in terms of integral equations, and its interpretation in terms of Fourier transforms were both carried out in America by industrial mathematicians.

[4] This method was developed in The National Physical Laboratory of England, in the course of studies which in America would probably have been undertaken by a government or industrial laboratory.

[5] Mr. Hall C. Hibbard of the Lockheed Aircraft Corporation comments on this remark as follows: "It is possible that the usefulness of this principle of mathematics has been overlooked to a large extent in certain fields where it might be applied to advantage. In particular, that phase of engineering known as 'lofting,' which deals with the development of smooth curved surfaces, might offer an interesting field for certain types of advanced geometry. Practically all of this work is now done by 'cut and try' methods and the application of mathematics would no doubt save a great deal of time. The same thing is true in the field of stress analysis, where a great deal of time is absorbed in determining the location and direction of certain structural members. It is even possible that the application of vector analysis technique would greatly simplify certain forms of structural analysis, particularly space frameworks. The lack of application of geometry in these fields is probably due to the wide gap that exists between the mathematician and the 'practical' designer and draftsman. Advanced geometry might also turn out to be a very useful tool in connection with problems that we are now encountering in the forming of flat sheet into surfaces with double curvature, an operation that is extensively employed in aircraft manufacture."

[6] In this connection, see the quotation from Dr. E. C. Williams on pages 30–31.

Bicircular Coordinates



$$(x + \coth u)^2 + y^2 = \operatorname{csch}^2 u; \; x^2 + (y - \cot\Theta)^2 = \csc^2\Theta$$
$$u = \log (r_2/r_1)$$

Using the bicircular system of coordinates facilitates finding the distribution of electric charge on two parallel conductors, and thence their capacity. Rotating the bicircular system about the vertical axis generates a toroidal coordinate system which facilitates determining the capacity of a torus.

Finally, we must remark upon two facts: (1) that approximate solutions of problems, and hence methods of iteration (successive approximation), play a much more conspicuous role in applied mathematics than in the pure science; (2) that the highly convenient assumption that linear approximations to natural laws (such as Hooke's law and Ohm's law) are sufficiently exact for practical purposes is less often true than formerly was the case, so that nonlinear differential equations are of great importance to the modern engineer.

Types of Service Performed by Mathematics

Leaving aside the important but rather trite observation that mathematics is a language which simplifies the process of thinking and makes it more reliable, and that this is its principal service to industry, we may distinguish certain less inclusive, but perhaps for that reason more illuminating, categories of usefulness.

First: It provides a basis for interpreting data in terms of a preconceived theory, thus making it possible to draw deductions from them regarding things which could not be observed conveniently, if at all.

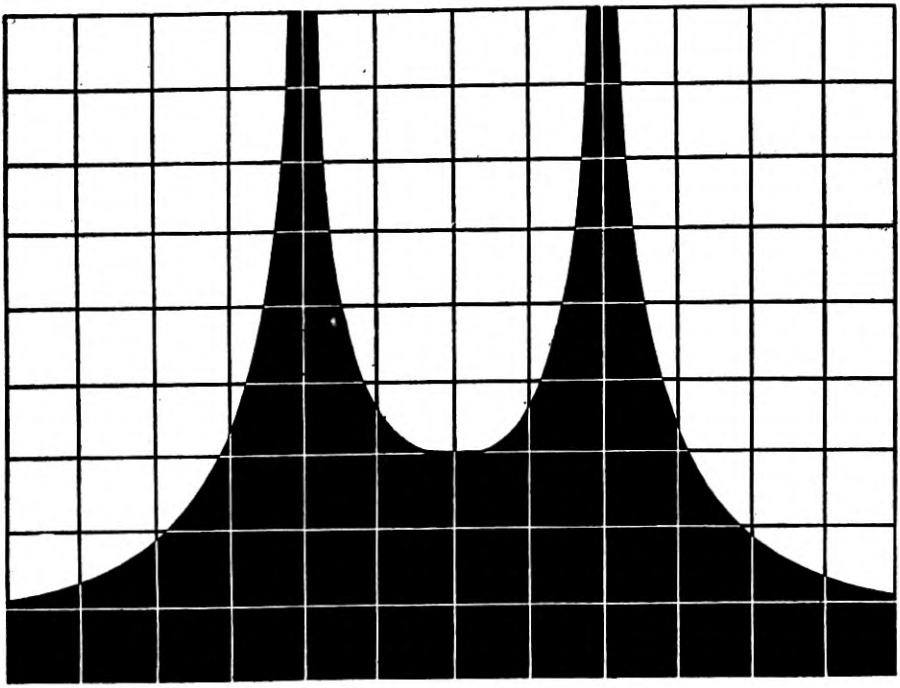
(*a*) An illustration is the standard method for locating faults on telephone lines. Mathematical theory shows that a fault will affect the impedance of the line in a way which varies with frequency, and that the distance from the place of measurement to the fault can be deduced at once from the frequencies at which the impedance is most conspicuously affected. This is obviously much more convenient than hunting the fault directly.

(*b*) A second illustration is the mapping of geological strata by means of measurements made upon the surface of the earth. One method extensively employed uses a large number of seismographs, each of which records the miniature earthquake shock produced at its location by a charge of dynamite set off at a known place. A theory of reflection and refraction similar to that used in geometrical optics shows that certain observable characteristics of these records are related to the depth and tilt of the underground layers, and hence enables the situation of these layers to be plotted. By this means the location of the highest point of an oil-bearing stratum can be found, and the most favorable position for drilling determined.

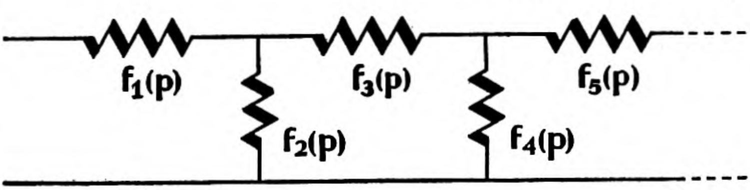
Underground geology is also studied by means of gravity, electrical or magnetic measurements upon the surface. In this case the basic theory is that of the Newtonian potential field, and the interpretation of the data leads into the subject of inverse boundary value problems, which is still insufficiently understood. Enough progress has been made in several geophysical laboratories, however, so that the gravity method is now being widely used, and the electrical methods appear promising for some applications.

Second: When data are incompatible with the preconceived theory, a mathematical study frequently aids in perfecting the theory itself. The

CONTINUED FRACTIONS



$$Z = f_1(p) + \cfrac{1}{\cfrac{1}{f_2(p)} + \cfrac{1}{f_3(p) + \cfrac{1}{\cfrac{1}{f_4(p)} + \cdots}}}$$



A mathematical method of systematically designing a circuit of predetermined impedance has been developed in Bell Telephone Laboratories. The given impedance, as a function of frequency, is expanded in a Stieltjes continued fraction, whose terms give the electrical constants of the desired network.

classical illustration in pure science is the discovery of the planet Neptune. The motion of the planet Uranus was found to be inconsistent with the predictions of the Newtonian theory of gravitation, if the solar system consisted only of the seven planets then known. Mathematical investigation indicated, however, that if an eighth planet of a certain size was assumed to be moving in a certain orbit, these discrepancies disappeared. Upon turning a telescope to the spot predicted, the new planet was found.

An illustration comes from the aircraft industry. I quote it from a report sent me by Mr. C. T. Reid, Director of Education of the Douglas Aircraft Company:

(*c*) "The behaviour of airplanes with 'power on' did not check closely enough with stability predictions which had been made without consideration of the effects of the application of power; therefore, a purely mathematical analysis of the longitudinal motion of an airplane was carried out, involving the solution of three simultaneous linear first-degree differential equations. The results led to the development of equations for dynamic longitudinal stability with 'power on' which enable the aerodynamicist more accurately to predict the stability characteristics of a given design. 'Power-on' dynamic longitudinal stability is an important design criterion in aircraft construction."

(*d*) Another illustration arises in communication engineering. Theoretical studies had established the fact that vacuum tubes would spontaneously generate noise because of the discrete character of the electrons of which the space current is composed. The theory predicted how loud this noise would be in any particular type of vacuum tube, a most significant result since it established a limit to the weakness of signals which could be amplified by this type of tube. The predictions of the theory were supported by experimental data so long as the tubes were operating without appreciable space charge. But it was found that when space charge was present the noise level fell far below the predicted minimum. In this case the missing factor in the theory was immediately obvious, but an understanding of the mechanism by which the reduction was affected and its incorporation into the theory in a workable form, required an extensive and difficult mathematical attack.

Third: It is frequently necessary in practice to extrapolate test data from one set of dimensions to a widely different set, and in such cases some sort of mathematical background is almost essential.

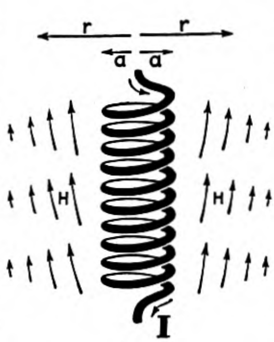
An example of this kind of service, concerned with the theory of arcs in various gases, is furnished me by Mr. P. L. Alger, Staff Assistant to the Vice President in Charge of Engineering, of the General Electric Company:

(*e*) "An example of this kind of problem is that of the theory of arcs in various gases. It has been experimentally known that the duration, stability and voltage characteristics of electric arcs in different gases and under different pressures vary very widely. The behaviour of such arcs is of great importance, both in

Elliptic Integrals

$$H=\frac{4I}{r}\left[\frac{1}{1-k^2}\int_0^{\pi/2}\sqrt{1-k^2\sin^2\lambda}\,d\lambda-\int_0^{\pi/2}\frac{d\lambda}{\sqrt{1-k^2\sin^2\lambda}}\right]$$

Some simple engineering problems require advanced mathematics in their solution. This is true, for example, in the computation of the magnetic field outside the spiral grid of a vacuum tube, a problem of interest to Bell Telephone Laboratories. If the grid is closely coiled, the current can be treated as a continuous cylindrical sheet, of radius *a*. Then the component of the magnetic field parallel to the axis of the grid at a distance *r* from the axis is given by the above function of two Elliptic Integrals whose "modulus" is *k=a/r*.



welding and in the design of circuit breakers and other protective devices. Recently a mathematical theory has been developed which relates the arc phenomena to the heat transfer characteristics of different gases. This theory has given excellent correlation between the known experimental results, and has enabled very useful predictions of performance under new conditions to be made. The theory has been applied in the design of high voltage air circuit breakers, which are of important commercial value, and it is also greatly curtailing the time and expense necessary to develop many other devices in which arc phenomena are of importance."

A second example, furnished me by Mr. Reid, has to do with the interpretation of wind-tunnel data in aerodynamics:

(*f*) "Here it is obviously impracticable to perform full-scale tests of such parts as wings or fuselage, much less of entire aircraft, and the extrapolation from the results of wind-tunnel measurements to the full-scale characteristics of airplanes must be based on theoretical considerations."

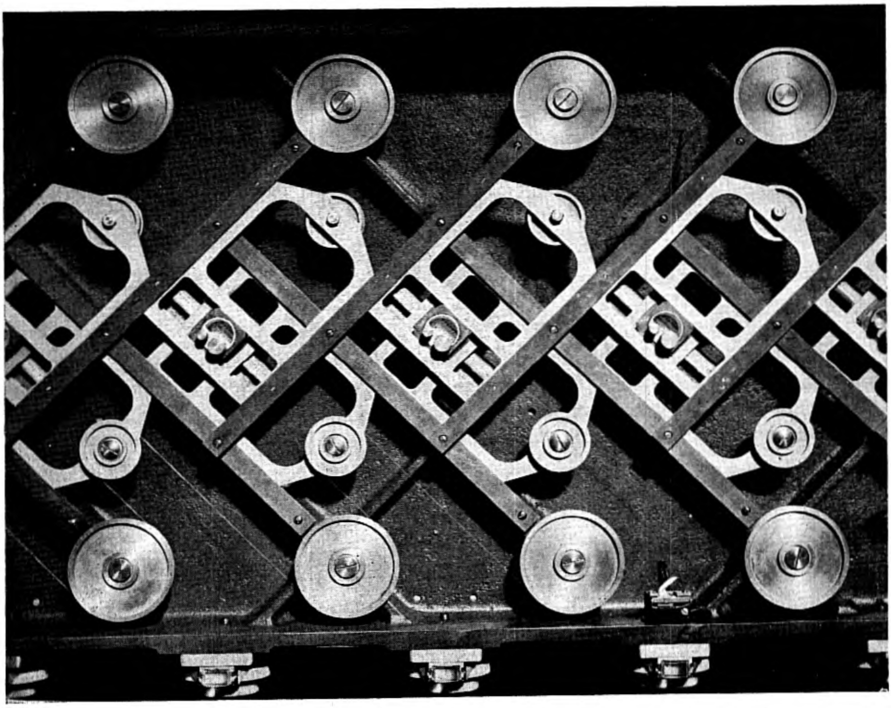
Fourth: Mathematics frequently aids in promoting economy either by reducing the amount of experimentation required, or by replacing it entirely. Instances of this kind are met everywhere in industry, not only in research activities, but in perfecting the design of apparatus and in its subsequent manufacture as well.

Mr. Alger describes in general terms one situation frequently met in research activities as follows:

"The first type of problem is one in which there are so many different independent dimensions of a proposed shape to be chosen, or in general so many independent variables, that it is hopeless to find the optimum proportions by experiment. The truth of this can readily be seen when it is realized that the number of test observations to be made increases exponentially with the number of variables. If 10 points are required to establish a performance curve for one variable, 1,000 observations will be required if there are 3 independent variables, and a million if there are 6 variables."

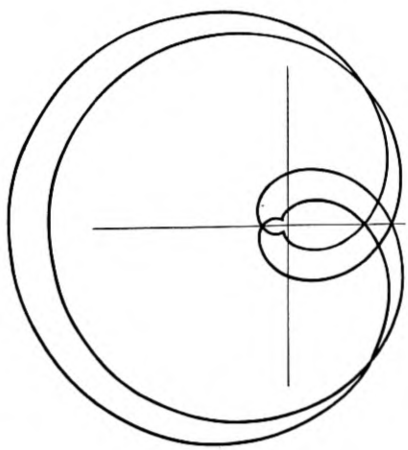
As an illustration he cites the following problem:

(*g*) "An example of this kind of problem is that of designing a *T* dovetail to hold the salient poles in place on a high speed synchronous generator. A large machine of this type may have 10 or more laminated poles carrying heavy copper field coils, each assembled pole weighing several tons and traveling at a surface peripheral speed of 3 miles a minute. The centrifugal force on each pound of the pole then amounts to approximately 500 pounds. The problem of designing dovetails to hold these poles in place, even at over speed, is, therefore, one of great importance and technical difficulty. For each such dovetail, there are 7 different dimensions which may be independently chosen. While empirical methods have enabled satisfactory results to be obtained in some cases, application of mathematics has recently enabled marked improvements in dovetail



THE ISOGRAPH

The Isograph was developed in Bell Telephone Laboratories to find mechanically the complex roots of polynomials of high degree. Let the polynomial to be factored be $p(z) = \sum_0^n a_j z^j$ or $\sum_0^n a_j r^j \cos j\Theta + i\sum_i^n a_j r^j \sin j\Theta$ if $z = r(\cos\Theta + i\sin\Theta)$. The isograph maps the complex values of $p(z)$ as the variable describes the circle $|z| = r$. This graph loops the origin once for each root smaller in absolute value than r. The number of roots between trial values of r is determined by counting loops, and by interpolation a value of r is found for which the graph passes through the origin. This value of r and the corresponding value of Θ define the real and imaginary parts of a root.



designs to be made. Generally speaking, these improvements have permitted an overall strength increase of 20 per cent to be obtained under steady stresses, and much higher gains to be made under fatigue stress conditions; while at the same time the certainty of obtaining the desired results on new designs has been very greatly enhanced."

A second example was brought to my attention by Mr. L. W. Wallace, Director of the Engineering and Research Division of the Crane Company:

(*h*) "A pipe fitting weighing several hundred pounds and intended for high pressure service had a neck of elliptical cross-section. As originally designed, the thickness of the casting was intentionally not uniform, the variations having been introduced empirically to strengthen it where strength was supposed to be most needed. A redesign carried out on the basis of the theory of elasticity showed the distribution of metal to be inefficient and resulted in a new casting in which the weight was reduced by half, while at the same time the bursting strength was doubled. The method used in arriving at this result is an interesting illustration of sensible mathematical idealization. The casting was regarded as an elliptical cylinder under hydrostatic pressure. As the stresses for this idealized structure were already known, the design problem reduced at once to the simple matter of establishing thicknesses sufficient to withstand these stresses."

Another example from the field of geophysical prospecting is furnished by Mr. Eugene McDermott, President of Geophysical Service, Inc.:

(*i*) "A specific case of mathematical research in instrument design was recently encountered. The instrument in question was intended for the measurement of gravity. After the machine had been completely built it was found to be unexplainably inaccurate. After weeks of trial and error it was turned over to a mathematician to try to find the trouble. He soon showed by simple trigonometry that the axis of the instrument would have to be located on its pivot with an accuracy which is not attainable. He also pointed out a means of avoiding this feature by a relatively simple change in design, and this appears to have remedied the trouble."

Another illustration from the petroleum industry, but this time concerned with the production of oil rather than prospecting for it, comes from Dr. E. C. Williams, Vice President in Charge of Research of the Shell Development Company:

(*j*) "The petroleum industry has one important problem not found in other fields; it has to do with oil production from the ground. A mathematical problem arising from this subject is the following: The oil-gas mixture underground flows under pressure through porous media; with a certain spacing of wells, determine the most economical way to recover this mixture. This is sometimes equivalent to asking: 'In what way can the largest fraction of the oil be obtained over a certain period of time?' Simplified problems of this kind have been solved by potential theory methods, since classical hydrodynamics becomes too involved, and in the general problems where the flow constants vary with liquid-gas composition, etc., partial differential equations are found which can be solved by approximate methods. On the basis of the solution of this mathematical problem, aided by extensive laboratory determinations of the required constants, one is able to find the best of several ways of producing from a given oil field."

As a final example under the heading of economy, we may mention the flight testing requirements imposed upon the aircraft industry by the Civil Aeronautics Authority. Of these, Mr. E. T. Allen, Director of Flight and Research of the Boeing Aircraft Company, says:

(*k*) "It was formerly required that each type of transport plane must be tested at all the altitudes at which it was intended to be flown, and at all flying fields where it was expected to be used. The cost of such testing was extremely high. A mathematical study of steady flight performance has, however, identified the basic parameters and established their relations to one another. This has made possible a scientific interpretation of flight test data taken at any suitable location convenient to the aircraft factory, and a reliable conclusion therefrom as to the performance to be expected under other conditions. This has greatly reduced both the cost and the time necessary to establish performance figures."

Fifth: Sometimes experiments are virtually impossible and mathematics must fill the breach. An example comes to me from Mr. Hall C. Hibbard, Vice President and Chief Engineer of the Lockheed Aircraft Corporation:

(*l*) "An unfortunate phenomenon that must be dealt with in aircraft design is a type of violent vibration which may be set up in the wings if the plane is flown too fast. It is known as flutter, and is highly dangerous, since the vibrations may be of such intense character as to cause loss of control or even structural failure. The technical problem is therefore to be sure that the critical speed at which flutter would occur is higher than any at which the craft would ever be flown. It is a phenomenon with respect to which wind tunnel experimentation is difficult, and flight testing very dangerous. It has been the subject of a number of mathematical investigations, the results of which have reached a sufficiently advanced stage that they are now being used to predict the critical speeds and flutter frequencies of aircraft while still in the design stage. Even more important, the mathematical investigation of this problem points the way to modifications of design which will insure that flutter cannot occur in the usable speed range."

Telephony provides a second example:

(*m*) The equipment in an automatic telephone exchange must be capable of connecting any calling subscriber with any called subscriber. It consists of several stages of switches, each of which can be caused to make connection with a number of trunks which lead in turn to switches in the next succeeding stage. Enough switches must be provided so that only a very small proportion of subscribers' calls will fail to be served immediately. Since the demands made by the subscribers fluctuate from moment to moment, the number of switches required depends in part upon the height to which the crests occasionally rise in this fluctuating load. It is also influenced, however, by the way the trunks are arranged, by the order in which the switches choose them, and by many other factors. Experimental appraisal of the effect of these various factors is impossible, both because it would be very costly, and because it would be exceedingly slow. Mathematically, however, they have been studied by the theory of *a priori* probability,[7] which is used not only in determining how much apparatus to install in a working exchange, but also in comparing the relative merits of alternative arrangements while in the development stage.

Sixth: Mathematics is frequently useful in devising so-called crucial experiments to distinguish once for all between rival theories. A famous example in the field of physics was the study of the refraction of starlight near the sun's disc, which afforded a means of deciding between Newtonian and relativistic mechanics. In this case, mathematical investigation showed that the result to be expected was different according to the two theories, and astronomical observations confirmed the prediction of relativistic mechanics. In the industrial field, an example of this kind comes to me from Dr. Joseph A. Sharpe, Chief Physicist in the Geophysical Laboratory of the Stanolind Oil and Gas Company:

(*n*) "As an example of the second sort of use of analysis there is the case of our study of 'ground-roll,' the large amplitude, low frequency surficial wave which caused so much grief in the early days of seismic reflection prospecting when filters were not used as extensively as at present. We hope to use our study of this wave motion as an aid to a better understanding of the properties of the surficial layers of soil and their effects on the reflected waves in which we are primarily interested.

Two views on the ground-roll are current, although neither is based on very much observation, and this of an uncontrolled sort. One view states that the ground-roll is an elastic wave. Analysis predicts that this wave will have a certain velocity in relation to the velocities of other waves, that it will have a certain direction of particle motion and relation of maximum horizontal to maximum vertical component of displacement, that it will attenuate with distance according to a certain law, that it will attenuate with depth in a certain way, and that its velocity will follow a certain dispersion law. The second view maintains that the 'ground-roll' is a wave in a viscous fluid, and analysis predicts a behavior which is similar in certain cases, and different in others, to that of the elastic wave. Having the predictions of the analysis at hand, we are enabled to devise a group of observations, and the special equipment for their prosecution, which will provide crucial tests of the two hypotheses."

Seventh: Mathematics also frequently performs a negative service, but one which is sometimes of very great importance, in forestalling the search

[7] *Not* statistics, which is *a posteriori* probability. This is one of the few cases in industry where the *a priori* theory finds application.

for the impossible; for many desirable objectives in industry are as unattainable as perpetual motion machines, and frequently the only way to recognize the fact is by means of a mathematical argument.

(*o*) A certain type of electric wave filter which is usually referred to as an "ideal" filter would be very useful if it could be produced. However, it has been shown mathematically that such a structure would respond to a signal before the signal reached it; in other words, that it would have the gift of prophecy. Since this is absurd, it follows that no such filter can be built, and consequently no one tries to build it.

Still another example from the field of communication deals with the design of feedback amplifiers.

(*p*) In practice, any amplifier is intended to handle signals in a given frequency band. For various reasons, it is preferable not to have it amplify disturbances outside this band, and hence its gain characteristic is made to drop off as rapidly as possible outside the limits of the useful band. It has been shown theoretically, however, that the gain cannot decrease at more than a certain rate, which can easily be computed, without causing the amplifier to become unstable. As a matter of fact, the allowable rate at which the gain may fall is often surprisingly low, and a great deal of design effort would be wasted in the attempt to obtain an impossible degree of discrimination if the theoretical limitations were unknown.

Eighth: Finally, mathematics frequently plays an important part in reducing complicated theoretical results and complicated methods of calculation to readily available working form. So many and so varied are the services falling in this category that it is difficult to illustrate them by means of examples. We arbitrarily restrict ourselves to two, chosen primarily for the sake of variety. The first comes from Mr. Hibbard:

(*q*) "In aircraft design the metal skin, though thin, contributes a large part of the structural strength. Nevertheless, such thin metallic plates will buckle or wrinkle after a certain critical load is exceeded. Beyond this point the usual structural theories can not be applied directly and it is therefore necessary to introduce new methods of attack to predict the ultimate strength of the structure. These stiffened plates are difficult to deal with theoretically, but by interpreting the effect of the stiffeners as equivalent to an increase in plate thickness or a decrease in plate width, the calculations can be brought within useful bounds."

The reduction of electric transducers to equivalent T or Π configurations, the interpretation of the elastic reaction of air upon a microphone as equivalent to an increase in the mass of its diaphragm, the postulation of an "image current" as a substitute for the currents induced in a conducting ground by a transmission line above it, and a host of other common procedures could be cited as similar instances of simplification based upon more or less valid mathematical reasoning.

The second example is furnished by Dr. E. U. Condon, Associate Director of the Research Laboratories of the Westinghouse Electric and Manufacturing Company:

(r) "In the manufacture of rotating machinery it is of extreme importance to have the rotating parts dynamically balanced, in order to reduce to a minimum the vibration reaction on the bearings which unbalance produces. Theory shows the phases and amplitudes of the bearing vibrations produced by excess masses located at various places on the rotor; conversely, by solving backward from observed vibration data, one can compute what correction is needed to eliminate the unbalance. Recently a most valuable machine has been developed which not only measures the unbalance, but also automatically shows what correction should be made, thus eliminating the necessity for these calculations.

The rotor to be balanced is whirled in bearings on which are mounted microphones that generate alternating voltages corresponding to the vibrations of the bearings. These voltages are fed into an analyzing network, which automatically indicates the correction needed in order to achieve dynamic balance. In some cases the output of the balancing machine has been arranged to set up a drilling machine so it will automatically remove the right amount of metal at the right place. These machines are finding application in the manufacture of small motors, of automobile crankshafts, and in the heavy rotors of power machines."

In the same class would come the isograph, by means of which the complex roots of polynomials can be located; the tensor gauge which registers the principal components of strain in a stressed membrane without advance knowledge of the principal axes; and slide rules for a great variety of special purposes such as computations with complex numbers, the calculation of aircraft performance, aircraft weight and balance, and the like. Perhaps we ought also include in the same category the use of soap-bubble films for the study of elastic stresses in beams, the use of current flow in tanks of electrolyte for the study of potential fields, and the use of steel balls rolling on rubber membranes stretched over irregular supports as a means of studying the trajectories of electrons in complicated electric fields. These are all mechanical methods for saving mathematical labor, but they are more than that, for they all rest upon a foundation of mathematical theory. They are, in fact, examples of the use of mathematics to avoid the use of mathematics.

Mathematics in Some Particular Industries

Communications

The communication field is the one in which mathematical methods of research have been most freely used. This is due partly to the fact that the transmission of electric waves along wires and through the ether follows laws which are particularly amenable to mathematical study; partly also to the fact that so much of the research has been centralized in a single laboratory, thus bringing together a large number of engineers into a single compact group, and justifying the employment of consultative specialists. Most important of all, however, is the fact that there are two devices—vacuum tubes and electrical networks—without which modern long-distance telephony would be impossible; and one of these, the electrical network, is and has been since its earliest days almost entirely a product of mathematical research. Mathematics has thus been as essential to the development of nation-wide telephony as copper wire or carbon microphones.

Number of Mathematicians. The Mathematical Research Department of the Bell Telephone Laboratories contains 14 mathematicians. Perhaps an equal number of men scattered through various engineering departments should also be classified as mathematicians according to the definition adopted for this report. Say a total of 25 or 30 for the Bell Laboratories, a few more for the Bell System as a whole, and perhaps 40 or 50 for the entire communication field including the companies interested in radio and television. A few of these men carry on a considerable amount of experimentation, but their significant work is theoretical.

In addition, there is a much larger number of men who use mathematical methods extensively in their daily work, but whose mental type is not that which we have described as mathematical, and who are therefore not included in the numbers quoted above. This is true in particular of the engineers who have the responsibility for designing networks.

Uses of Mathematics. Mathematical activity is most intense: (1) in designing wave filters and equalizers, (2) in studying transmission by wire and ether, the concomitant problems of antenna radiation and reception, inductive interference between lines, etc., (3) in studying various problems related to the standard of service in telephone exchanges, such as the amount of equipment required, the probability of delays and double connections, the hunting time of switches, etc., (4) in providing a rational basis for the design of instruments, such as transmitters and receivers, vacuum tubes, television scanning devices, etc., (5) in developing efficient statistical methods for the planning and interpretation of experiments, and for controlling the quality of manufactured apparatus.

Future Prospects. During the last 20 years the number of men employed in communication research has increased with great rapidity, but this rapid expansion appears to be about over. A large increase in the mathematical personnel of the industry therefore appears unlikely. It seems inevitable that the problems will increase in complexity, and that theoretical methods will become increasingly important, but it is believed that this trend will be matched by progressively better trained engineering personnel, rather than by an increased number of mathematicians. Indeed, unless the qualifications of the mathematicians rise progressively with those of the engineers, it may turn out that less rather than more will be employed.

Electrical Manufacturing

Substantially all the research in the power fields is carried on by a few electrical manufacturers. The power companies usually accept and exploit such equipment as the manufacturers supply, and contribute to improved design principally through their criticisms of past performance. Many of their engineers, however, are individually active in the invention and development of improved equipment.

Number of Mathematicians. The number of mathematicians in the industry is smaller than in communications, and is not easy to estimate because their work is less segregated from other activities. The total number who would here be rated as mathematicians is probably about 20.

As in communications, some are engaged partly in experimental work. There are some, however, whose relationship as consultants is clearly recognized, and there is evidence that management is becoming increasingly conscious of the nature and value of their services.

Uses of Mathematics. Mathematical activity is most intense: (1) in studying structural and dynamic problems, such as the strain, creep and fatigue in machine parts, vibration and instability in turbines and other rotating machinery, etc., (2) in appraising the evil effects of suddenly applied loads, lightning or faults upon power lines, and their associated sources of power, and devising methods to minimize these effects, (3) in studying system performance, particularly the most effective or economical location of proposed new equipment, and the evaluation of performances of alternative transmission or distribution systems, (4) in refining the design of generators, motors, transformers and the like, so as to improve their electrical efficiency and reliability, and in similar improvement of the thermal efficiency of turbines, (5) in the design of miscellaneous instruments and apparatus.

Statistical methods are being introduced into manufacturing and research, but are not yet utilized to the same extent as in telephony.

Future Prospects. The amount of money spent on development in these industries is gradually increasing, and as in other fields the problems are becoming more complex. Hence a slow increase in the number of mathematicians seems probable, with rising standards in the qualifications required, not only as to mathematical training, but as to temperament and personality as well.

The Petroleum Industry

The petroleum industry consists of many producing units of various sizes, highly competitive in character, and surrounded by a number of consulting service organizations, all of which are small. The larger producing companies—and within their resources, the service units also—maintain research laboratories. They tend to be secretive about the developments which take place in these, sometimes to a surprising degree. Hence there is much duplication of effort, particularly in such matters as the design of instruments for geophysical prospecting, and in methods of interpreting the data derived from them.

Number of Mathematicians. The industry employs more mathematicians than is generally appreciated, some of them men of very considerable ability. The total of first-rank men is perhaps 15 or 20. Due to the small size of the individual research staffs, however, most of these men carry considerable project responsibility along with their theoretical work. This is the normal state of affairs in small groups: the abnormality is the lack of contact with, and stimulus from, similar men in other companies.

Uses of Mathematics. Petroleum research extends in three directions: prospecting for oil, producing it, and refining it.

There are five recognized methods of prospecting: gravity, seismic, electric, magnetic and chemical. In the first four, important mathematical problems arise in designing sufficiently sensitive instruments and in interpreting data. The fifth requires the use of statistical methods.

Research on methods of producing a field has led to a few mathematical studies of underground flow, and would undoubtedly give rise to others if the results of these studies could be profitably applied. However, since the rate at which oil is brought to the surface is almost entirely determined by law, and the same is indirectly true of well location also, mathematical consideration of the subject is largely sterile, at least so far as American oil fields are concerned.

The third activity—refining—is essentially a chemical industry. Hence the following remarks by Dr. E. C. Williams, Vice President in Charge of Research of the Shell Development Company, presumably apply not only to the petroleum business, but to manufacturing chemistry in general:

"The two chief problems in chemistry are (aside from the identification of substances): The calculation of chemical equilibrium and the calculation of the rates of attainment of these equilibria. The first problem, involving thermodynamics and statistical mechanics, is rather well understood and usually, by very simple computations, information sufficiently accurate for industrial application, at least, can be found. Frequently, when several equilibria are possible simultaneously, complicated equations arise, but we rarely solve them directly, but rather set up tables of the dependent variable (the per cent conversion possible) as a function of the independent variables (temperature, pressure concentration). The sources of these data, however, are numerous and at times require complicated mathematics, as in the calculation of thermodynamic properties from spectroscopic data via quantum statistics.

The situation is much less favorable in the calculation of the rates of chemical reactions. A semi-empirical method, based on quantum mechanics, has been applied with a little success to some of the simplest reactions taking place in the gas phase, but virtually no progress has been made in the more important field of heterogeneous reactions (reactions of gases on surfaces, for example). We may say that no satisfactory mathematical theory for such calculation exists at the present time. Some progress is being made, but we are far from being able to predict a suitable catalyst for any desired reaction. For the present we are happy to be able to account for observations made on some simple reactions."

Future Prospects. It is inconceivable that research in the industry will not continue at at least its present level. Hence more, rather than less mathematical work will probably be undertaken in prospecting and in refining. A demand of moderate proportions should exist for able mathematicians with a suitable background of geology and classical physics for the geophysical work, and of physical chemistry and molecular physics in the chemical field.

Aircraft Manufacture

The aircraft industry also consists of a number of independent units, and is highly competitive. It is a new industry in which rapid technical development and rapid increase in size has been the rule. It has depended primarily upon government-supported laboratories and, to a lesser extent, upon the universities for its research, and has busied itself with the exploitation of that research in the advancement of aircraft design. No unit of the industry has had or, for that matter, now has a research laboratory, in the sense in which the words would be used in older and larger businesses, but the beginnings of research departments have appeared, and individual researchers and research projects are clearly recognizable.

Numbers of Mathematicians. Some men in the engineering departments of these companies should undoubtedly be classed as mathematicians, but it is impossible to make even an approximate estimate of their number. It is possible, however, to cite pertinent information which bears on the importance of mathematics to the industry.

The design of a modern four-engine transport plane requires about 600,000 hours of engineering time up to the point where complete working drawings have been prepared. About 100,000 hours are spent on mathematical analysis of structures, performance, lift distribution and stability. Most of this work is routine, but some is fundamental in character, as is evident from several of the examples mentioned earlier in this report.

Of 670 men in the engineering department of one of the larger companies, about 25 have mathematical training beyond that usually obtained by engineers, and 10 or so of these are using this advanced training to a significant extent.

Uses of Mathematics. In designing an airplane, five factors are of particular importance. These may be used to indicate the directions in which mathematical research may be expected.

(1) Performance (that is, pay-load, range, speed, climbing rate, etc.)

In the past, forecasts of performance have been based almost entirely on empirical data. Mathematical methods of estimation are now being developed from hydrodynamic theory, however, and are being used to an increasingly greater extent.

(2) Lift and Drag (i.e., the force variation over the wings)

This is the principal objective in the aerodynamic design of the wing. The technique of prediction rests on two supports: wind tunnel experiments and airfoil theory, by means of which experimental data are interpreted and applied. For example, airfoil theory suggests the shape of airfoil to avoid unfavorable pressure distributions and is leading to improved wing sections. This part of aircraft design is already highly mathematical, but a number of fundamental problems still remain unsolved. For example, the theory is still unable to predict stall, and too little is known about optimum shapes or about turbulence, though the recently developed statistical theory of turbulence has contributed to the understanding of the airflow over an airplane and resulted directly in a decrease in airplane drag and consequent improvement in performance.

(3) Stability (inherent steadiness of motion)

The stability of an airplane in flight is inherent in its aerodynamic design and quite distinct from its control or maneuverability. The theory of "small oscillations" has been successfully applied to rectilinear flight. More recently the problem of predicting the response of an airplane to control maneuvers has used the Heaviside operational calculus. Current problems of dynamical stability in which applied mathematicians are interested are the behavior of an airplane when running on the ground and the behavior of seaplanes when running on the water (porpoising).

(4) Structural Safety

Very precise appraisal of structural strength is required in aircraft design. In most industries inaccuracy can be compensated by increased factors of safety, but the pay-load of an airplane is so small a proportion of its total weight that slight increases in factors of safety would seriously reduce its carrying power or even make it unable to get off the ground. Mathematical methods have always been used in this phase of aircraft design in so far as they were available. The standard technique is first to design a part on the basis of calculated strength, then build and test it, and if the tests do not agree with predictions, revise the design and build and test the modified part. This process is continued as many times as necessary to attain a satisfactory result. It is slow and expensive. Theoretical methods are now reliable enough that the majority of structural tests confirm predictions with sufficient accuracy to require no revision. However, new problems constantly present themselves—the introduction of pressurized cabins recently gave rise to several—and hence continual mathematical study is required. A beginning has also been made in the use of the principles of probability in setting up structural loading factors.

(5) Flutter

We have already commented upon the impracticability of studying this phenomenon by any means other than the mathematical. The general equations are complicated, and have only been solved by making important simplifying assumptions. The results are serviceable for check purposes, but need further elaboration. The importance of the problem increases progressively as more efficient planes are designed, and the necessity for an adequate mathematical theory is becoming critical.

Future Prospects. It appears inevitable that from motives of economy the industry will rely increasingly upon theoretical methods of design, and that mathematics will play a larger part in the future than at present. It is also probable that for competitive reasons the various companies will supplement government research by fundamental studies of their own. Furthermore, in view of the present fragmentary state of aerodynamic theory, it would not be surprising if part of the research effort was devoted to the improvement of the basic theory itself.

The reliability of these predictions is, of course, conditioned by the financial prospects of the industry. Just now, war orders are causing abnormal inflation of earnings; when these cease, retrenchment will be inevitable. The industry is not highly mechanized, however, and hence its present cycle of inflation does not imply so large an expenditure for plant as would be true in most manufacturing fields. For this reason, the period of deflation may prove to be one of large war profits in the bank, but insufficient orders to occupy the time of many competent technical men whom the management would be reluctant to let go. If this should occur, an almost explosive development of research may take place.

Whether the development is explosive or not, however, it is probable that the industry will soon become one of the largest employers of industrial mathematicians.

Industrial Statistics and Statisticians

The subject of statistics enters the business world at points quite distinct from those touched by the rest of mathematics. Moreover, the types of business activity to which it most frequently applies—insurance and finance, economic forecasting, market surveys, elasticity of demand against price, benefit and pension plans, etc.—belong to the field of economics which is the subject of a separate report, and need not be touched on here.

There are certain other respects in which statistical theory could be of great service in industry, but they have been exploited to only a limited extent. This report must therefore point out these hopeful fields rather than record achievements in them.

Statisticians in Industry

By "statistician" we mean a person versed in and using the mathematical theory of statistics, not one who collects, charts and scrutinizes factual data. In the business world the word is more often used in the latter sense.

There is a very great difference between the number of statisticians in industry, and the number of men interested in some form of statistics. How great the discrepancy is will be clear from a comparison of the membership of the American Statistical Association, which devotes itself to the application of statistics in its broadest sense, and of the American Institute of Mathematical Statistics, which confines itself narrowly to the development of statistical technique. The former lists 277 names with industrial addresses; the latter only 10.

Statistics in Industry

Dr. W. A. Shewhart, Research Statistician of the Bell Telephone Laboratories, has delineated broadly and succinctly the field in which statistics may be expected to find application as follows:

"Since inductive inferences are only probable, or, in other words, since repetitions of any operation under the same essential conditions cannot be expected to give identical results, we need a scientific method that will indicate the degree of observed variability that should not be left to chance. Hence it appears that the use of mathematical statistics is essential to the development of an adequate scientific method, and that mathematical statistics may be expected to be of potential use wherever scientific method can be used to advantage."

More specifically, there are five recognizable types of industrial engineering activity in which statistical theory either is, or should be used.

(*a*) In studying experimental data to determine whether the observed variations should be regarded as accidental or significant. An example is found in the field of geochemical prospecting. The surface soil overlying regions in which there is oil contains a higher proportion of hydrocarbons and waxes than occur in other locations. Chemical analysis of surface soil therefore affords a means of prospecting for oil. Mr. Eugene McDermott writes:

"In the geochemical method, it was found necessary to determine between samples showing significantly high analysis values, and those which were normal values. These normal sample values, of course, had considerable variation between themselves, due to analysis and in larger part sampling errors. After examining these data for a long period of time, it was decided to approach the problem statistically. This disclosed at once that areas surveyed could be divided into positive (having significant values, and hence favorable from the standpoint of petroleum possibilities), negative (no significant values and unfavorable for petroleum) and marginal (indeterminate). The latter case is always the most difficult one in surveying, and while we are now able to recognize it, further work is needed to fully interpret it. This kind of mathematics is being applied at the present moment, and bids fair to solve the problem."

(*b*) In planning the kind of experiments from which such data arise. Whether variations are or are not significant depends in no small degree upon the fashion in which the data were taken. Consideration of the experiment in advance from a statistical point of view often results in economy of procedure, or even points the difference between a trustworthy and a meaningless result.

The following example is quoted from an address by Dr. R. H. Pickard, Director of the British Cotton Research Association:

"To illustrate the advantage of good experimental design I may refer to some experiments carried out at the Shirley Institute to find the effect of various treatments on a quality of cloth. This quality varies considerably at different parts of the same piece of cloth, and in order to measure the effect of the treatments the tests are repeated systematically so that the variations are 'averaged out.' Some of the natural variation, however, is systematic, and by adopting a 'Latin Square' arrangement of treatments on the cloth (such as is much used in agricultural yield trials), these systematic variations are eliminated from the comparison, and in the instance quoted the result was to reduce by one-half the number of tests necessary for a given significance as compared with a random arrangement."[8]

To the extent to which biology becomes an important element in industrial research—and it would appear to be on the point of doing so in such fields as food manufacturing—it can be expected that the type of statistical work listed under (*a*) and (*b*) will rapidly increase.

(*c*) In laying out an inspection routine. Manufacturing inspection frequently yields data which are best interpreted statistically, either because only spot-checks are taken, or because the method of inspection gives measurements which are themselves subject to accidental fluctuation. In such cases statistical theory is of great advantage in setting up an effective and economical inspection program. It is being so used in certain industries, notably in electrical manufacturing and textiles, but the potential field of usefulness is far from covered.

The following example is quoted from an address by Mr. Warner Eustis, Staff Officer on Research of the Kendall Company:

"Surgical sutures are twisted strands of sheep intestine, which has been slit lengthwise. . . . After a stated number of days a sewing with such material, implanted in the body during a surgical operation, will be digested and disappear as the healing processes progressively take up the load originally held by the suture. . . . Here is a product which it is impossible to test in any way without destroying the product, especially as each suture is sealed in an individual, sterilized tube. Our final product tests must all be conducted by breaking open a sterile tube and testing the product therein. The quality appraisal of such a product naturally rests upon probability, rather than upon an actual testing of each item. Due to the nature of such a product, in which a single failure may destroy human life, the need for accurate quality appraisal is superlative."[9]

(*d*) In the control of manufacturing processes. Inspection is not merely a means of discarding bad product; it is also a means of detecting trouble in the factory. This is obvious in the extreme cases when the product is

[8] "The Application of Statistical Methods to Production and Research in Industry," by R. H. Pickard, Supplement to the *Journal of the Royal Statistical Society*, Vol. 1, No. 2, 1934, pp. 9-10.

[9] "Why the Kendall Company is interested in Statistical Methods," by Warner Eustis, *Proceedings of the Industrial Statistics Conference* held at M. I. T., Cambridge, Mass., Sept. 8-9, 1938, pp. CXLIII-CXLIV, published by Pitman Publishing Corporation.

unusually bad. By the use of suitable routines set up in accordance with statistical theory, the day-to-day results of inspection can be used to detect incipient degradation in the process of manufacture which might otherwise escape notice. This procedure is used extensively by the Western Electric Company in assuring uniform quality in many items of manufacture, and to a lesser extent in other industries. Of it, Mr. J. M. Juran, Manufacturing Engineer of the Western Electric Company, says:

> "Too frequently we have seen an inspection group grow lax in vigilance until a complaint from the customer wakes them up. They promptly swing the pendulum a full stroke in the opposite direction, and the factory groans in its effort to meet the now unreasonable demands. A sound and steady control, like a sound currency in commercial relations, gives factory foremen a feeling of confidence and gives the consumer a feeling that control is being exercised before the product reaches him."[10]

(*e*) In writing rational specifications. Obviously, if such a procedure helps the manufacturer to assure uniform quality, it is also of value to the purchaser of his products. Hence the subject of statistics enters into the writing of the buyer's specifications. It has been so used to a limited extent in the Bell System in connection with telephone apparatus, and by the United States Government in the purchase of munitions. However, it must still be rated as a relatively undeveloped field. Of it, Captain Leslie E. Simon, Ordnance Department of the United States Army, says:

> "Statistical methods have proved to be a powerful tool in the critical examination of some ammunition specifications prior to final approval. Their use, either directly or indirectly, is almost essential in determining a reasonable and economic standard of quality through the method of comparing the quality desired with that which can be reasonably expected under good manufacturing practice. In like manner, the statistical technique renders a valuable service in framing the acceptance specification. Through its use the quantity and kind of evidence which will be accepted as proof that the product will meet the standard of quality can be clearly expressed in a fair, unequivocal and operationally verifiable way."

Conclusion

It is perhaps unusual to conclude a survey of this sort by stating the impressions which it has made upon its writer. In the present instance, however, the element of self-education has been so large that these impressions may summarize the report better than any more formal recapitulation. They are:

[10] "Inspectors' Errors in Quality Control," by J. M. Juran, *Mechanical Engineering*, Oct., 1935, pp. 643–644.

(1) Because of its general significance as the language of natural science, mathematics already pervades the whole of industrial research.

(2) Its field of usefulness is nevertheless growing, partly through the development of new industries such as the aircraft business, and partly through the incorporation of new scientific developments into industrial research, as in the application of quantum physics in chemical manufacturing and statistical theory in the control of manufacturing processes.

(3) The need for professional mathematicians in industry will grow as the complexity of industrial research increases, though their number will never be comparable to that of physicists or chemists.

(4) There is a serious lack of university courses for the graduate training of industrial mathematicians.

(5) Management, which is already keenly alive to the importance of mathematics, is also rapidly awakening to the value of mathematicians and the peculiar relationship which they bear to other scientific personnel.

This last observation is not trivial. There was a day when, in engineering circles, mathematicians were rather contemptuously characterized as queer and incompetent. That day is about over. Just now, an attitude more commonly met is one of amazed pride in pointing to some employee who "isn't like most mathematicians; he gives you an answer you can use, and isn't afraid to make approximations." As the proper function of the industrial mathematician becomes better understood, these proud remarks will no doubt cease. Those who are adapted to the job will be taken for granted; the others will be recognized as personnel errors and not mistaken for the professional type. Perhaps the present report may speed this day. If so, it will have been a service to the profession and to industry.

The Transmission Characteristics of Toll Telephone Cables at Carrier Frequencies

By C. M. HEBBERT

IN THE design of a new telephone transmission system a knowledge of the characteristics of the medium over which the waves are to pass is, of course, a prerequisite. What painstaking experimentation is necessary to accumulate such knowledge, however, what voluminous data are involved, what minutiae of detail, and what extremes of accuracy, are things far less obvious.

Recent papers have described a new 12-channel carrier telephone system for operation over cable pairs.[1] For this system a knowledge of the maximum cable losses is needed in order to determine the necessary repeater gains. Accurate data on the insertion loss slope versus frequency are required so that compensating equalizers can be designed to give uniform transmission over the frequency band. In order to design a regulating system to compensate for the variations in attenuation which result from changes in cable temperature, precise knowledge of these variations as a function of frequency is essential. It is necessary to know the impedance of the cable pairs in order that the amplifier impedance may be matched to it, thereby avoiding reflections which would aggravate cross-talk effects. For various purposes, e.g., testing the cables, designing the coils to balance out crosstalk, etc., it is also necessary to know the fundamental parameters (resistance, inductance, capacitance and conductance) or so-called primary constants of the pairs. The velocity of transmission also plays a part in determining the characteristics of the channels. In addition to all these transmission characteristics, it is, of course, essential to know the cross-talk couplings between different pairs. This subject has been treated elsewhere[2], however, and is not considered herein.

In order that the cable carrier systems may be applied in the plant without requiring extensive transmission measurements on each individual carrier pair in each repeater section, it is important that the differences in the transmission characteristics between different pairs be known. The problem therefore becomes one of statistical analysis. In most cases the

[1] "A Carrier Telephone System for Toll Cables," C. W. Green and E. I. Green, *B.S.T.J.*, Vol. 17, January 1938, page 80. "Experience in Applying Carrier Telephone Systems to Toll Cables," W. B. Bedell, G. B. Ransom and W. A. Stevens, *B.S.T.J.*, Vol. 18, October 1939, page 547.

[2] "Crosstalk and Noise Features of Cable Carrier Telephone System," M. A. Weaver, R. S. Tucker and P. S. Darnell, *B.S.T.J.*, Vol. 17, January 1938, page 137.

effects involved are cumulative with distance and the accuracy involved in the determination of the various characteristics is therefore set by the maximum distance over which the system is designed to operate. For a distance of 4000 miles the total loss at the top frequency of 60 kc. will be approximately 16,000 db, the attenuation difference between the top frequency of 60 kc. and the bottom frequency of 12 kc. nearly 6000 db if the cable is at about the average temperature, 55°F. The range of variation in loss with temperature, assuming aerial cable over the whole distance, will be about ±8 per cent of the total at 60 kc. It is desired to correct these frequency differences in loss and variations with temperature so accurately that individual channels will be constant to within ±2 db.

Prior to the beginning of experimentation with cable carrier systems limited use had been made, in connection with carrier systems operated over open-wire lines at frequencies up to 30 kc., of conductors in relatively short entrance and intermediate cables. The available data, however, were quite inadequate for the cable carrier problem. Accordingly, an extensive series of tests was undertaken. Reels of standard toll cable were placed in temperature controlled rooms where the extreme temperature variations of the mid-west could be substantially duplicated (the actual laboratory temperatures ranged from just below 0° F to 120° F) and measurements were made to determine the changes in the parameters of the cable accompanying these wide temperature variations at frequencies from 1 kc. to 100 kc. and higher in some cases. Certain of the tests even studied the effect of varying the humidity content of the cables. Further measurements were then made on suitable lengths of pairs in actual commercial cables in which carrier systems were to be installed. These results corroborated and extended the data from the laboratory measurements; the subsequent operation of equalizers, regulators, etc., based upon these data, showed no essential discrepancies.

The present paper, after referring to the types of toll cables employed for the new carrier systems, outlines the methods employed in determining their characteristics both in the laboratory and in the field, summarizes these characteristics for typical 19-gauge cable at frequencies up to 100 kc. and finally extends them to frequencies as high as 700 kc. for 16 and 19-gauge cables.

Types of Carrier Toll Cables

The type K carrier system has been designed so that it may be applied to existing cables, thus in many cases avoiding the installation of expensive new cables. Most of the standard toll cable in the Bell System contains chiefly 19-gauge paper insulated conductors in "multiple twin quads," i.e., two conductors are twisted together to form a pair and two pairs twisted together to make a quad. The nominal capacitance of a pair is .062 mf. per mile. There are various twist lengths of both pairs and quads in a given cable as well as cables ranging in size from 12-quad cable to the oversize 19-gauge cable containing 225 quads. Some type K is operating over "paired" cable, i.e., cable in which only the wires of each pair are twisted together.

Operation in two directions is accomplished by using either a separate cable for each direction or a single cable with two groups of conductors separated by a layer shield. This avoids serious near-end crosstalk effects which would result from the large level difference existing between opposite directions at a repeater point.

Methods of Measurement

As mentioned above, 250-foot reels of standard toll cable were placed in a special room which could be accurately maintained at any desired temperature from about zero to 120 degrees, Fahrenheit, and measurements made for various frequencies and temperatures. For the most part, these consisted of open-circuit admittance and short-circuit impedance measurements on part of the pairs in the cable at temperatures of about zero, 30, 50, 90 and 120 degrees F., over the frequency range from 4 to 100 kc. From these measurements computations could then be made to determine the resistance, inductance, capacitance and conductance as well as the attenuation, phase constant and characteristic impedance of this type of cable at the different temperatures and frequencies. Detailed data on frequency and temperature variations of these quantities are given below. Most of these data were obtained from measurements made on 16- and 19-gauge pairs in a typical reel-length of standard toll cable. The temperature is difficult to maintain at a constant level and d-c. resistances of certain pairs were measured at frequent intervals during the process in order to get accurate temperature readings by comparing with resistance-temperature curves of these pairs. Thermocouples were also attached to the cable at various points along its length and sheath temperatures determined from them. After stabilizing the room temperature as closely as possible, the variations in cable temperature took place slowly enough to be allowed for in the computations.

After the selection of the Toledo-South Bend route for a trial installation, further measurements were made on certain of the pairs in this cable. The test sections, extending out of Lagrange, Indiana, were made about 10 miles long in order to obtain the averaging effect of length. For this distance it was not possible to use open and short-circuit measurements as was done in the laboratory, and a substitution method[3] was devised (Fig. 1).

[3] This was devised by H. B. Noyes and will be described by him in a paper in the *Bell Laboratories Record.*

This consists essentially of first measuring the input and output a-c. currents at the two ends of the test pair by means of thermocouple-milliammeter arrangements and then immediately sending d-c. over another pair (called reference pair in Fig. 1) built out to a convenient fixed d-c. resistance, the same for all measurements, and adjusting resistance networks at both ends of the line until the meter readings are the same as for the a-c. Suitable charts then enable readings of attenuation (insertion loss) corresponding to the d-c. (and therefore also to the a-c.) readings to be made very rapidly.

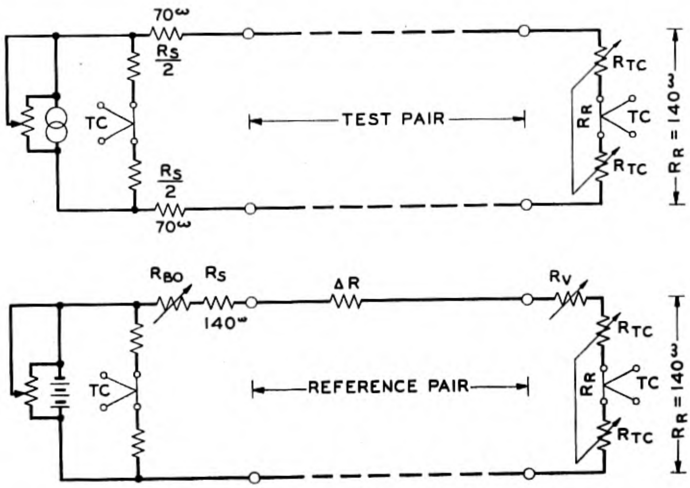


Fig. 1—Simplified attenuation measuring circuit

Toll Cable Characteristics Below 100 Kc.

Primary Line Parameters

The four primary line parameters, R, L, G and C—series resistance, series inductance, shunt leakage and shunt capacitance—are of the same sort for all kinds of transmission lines, but the relative importance of the various elements changes considerably with frequency and the type of structure considered. The old name primary "constants" is obviously a misnomer, and it is simpler to speak of them as line "parameters," since this does not necessarily imply anything regarding their constancy or inconstancy under various conditions.

The "true" or distributed values of these parameters are usually obtained from measurements of the open circuit admittance, $G' + j\omega C'$, and the short-circuit impedance, $R' + j\omega L'$ of the actual pair in a short length of cable, by means of the following formulas:

$$R \text{ (resistance)} = R'(1 - \tfrac{2}{3}\,\omega^2 L'C') + \cdots$$

$$L \text{ (inductance)} = L'(1 - \tfrac{1}{3}\,\omega^2 L'C' \cdots) + \tfrac{1}{3}\,R'^2C' + \cdots$$

$$G \text{ (conductance)} = G'(1 - \tfrac{2}{3}\,\omega^2 L'C' \cdots) - \tfrac{1}{3}\,R'\omega^2C'^2 + \tfrac{2}{5}\,R'\omega^4L'C'^3 \cdots$$

$$C \text{ (capacitance)} = C'(1 - \tfrac{1}{3}\,\omega^2 L'C' + \tfrac{2}{3}\,R'G' \cdots) \qquad (1)$$

These formulas give accuracies within one per cent for reel-lengths of 500 feet or less and frequencies up to 100 kilocycles for 19-gauge cables having a capacity of .062 mf per mile. All the curves of R, L, G, C herewith are based on true values obtained from such computations.

Resistance

The quantity R, series resistance in ohms per mile, has a large variation with frequency produced by the well-known phenomenon called skin effect and another large increment, resulting from the closeness of the wires in cables, known as the proximity effect.[4–7]. The magnitude of the proximity effect varies with the diameter of the conductors as well as with their separation. The curves in Fig. 2 show the increment in resistance resulting from skin effect and the total increase including proximity effect as computed for a pair of wires separated by various multiples of their diameters. The abscissa, B, in Fig. 2 is a sort of universal parameter used in data on skin effect so that a single curve will suffice for various gauges. If f is frequency in cycles per second and R_0 is the d-c. resistance for 1000 feet of the wire (not a 1000-foot loop), the parameter B is given by the equation

$$B = \sqrt{f/R_0} \doteqdot \sqrt{f/8} \qquad (2)$$

for 19-gauge wire so that $B = 80$ corresponds to 51,200 cycles. According to the curves at $B = 80$ (51.2 kc), the skin effect increases the a-c. resistance to about 12 per cent more than the d-c. resistance.

For a separation of two diameters between centers of the wires of a pair ($k = .25$) the proximity effect adds another 6 per cent to the resistance ratio making the total a-c. resistance about 1.18 times the d-c. resistance at 51.2 kc. If the wires are closer together ($k = .4$) the a-c. resistance is computed to be about 1.30 times the d-c., which is about the ratio actually measured. The effects caused by the presence of the adjacent pair in a

[4] J. R. Carson, "Wave Propagation over Parallel Wires—The Proximity Effect," *Phil. Mag.*, Vol. 41, April 1921, pp. 607–633.

[5] A. E. Kennelly, F. A. Laws and P. H. Pierce, "Experimental Researches on Skin Effect in Conductors," *A.I.E.E. Trans.*, Vol. 34, Part 2, 1915, pp. 1953–2021.

[6] A. E. Kennelly and H. A. Affel, "Skin Effect Resistance Measurements of Conductors at Radio Frequencies," *I.R.E. Proc.*, Vol. 4, No. 6, Dec. 1916, pp. 523–574.

[7] Günter Wuckel, "Physics of Telephone Cables at High Frequencies," *EFD* **47**, (Nov. 1937) pp. 209–224.

quad, the surrounding wires and the lead sheath are not included in these computations.

These values assume a temperature of 20° Centigrade (68° Fahrenheit) but if the temperature varies, so also does the resistance. Figure 3 shows the a-c. temperature coefficient of resistance and its variation with temperature for 19-gauge pairs in ohms per ohm per degree, Fahrenheit, i.e.,

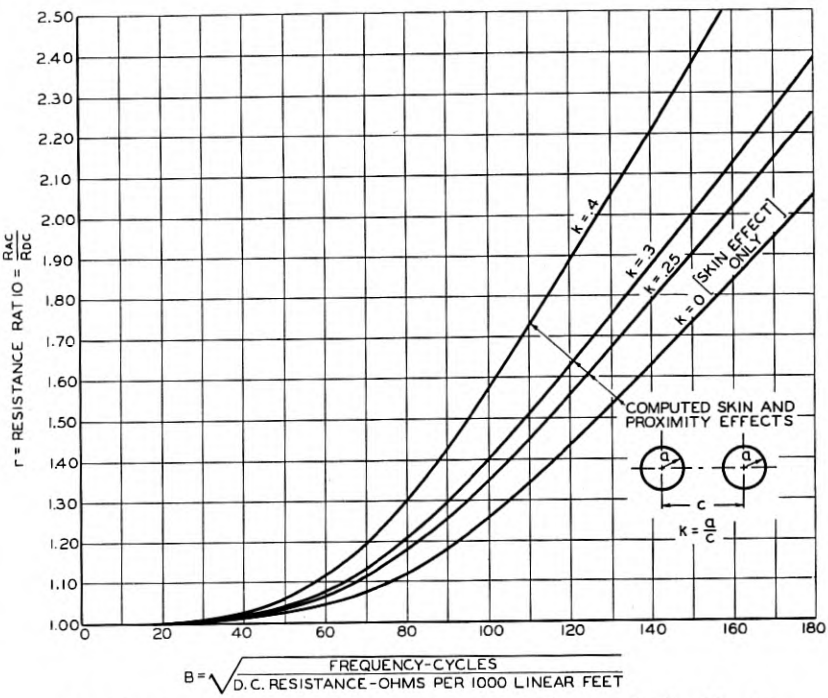


Fig. 2—Skin effect and proximity effect on a.-c. resistance of toll cable pairs

$$A = \frac{1}{R_1} \frac{dR}{dt} \tag{3}$$

where A is the a-c. temperature coefficient of resistance of copper at t_1 degrees, Fahrenheit. The a-c. resistance R at temperature t is given by the formula

$$R = R_1 [1 + A(t - t_1)] \tag{4}$$

where R_1 is the a-c. resistance at temperature t_1 degrees, Fahrenheit. The coefficient A decreases with increasing frequency, but not indefinitely; it approaches 1/2 the d-c. coefficient as its asymptotic limit with frequency.

The normal variation of air temperatures in the middle western part of the country is from about 1° Fahrenheit, (−17° Centigrade) to plus 109° Fahrenheit (43° Centigrade). Extremely hot summers like that in 1936, which was preceded by a severe winter, show even higher temperatures and there are occasional periods in mid-western winters when the temperature hovers continuously around −30° F., for a week or two. These temperatures are almost the temperatures assumed by open wires, but wires inside a lead sheath like those in an aerial cable are subjected to much higher than air temperatures in hot weather when exposed to direct sunlight in the absence of wind. Some observations made at Lagrange, Indiana, in 1936

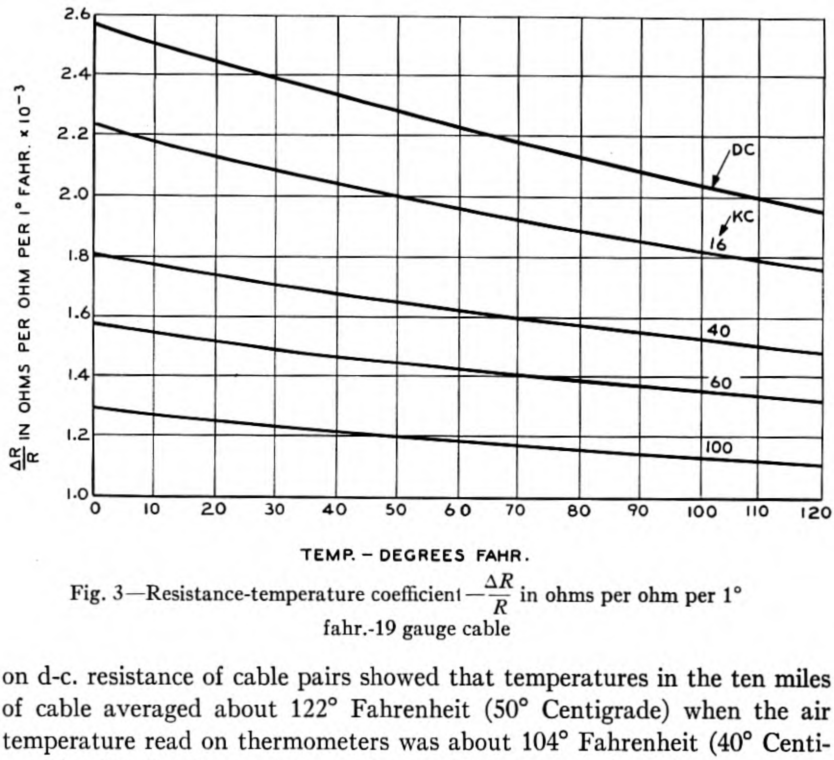


Fig. 3—Resistance-temperature coefficient—$\frac{\Delta R}{R}$ in ohms per ohm per 1° fahr.-19 gauge cable

on d-c. resistance of cable pairs showed that temperatures in the ten miles of cable averaged about 122° Fahrenheit (50° Centigrade) when the air temperature read on thermometers was about 104° Fahrenheit (40° Centigrade). Similar data taken at Chester, New Jersey, showed temperatures in the cables 20° to 25° Fahrenheit higher than the air temperature on hot bright days.

The actual observed cable temperature range in that season (1936) as indicated by the d-c. resistance measurements was thus from −4° Fahrenheit (−20° Centigrade) to 122° Fahrenheit (50° Centigrade). In terms of a-c. resistance changes, this amounts to a resistance change of about 20 ohms per mile at 50 kc., the resistance at the lower temperature being about 96 ohms per mile and at the higher temperature about 116 ohms per mile. This amounts to about ± 10 per cent variation from the mean.

In addition to the wide annual variations, there are daily variations of as much as 50° Fahrenheit at times, that is, almost half as much as the normal annual variation. The practical importance of these large resistance changes lies in their large contribution to changes in attenuation as will be brought out more fully in connection with variations of attenuation with temperature.

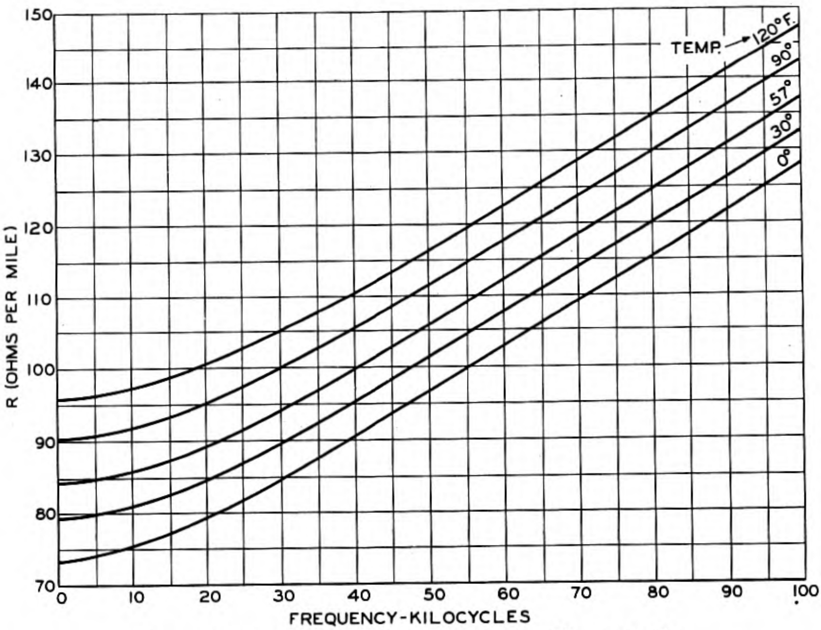


Fig. 4—Resistance per mile vs. frequency—19 gauge pairs

Underground and buried cable are, of course, not subjected to such wide annual variations and daily variations are almost entirely eliminated by the attenuation of heat changes by the soil. Cable in ducts usually lies well below the freezing line and this depth at the same time protects it from the summer's heat. The normal range for cable in ducts is from about freezing to about 70 degrees, F. Cable buried only a foot or so underground would have a considerably larger annual temperature range but a great deal of such cable is buried two to three feet deep.

Curves in Fig. 4 show the actual a-c. resistance variation with frequency and in Fig. 5 are shown temperature variations of resistance at typical frequencies for 19-gauge toll pairs in a reel-length of standard toll cable.

In addition to the variations with frequency and temperature there are the initial differences between pairs on account of manufacturing processes,

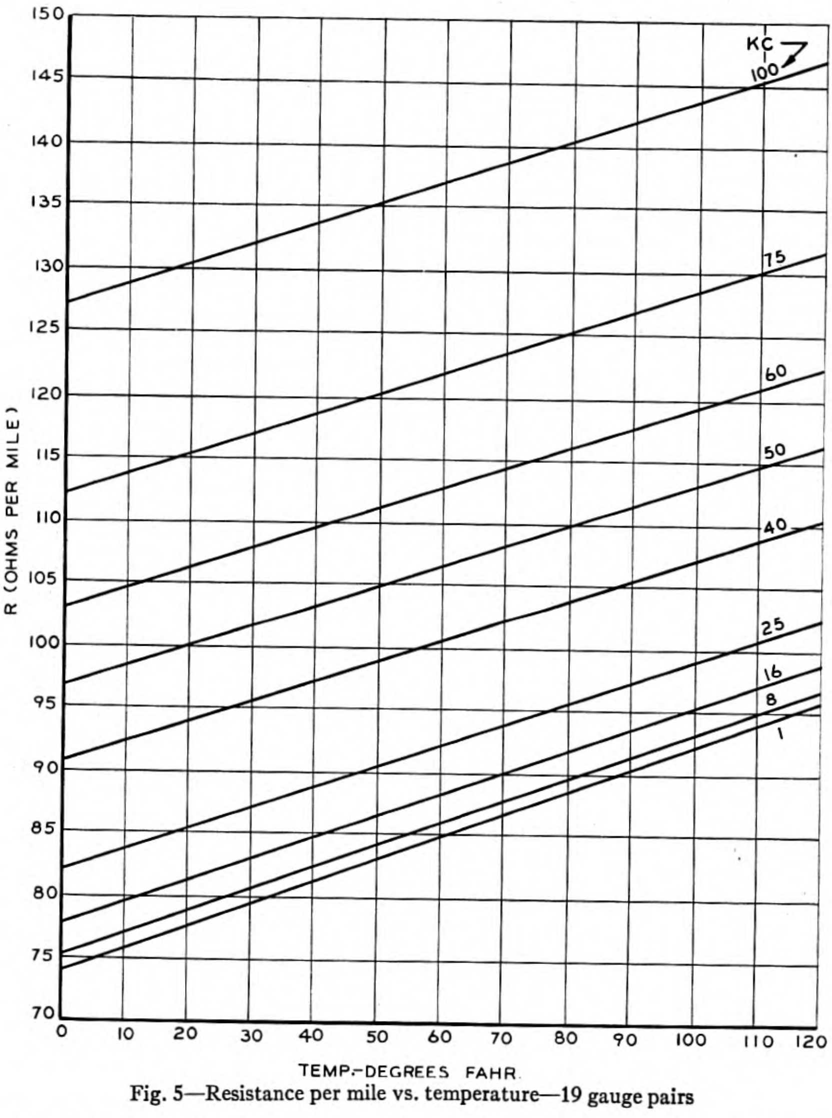


Fig. 5—Resistance per mile vs. temperature—19 gauge pairs

etc. One such source of variation in resistance of pairs is the difference in wire diameters caused by wear of the dies used in drawing wire.[8] The

[8] John R. Shea and Samuel McMullan, "Developments in the Manufacture of Copper Wire," *B.S.T.J.*, VI (April 1927), pp. 187–216.

permissible variation in diameter for ordinary toll cable is ± 1 per cent which means a d-c. resistance variation of about ± 2 per cent.

Still another cause of resistance variation is the presence of small quantities of impurities in the copper which show up as a reduction of as much as 2 per cent in the conductivity. This causes trouble in calibrating temperature-resistance curves in the laboratory setup.

Finally, in a single reel length the outside pairs are longer than the inside pairs. The total pair-to-pair variation in resistance from the average of the reel caused by all these factors amounts to about ± 3 per cent with a standard deviation of about 1.5 per cent.

Inductance

The inductance of a circuit formed by two parallel wires closely spaced relative to their length is

$$L = 0.64374\left[2.3026 \log_{10} \frac{2D}{d} + \mu\delta\right] \times 10^{-3} \text{ henrys per loop mile} \qquad (5)$$

where d, the wire diameter, and D, the separation of the wires, are measured in the same unit; μ is the permeability, and δ is a frequency factor.

As is well known, the tendency of alternating currents to concentrate on the surface of a wire reduces the magnetic flux within the wire and decreases the internal inductance of the wire. This internal inductance is given by the term $\mu\delta$ in Equation (5). In like manner, the "proximity effect" produces a concentration of current density in the adjacent portions of the two wires of a pair.

Another term might well be added to formula (5) to represent this proximity effect. The procedure outlined by J. R. Carson on pages 625 and 626 of the *Philosophical Magazine* paper[4] of 1921 has been carried out with the results given in an Appendix to this paper. Formula (11a) of the Appendix gives the ratio, K, of the a-c. inductance of the pair (less the "geometric inductance") to the a-c. inductance of a wire with concentric return, which is given by a well-known formula (7a in the Appendix). It will be seen that the factor introduced by proximity effect decreases with frequency but is asymptotic to a definite value, depending upon the separation of the two wires, as the frequency increases indefinitely. Similar curves are given in an extensive study of the mutual inductance of four parallel wires of a quad by R. S. Hoyt and Sallie Pero Mead[9]. Their theoretical studies agree closely with experimental values given by R. N. Hunter and

[9] Ray S. Hoyt and Sallie Pero Mead, "Mutual Impedances of Parallel Wires," *B.S.T.J.*, XIV (1935), pp. 509–533.

R. P. Booth[10] who made measurements on 18-gauge and 20-gauge pairs in various arrangements and on a 55-foot length of 19-gauge quadded cable.

Overall inductance variations of 19-gauge pairs with frequency and temperature are shown by the curves of Figs. 6 and 7.

The magnitude of inductance variations from pair to pair in reel lengths of cable is about ±3 per cent from the mean with a standard deviation of about 1.5 per cent.

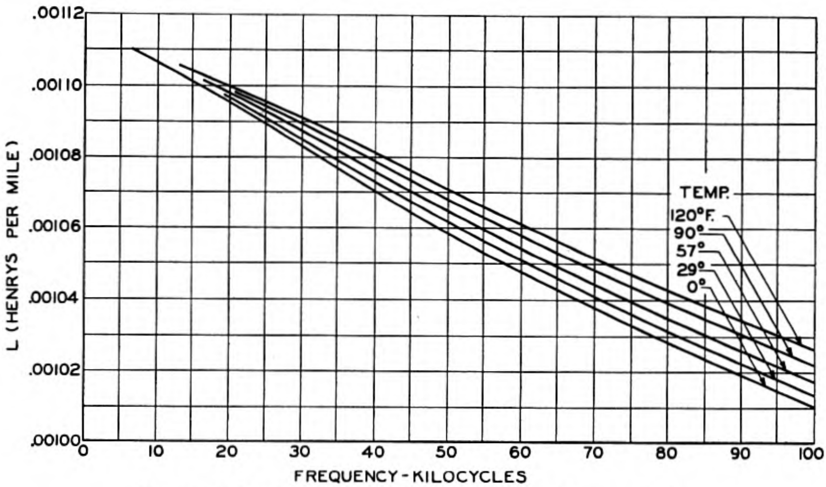


Fig. 6—Inductance per mile vs. frequency—19 gauge pairs

Capacitance

The usual formula for capacitance of two parallel wires in space, separated by a distance negligible compared with their length, is

$$C = \frac{0.019415}{\log_{10} \frac{2D}{d}} \times 10^{-6} \text{ farads per loop mile} \tag{6}$$

Conditions in a cable are vastly different from those assumed in this formula which assumes that the two wires are at a great distance from other wires and from the ground. In the cable, pairs are twisted and, in addition, other wires are very near and the sheath is effectively at ground potential, resulting in a considerable modification of the capacitance. Moreover, the formula assumes that the wires are in air, which has a dielectric constant almost equal to unity. (1.00059 at 0° Centigrade) The dielectric constant of

[10] R. N. Hunter and R. P. Booth, "Cable Crosstalk—Effect of Non-Uniform Current Distribution in the Wires," *B.S.T.J.*, XIV (1935) pp. 179–194.

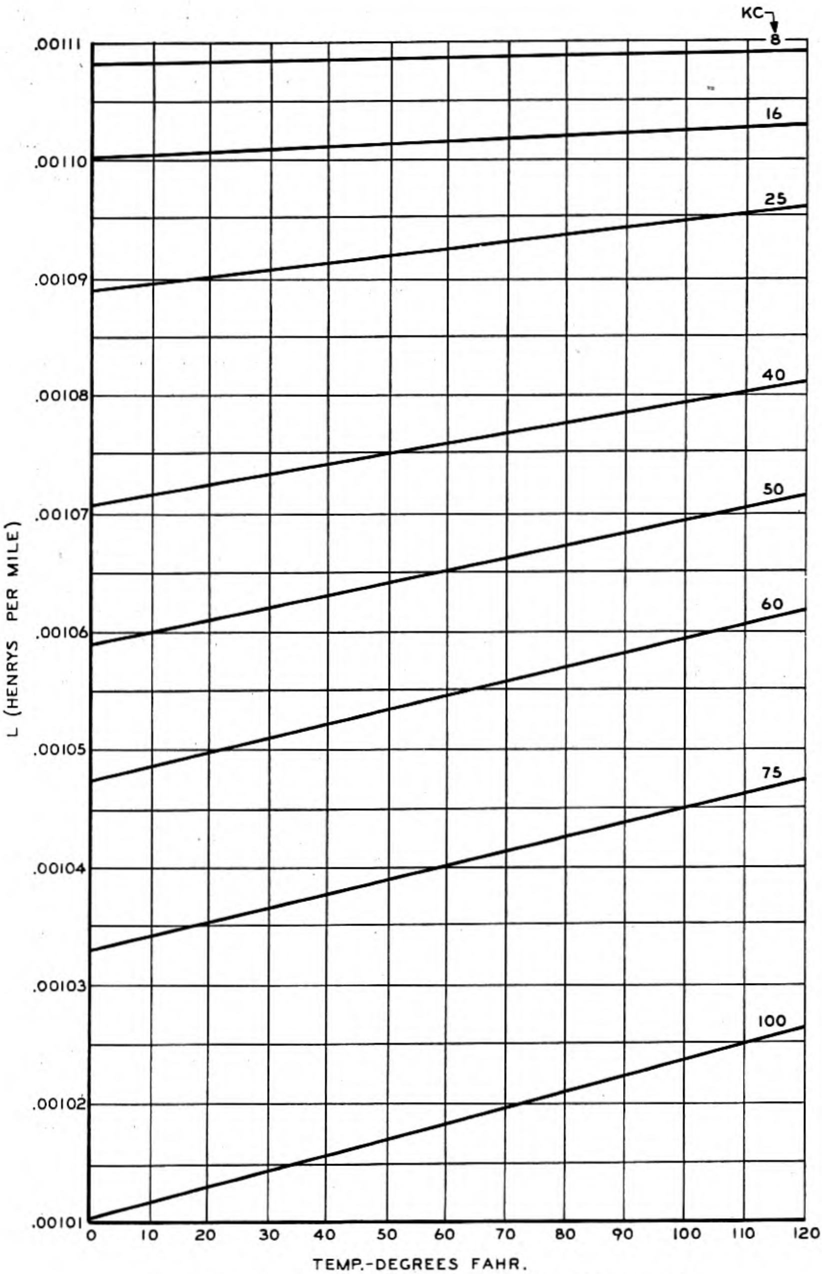


Fig. 7—Inductance per mile vs. temperature—19 gauge pairs

the paper in cables varies from 1.7 to 1.9 depending upon the amount of air and impurities contained in the paper. Of the space around the wires inside the sheath about 40 per cent is filled with paper and the remaining 60 per cent with air. Frequency and temperature of the cable affect the true dielectric constant in a complicated way. Slight amounts of moisture remaining in the cable even after drying affect the dielectric constant and the capacitance as well as the leakage conductance and introduce further changes in the frequency-temperature characteristics.

Results of an extensive study of the dielectric constant were given by E. J. Murphy and S. O. Morgan in a series of recent papers[11]. They point out (first paper, p. 494; second paper, p. 641) that a dielectric may be thought of as an assemblage of *bound charges*, that is charged particles which are so bound together that they are not able to drift from one electrode to the other under the action of an applied electric field of uniform intensity. But the applied field disturbs the equilibrium of the forces acting on the bound charges and they take up new equilibrium positions, thereby increasing their potential energy when the applied field is removed. Then when the applied field is removed, some of this energy is dissipated as heat in the dielectric. If the applied field is alternating, the bound charges swing back and forth with certain amplitudes and the sum of the product of the amplitude by the charge extended over all the bound charges in a unit volume determines the *dielectric constant* of the material. The energy dissipated as heat by the motions of the bound charges is the *dielectric loss*, which is proportional to the a-c. conductivity after the d-c. conductivity has been subtracted from it.

Considering the fact that positive and negative charges will be displaced in opposite directions and such a motion constitutes an electric current, there is thus what is called a *polarization current* or *charging current* flowing while the polarization (or displacement of charges) is being formed. If the current alternates too rapidly for the polarization to form completely before the field reverses its direction, the magnitude of the dielectric polarization and the dielectric constant will be reduced. The result of this lag, therefore, is that the dielectric constant (and likewise the capacity) decreases with increasing frequency. This is the phenomenon known as anomalous dispersion from its relation to the anomalous dispersion of light, i.e., at visible frequencies.

A further important concept in dielectric theory is that the molecules of all dielectrics except those in which the positive and negative charges are symmetrically located, possess a permanent electric moment characteristic

[11] E. J. Murphy and S. O. Morgan, "The Dielectric Properties of Insulating Materials," *B.S.T.J.* XVI (1937) pp. 493–512; XVII (1938) pp. 640–669; XVIII (1939) pp. 502–537. These are referred to as "First Paper," etc.

of those molecules. These polarized molecules are called *dipoles* and when an electric field is applied the dipole axes tend to line up in the direction of the applied field. It is probable that for a combination dielectric such as the paper and air in cables with possible traces of moisture, in spite of oven-drying, the dipoles constitute only part of the charges. The frequency also is too low, in most of the data, to emphasize the effects due to dipoles. The paper-air combination introduces another slowing up of the polarization process on account of *interfacial polarization.* Maxwell showed that if the dielectric in a condenser consisted of two layers of materials having different constants, the capacity depends upon the charging time because of time required in charging the interface between the two dielectrics. For a-c. this means a decreasing capacity with increasing frequency and, since there

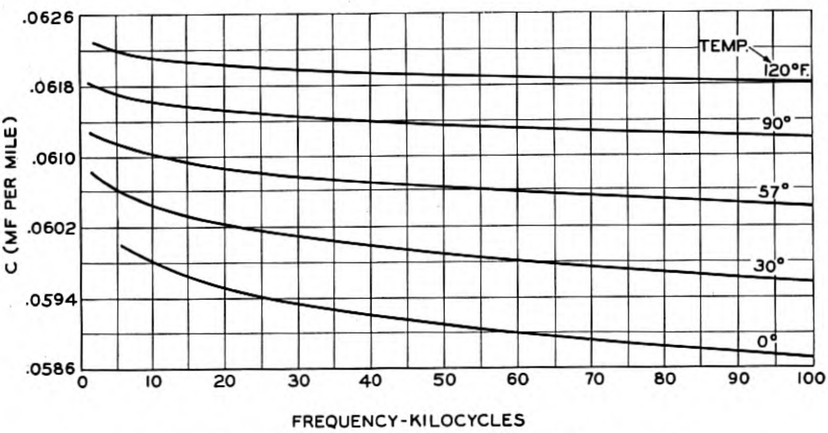


Fig. 8—Capacitance per mile vs. frequency—19 gauge pairs

are effectively an immense number of interfaces between paper and air in the cable, this effect must be of some importance.

Increasing the temperature increases the thermal energy of the molecules and their consequent thermal motion which helps maintain the random orientation of the molecules. Thus, the thermal motion opposes the action of the electric field in maintaining the alignment of the dipoles so that as the temperature rises, the polarization is reduced. But in the cable there are unequal expansions of the copper and the lead sheath which may act to increase the internal pressure as the temperature rises, increasing dielectric densities as well as bringing the wires closer together.

The final result of all these effects on the capacitance of the cable pairs is shown by the curves of Fig. 8, which give the 19-gauge capacitance-frequency relations for several temperatures. Figure 9 shows the variation of capacitance with temperature for several frequencies. The largest change

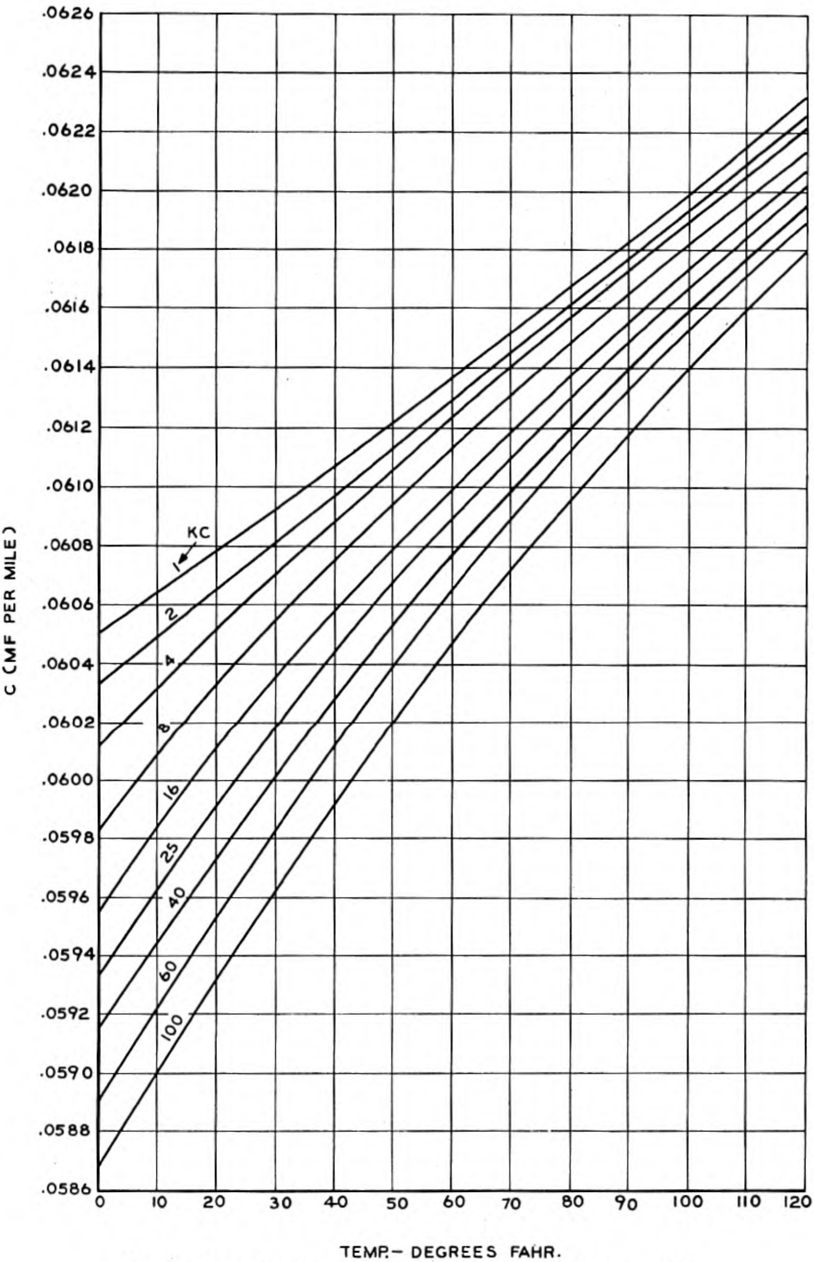


Fig. 9—Capacitance per mile vs. temperature—19 gauge pairs

shown is at 100 kc. and amounts to about 6 per cent increase for 120 degrees increase in temperature.

Leakage Conductance

The variation in the dielectric constant of the insulating layers of paper is further reflected in the leakage conductance, G. This is probably the most inconstant of the parameters and is a function of separation of the wires and their diameters, as well as frequency and temperature and the nature of the dielectric. Humidity, if present, is a highly important contributor to high leakance, but in properly dried cables the humidity is not very great. It will be seen in the later discussion of attenuation and the factors affecting it that conductance is a much less important factor, relatively, than it is for open-wire lines.

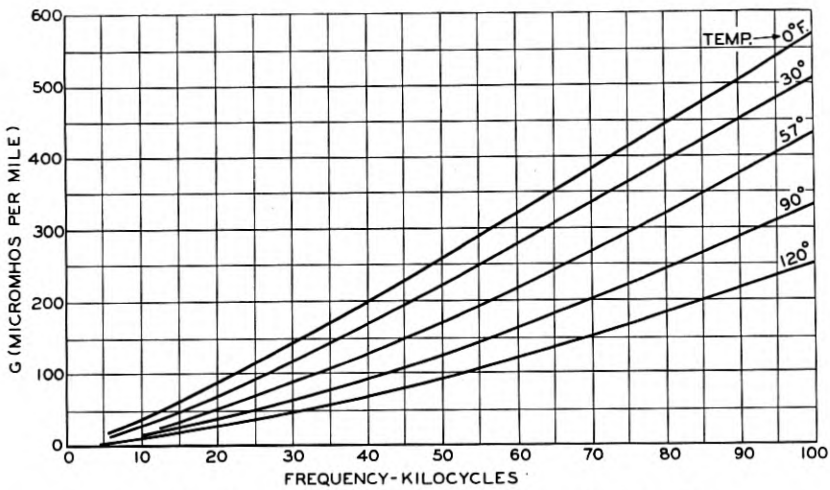


Fig. 10—Conductance per mile vs. frequency—19 gauge pairs

The curves of Fig. 10 show the variation of conductance of 19-gauge pairs with frequency at five temperatures from zero to 120° F. When plotted on log-log paper these curves are nearly linear, showing that conductance varies with frequency approximately according to a formula of the type

$$G = aF^k \qquad (7)$$

where a is about $.0001 \times 10^{-6}$ and k is about 1.33 for the 57° data. F. B. Livingston in a paper[12] on conductance in cables stated that for the data there given k averaged about 1.3.

The range of conductance from pair to pair in a reel is about ± 11 per cent from the average and the standard deviation about 5.5 per cent.

[12] F. B. Livingston, "Conductance in Telephone Cables," *Bell Laboratories Record*, Vol. XVI (Dec. 1937) p. 141.

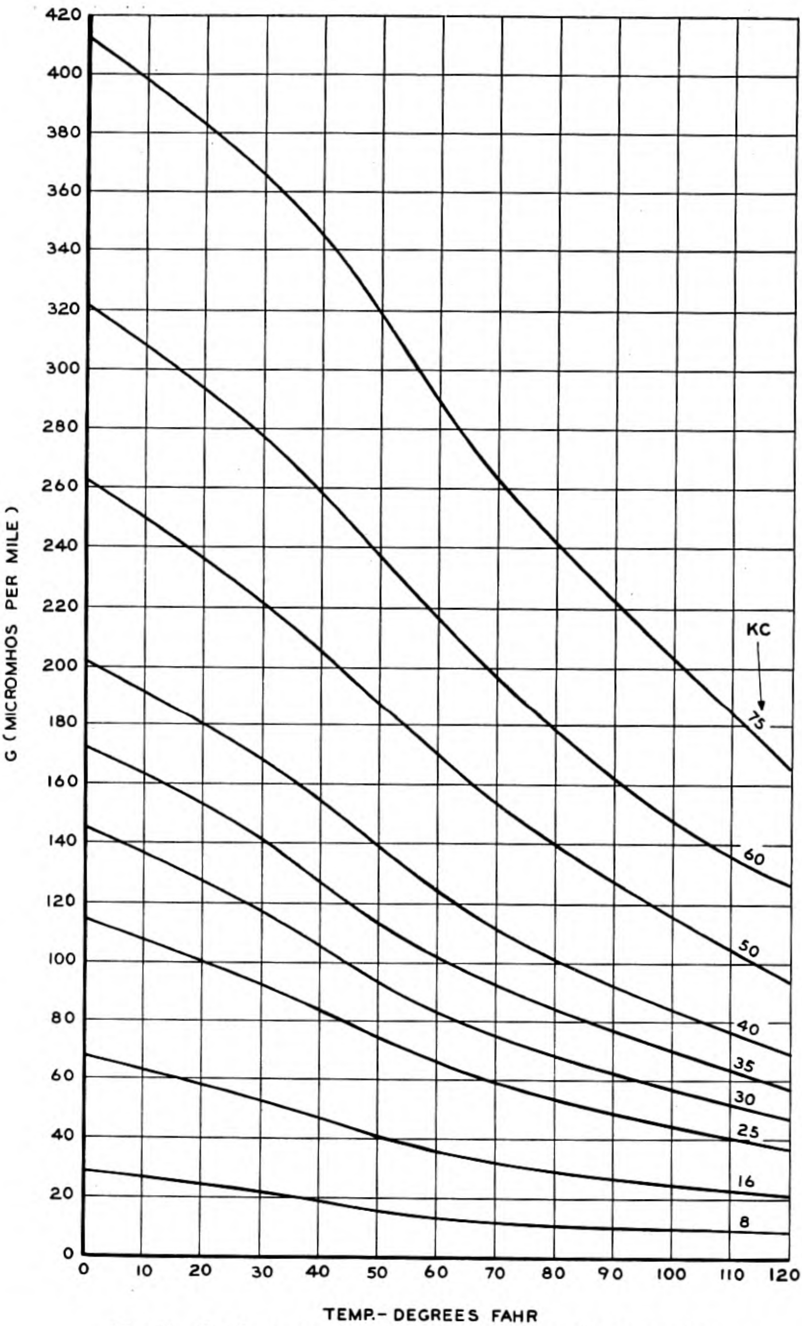


Fig. 11—Conductance per mile vs. temperature—19 gauge pairs

As might be expected, temperature has a great deal to do with the value of G. Variations with temperature shown by the curves in Fig. 11 may be expressed for small temperature ranges by the equation

$$G = G_1 [1 + k(t - t_1)] \tag{8}$$

where G_1 is the value of G at the temperature t_1 and k is the temperature coefficient of leakage conductance. Curves of k based on measurements on a 61-pair, 16-gauge cable are given in Fig. 12 in the neighborhood of 70 degrees Fahrenheit. It will be noticed that k is negative below 1200 kc. but at high frequencies the coefficient increases rapidly from its minimum value reached at about 500 kc.

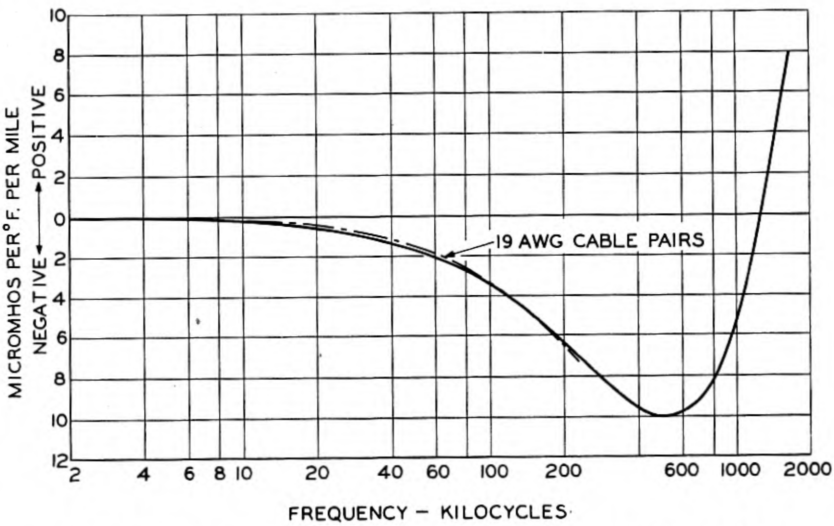


Fig. 12—Conductance-temperature coefficient; micromhos per mi. per 1°F. 16 AWG 61-pair paper insulated cable at 70°F.

It was mentioned above that moisture in the cable has a pronounced effect on the conductance. To drive out excess moisture during manufacture the reels of paper covered cable (or cores) are placed in vacuum driers[13] and then stored in a room maintained at about 110° F. and at a relative humidity of 1/2 of 1 per cent or less until the cables are covered with their lead-antimony sheaths. The lead presses are adjacent to the ovens and the cable is fed through the wall directly into the press so that it emerges at the opposite side covered with the sheath. This procedure minimizes the amount of moisture entering the paper of the cables after they have been dried. The practical measure of the moisture content and the effectiveness of the drying

[13] C. D. Hart, "Recent Developments in the Process of Manufacturing Lead-Covered Telephone Cable," *B.S.T.J.*, VII (1928) pp. 321–342.

is the value of the quantity $G/2C$ = conductance divided by twice the capacitance, both measured at the room temperature in the factory. The quantity $G/2C$ is used because it is the coefficient in the leakage component of attenuation as explained in connection with the formula (12) below for high-frequency attenuation. The average value of $G/2C$ for 1000 cycles at 70° F. is about 8.3. This quantity increases with frequency and at the same time decreases with temperature in the same way G changes, since the capacitance changes are relatively so much smaller than the conductance changes.

Layer to Layer Variations of Primary Parameters

The values of the primary parameters vary from inside layers to outside layers of cables, in addition to variations mentioned under specific parameters above. There are three basic reasons for this variation with location in the cable. The first is that the length of an outside pair is usually considerably greater than the length of an inside pair. Unusual twisting of the inside layers might make up for this difference but in the ordinary construction this is not done. This increase in length amounts to as much as 1 or 2 per cent and is reflected at once in the d-c. resistance as well as in the a-c. parameters.

The second reason is that, particularly in the outside layer, the sheath being made of lead-antimony, has electrical properties considerably different from the properties of copper wires. The large size of the sheath relative to the wires is an important factor. High-frequency currents in the wires near the sheath produce fields cutting the sheath which affects the fields in a different fashion from the way adjacent copper wires affect the field of a pair of conductors near the center.

The third reason, closely allied to the second, is that the conductors in the core of the cable are surrounded by a practically symmetrical mass of copper conductors and paper plus the sheath, while conductors in any other layer are surrounded by an unsymmetrical arrangement of conductors and paper.

A fourth factor is the variation in the amount of space allowed pairs in the core by the pressure of the outside layers.

The magnitude of these effects is indicated by the curves of Fig. 13, showing layer-to-layer variations in per cent for Resistance, Inductance, Conductance and Capacitance. Such large variations would be of considerable importance were it not for the fact that in the process of splicing pairs are made to pass, in effect, from inside to outside of the cable and vice versa. A long study of these variations will be found in the paper by Wuckel[7]. The effects of splicing together sections slightly different in their character-

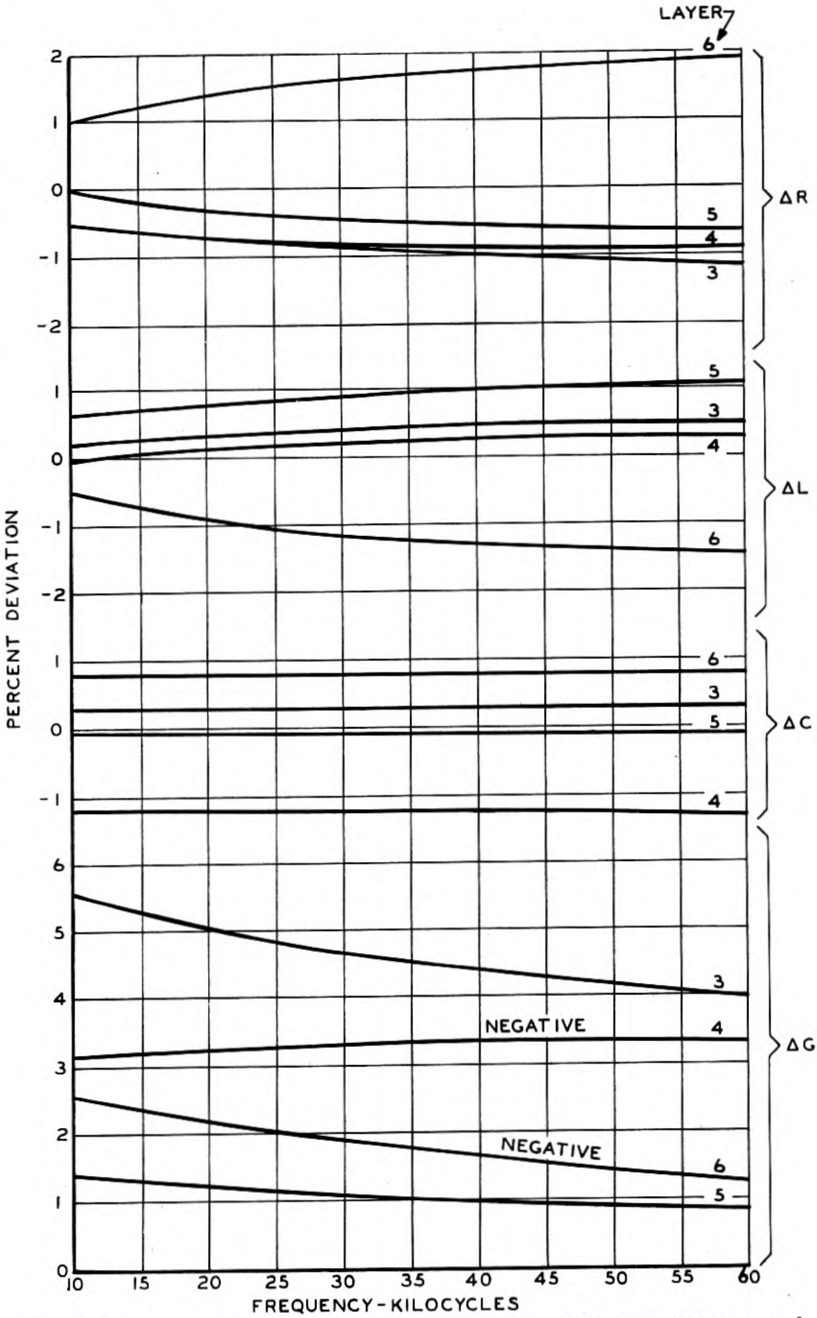


Fig. 13—Percentage deviations of layer average values of *R*, *L*, *G*, *C* from grand average—19 gauge pairs

istics were given for impedance, attenuation and delay distortion by Pierre Mertz and K. W. Pfleger[14].

Closely allied to these effects is the possibility of temperature differences across the section of cable in actual installed cables. Splicing usually takes care of this, too, but there are traces of such a lag in cases where the pairs remain in the outer part of the cable for a long distance and then pass to the inner group for the remaining part of the line. No such cross-sectional variation entered into the laboratory measurements as the temperatures were sufficiently well maintained close to given desired values.

Attenuation

The propagation constant γ is given by the familiar formula

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= j\omega\sqrt{LC}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)} \quad (9)$$

The real part, α, is the attenuation in nepers and the imaginary part, β, is the phase in radians. Expressing the attenuation in terms of reals, gives

$$2\alpha^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - (\omega^2 LC - RG)$$

$$= \omega^2 LC\sqrt{(1 + R^2/\omega^2 L^2)(1 + G^2/\omega^2 C^2)} - (\omega^2 LC - RG) \quad (10)$$

In cables, $G/\omega C$ is small as compared to unity, in which case (10) may be reduced to the approximate form

$$2\alpha^2 = \omega^2 LC\sqrt{1 + R^2/\omega^2 L^2} - (\omega^2 LC - RG) \quad (11)$$

The formula for β^2 is the same as for α^2 except for the sign of the last two terms in (10) and (11), that is, the sign in front of the parenthesis is + instead of −.

By expanding the square root term in (10) and using only first order terms in the expansion, another approximate form, frequently found useful in checking high-frequency values, is obtained, viz.,

$$a \doteq \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} = \left(\frac{R}{2L} + \frac{G}{2C}\right)\sqrt{LC} \quad (12)$$

(Terms neglected in this approximation all include powers of ω^2 in their denominators and so become negligible at high frequencies.) The first term is commonly called the "resistance component of attenuation" and represents series losses. The second term represents shunt losses and is called the "leakage component of attenuation". The quantity $\sqrt{L/C}$, as is well known, represents the nominal characteristic impedance of the circuit.

[14] Pierre Mertz and K. W. Pfleger, "Irregularities in Broad-Band Wire Transmission Circuits," *B.S.T.J.*, XVI (Oct. 1937) pp. 541–559.

The shapes of attenuation-frequency curves at high, low and intermediate temperatures are shown by the curves of Fig. 14. These curves do not appear to be strikingly different in shape but more detailed study of the variations with temperature show the rate of change (decibels per degree per mile) to vary with frequency according to the curve of Fig. 15. The frequency of maximum rate of variation depends upon the gauge, as does the

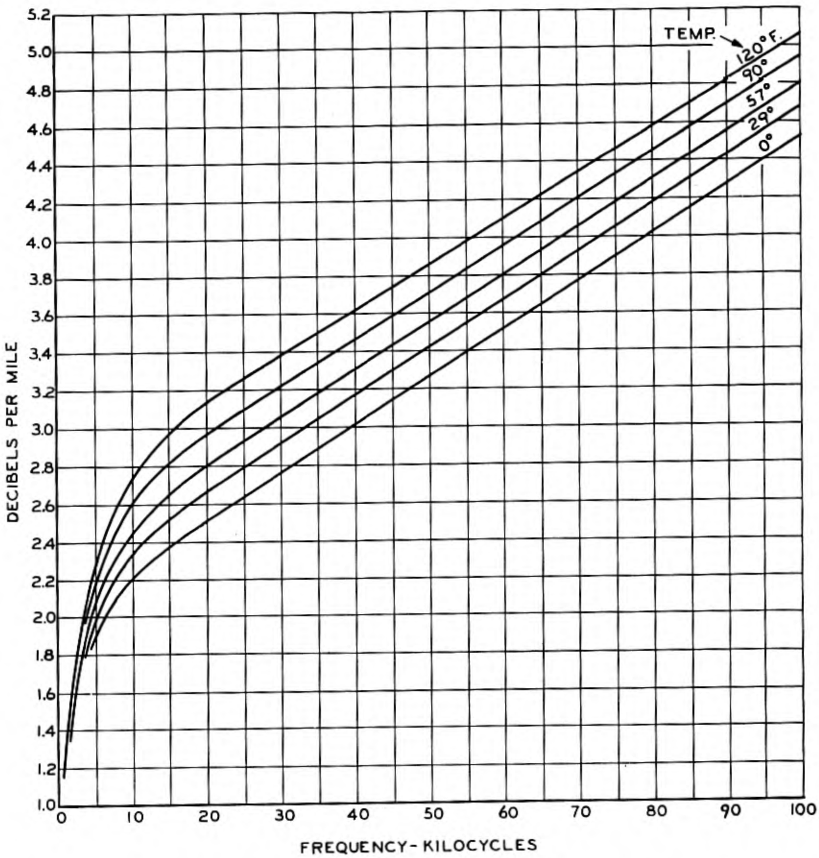


Fig. 14—Attenuation; decibels per mile—19 gauge pairs

actual rate of change. If the curves are plotted as attenuation coefficients (db per db per degree, Fahrenheit) with the abscissa

$$B = \sqrt{\frac{\text{frequency}}{R_{dc} \text{ per } 1000 \text{ ft}}},$$

the same as for skin and proximity effects in Fig. 2, the peaks are brought together as indicated in Fig. 16.

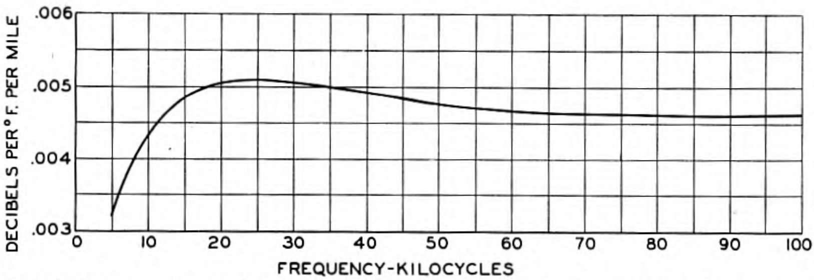


Fig. 15—Temperature variation of attenuation; decibels per degree Fahrenheit per mi!e vs. frequency—19 gauge pairs

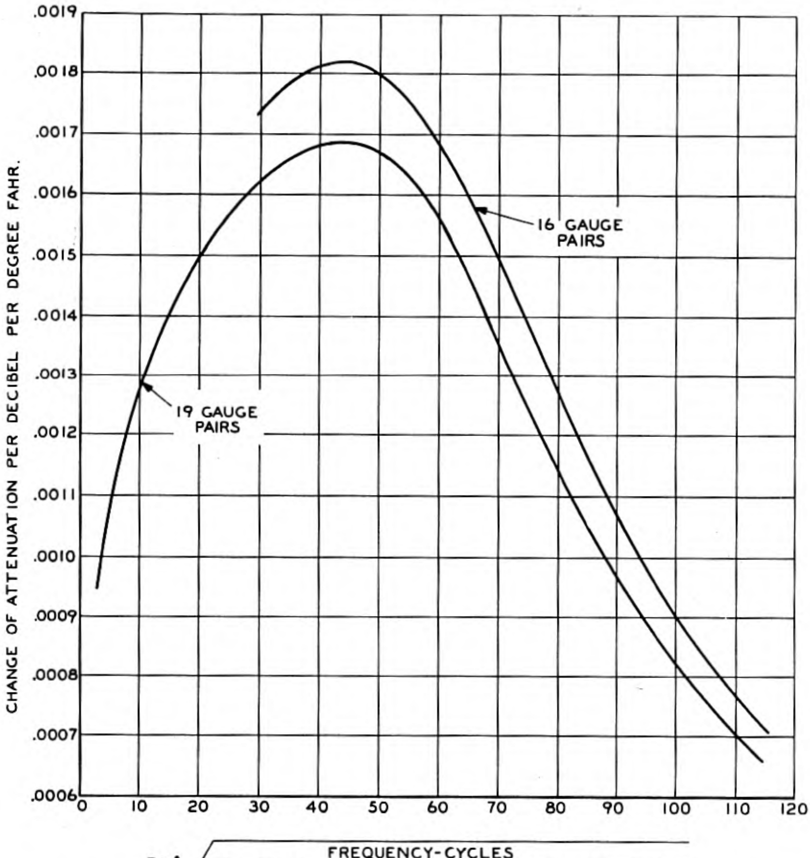


Fig. 16—Temperature coefficient of attenuation, α. $A = A_{50}[1 + \alpha(T - 50)]$

The change in attenuation with temperature may be formulated in various ways as a function of its component variables, R, L, G, C, the most obvious way being to take the partial derivatives of one of the equations (9), (10), or

(11) with respect to R, L, G and C, in order to get a differential expression $d\alpha$ in terms of dR, dL, dC and dG. This may then be interpreted as a change with temperature or a manufacturing variation, or a variation of attenuation from pair to pair in the cable. This procedure applied to (10) results in the formula

$$\begin{aligned} 4\alpha\cdot d\alpha = & \left[G + R\sqrt{(G^2 + \omega^2 C^2)/(R^2 + \omega^2 L^2)}\right] dR \\ & + \left[R + G\sqrt{(R^2 + \omega^2 L^2)/(G^2 + \omega^2 C^2)}\right] dG \\ & - \left[\omega^2 C - \omega^2 L\sqrt{(G^2 + \omega^2 C^2)/(R^2 + \omega^2 L^2)}\right] dL \\ & - \left[\omega^2 L - \omega^2 C\sqrt{(R^2 + \omega^2 L^2)/(G^2 + \omega^2 C^2)}\right] dC \qquad (13) \end{aligned}$$

Curves of Fig. 17 show the components of the temperature variation produced by changes in R, L, G and C for standard 19-gauge cable.

A better formula from the point of view of equalizer design results from applying Taylor's series expansion to equation (9) and taking the real part of the resulting expressions. In this method, the variables are taken to be LC, R/L and G/C which effectively reduces the number of variables by one. There is a further advantage which appears in equation (14), below, namely, that the coefficient of the per cent variation in LC is just 1/2 the attenuation constant α and this means, therefore, only a slight addition to the basic equalizer which matches the curve for α vs frequency. There are thus added only two new types of temperature equalizers, one for R/L and one for G/C correction. Since equation (10) is already in the real form, it is more straightforward to expand it by Taylor's series and use the required number of terms. The formula thus obtained is naturally the same as that obtained from (9) and is as follows:

$$\begin{aligned} \Delta\alpha = & \frac{\alpha}{2}\frac{\Delta(LC)}{LC} + \frac{1}{2}\frac{R}{\omega L}\sqrt{\frac{\alpha^2 + \omega^2 LC}{1 + R^2/\omega^2 L^2}}\frac{\Delta(R/L)}{R/L} \\ & + \frac{1}{2}\frac{G}{\omega C}\sqrt{\frac{\alpha^2 + \omega^2 LC}{1 + G^2/\omega^2 C^2}}\frac{\Delta(G/C)}{G/C} \\ & - \frac{1}{8}\frac{R^2}{\omega^2 L^2}\frac{(R/\omega L)\sqrt{\alpha^2 + \omega^2 LC} + \sqrt{\alpha^2 - RG}}{\sqrt{(1 + R^2/\omega^2 L^2)^3}}\left[\frac{\Delta(R/L)}{R/L}\right]^2 + \cdots \qquad (14) \end{aligned}$$

Application of this formula gives slightly different values for the temperature-attenuation coefficient at different parts of the temperature range for most frequencies. This means that the change of attenuation with temperature is not quite linear. The nonlinearity is so small below 100 kc. that it has not been measured with any certainty on lengths of cable varying from 500 feet up to 10 miles, but on long cable carrier circuits corrections for it may become necessary.

The formula has another use, however, in determining the effects of small manufacturing variations on the probable attenuation of cables made up of the resulting product, as well as computing fairly accurately the attenuation of all the pairs in a layer by means of the average values of the constants and the departures of the values of the constants of the individual pairs from the average values. Actual attenuation variations of pairs in a reel

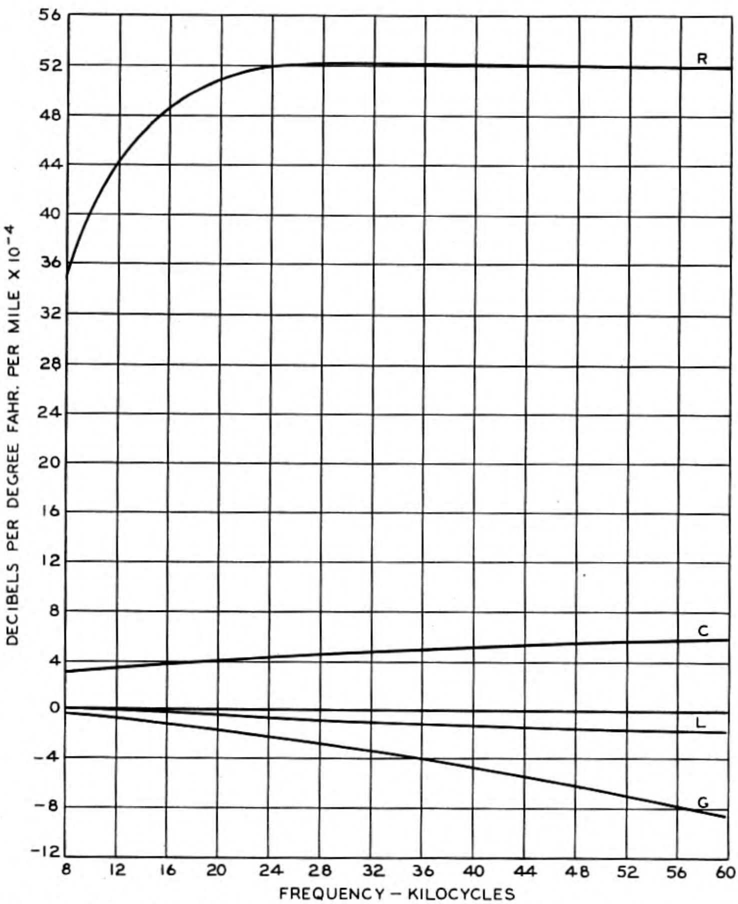


Fig. 17—Analysis of variation of attenuation with temperature; variations due to the components R, L, G, C—19 gauge pairs

are about ±5 per cent and the standard deviation of the variations is about 2.5 per cent.

Practically, since $G/\omega C$ is small, formula (11) may be used in the Taylor series expansion with the variables R/L, LC and RG. This gives the formula

$$4\alpha\cdot\Delta\alpha = \frac{RC}{\sqrt{1 + R^2/\omega^2 L^2}}\cdot\Delta(R/L) + \Delta(RG) + \omega^2(\sqrt{1 + R^2/\omega^2 L^2} - 1)\Delta(LC) \quad (15)$$

Components of $\Delta\alpha/\Delta T$ computed by formula (15) are given in Fig. 18 It is evident that changes in R/L are responsible for most of the change in attenuation since the small changes in attenuation introduced by ΔLC and ΔRG tend to annul each other over most of the frequency range shown. This is to be expected from the approximate high-frequency formula (12) in which $G/2C$ is much smaller than $R/2L$.

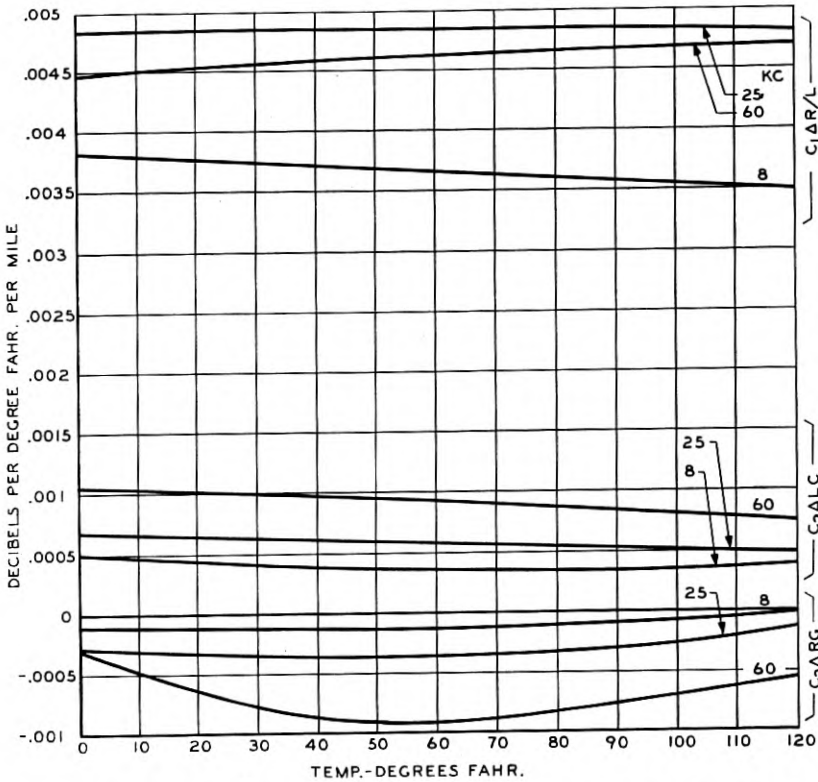


Fig. 18—Components of $\frac{\partial\alpha}{\partial T} = DB$ per °F. per mile $= C_1\Delta R/L + C_2\Delta LC + C_3\Delta RG$ vs. temperature—19 gauge pairs

Phase Change and Velocity

As pointed out above, merely changing the signs of the last parenthetical expression in equations (10) and (11) gives corresponding formulas for phase angle in radians. Fortunately, the phase change is nearly, though not quite, linear with frequency (Fig. 19). The velocity, $V = \omega/\beta$, for 19-gauge pairs is about 105,000 miles per second at 10 kc. and increases slowly to about 125,000 mps. at 100 kc. At high frequencies the internal inductance is small and, if the inductance L is expressed in abhenries and capacitance C in abstat-microfarads, then $V^2 = 1/LC = 1/k$, where k is the dielectric constant[15].

Impedance

The characteristic impedance is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{16}$$

which has a large reactive component at low frequencies as shown by the curves of Fig. 20, based on the same reel-length measurements as the curves

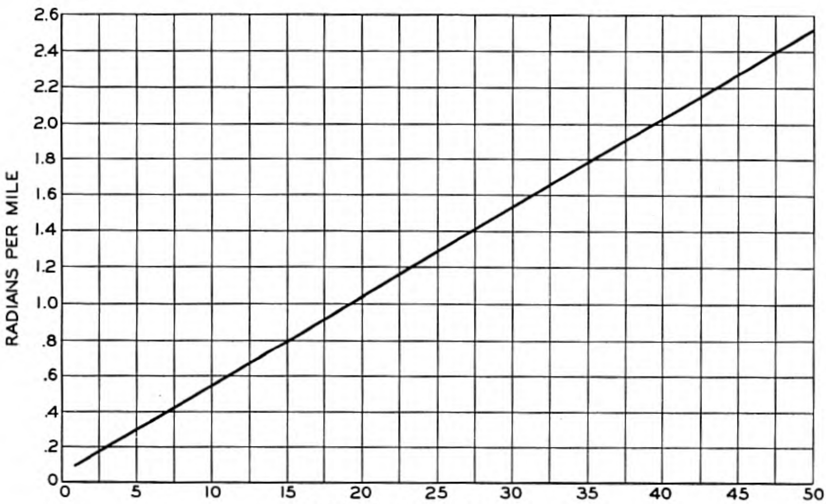


Fig. 19—Phase; radians per mile—16 gauge pairs 36°F.

for R, L, G and C in Figs. 3 to 11. In actual long cables the curves are irregular with frequency as a consequence of small irregularities along the line[13]. (See, for example, Fig. 26.) There are also small variations with temperature; for the resistance component about ± 1.5 per cent from the average for the temperature range zero to 120° F. at 10 kc., and about ± 1 per cent at 100 kc. The reactive component varies ± 10 per cent at 10 kc. over the same temperature range*.

[15] G. H. Livens, "The Theory of Electricity," p. 456 and p. 539.

* K. Simizu and I. Miyamoto, "Effect of Temperature on the Non-Loaded Carrier Cable, *Nippon Elec. Comm. Eng.*, May 1939, p. 596–599. Give similar data on the variation of parameters and attenuation for spiral-four cable at frequencies 0-30 kc. and temperatures 0–50°C. They do not specify the length measured but state that the wire diameter was 1.5 mm. From their d-c. resistance data the length appears to have been about 160 feet.

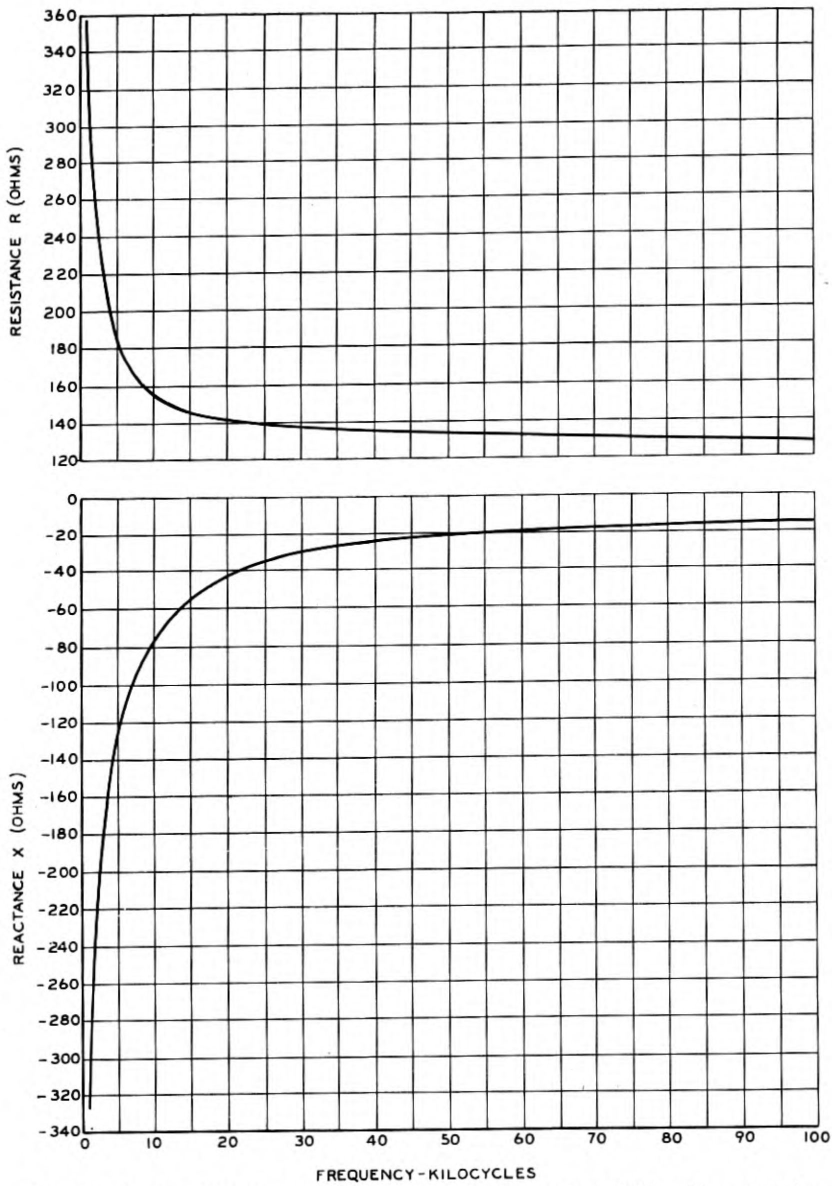


Fig. 20—Characteristic impedance, $Z = R - jx$; temperature 90°F.—19 gauge pairs

Characteristics of Toll Cable Above 100 Kc.

The preceding discussion has dealt largely with the characteristics of toll cables up to 100 kc. However, some measurements have been made extending to much higher frequencies. In the laboratories measurements were made on 16 and 19-gauge pairs in reel-lengths at frequencies up to about 3000 kilocycles. Field data at frequencies from 100 kc. to 2000 kc. were obtained on 16-gauge and 19-gauge pairs in cables about 3 miles long at

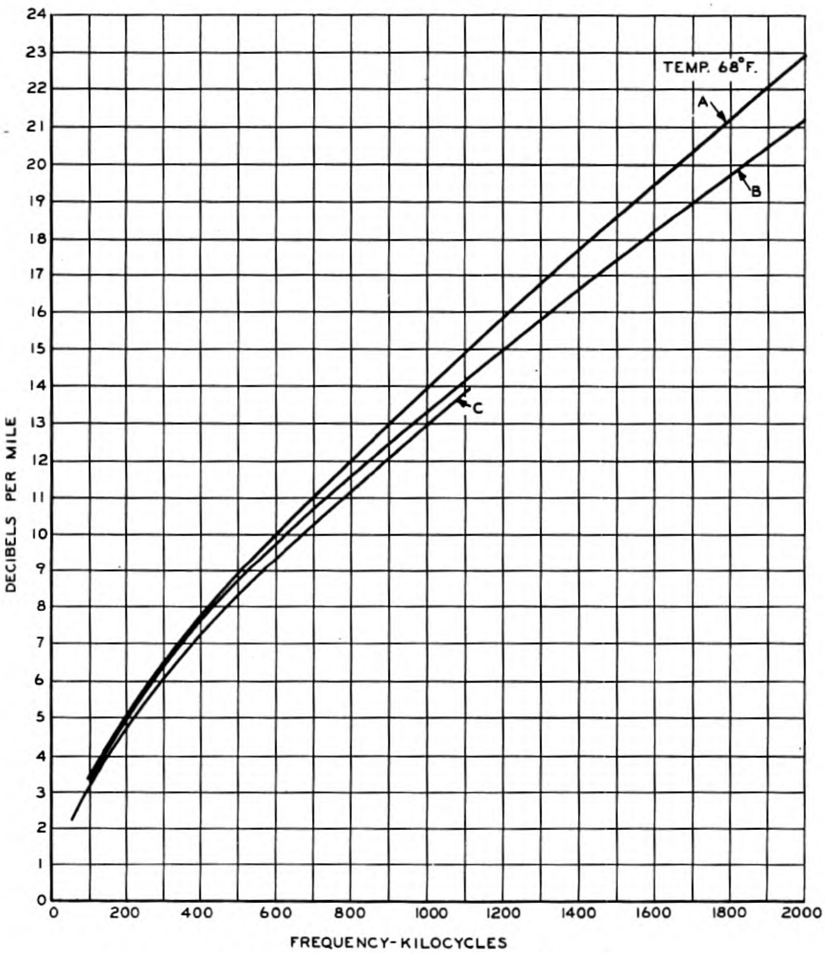


Fig. 21—Attenuation of 16 gauge cable pairs

A, on reel, 247 feet; B, fourteen reels, 1.3 miles; C, aerial cable, 3.6 miles, Ticonderoga, N. Y.

Ticonderoga, New York. A third set of data was obtained from measurements on 7000 feet of a special type (61-pair) of 16-gauge cable on reels in the laboratory under controlled temperature conditions. Figure 21 shows the attenuation values to 2000 kc. obtained in the three sets of 16-gauge

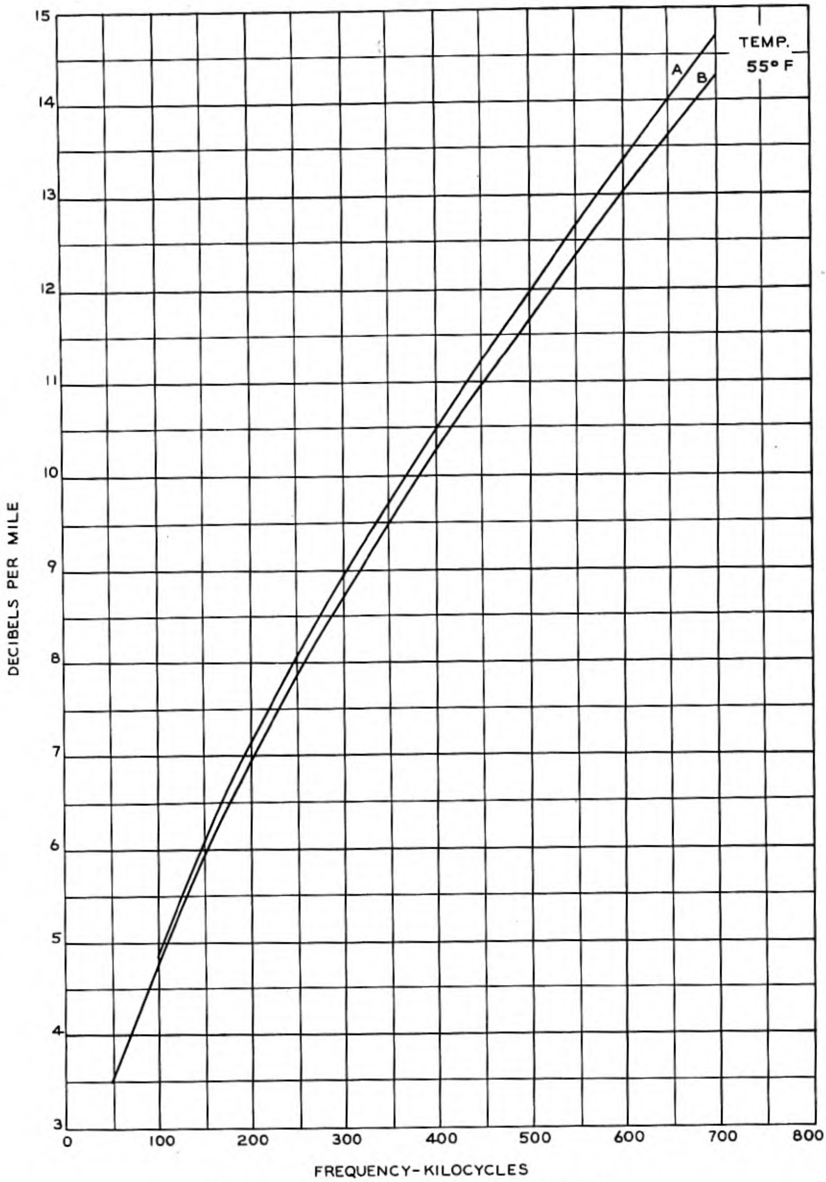


Fig. 22—Attenuation of 19 gauge cable pairs

A, on reel, 247 feet Bell Tel. Labs., Inc.; B, aerial cable, 3.6 miles, Ticonderoga, N. Y. Curves A and B show average of 10 pairs.

data at 68 degrees Fahrenheit. Figure 22 shows results on 19-gauge pairs at 55 degrees from field and laboratory data up to 700 kc.

The curves of the change in attenuation per degree F. per mile (db/1°F./mi) as shown by Figs. 23 and 24 are highly dependent upon the temperature, showing that at these high frequencies the attenuation is decidedly nonlinear with temperature in the toll cables.

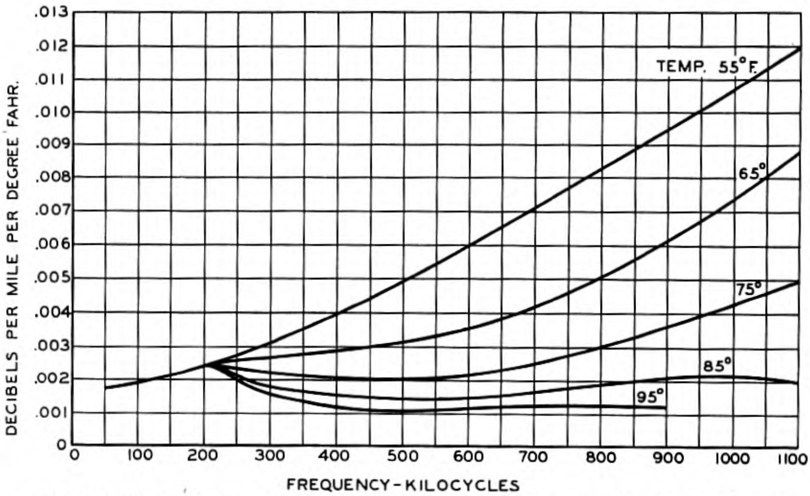


Fig. 23—Variation in attenuation at different temperatures for 1°F. change in temperature; aerial cable—16 gauge pairs

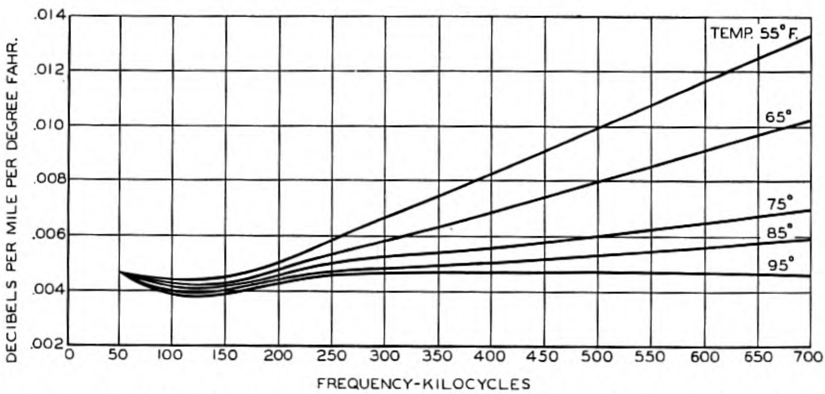


Fig. 24—Variation in attenuation at different temperatures for 1°F. change in temperature; aerial cable—19 gauge pairs

Toll Entrance Cable

The insertion losses measured between 125-ohm resistances on various lengths of 13, 16 and 19-gauge toll entrance cables at Denver, Colorado, are shown in Fig. 25. The data have been reduced to a per-mile basis by

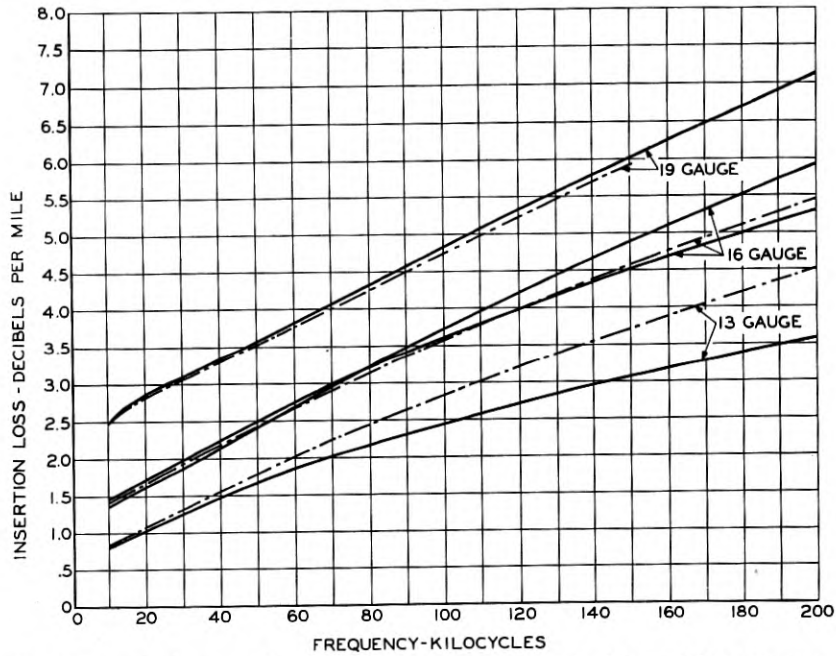


Fig. 25—Carrier frequency loss* of toll entrance cable; non-loaded, quadded—temperature 60°F., approx.

*Insertion loss between 125-ohm resistances

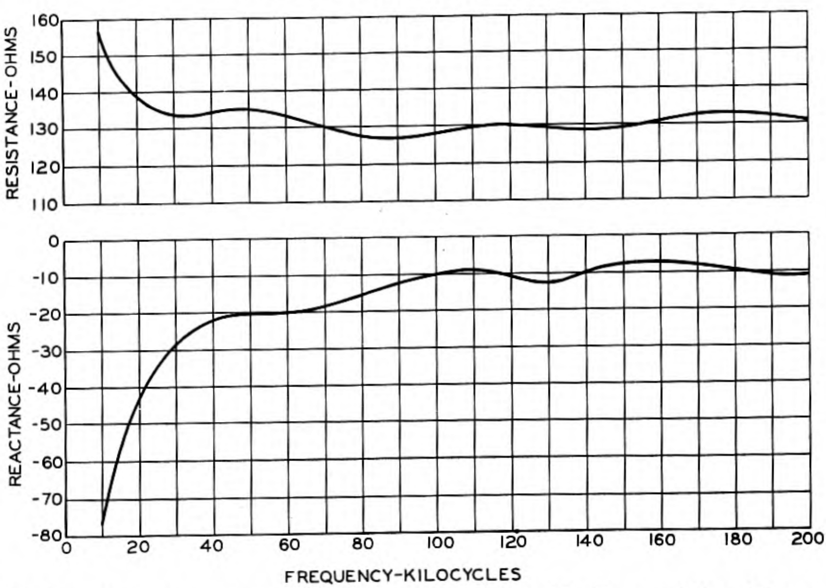


Fig. 26—Carrier frequency impedance of toll entrance cable, Denver, Colo.—19 gauge, quadded, non-loaded—terminated in 125 ohms

direct division of the measured attenuations by the lengths. This is, of course, not strictly accurate, but the errors are very small at these frequencies. This is nonloaded cable and frequencies measured were from 10 kc. to 200 kc. The values check closely the values shown in the previous figures for frequencies below 100 kc. The 13-gauge figures are the first of such data given herein but the first laboratory measurements on reel-lengths (begun in 1921) included reels of 13-gauge cable, and curves of 13-gauge attenuation and impedance were given in a paper[16] by E. H. Colpitts and O. B. Blackwell.

Corresponding data on impedance show the values given on Fig. 26. The wavy characteristic of these curves, as mentioned in the section on Impedance above, is caused by small irregularities in the pairs, particularly differences between pairs in successive reel-lengths giving rise to reflection currents at certain frequencies[14]. In new construction smoother impedance characteristics can be obtained when it is important to do so, by close control of the product during manufacture, followed by suitable splicing methods.

Acknowledgment

It is practically impossible to name all my associates in the Bell Telephone Laboratories whose work has been drawn upon in assembling these data, but I am especially indebted to Mr. Pierre Mertz and Mr. E. I. Green for their helpful suggestions and continued encouragement.

APPENDIX

Wave Propagation over Two Parallel Wires: The Proximity Effect—Inductance*

In his paper[4] on the Proximity Effect, J. R. Carson carried out the detailed computations for the ratio C of the a-c. resistance of two parallel wires to the a-c. resistance of a wire when the return conductor is concentric. He gave a formal expression for the impedance (equation 64 of his paper), viz.,

$$R + iX = 2Z + ipL \qquad (1a)$$

This simple equation is complicated by the fact that Z and L are given by two complex expressions involving Bessel functions and the set of harmonic coefficients of the Fourier-Bessel expansion for the axial electric force in

[16] E. H. Colpitts and O. B. Blackwell, "Carrier Current Telephony and Telegraphy," *Jour. A. I. E. E.* XL, Feb. 1921, pp. 205–300.

* This work, done under the direction of Mr. J. R. Carson, was completed in April, 1922. For the general theory of wave propagation on parallel conductors see a paper by Chester Snow, "Alternating Current Distribution in Cylindrical Conductors," Proc. Int. Math. Congress, Toronto (1924) Vol. II, pp. 157–218.

one of the wires and the separation of the wires. The Bessel functions are of order zero to infinity and the argument, b, is given by

$$b = ia\sqrt{4\pi\lambda\mu i p} \tag{2a}$$

where

a = radius of the wire in cm.
λ = conductivity of wire in c.g.s. units
μ = permeability of wire in c.g.s. units
p = 2π times the frequency in cycles per second
i = $\sqrt{-1}$

The separation comes in by way of the quantity k, the ratio a/c of the radius to the interaxial separation of the wires, and a function s which can be expressed as a continued fraction in k^2, viz.,

$$s = \cfrac{1}{1 - \cfrac{k^2}{1 - \cfrac{k^2}{1 - \cdots}}} \tag{3a}$$

which results in

$$s = \frac{1}{1 - k^2 s} \tag{4a}$$

from which

$$s = \frac{1 - \sqrt{1 - 4k^2}}{2k^2} \tag{5a}$$

as given by Carson's equation (38).

The actual expression for $R + iX$ is as follows:

$$\begin{aligned} R + iX &= 2Z + ipL \\ &= -4ip \log ks + 2Z_0\left[1 + \sum_{n=1}^{\infty} (-ks)^n h_n J_n/J_0\right] \end{aligned} \tag{6a}$$

where

$$\begin{aligned} Z_0 &= R_0 + iX_0 \\ &= \frac{2p}{b}\,\frac{u_0 v_0' - u_0' v_0}{u_1^2 + v_1^2} + i\,\frac{2p}{b}\,\frac{u_0 u_0' + v_0 v_0'}{u_1^2 + v_1^2} \end{aligned} \tag{7a}$$

which is the impedance of a wire with concentric return expressed as usual[17] in terms of the ber and bei functions related to the Bessel functions by the formula

[17] Russell, "Alternating Currents," Edition 1904, Vol. I, p. 370.

$$u_n + iv_n = J_n(b\,i\,\sqrt{i}) \tag{8a}$$

and primes denote derivatives with respect to b.

Substituting Z_0 from (7a) in (6a) and carrying out the algebraic processes involved gives finally

$$R + iX = 2\,R_0\,C + i\,(-4\,p\log ks + 2\,K\,X_0) \tag{9a}$$

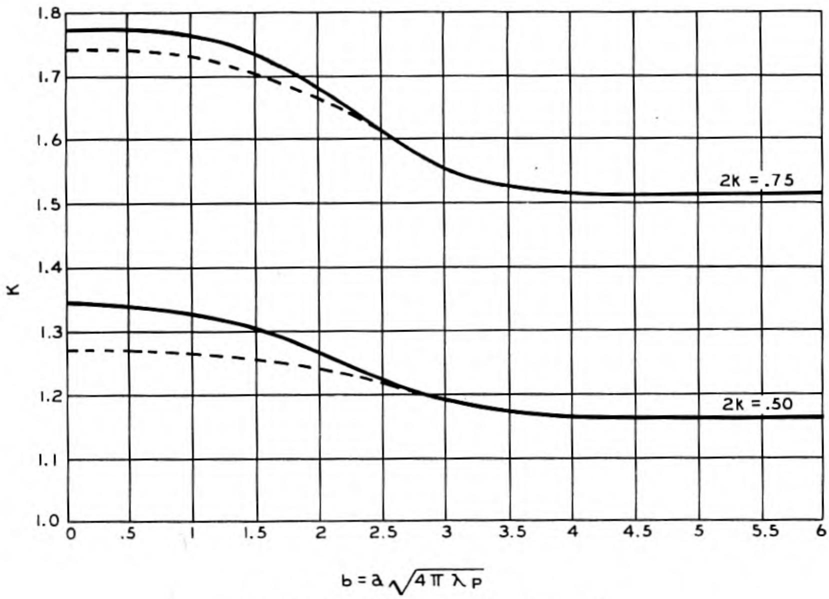


Fig. 27—Values of correction factor, K
For $2k = .75$ and $.50$

where

$$C = 1 + \frac{4p}{bR_0}\sum (k^2s^2)^n w_n + \frac{4p}{bR_0}\sum n(k^2s)^{n+1} gw_n$$
$$- \frac{4}{b}\frac{X_0}{R_0}\frac{u_1^2 + v_1^2}{u_0^2 + v_0^2}\sum n(k^2s)^{n+1}\frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u_{n-1}^2 + v_{n-1}^2} \tag{10a}$$

and

$$K = 1 + 2\sum (k^2s^2)^n \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u_{n-1}^2 + v_{n-1}^2}\cdot\frac{u_0'^2 + v_0'^2}{u_0u_0' + v_0v_0'}$$
$$+ \frac{4p}{b^2X_0}\sum n(k^2s)^{n+1} bg\,\frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u_{n-1}^2 + v_{n-1}^2} + 2w_n\frac{u_0u_0' + v_0v_0'}{u_0^2 + v_0^2}$$

$$= 1 + \frac{4p}{bX_0} \sum (ks)^{2n} \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

$$+ \frac{4p}{bX_0} \sum n(k^2 s)^{n+1} g \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

$$+ \frac{4}{b} \frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} \sum n(k^2 s)^{n+1} w_n \tag{11a}$$

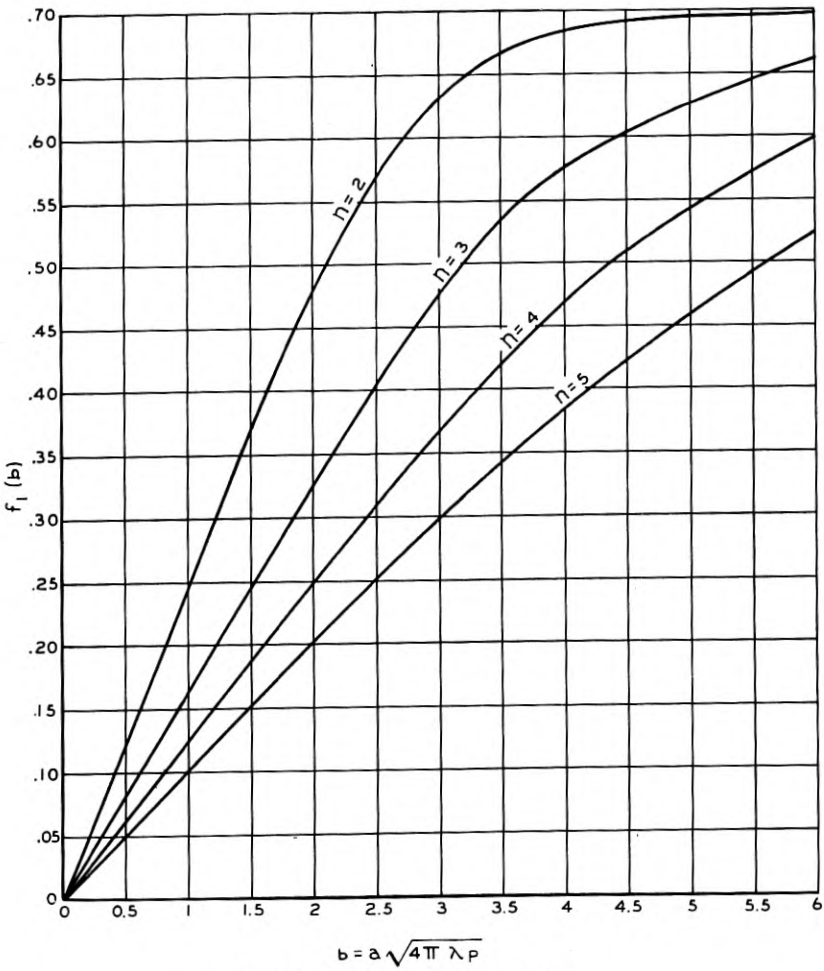


Fig. 28—Values of auxiliary functions

$$f_1(b) = \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

In these formulas

$$g = \frac{2}{b}\frac{u_0'v_0 - u_0v_0'}{u_0^2 + v_0^2} = -\frac{R_0}{p}\frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} \tag{12a}$$

$$w_n = \frac{u_n v_n' - u_n' v_n}{u_{n-1}^2 + v_{n-1}^2} \tag{13a}$$

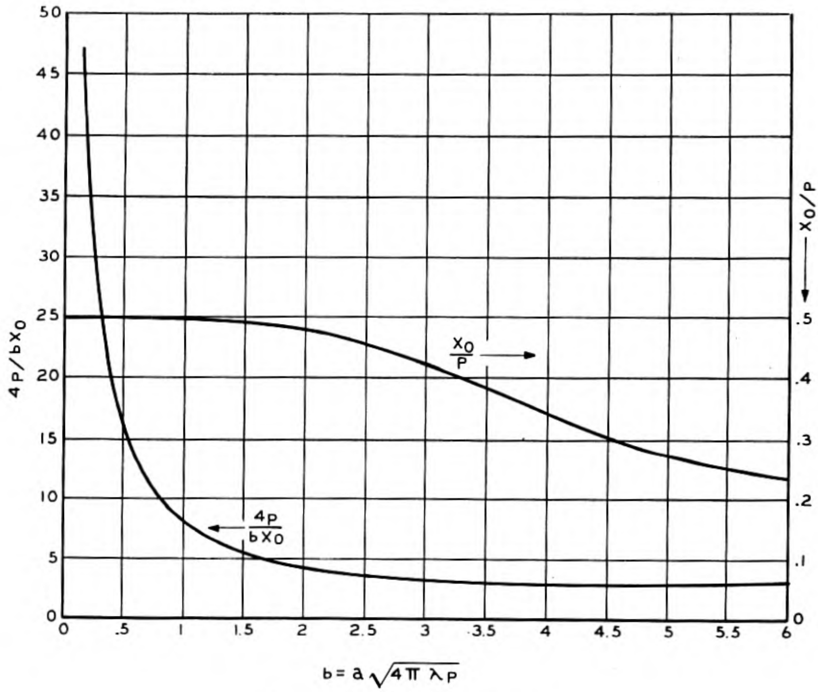


Fig. 29—Values of functions

$$\frac{4p}{bX_O} \quad \text{and} \quad \frac{X_O}{P}$$

The curves of figures 28–30 show the auxiliary functions vs b and figure 27 the correction factor K. The dotted curve for K is computed from Mie's formula[18] (14a) for small b. Two values of $2k$ are shown, .75 and .50, respectively.

G. Mie[18] gave formulas for small and large values of b, as follows:

For small b,

[18] G. Mie, *Annalen der Physik*, Vol. II, (1900) pp. 201–249.

$$L = 1 - 4 \log k - l_n - l'_n \tag{14a}$$

where $l_n = .417\ b^4/16 - .003\ b^8/256$

$$l'_n = b^4(1.33k^2 - .917k^4 - .652k^6 - .496k^8 \cdots)/16$$
$$- b^8(.633k^2 - 1.354k^4 + .539k^6 + .584k^8 \cdots)/256$$

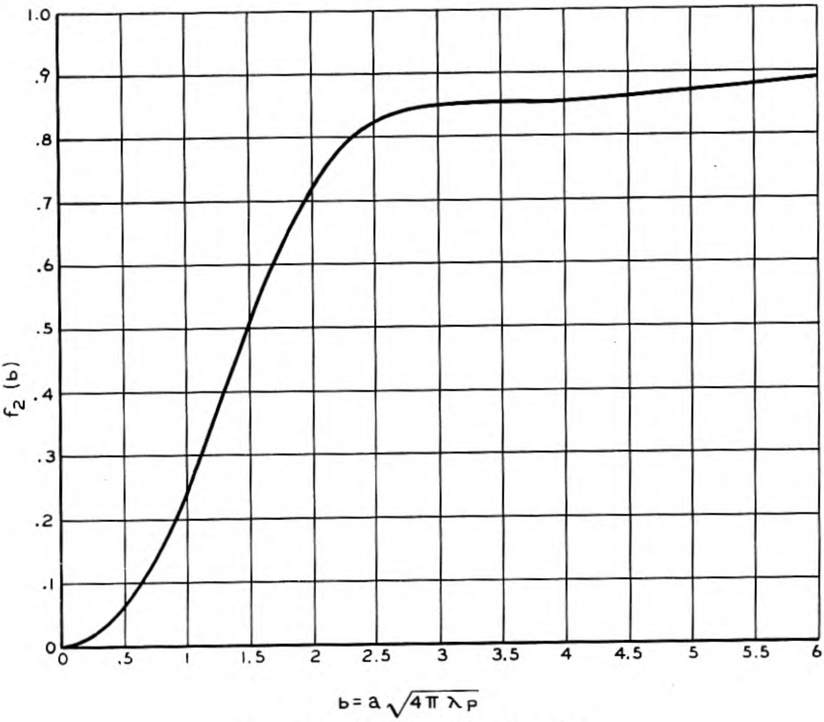


Fig. 30—Values of auxiliary functions

$$f_2(b) = \frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} = \frac{ber'^2 + bei'^2}{ber^2 + bei^2}$$

For large b,

$$L = -4 \log ks + \frac{2\sqrt{2}}{b} C_m - \frac{3}{2b^2} \tag{15a}$$

where $C_m = s/(2 - s)$.

Some Analyses of Wave Shapes Used in Harmonic Producers

By F. R. STANSEL

Analyses by Fourier's Series have been made of waves consisting of sinusoidal, rectangular and trapezoidal pulses and also waves of the type found in multivibrator circuits. The method of increasing harmonic content by modulating a wave with a submultiple is treated mathematically.

THE heterodyne method of frequency comparison requires, except in the case of the comparison of nearly identical frequencies, the generation of harmonics of either the unknown, or of the standard frequency or of both. These harmonics may be generated directly in the modulator which produces the difference frequency, or "beat note", or may be generated in an entirely separate circuit before the frequency is applied to the modulator. An example of the latter is the multivibrator circuit often used in connection with a frequency standard to produce a series of harmonics of this standard frequency.

The design of harmonic generators for frequency measuring equipment presents a different problem from the design of equipment for producing a single harmonic such as doubler or tripler stage in a radio transmitter. In the latter case the amplitude of the one harmonic and the efficiency are of primary importance. In frequency measuring equipment, although a large amplitude of each harmonic is desirable, it is of greater importance that each harmonic within the range to be used, which may be up to the 100th or 150th harmonic or even higher, be present and that the amplitude of nearby harmonics be of the same order of magnitude. Unless the latter conditions are met, there is a danger that the beats obtained with a weak harmonic will either be entirely overlooked or mistaken for a higher order modulation product.

The generation of harmonics is usually accomplished by the distortion of the wave shape in some nonlinear circuit element such as a vacuum tube. One such harmonic generator consists of a vacuum tube biased so that there is no output for a portion of the cycle. The plate current of such a tube may be approximated by a sine wave shaped pulse such as shown in Fig. 1. Any such periodic wave can be resolved into its harmonic components[1] and in the case of this wave the amplitude of the nth harmonic is found to be

[1] This and the subsequent analyses were made by application of Fourier's Series. See I. S. Sokolnikoff and E. S. Sokolnikoff, "Higher Mathematics for Engineers and Physicists," Chapter VI.

$$h_n = \frac{A}{n\pi(1 - \cos b/2)} \left[\frac{\sin (n - 1)b/2}{n - 1} - \frac{\sin (n + 1)b/2}{n + 1} \right] \quad (1)$$

in which A is the amplitude of the pulse and b the pulse width as shown in Fig. 1.

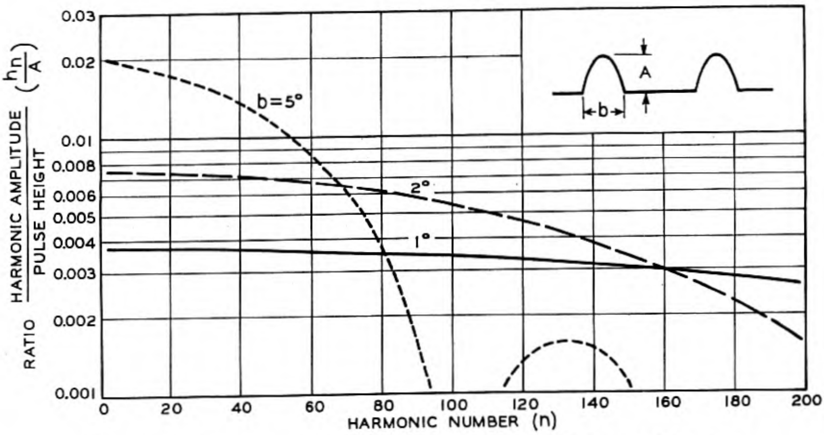


Fig. 1—Harmonic content of a wave consisting of sinusoidal pulses

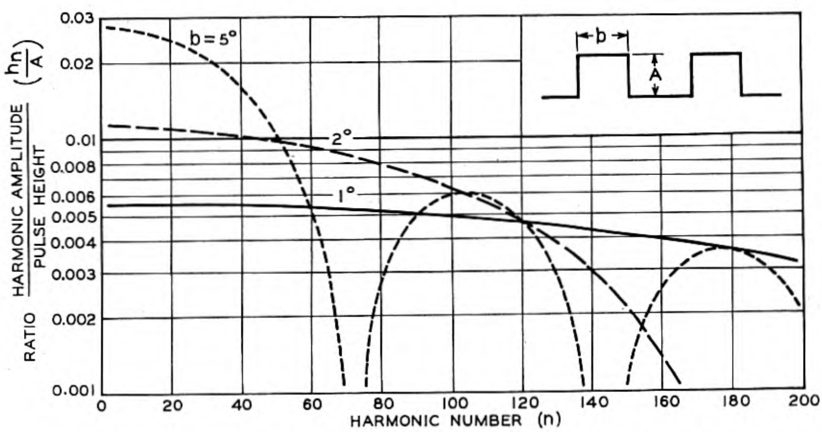


Fig. 2—Harmonic content of a wave consisting of rectangular pulses

The form of this expression immediately suggests that for some harmonics the terms

$$\frac{\sin (n - 1)b/2}{n - 1} - \frac{\sin (n + 1)b/2}{n + 1}$$

may become equal to zero causing these harmonics to vanish. That this is the case is shown in the curves of Fig. 1 in which the harmonic amplitudes are plotted against n for pulse widths of 5°, 2° and 1°. With a 5° pulse harmonics in the vicinity of the 105th and again the 150th become negligibly small. For a shorter pulse width the amplitude of the lower harmonics decreases but all harmonics up to beyond the 200th are present.

The wave shown in Fig. 1 can only be considered as a first approximation of the plate current in such a harmonic generator as it implicitly assumes that the tube is linear to cut-off. More frequently sufficient excitation is placed on the grid of the tube to saturate it and the resulting current wave may better be represented by a series of rectangular pulses such as shown

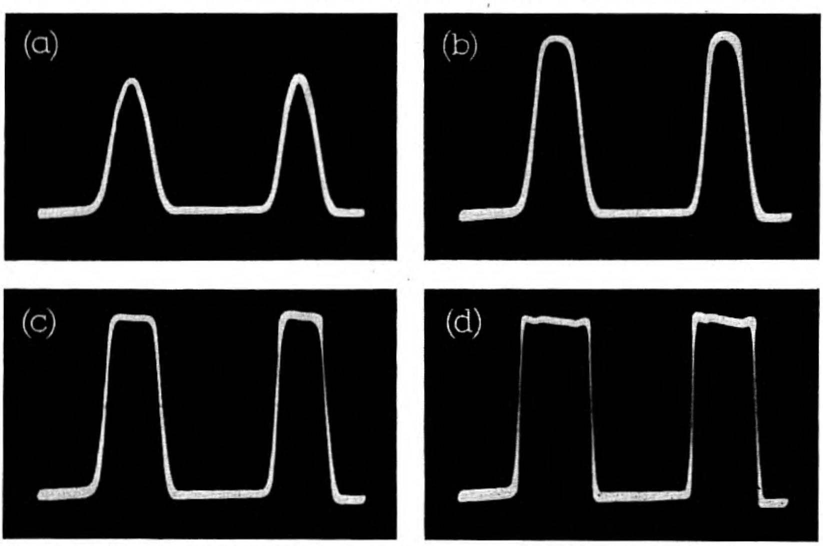


Fig. 3—Oscillograms of the plate current of a vacuum tube showing the transition from sinusoidal to rectangular pulses as excitation is increased

(a) Excitation 6 volts
(b) Excitation 8 volts
(c) Excitation 10 volts
(d) Excitation 20 volts

in Fig. 2. This transition from sine wave pulses to rectangular pulses as the grid excitation is increased is shown in the series of oscillographs in Fig. 3.

The analysis of a wave consisting of rectangular pulses such as the one in Fig. 2 shows the amplitude of the nth harmonic to be

$$h_n = \frac{2A}{n\pi} \sin \frac{nb}{2} \tag{2}$$

From this equation it is seen that certain of the harmonics are not present as the expression (2) becomes equal to zero whenever

$$n = \frac{2\pi}{b} m \tag{3}$$

$$m = 1, 2, 3, 4, \cdots$$

Thus for a rectangular pulse of 5° ($\pi/36$ radians) pulse width the 72nd, 144th, 216th, etc. harmonics vanish, and harmonics in the vicinity of these missing harmonics have lower amplitudes as can be seen from the curves of Fig. 2.

As the pulse width of a rectangular wave increases, the number of harmonics which vanish increases. For a pulse width of 90° every fourth harmonic is missing. For a pulse width of 180°, the familiar square wave,

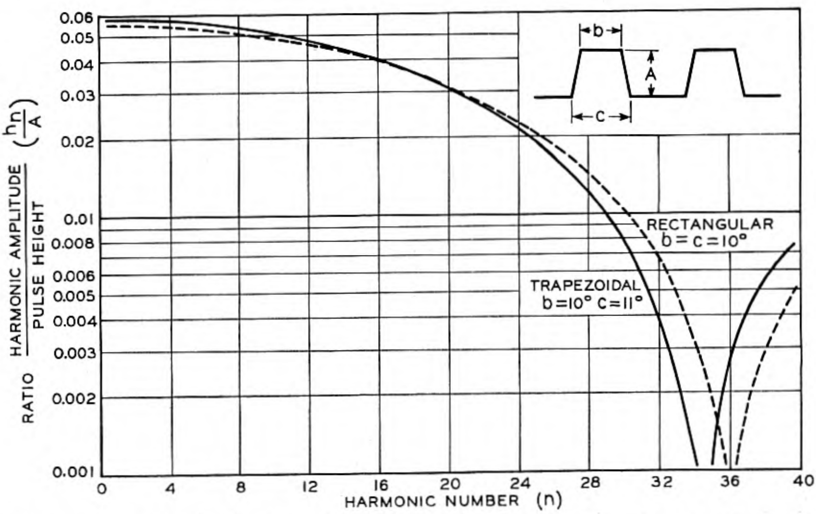


Fig. 4—Comparison of the harmonic content of waves consisting of rectangular and o trapezoidal pulses

every even harmonic vanishes and the wave contains only odd harmonics. As the pulse width is increased beyond 180° the number of harmonics increases and it can be shown that a wave having a pulse width greater than 180° will have the same harmonic content[2] as a wave of pulse width (360°—b). Thus for a large harmonic content it is desirable to have a wave having either extremely narrow pulses or pulses lasting nearly 360°.

True rectangular pulses are never obtained in practice. One common type of distortion in such pulses when obtained by the "limiter" action of a vacuum tube consists in the pulses having sloping rather than vertical sides. The sloping sides arise from the fact that the pulses are essentially sine waves

[2] This statement is correct for absolute magnitude of the harmonics only. Certain of the harmonics in the two waves will be 180° out of phase.

with their tops chopped off. The analysis of a pulse of the dimensions shown in Fig. 4 shows that the amplitude of the nth harmonics is given by the expression

$$h_n = \frac{4A}{n^2\pi(c-b)}\left[\cos\frac{nb}{2} - \cos\frac{nc}{2}\right] \tag{4}$$

In order to show better the relationship between a wave of rectangular pulses and one of trapezoidal pulses, consider the ratio of the nth harmonic for these two waves. From (2) and (4)

$$\frac{h_n \text{ for trap. pulse}}{h_n \text{ for rect. pulse}} = \frac{2(\cos nb/2 - \cos nc/2)}{n(c-b)\sin nb/2} \tag{5}$$

Substituting $c - b = \delta$ and expanding $\cos nc/2 = \cos\,(nb/2 + n\delta/2)$, the right hand side of (5) becomes

$$\frac{2}{n\delta}\left[\frac{\cos nb/2}{\sin nb/2} - \frac{\cos nb/2 \cos n\delta/2}{\sin nb/2} + \sin n\delta/2\right] \tag{6}$$

For small values of $n\delta/2$, that is for trapezoidal waves whose base is only slightly wider than the top, $\cos n\delta/2$ may be replaced by unity and $\sin n\delta/2$ by $n\delta/2$. The first two terms then cancel and the approximation

$$\frac{h_n \text{ for trap. pulse}}{h_n \text{ for rect. pulse}} \cong 1 \tag{7}$$

is obtained showing that a slight slope in the sides of the pulse has only a second order effect on the harmonic content of the wave.

The curve in Fig. 4 shows the harmonic content of a rectangular wave having a pulse width of 10° compared with that of a trapezoidal wave having a pulse width of 10° at the top and 11° at the bottom. For lower harmonics the amplitudes are nearly the same, but in the vicinity of the 36th harmonic there is an essential difference. For the rectangular pulse, the 36th harmonic vanishes, while the trapezoidal pulse has a minimum at a somewhat lower value of n and all harmonics have finite values.[3] This is shown in Table 1 which tabulates the amplitude of the harmonics in this case.

A second form of distortion in rectangular pulses is the rounding of the corners at both the top and the bottoms of the pulse. This type of distortion is more difficult to analyze and while no complete analysis has been made the effect of such distortion is known to be, in general, to reduce the amplitude of the higher harmonics.

[3] In discussing the curves in Fig. 1 thru 5 it must be remembered that while these are drawn as solid lines, the lines have a meaning only for integral values of n. Fractional values of n are meaningless.

From the examination of these cases it is evident that in the design of a harmonic generator of the type here considered the decision as to the pulse

TABLE 1

Harmonic Content of Rectangular and Trapezoidal Pulses Shown in Figure 4

Harmonic	Harmonic amplitude / Pulse height $= \frac{h_n}{A}$	
	Rectangular	Trapezoidal
Fundamental	.0555	.0581
2	.0554	.0580
3	.0550	.0576
4	.0545	.0570
5	.0540	.0563
10	.0488	.0505
15	.0411	.0416
20	.0314	.0307
25	.0209	.0191
30	.01061	.00811
32	.00681	.00411
33	.00501	.00226
34	.00325	.000489
35	.001901	−.001085
36	0	−.00276
37	−.00180	−.00412
38	−.00291	−.00557
40	−.00545	−.00791

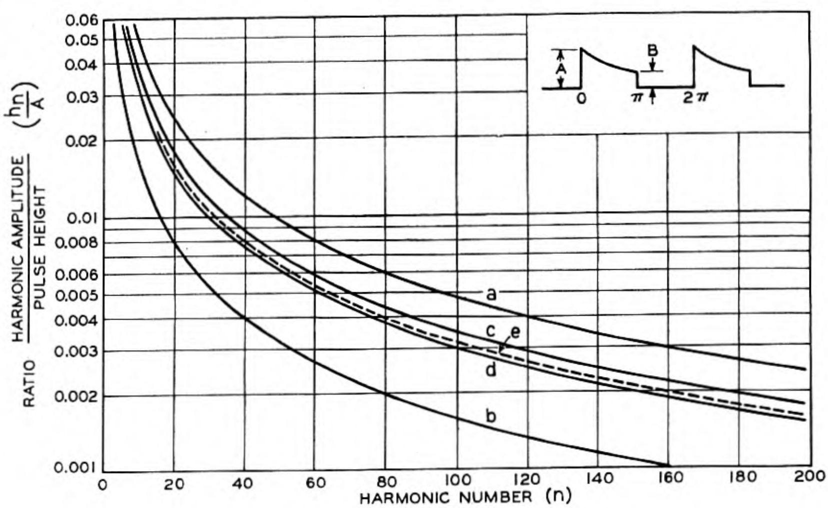


Fig. 5—Harmonic content of multivibrator wave
(a) Odd harmonics $\tau = 1/2$
(b) Even harmonics $\tau = 1/2$
(c) Odd harmonics $\tau = 1/10$
(d) Even harmonics $\tau = 1/10$
(e) All harmonics $\tau = 0$

width must be based on the type of service to which it is to be put. If only a few harmonics are required, a considerable gain in the amplitudes of the harmonics can be obtained by using a wider pulse width. When a wide range of harmonics is required, the band width must be greatly reduced to avoid blank intervals in the frequency spectrum.

A second type of harmonic generator is the multivibrator. The output wave of such a harmonic generator has a shape similar to that shown in Fig. 5. The current pulse lasts for a complete 180° rising abruptly to the peak value, then falling more or less exponentially to a lower value and finally breaking abruptly to zero. Assuming an exponential decay this wave will be found to contain the following harmonics

$$h_n = \frac{A(1-\tau)}{\sqrt{n^2\pi^2 + (\ln \tau)^2}} \text{ for even harmonics} \tag{8}$$

$$h_n = \frac{A(1+\tau)}{\sqrt{n^2\pi^2 + (\ln \tau)^2}} \text{ for odd harmonics} \tag{9}$$

Except for small values of n, the $(\ln \tau)^2$ term is negligible and these equations can be written

$$h_n = \frac{A(1-\tau)}{n\pi} \text{ for even harmonics} \tag{10}$$

$$h_n = \frac{A(1+\tau)}{n\pi} \text{ for odd harmonics} \tag{11}$$

In all of the above equations $\tau = B/A$, the ratio of the amplitude at the end to the amplitude at the beginning of the pulse.

The curves in Fig. 5 show the harmonic content of such a wave for $\tau = \frac{1}{2}$ and $\tau = \frac{1}{10}$. In the first case the amplitudes of the odd and even harmonics differ by approximately 9.5 db while in the second case the amplitudes are not greatly different. The dotted curve shows the limiting condition which all harmonics approach as τ approaches zero, that is as the current at the end of the pulse approaches zero.

The analysis of such a pulse except assuming a linear rather than exponential decay yields the following equations

$$h_n = \frac{A\ (1-\tau)}{n\pi} \text{ for even harmonics} \tag{12}$$

$$h_n = \frac{A}{n\pi}\sqrt{(1+\tau)^2 + \frac{4(1-\tau)^2}{n^2\pi^2}} \text{ for odd harmonics} \tag{13}$$

As n becomes large the second term under the radical becomes small and (13) becomes

$$h_n = \frac{A\,(1 + \tau)}{n\pi} \text{ for odd harmonics} \tag{14}$$

Equations (12) and (14) are identical with (10) and (11) showing that in harmonic generators of this type the harmonic content of the output wave is primarily a function of the initial and final values of the current rather than of the shape of the decay curve.

All of the foregoing curves show that the amplitudes of the higher harmonics are quite small so that in many applications some method of increasing their amplitudes may be required. This can be accomplished by the use of tuned amplifiers. An alternative method is to modulate a standard frequency wave with a lower derived frequency.

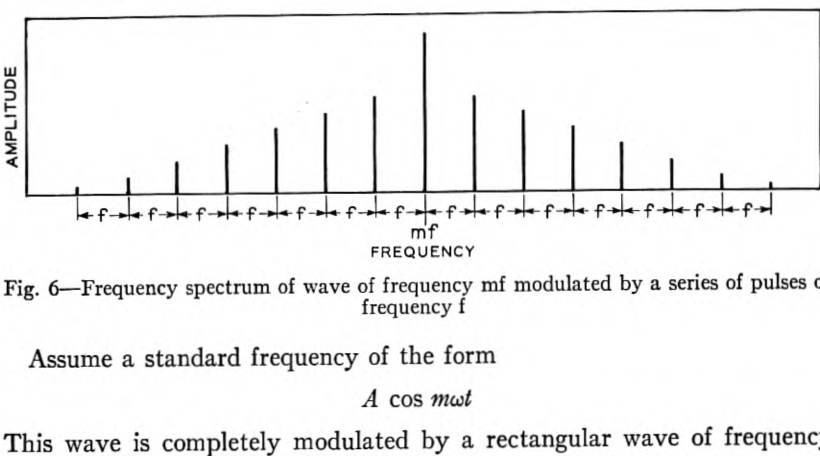


Fig. 6—Frequency spectrum of wave of frequency mf modulated by a series of pulses of frequency f

Assume a standard frequency of the form

$$A \cos m\omega t$$

This wave is completely modulated by a rectangular wave of frequency $\omega/2\pi$ and pulse width b. The modulated wave will then be of the form

$$I = A[1 + Kf(t)] \cos m\omega t \tag{15}$$

As shown previously the modulating wave is of the form

$$f(t) = \frac{b}{2\pi} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{nb}{2} \cos n\omega t \tag{16}$$

For 100 per cent modulation $K = 1$. Since

$$\cos (m\omega t) + \cos (n\omega t) = \tfrac{1}{2} \cos (m + n)\, \omega t + \tfrac{1}{2} \cos (m - n)\, \omega t \tag{17}$$

the modulated wave is

$$i_p = \frac{Ab}{2\pi} \cos m\omega t + \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin \frac{nb}{2} \cos (m + n)\, \omega t + \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin \frac{nb}{2} \cos (m - n)\, \omega t \tag{18}$$

The frequency spectrum of this wave is shown in Fig. 6. The original standard frequency $m\omega/2\pi$ is present and on either side above and below $\omega/2\pi$ cycles apart are additional components. The rate at which the amplitude of these frequencies dies out depends on the modulating pulse width and is equal to half the amplitude of the corresponding harmonic in Fig. 2.

If the standard frequency is not a pure wave but contains harmonics each of these harmonics will be modulated by the rectangular pulses, that is the function (16). The result will be a series of frequency spectra similar to the one in Fig. 6, each centered at one of the harmonics of the standard frequency. By proper choice of the frequency of the modulating wave these spectra may be made to overlap giving a continuous series of harmonic of the modulating frequency with much larger amplitudes than can be obtained from a straightforward harmonic generator. As an example, a one-megacycle wave heavily modulated with 100 kc was found to give strong 100 kc harmonics up to well over 35 mc.

ACKNOWLEDGMENT

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Forces and Atoms: The World of the Physicist*

By KARL K. DARROW

ONE of the signs whereby a physicist may be known is a fondness for putting dots upon blackboards. This is not an irrational habit, but a symbolic practice. It is a symbol of his manner of regarding the world as a multitude incredibly enormous of particles incredibly small. The dots stand for the particles, and the bare regions of the blackboard for the empty spaces between them. The habit has not indeed been universal. Many a thinker has preferred to consider the world as a continuum, a solid or jelly or fluid; and we shall see that this alternative has always been very near in the background, even when the "atomists" were at their most triumphant. Let me however defer this other idea, and derive as much as possible from the notion of particles in a void.

But when the dots are set down on the otherwise clean board with regions of black emptiness between, the story is far from completed. It is, in fact, only begun, for the major part is yet to be written: the account of the forces among the particles. Though these last be separated from each other by spaces apparently empty, yet they are not unconscious of each other, for each of them is subject to a force—the resultant of many forces, due to all rest.

One might attach an arrow to each dot, to signify the strength and the direction of the force which acts upon it. One might draw wandering curves all over the board, to intimate at every point the direction and strength of the force which a particle would feel, were it to be at that point. This is accepted practice, but it would be worth the doing only if our assumptions and our ambitions were much more specific than for the present they are. Perhaps at least the blackboard should be smeared with a uniform coating of chalk, to signify that a particle in space is not left entirely to itself, but feels the influence of the others. Among our not-so-distant ancestors there seems to have been a psychological need for a gesture of the sort; they talked about space as though it were filled with a "medium" or "aether", because it seemed wrong to them to say that space is empty if the particles which wander in it are subject to forces. Our generation has nearly lost the need, whether of an aether to occupy actual space or of a

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smear of chalk to symbolize it on the blackboard. Let us say that space is empty and leave the blackboard black between the dots, without deeming ourselves deprived of the right of saying that the particles exert and suffer forces on and from one another.

What is there to be said about the forces? A very great deal! for most of theoretical physics is made up of beliefs or ideas about the forces, augmented by the mathematical operations—very hard and very long-winded, in far too many cases—required for making the ideas really useful. So great a programme is indicated by that sentence, that I am wasting words in adding that it will not be fulfilled in one or two lectures, nor in the whole of the course. Only the most general of statements can be made in what follows. Of these I lay down at once the first, which is negative and self-evident:

The forces cannot be purely repulsive. For if they were, all of the particles would rush off into the uttermost depths of space, and we should have no model at all for a universe which, with all its faults, does manage at least to stick together.

Therefore there must be attractive forces, and these by and large must overpower the repulsive ones, if any such there be.

But need there be any repulsive forces at all? (Let the sophisticated reader now forget for a little that there are electrical forces which are repulsive, so that he may enquire with an open mind as to whether such could be avoided.) At first, it may not seem so; and one may invoke the great authority of Newton, who is often thought to have contented himself with assigning to all bodies the power of attracting one another with the force of gravity. He did not so content himself, and we shall learn this shortly. For the moment, let it be remembered that forces of attraction unopposed would tend to draw all of the particles of the universe into a single compact clump. If the volume of each particle were infinitely small, so also would be that of the ultimate clump; if the volume of each particle were irreducible below a certain minimum—but we shall ere long find what *that* idea can involve us in! Briefly, there must be something to oppose the attractive forces. To call this something by the name of "force", or even to call it by any single name, would be to limit it unduly. So to the second general statement I give the form:

There must be attractive forces, but there must also be antagonists to them. If someone wanted a particular problem of the theory of physics identified to him as the profoundest, the problem of these antagonists might well be selected as such.

There is indeed one famed and spectacular case, which makes one antagonist clear. It is the case of the heavenly bodies: the planets revolving around the sun, the satellites around the planets. Why does not the moon fall onto the earth and the earth fall into the sun? Newton's laws of motion tell the answer. The antagonist is *motion*—or, to speak more precisely, momentum—or, to speak yet more precisely, angular momentum. If two particles attract one another but are moving with a relative motion which is not along the line that joins them, they never will meet. However great the attraction between them, it cannot draw them together. Attraction can do no more than constrain them to swing in permanent orbits around their common centre of mass. Therefore,

The celestial bodies exhibit to us a system kept stable by the attraction of gravity, with motion for the antagonist thereto.

However natural this statement may now seem, it is by no means an idea inborn in the human mind. There was an era when it was believed that motion dies out of itself, unless continually sustained by a never-ceasing stimulus. Were motion to die out of itself, it could not be an eternal antagonist to gravity. Newton cleared the way for the new idea by abolishing the old one.

May we now assume that the ultimate particles of the world act on each other by gravity alone, with motion as the sole antagonist to keep the universe from gathering into a single clump?

The answer to this question is a forthright and irrevocable NO.

That the answer should be *no* is not at all surprising to this generation, which is familiar with other forces than gravity, the electromagnetic forces especially. Those who underrate the prowess of our forerunners may feel surprise on hearing that the negative answer was quite as apparent to Newton. No apology is ever needed for quoting verbatim what Newton wrote in English, though it is a dangerous act for the quoter, whose writing must suffer by contrast with the simple elegance of the seventeenth century. Incurring the danger, I cite from the *Opticks* (a book of which the name falls decidedly short of the scope):

"The attractions of gravity, magnetism and electricity reach to very sensible distances, and so have been observed by vulgar eyes, and there may be others which reach to so small distances as hitherto escape observation. . . . The parts of all homogeneal hard bodies which fully touch one another stick together very strongly. And for explaining how this may be, some have invented hooked atoms, which is begging the question; and others tell us that bodies are glued together by rest, that is, by an occult quality, or rather by nothing; and others, that they stick together by conspiring motions, that is, by relative rest among themselves[1]. I had rather infer from their cohesion, that their particles attract one another by some force, which in immediate contact is exceedingly strong, at small distances per-

[1] These remarks seem to be aimed at Lucretius, or else at the Greeks from whom Lucretius took some of his ideas.

forms chemical operations, and reaches not far from the particles with any sensible effect."

Further along in the *Opticks* we read:

"Thus Nature will be very conformable to herself and very simple, performing all the great motions of the heavenly bodies by the attraction of gravity which intercedes those bodies, and almost all the small ones of their particles by some other attractive and repelling powers which intercede the particles."

With the powerful aid of Newton we have now distinguished between the attractive force of gravity and another attractive force, for which I retain the old-fashioned name "cohesion". I give another basis of distinction, one which could not have been found until in the mid-nineteenth century the equivalence of heat with mechanical work was established. Consider a piece of solid or liquid matter, and put the question: how much work must be done to tear its atoms apart and dissipate them into the infinite reaches of space, if the only force whereby they act on one another is the attraction of gravity? The question is answerable, if it is known how massive the atoms are and how far apart (on the average) they are. These things are known. The result of the computation is to be compared with the amount of work which is actually expended—in the form of heat—when the solid or liquid is volatilized into vapor. It is found that only about the billionth part of a millionth part of the heat so spent is devoted to "breaking down the gravitational bond", to doing work against the attraction of gravity which is overcome when the atoms are dispersed[2]. All the rest is required for overcoming that more intimate force of cohesion.

Gravity now is pushed into the background, and sinks into the relative insignificance which may be gauged from the fact that in the endless speculations of physicists and chemists as to how matter is built up and joined together, it is completely left out. The force which dominates the planets, which makes a hill so hard to climb and a height so dangerous to fall from—how amazing that it should be trivial, compared with others which the flame of the gas-jet vanquishes as the water boils out of the kettle! Trivial of course by comparison only, and at small distances, not at great; or to phrase the situation better, it is the force of cohesion which is trivial at great distances, gigantic at small. This is the contrast which is implied by the technical terms of physics, "long-range forces" versus "short-range forces".

[2] The computation for mercury was made by my colleague Dr. L. A. MacColl, on the basis most favorable to gravity: by assuming mercury to be a continuum, or in other words, to be made up of infinitesimal atoms infinitely close together—an assumption giving the greatest possible value to the work required for spreading the mercury through infinite space, if gravity be the only restraint. The latent heat of vaporization of mercury is found by experiment to be $1.88.10^{16}$ times this value. Thus the contrast mentioned in the text is not contingent upon knowledge of the mass and spacing of the atoms, though the knowledge is available if wanted.

Gravity is long-range, because it falls away gently with increase of distance; cohesion is short-range, because it falls away precipitately. We shall soon be meeting with other examples of either character.

One other fact to illustrate the short-range quality of the cohesive forces: When a kettle of water is boiling away on the stove, the amount of heat consumed in dispersing the first cubic inch that departs is the same as is spent in dispersing the second, and the third, and each of the others down to and including the last. This could not be so, if the particles were drawn together by important long-range forces; for then each cubic inch would be easier to drive off than that which last preceded it into the vaporous state, since there would be less of the liquid remaining behind to attract it.

The celestial bodies—useful as they have been in showing us the laws of motion—have therefore served us badly by hinting that gravity is the sole attractive force, a hint which is quite misleading. In another important respect they fail to give us a lead: they show us no examples of collision. Collision, more commonly known as impact, is one of the most important of earthly phenomena, as it is one of the most uncomfortable. The apple which fell in the orchard of Newton, and inspired him with the law of gravitation, may have been a legendary apple; if it was real, we may be sure that it ended its fall in a collision—ended its fall, not its existence. It did not pass through the globe and pop out of the ground in the Antipodes; it did not instantly merge with the grass or the soil of the orchard; it bounced and rolled a little, perhaps, and then lay quietly pressing against the earth, entire and whole. The earth was impenetrable to the apple, as the apple to the earth.

We do not even have to look to impact, to be taught this lesson about the impenetrable. Not less impressive than the fact that the piece of iron sticks together, is the fact that it does not shrink. For any particular choice of temperature and pressure, it has a particular volume which is its own. Work or heat must be expended to dilate it or tear it apart altogether, but also work must be expended to make it denser.

Having ascribed to attractive forces the fact that it takes heat—or let me say henceforward, energy—to vaporize a piece of matter solid or liquid, we now ascribe to repulsive forces the fact that it takes energy to squeeze the piece. The forces must be short-range—still more short-range than are the cohesive forces, inasmuch as these come into play to capture the atoms and hold them together, before those get their opportunity of crying "hold, enough!" They must be very potent, for the most terrific pressures which have been achieved by man do not avail to squeeze the most compressible solid into half of its original volume. Why talk of artificial pressures? everywhere in the globe of the earth, except within a hundred miles of the surface, the pressure is greater by far than any of them; and yet, the average density of the earth is less than double that of its superficial crust.

We have imagined that as two atoms approach each other, the gravitational force between them rises gently, the cohesive force remaining undetectable till they come very close together, when at some critical distance it begins a sharp and sudden rise which quickly carries its value far over that of gravity. Now we are to conceive of yet a third force, repulsive, undetectable till they come still closer together, then at a lesser critical distance entering on a sharper more sudden rise which rapidly carries its value far over those of both of the other two.

This essential and powerful force has no name of its own. This is because it is usually described in words not conveying directly the notion of force. What we have now encountered is the concept of the incompressible atom, the particle of irreducible volume—the doctrine that the atoms are to be pictured not as infinitely small like the points of geometry, but as hard impenetrable elastic pellets, minute indeed but not inconceivably so. This is a doctrine frankly expressed by many a thinker of the past, who perhaps was more unwilling than we to receive uncritically that difficult dogma of the point of infinite smallness. Hearken again to Newton: "It seems probable to me that God in the beginning formed matter in solid, hard, massy, impenetrable, moveable particles . . . incomparably harder than any porous bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary power being able to divide what God himself made one in the first creation."

The completely unsqueezable atom corresponds to a force of repulsion which passes suddenly from zero to an infinite strength at a certain critical distance. The critical distance is the "radius of the atom." Reversely the idea of a force of repulsion rising rapidly indeed, but always continuously, as two particles draw nearer—this corresponds to a squeezable atom, without a definite radius. Solids and liquids in bulk are compressible, and this seems to rule out the former idea, which anyhow is more drastic than one likes to accept. It is not ruled entirely out, for there may be interstices among the particles, and the shrinkage entailed by pressure may be ascribed to the atoms so setting themselves that the cavities lessen in size. However, this does not seem adequate, and it is better to accept a compressible atom and make it share with the cavities the responsibility for the shrinkage. Then there is also the fact that solids expand when warmed. This is ascribed to the atoms dancing around with the heat, and so we approach a new situation in which repulsion and motion are allied as the two antagonists to cohesion.

Instead of exploring this situation further, let us ask whether there is a difference between the concept of the more-or-less squeezable atom and that of the force-field curiously devised which I have been describing? Formally, there is not. But in respect of the path which the mind next tries to follow, there is a difference, and a great one.

The compressible atom being accepted, one asks, of what is it made? and finds that one is thinking of a continuous substance, elastic and dense. One who is trying to become a thoroughgoing atomist is hardly pleased to discover a continuum at the base of the theory. The displeasure would not be long-lasting, if by assigning a few simple qualities to the continuum one could arrive at the right numerical values for things that can be measured—if one could infer, for instance, that the continuum by its nature divides itself into globules of just the same radii as the structure of crystals demands for the atoms. We are to meet in nuclear physics with a calculation singularly like this—but in general, the feat has not been done. It is not an adequate retort to say that the thoroughgoing atomist is obliged to assign to his atoms the sizes and the masses which they actually have, without giving any deeper reason. He manages to avoid the question; it becomes imperious, when the continuum is brought upon the scene. The road to success may lie by way of the continuum, but it is a road that has not been successfully trodden.

The force-field around the point-particle being accepted, one asks, why this so curious force-field? An inverse-square field would seem so natural as not even to ask for further explanation (but this is probably because the human mind has had two and a half centuries for getting accustomed to it). This combination of a short-range attraction with a repulsion still shorter in range cries out from explanation. Could one but somehow reduce it all to inverse-square forces, one would be more contented. This road seems impassable, but already it has been trodden—built and trodden—to splendid successes. Therefore I lay aside the compressible atom scooped out of a continuum, mentioning that even now we have not heard the last of it. Two stages of preparation are now required.

First, I must take more care henceforward in using the words "atom" and "particle". Hitherto I have used them interchangeably; from this moment on, "atom" is to have one meaning and "particle" another. Of the two, it will be "atom" which comes the closer to meaning what both words have meant up to now. Atom will attract atom by the force of cohesion; atom will repel atom by the nameless short-range force. The atoms in their turn will be made up of more elementary particles, bearing such names as "nucleus" and "electron". As to the forces between them,—that is the topic to which we are coming.

Second, I must introduce at long last the forces which the reader has so long been missing from this discourse: the electromagnetic.

Of these, it is the "electrostatic" force which stationary charges exert on one another which concerns us the most. Newton spoke of it in one of the passages which I have just been citing, but the pleasure was denied him of knowing how it resembles gravity. Both follow the law of the inverse-square; yet two centuries were to elapse between the years when Newton proved this for the one and Coulomb for the other. The electrostatic force is broader though than gravity, for it includes an attraction and a repulsion. There are two categories of charge, the positive and the negative: any charge repels those of its own category, attracts those of the other.

This entry upon the scene of a long-range repulsion modifies the prospects of a successful picture of the world as a congeries of particles, and seems at first glance to brighten them greatly. Dismiss gravity—forget about cohesion—put the question: in an imaginary universe made up of electrified particles some positive and some negative, acting on one another by electrostatic forces only, is it possible to have stability with all of the particles standing still?

Again the answer is no. This is not, however, too disappointing: we are accustomed to motion as the antagonist of gravity in the celestial case; shall we not now introduce it to be an ally to the electrostatic repulsion, the two of them conjointly being the antagonists of the attraction?

Now with real surprise and disappointment, one stands confronted again by the ruthless negative answer. The past revives: I have said that a pre-Newtonian philosopher would scarcely have accepted motion as the deathless antagonist to gravity, because he would have believed that motion dies out of itself. Well, the motion of an electrified particle *does* die out of itself —so says the electromagnetic theory. A proviso must here be inserted for correctness' sake, though it does not alter the situation. Uniform motion does not tend to die out—but uniform motion is useless to our ambitions. The orbital motion of a planet, the swing of a pendulum,—on these the theory must be built; but these are accelerated motions; and accelerated motions destroy themselves, when the moving body is electrified. Their energy passes into light, and the body sinks to rest. Aristotle was avenged in the nineteenth century on those who sneered at him; for what he had believed of motion generally, was in effect what they believed of the motion of electricity. Still, as nearly everyone knows, there is, after all, an electrical theory of matter; the elementary particles are deemed to be electrified, and the forces between them are deemed to be electromagnetic.

How is all this to be reconciled? By a statement which is the prelude to the final one—provided, that is, that all works out as well as physicists now hope, and provided also that we avert our eyes from the phenomena called "nuclear". Having imagined the elementary particles as points possessed of mass and bearing charges, and acting upon one another by electromagnetic forces, we are to treat their motions by the method of quantum mechanics, and not by the method of classic mechanics.

I will not pretend that this is a slight innovation, nor try to represent it as anything less than a great and difficult revolution in some of our most cherished habits of thought. Concepts formerly sharp, even those of position and motion themselves, become hazy; there are pitfalls and labyrinths; the mathematical technique is novel and hard. Yet in the picture of the universe as now presented, there are particles possessed of charge and mass; there are electromagnetic forces between the particles; there is motion of the particles; there is radiation, which it is just barely permissible to disregard in an outline like this one, and which I am disregarding; and outside of the realm of "nuclear" phenomena, there is nothing else. The stability of the world, that is to say, of the picture, is assured by attractions and repulsions electrical in nature, and by motion, with radiation playing an essential part.

The hydrogen atom appears before the eye of the mind as a system of a nucleus and an electron: two particles of known, equal and opposite charges, of known unequal masses, attracting one another by electrostatic force. The force draws them together, but there is kinetic energy and there is motion, and so they stay apart. It takes a definite amount of energy to separate them, and the theory derives its actual value very exactly from a basic principle. Any other atom appears before the mind as a system of a nucleus and two or more electrons. The nucleus bears a positive charge, the electrons are negative; the nucleus attracts the electrons, but they repel one another; there is motion; between the attraction and the motion and the repulsion, there is stability. A molecule is a system of two or more nuclei positively charged and two or more electrons negatively charged, and the same three qualities hold the balance. A tangible piece of metal is an enormous multitude of nuclei and electrons, these latter enjoying a very wide variety of motions, some moving almost as freely as though the metal were a vacuum: again the balance is held, the metal tending neither to shrink nor to explode.

All this is a programme for the explanation of Nature; and it is a programme which has been largely fulfilled—wherefore this lecture and a portion of the course. Not everything has been explained, nor ever will be. Quite apart from the phenomena called nuclear, there are countless things and happenings on earth which are so complicated, that they may well obey our fundamental laws without ever giving us the chance to prove it. If we should apply our assumptions to them and start to work out the consequences, it would take a geological era to finish the job. Perhaps all phenomena of life are of this type. The most that can be asked for is, that the theory should deal capably with all the phenomena for which it cannot reasonably be claimed that they are so complex as to defy any theory. I do not allege that our theory of massive particles, electromagnetic forces and quantum mechanics has done even this. It has, however, done a great deal, so much that it takes a rather skeptical physicist to deny it in the realms to which it lays claim.

In the light of this theory, let us consider the situation of the several forces.

Gravity remains apart and inaccessible, one of the ultimate forces, quite probably a quality of space as Einstein has proposed.

The *electromagnetic forces* remain ultimate, not explained in terms of anything else, united among themselves by the theory of relativity, responsible for the incessant passage of energy to and fro between matter and light which is one of the major features of the world. The ionization of atoms, the generation and the absorption of light, show us these forces at work within the atoms, holding together the electrified particles of which the atoms are made, balanced by motion and by their own dual character of attractions and repulsions.

Cohesion, and the chemical forces which bind atoms into molecules and grade insensibly into cohesion, and the nameless *repulsive* force which holds the balance to them and led many to the concept of the more-or-less-compressible atom: these are derivable from the electromagnetic forces between the elementary particles whereof the atoms are made up. I repeat: *derivable from the electromagnetic forces, with the aid of quantum mechanics*,—without which aid they would not have been derived. In the literature one finds incessant reference to "exchange forces"; these are not a novel category, but a step in the derivation.[3] Here are the fields of research where work is the most active. The theory of chemical forces, which some call "quantum chemistry", is well advanced; the theory of metals, not so well. Much earlier and much more often than we like, do we impinge on the class of phenomena, for which it can all too reasonably be claimed that they are so complex as to defy the theorist probably for all time. Yet there are many simple ones which have brilliantly been explained, and there is satisfaction on the whole—until one raises the eyes and looks ahead: for the nuclear phenomena are still before us.

As a prelude to these we may view the electron itself. Hardly have we begun to "look narrowly" upon it, before we see the spectre rising up of that old antithesis between the point-atom and the atom carven out of a continuum; nor is it long before the spectre grows more frightful than it was in the earlier case. If the point-electron is adopted, all the old conceptual

[3] There is also a strange quality of Nature bearing in quantum-mechanics the name of "the exclusion-principle of Pauli," which to some extent resembles a repulsive force acting between similar particles such as electron and electron or proton and proton, under very special conditions.

troubles return in the company of a new one. The intrinsic energy of this point-particle is infinite—so says the electromagnetic theory; the mass must therefore be infinite—so says the relation of Einstein of which I will presently show the power. If from this alternative we rebound to that of a globule of continuous electrical fluid, the old difficulties come back in the company of another new one. The parts of the globule of negative electricity repel each other, so our electron-model turns out to be a high-explosive bomb. The reader if he wishes may seek in Lorentz' "Theory of Electrons", a classic of some thirty years ago, the details of a scheme for preventing the electron from exploding by means of nonelectrical forces—a surrender, therefore, of the viewpoint that the ultimate forces are electrical.

Leaving these difficulties still unmastered, I turn to nuclear physics. This is a term which covers two fields: on the one hand, the structure and the qualities of atom-nuclei; on the other, some remarkable attributes of electrons, which they display either when they have tremendous energies, or under conditions which it takes tremendous energies to create. "Tremendous" energies are enjoyed by electrons fresh from radioactive substances, are obtained from the cyclotron and the electrostatic generator, and are found at their extremest in the cosmic rays. Of these attributes the only one which I will mention is mortality.

Mortality: this is a very obnoxious attribute for an elementary particle. All atomists heretofore have devised their atoms specifically to be immortal, to be *the* immortal things, to be the one thing permanently changeless under the flux of phenomena. But the electron is mortal, subject to birth and to death. Electrons are born in pairs, a positive and a negative springing together into existence. Electrons die in pairs, a positive uniting with a negative and the two of them passing out of existence.

These are not exactly cases of something coming out of nothing and something turning into nothing. Energy, mass and momentum are all conserved. Corpuscles of light disappear where and when an electron-pair is born, are born where and when a pair of electrons vanishes. So far as can be told, the corpuscles of light possess just the energy, just the mass and just the momentum which is destined to go to the nascent electrons or to be left unpossessed by those about to die. Now I have to admit my fault in not elevating earlier the corpuscles of light to a parity with the electrons and the atoms. They have the singular attribute of moving always with the same speed (when in a vacuum); they do not collide with one another, or rather such collisions have not been detected, though collisions with electrons are known; and they suffer from mortality, very much more so than do electrons. (Positive electrons are so rare, that negative electrons enjoy an almost perfect security.) Immortality is reserved for energy and mass and momentum. Now we feel ourselves swerving again toward a continuum-theory. The ground is slippery, and I step hastily from it into the last section of this lecture, into nuclear physics proper.

All of the theory of nuclei is firmly grounded on one basic statement, which is this: the masses of all nuclei are *nearly* integer multiples of a common unit, this being *slightly less than* the mass of the lightest among them.

Here is a statement bitterly disappointing! the little word "nearly" and the three little words "slightly less than" conjointly make a bright hope stillborn. Were it not for those words, we should already have joyously leaped to the conclusion that all nuclei are clusters of a single kind of fundamental particle, different clusters differing only in how many of the particles they comprise. The conclusion is so tempting that one is quite unable to resist it, hoping against hope that the words of frustration can somehow or other be cancelled. Soothing the reader with this veiled assurance, I adopt the conclusion.

The conclusion itself must be tempered at once, for there is a second basic statement coequal with the first: the charges of all nuclei are integer multiples of a common unit of charge. No pernicious adverbs here! this statement is an exact one, to the best of our knowledge and belief. The common unit of charge, as nearly everyone knows, is equal to the electron-charge and positive in sign.

The conclusion would still be sound, if the charges of all nuclei were proportionate to their masses (we should merely attribute an equal charge to every particle). Definitely this is not so, being most strikingly denied by the fact of "isobars": there are nucleus-types agreeing in mass, disagreeing in charge. We seek the next simplest assumption, and find that it suffices: Two types of fundamental particles—equal in mass—the one of them charged positively, the other neutral—each nucleus to be distinguished by two integers, one being the number of the charged component particles of the cluster, the other the number of the neutrals—"proton" and "neutron" for the names of the two.

This is the beginning of the programme for nuclear theory. Having taken the first step by writing it down, we enter upon the second—and find ourselves on the very road which our ancestors trod when atomic theory was new, facing the same ascents, the same passes and the same morasses. The long-range forces—the short-range forces—the cohesion—the repulsion—the more-or-less-incompressible particle—the troubles of the concept of the point-particle—the countervailing troubles of the continuum carven into globules—the dream of reducing everything to long-range forces and motion holding each other in balance—every one of these rejoins us on our journey. The mighty difference is, that the road still ends in the darkness, and the dream is still a dream. Therefore it is that the language of nuclear theorists wanders about in the most disconcerting way, so that often in a single article the wording in one place will be intelligible only to a few hundred (if so many) of the most advanced of specialists, and in another will sound like the voice of Newton speaking out of the *Opticks*, only in a much more cumbrous manner.

In the atomic world we have already seen how gravitation is neglected, being pushed into the background by the electromagnetic forces and the cohesions and repulsions derivable from these. Now in their turn the electromagnetic forces must recede into the background. This sounds extraordinary. Have we not all been told of the supreme importance of nuclear charges? Have we not been taught that by its charge a nucleus attracts electrons and organizes them into a family about itself and so creates an atom,—an atom which coheres with others, so that the world as we know it is organized by the charges of nuclei? All this is true, and very important from our viewpoint—but not so important, it seems, from the viewpoint of a nucleus. To this little cluster of protons and neutrons, the mass is more important than the charge; the total number of its component particles is more important than the number of protons separately or the number of neutrons separately; the cohesive forces are more important than the electrical. Perhaps a nucleus cares little about its charge, and nothing at all about the swarm of electrons which that charge coerces to swirl about it like a cloud of flies, though if it were not for those swirls the world would be barren and dead.

The cohesive forces certainly overpower the electrical. We are in no doubt of this, for the electrical forces are repulsive. Newton had gravity available for binding his atoms together; it was of the right type but inadequate, so he gave it cohesion as an ally. The electrostatic force between proton and proton is a repulsion, so to bind such particles together the Newtons of nuclear physics must overcome it with cohesion as an adversary. How greatly it is overcome is shown in much the same sort of way, as I followed when invoking the vaporization of solids to show how greatly the cohesion of atom with atom surpasses gravity. It is possible (at the end I will mention how) to compare the amount of energy required for tearing apart a cluster of two protons and a neutron with that required for tearing apart a cluster of two neutrons and one proton. The two amounts differ by only a few per cent; and more surprising yet, the former is the greater! Though the first-named of the clusters contains the inherent explosive power of two protons trying to drive themselves apart by the long-range repulsion, it is stuck tighter together than the other, which contains nothing of the sort. As a minor detail this shows that the cohesive forces depend to some extent on whether the particles are neutrons or protons; but the major conclusion is, that the cohesive forces are the masters.

Are they short-range or long-range? By calling them "cohesive" I have already committed myself, but correctly. There is an argument quite similar to the second which I drew from the vaporization of liquids. Think again of the kettle of water boiling away on the stove. It takes as much energy from the flame to disperse the last cubic inch of water that goes as it does to drive off the first, despite the fact that the first is exposed to all the long-range forces of attraction exerted on it by all the other cubic inches remaining in the kettle, and the last is not. Therefore the long-range forces which act between atom and atom are trivial, and cohesion is a force exerted by the atoms on their near neighbors only. Think now of the cluster of protons and neutrons which is a nucleus—a massive one by choice, built of two hundred particles or more. Imagine it taken to pieces by detaching one particle after another. I admit that this precise experiment is beyond the art of the physicist, but for a certain reason—the one which I have already promised to give, and will give at the end—he is as confident of its result, as he ever is of the result of any experiment which he has not actually performed. The result is, that it takes *roughly* as much energy to remove a particle when there are two hundred left behind to pull it back, as when there are but a dozen left behind, or any number in between. Therefore the long-range forces which act between the fundamental particles are minor, and the intra-nuclear cohesion is a short-range force.

I have carefully made these last statements rather weaker than their analogues for the water boiling away. The amount of energy required for taking away a particle does depend to some extent on the number left behind, and the long-range forces are therefore minor but not trivial. If the long-range forces are attractive, the binding-energy of a particle—this is the shorter name which is given to the "energy required for taking away a particle"[4]—must be greater, the greater the size of the cluster, *i.e.*, the greater the mass of the nucleus. Now for nuclei of some fifty particles or more, the contrary is the case. Therefore the long-range force, or the major one if there are more than one, is a repulsion. We already know of one long-range repulsion, to wit, the electrostatic force between proton and proton. Is this the force in question? The answer is oddly difficult to give with assurance, but at present is believed to be *yes*.

If the answer is definitely *yes*, then the electrostatic force has after all one role of supreme importance in nuclei. It fixes their maximum size and their maximum charge, therefore limits the number of chemical elements, and may indeed be all that prevents the universe from caving together into a single lump of protons and neutrons with the electrons fluttering help-

[4] It ought strictly to be called the "unbinding-energy" or "binding lack-of-energy," since it is given as positive when energy must be contributed to the system in order to detach the particle.

lessly around it. So long have the chemists been on the search for new elements, and so completely have they searched, that we may believe them when they say that apart from the works of the "atom-smashers," no nucleus exists having more than 238 particles altogether, 92 of which are protons. Even the atom-smashers or (as I should rather call them) the transmuters, for all the wonder and power of their art, have not forced the total number of protons upward by more than two or the total number of particles altogether upward by more than one. Moreover all of the two dozen or so most massive nuclei known are subject to explosion—to explosions quite terrific, some of them spontaneous, others touched off by what seems a very minor cause. It may therefore be taken as nearly certain that there is an upper limit to the size of nuclei, and probable that it is electrostatic force that sets the limit.

Now we come down to the short-range repulsion. Such a one there must be, for again we can rehearse the ancient argument. A piece of iron does not shrink into a point; therefore the iron atoms must either exert a force of repulsion or else be more-or-less compressible pellets. A nucleus does not shrink into a point, but offers an impenetrable front, measurable though small, to an oncoming neutron; therefore the nuclear particles—but why repeat the words?

Shall we interpret neutrons and protons alike as systems of particles still smaller, acting on one another by electromagnetic forces, to be treated by quantum mechanics? Alas, if there is one surety in this field, it is that we cannot play quite the same game twice. Quantum mechanics may not be used up (some think that it is) but the electromagnetic forces certainly are. In this direction we have as yet no leadership.

Shall we then adopt the compressible globule or the point-particle with a curious field of force surrounding it? Though the language of nuclear theorists verges sometimes on the former, it is the latter practice which is common—a fact which will hardly surprise the reader. In the specialized literature, one finds many a speculation and (what is of more moment) many an inference about the force-field which is drawn pretty directly from reliable data. As a rule the inferences are expressed in language very different from the phrases of this lecture: "interaction" is used instead of "force-field," and there are queer and slightly comic technical terms such as "potential-well." When you read of a "rectangular potential-well," interpret that what I have been calling the "cohesive force" becomes suddenly enormous at a certain specific radius; when of an "error-well" (!) understand that the cohesive force increases rapidly according to a certain law with decline of distance; when of a "Coulomb interaction" realize that it is the inverse-square force-field of the electrostatic repulsion between proton and proton. Of these interactions I will give only two facts: first, that the short-range attraction is confined within a very few times 10^{-13} cm of the centre of the proton or neutron, whereas the cohesive attraction of atom for atom spreads over a radius a hundred thousand times as great; second, that the three short-range attractions of proton for proton, neutron for neutron and neutron for proton are nearly the same.

Shall we adopt the force-fields as given to us by experiment, with some plausible assumptions added (for one cannot as yet do without them) and operate on them by the procedures of quantum mechanics, hoping to arrive at (say) values of binding-energies compatible with the data? This is the present, or perhaps I should say the recent, programme of nuclear theory. If one reads the theoretical papers of any one year out of the last ten, one may readily get the impression that success is just around the corner. But if one reads the papers of two or more years and takes note of the rapid changes, the prospect does not look quite so rosy—nor when one overhears the conversations of the theorists themselves. I will not conduct the reader down the paths which are as yet so tortuous and hazy; it will be better to fill in the picture with a few of the many remaining details.

Mass was the first of properties (along with hardness) to be assigned to the elementary particles; the second was charge; to these have lately been added angular momentum and magnetic moment. It is difficult to say when the idea of a spinning atom was first propounded (one recalls the vortices in a continuous fluid which Kelvin introduced as one of the most brilliant of all attempts to contrive a continuum and atoms as a part of it) but easy to fix the time when the idea of the spinning electron became so definite and sharp, as to be successfully used in explaining crucial data; this was 1925. The electron, the proton and the neutron all have equal angular momentum; its amount, common to these three which at present claim most strongly the rank of *elementary* particle, is one of the universal constants. When protons and neutrons are assembled in a nucleus, their axes of spin all point in an identical direction, though not by any means necessarily in the same sense in that direction. It is possible for a nucleus to have zero angular momentum, through half of its particles setting themselves in the one sense and half in the other; the lightest nucleus for which this happens is the alpha-particle, composed of two protons and two neutrons. The magnetic moments of the three elementary particles are very far from equal, that of the electron being some seven hundred times as great as that of the proton, which in turn is half again as great as that of the neutron. One of the tragedies of theoretical physics occurred in this connection. A principle of quantum mechanics had been proposed, superbly capable of serving as a basis for most of the incomplete principles which had already so well justified themselves in atomic physics, and including among its parts the actual values of the angular momentum and magnetic moment of the electron. Its empire would have been extended, had the ratio of the magnetic moments of proton and electron been equal to the reciprocal of the ratio of the masses of these two—actually the former ratio is too great by a factor of 2.78. This contretemps has led many to deny the title of "elementary" particle to the proton; while as for the neutron, the fact that it lacks an apparent electric charge while nevertheless displaying a magnetic moment leaves it also open to suspicion.

Few readers of these pages will be unaware that electrons are observed proceeding out of nuclei: it may well be a source of wonderment that they are denied a residence in these assemblages of protons and neutrons only. This is of course another example of the mortality of the electron. Having observed that it is subject to birth and to death, should we be deterred from supposing that it is born as it quits the nucleus from which it comes? This rhetorical question gives a false impression of the course of history. There was indeed an era when electrons were believed to inhabit nuclei, when nuclei were regarded as assemblies of protons and electrons only. It ended in 1932; but the observation of the birth and the death of electrons did not ensue for yet another year. What happened in 1932 was the discovery of the free neutron. Only when this particle had been discovered did a physicist (Heisenberg) think it worth while to begin to develop in detail the theory that the components of nuclei are protons and neutrons and no other particles but these.

Now I bring this article to a close by fulfilling my promise to speak of Einstein's relation between energy and mass, which on the one hand has been rigorously tested in the realm of nuclear physics, and on the other has extended that realm.

The relation may be worded in several ways; I will employ the shortest: *energy has mass.*

Now imagine an assemblage of particles sticking together. "Sticking together" is not the dignified phrase of a physicist; such a one would say, more abstractly but more exactly, that energy must be given the particles to take them apart. But energy has mass; therefore the mass of the assemblage must be augmented, when they are taken apart. Therefore the mass of the interconnected assembly is less than the sum of the masses of the particles when free.

Now with a single stroke this principle does away with what otherwise would have been a quite unsurmountable obstacle to the doctrine that all nuclei are made up of protons and neutrons. For "proton" and "neutron" are not merely the names of hypothetical particles whereof nuclei are made up; they are also the names of the two lightest of nuclei. These two lightest of the nuclei are so massive, that it could not possibly be said that the other nuclei are made up of them, were it not for the detraction of mass which occurs when they are bound up together. This deficit of mass corresponds to the unbinding-energy or, badly called, the binding-energy of which I earlier spoke. The binding-energy is the amount of energy which must be supplied to the nucleus, to break it up into protons and neutrons. The deficit of mass—the difference between the actual mass of the nucleus, and the masses of all of its neutrons and protons dispersed into freedom—is related to the binding-energy by Einstein's relation.

I have said that this relation has been tested in the realm of nuclear physics, and has served also to extend that realm. The possibility of testing arises from the fact that in certain cases the physicist is able to convert a system of two nuclei into a system of two other nuclei, the masses of all four being known. This seems a somewhat pedantic way of expressing the well-known fact that in performing an act of transmutation, the physicist causes one nucleus as "projectile" to impinge upon another as "target," whereupon the two merge and two others spring apart from the scene of the merger. The masses of the two initial nuclei do not as a rule add up to the same precise sum as the masses of the two final nuclei. But if to the first pair of masses we add that of the kinetic energy of the projectile, and if the second pair is augmented by that of the kinetic energies of the final nuclei—why, then, the equation balances, and Einstein's relation is justified.

As for the extensions of the realm of nuclear physics, or let me rather say, the realm of physics generally: no fewer than three have been stressed in these few pages. First, mass could not be conserved in the birth or the death of electron-pairs, were not the energy of the electrons accompanied by its mass when it passes out of or into the form of radiant energy. Then, we should not so soon have known that the system of two protons and one neutron requires less energy to unbind it, than the system of two neutrons and one proton; this was deducible from the masses of these two nuclei, before it was attested by the discovery that the former changes spontaneously into the latter. Then, we should not have the evidence that the binding-energy of the individual particle lessens, as the number of particles remaining behind in the nucleus increases; for this is a statement derived from observations on the masses of the nuclei.

So all seems well with the model of the nucleus as a system of protons and neutrons, and the particle-theory stands triumphant. Yet notice at what a price this triumph has been bought! Of all the attributes of the fundamental atom, of the elementary particle, constancy of mass was the earliest and the most firmly accepted. The elementary particle was a bit of immutable mass, set forever apart from change. Now it turns out that when the particle adheres to another, some of its mass departs. What has departed is not perished and gone. It is known sometimes to have passed into radiant energy, sometimes into energy of motion, sometimes into that mingling of the two which is known by the name of heat. Changelessness has ceased to be the quality of the atom, remaining that of the mass and the energy of the world as a whole. Immortality has gone from the atom back into the continuum. This is as good a place as any to step out from the incessant alternation, never yet ended and probably endless, between the particle and the continuum as the basis of thought about physics.

Abstracts of Technical Articles by Bell System Authors

Tentative Standards for Wood Poles Become Approved American Standards.[1] RICHARD C. EGGLESTON. The six American Tentative Standards covering specifications for wood poles, several of which were approved by the American Standards Association as tentative in 1931 and the rest in 1933, have now been reviewed by the ASA committee and approved by the ASA as full American Standards. In reviewing the standards, the committee found that the general principles of the standard requirements have been universally recognized as a satisfactory basis for the selection of poles. Covering as they do northern white cedar poles, western red cedar poles, chestnut poles, southern pine poles, lodgepole pine poles, and Douglas fir poles, the standards represent a rational uniform standardization system for the six major pole timbers of the United States.

The standards establish practical limits that can be applied economically in the production of poles for general use, but they are intended also to be flexible enough to cover the purchase of poles of high quality for special purposes. At the same time, it is not desired that they should be so restrictive that any considerable quantity of usable poles produced under normal production practices would be labeled substandard because of the specification restrictions.

The standard specifications include material requirements for shape and straightness of grain, limit defects such as knots, checks, insect damage and decay, and define the minimum quality of acceptable poles. In the standards, departures from straightness are held within practical limits for ordinary use. Decay and the presence of wood-rotting fungi are generally prohibited. Definite limitations on knots are set, and fire-killed poles are acceptable only by special agreement between producer and purchaser.

The standard dimensions now included with the specifications in one standard for each type of pole, were based on recommended fibre stresses contained in the American Standard for Ultimate Fiber Stresses of Wood Poles (05a-1930). They were approved as American Standards from their inception, and until they were included with the specifications in the present American Standards they were considered as separate standards. These standard dimensions have all been prepared according to the same principles for all types of poles. The sizes at six feet from the butt in all six standards have been so fixed with respect to ground-line resisting moments,

[1] *Industrial Standardization*, June 1941.

that, for any given class and length of pole, all six species are equal in strength. In calculating these six-feet-from-butt sizes, distance from the butt to the ground line for any given pole was assumed, by definition, to be as shown in the column in the tables headed "Ground Line Distance from Butt." The equality-in-strength principle holds good, however, for any reasonable depth of set required.

Approval of the six standards at this time followed a policy adopted by the Standards Council of the American Standards Association in April 1939. At that time the Standards Council decided to withdraw approval of standards having a tentative status, and requested the reconsideration of such standards with the idea of either discarding them or of advancing them to American Standards.

Equilibrium Relations in the Solid State of the Iron-Cobalt System.[2] W. C. Ellis and E. S. Greiner. There are important transformations in the solid in the iron-cobalt system. One of these originates from the A_3 transformation in iron. Cobalt in the binary system at first raises the A_3 transformation to a maximum in the region of 45 weight per cent cobalt. Further additions decrease the temperature of transformation which rapidly approaches room temperature in the region of 80 weight per cent cobalt. An extended two phase region from 76.5 to 88.5 weight per cent cobalt was established at 600 degrees Cent. (1110 degrees Fahr.).

An order-disorder transition occurs in the alpha phase in the region of 50 weight per cent cobalt. The critical temperature of order is in the neighborhood of 700 degrees Cent. (1290 degrees Fahr.) depending upon the composition. The ordered arrangement has the cesium chloride structure.

The lattice constants of the alpha phase deviate widely from a linear function of the cobalt content. The first additions of cobalt increase the cell size to a maximum at approximately 20 per cent cobalt. Further additions result in a contraction in the cell size to the limit of the alpha phase. Compositions in the region of 50 per cent cobalt exhibit an increase in cell size on ordering.

Determination of Microphone Performance.[3] F. L. Hopper and F. F. Romanow. Methods of determining the performance characteristics of microphones by acoustic measurements are described. Work factors involving the accuracy of the methods are discussed. The correlation between a microphone's performance as determined by acoustic measurement and by listening tests is reported. Application of both types of test to a studio type of cardioid microphone is given as an example.

[2] *Trans. Amer. Soc. for Metals*, June 1941.
[3] *Jour. Soc. of Motion Picture Engineers*, April 1941.

Room Noise Spectra at Subscribers' Telephone Locations.[4] DANIEL F. HOTH. That room noise can be a distinct handicap to conversation by masking the speech sounds in the ear of the listener and thus impairing the ease and accuracy of reception is of considerable concern to the telephone engineer. Room noise not only complicates the problems involved in the design and engineering of telephone systems capable of affording satisfactory service, but it is also one of the factors which affect the costs of the telephone plant. The effects of noise on telephone conversation depend, of course, upon the characteristics of the noises which occur at the places where telephones are being used. The arrangements and practices necessary for reducing the effects of noise depend upon a knowledge of these characteristics. As a result numerous measurements of room noise have been made from time to time over a period of many years by Bell System engineers. For the most part such measurements have involved the determination of a single figure to represent the noise measured, as in the recent survey of sound levels described by Mr. D. F. Seacord in the July 1940 issue of *The Journal of the Acoustical Society of America.* While such measurements are invaluable in providing information on the frequency of occurrence of different noise levels at telephone locations, their value is enhanced by additional measurements of the distribution of the noise energy throughout the frequency band involved in the reception of speech. The present paper describes such measurements and shows the effects of a number of contributing factors on the spectrum of the noise. It is shown that the spectrum of room noise has a characteristic shape.

Film Scanner for Use in Television Transmission Tests.[5] AXEL G. JENSEN. This paper describes the design and construction of a television film scanner primarily intended for use as a testing tool in designing circuits suitable for television program transmission.

The equipment employs electronic scanning and the image dissector is used as the electronic pickup device. The image dissector has a high degree of linearity between light input and signal output and the picture signal is not accompanied by any spurious shading signals. Furthermore, the direct-current component of the television signal is directly available at the output of the tube. The lower sensitivity of the dissector tube is not important in this case since a highly efficient optical projection system makes it possible to override noise to a high degree.

In film scanners for entertainment purposes it is desirable to use ordinary 24-frame motion pictures and such film scanners therefore include a me-

[4] *Jour. Acous. Soc. Amer.*, April 1941.
[5] *Proc. I.R.E.*, May 1941.

chanical or optical translating mechanism for translating the 24-frame film picture into a 30-frame interlaced television picture. In the present equipment it was found more expedient to simplify the construction by allowing the use of specially printed film. Ordinary 24-frame film is "stretched" by printing every other frame twice and the remaining frames three times in succession, thereby producing a film with a total of 60 frames instead of the original 24. Vertical scanning is then obtained by the continuous motion of this film at the rate of 60 frames per second and horizontal scanning by a simple electronic line sweep in the dissector tube.

Acoustic Design Features of Studio Stages, Monitor Rooms, and Review Rooms.[6] D. P. Loye. A survey was made of studio experience, and measurements were made of stages, review rooms, and other units. These data were correlated and used as a valuable guide in the determination of the optimum characteristics and dimensions recommended for major studio scoring stages, monitor rooms, dubbing rooms, review rooms, and studio theaters.

Information regarding Hollywood preview theaters is included in an Appendix.

A New Microphone Providing Uniform Directivity over an Extended Frequency Range.[7] R. N. Marshall and W. R. Harry. A new microphone is described which consists of a moving coil pressure element combined with an improved ribbon pressure gradient element to give a cardioid directional characteristic. The theory of operation is reviewed, and consideration is then given to variations in directivity caused by diffraction, separation of the elements, and disparities in their phase and response characteristics. It is then shown how these variations are largely eliminated by equalization in the electrical circuit so that the resulting directivity is practically independent of frequency throughout the range from 70 to 8000 cycles. The use of a moving coil pressure element makes high efficiency possible, while the design of an unusually rugged ribbon element provides a marked reduction in noise due to air currents. Several useful directional patterns in addition to the cardioid pattern are provided in the new microphone, and the theory and merits of these patterns are presented. Finally some of the results which were obtained in field trials of the new microphone are discussed.

The Magnetostriction, Young's Modulus and Damping of 68 Permalloy as Dependent on Magnetization and Heat Treatment.[8] H. J. Williams, R. M.

[6] *Jour. Soc. of Motion Picture Engineers*, June 1941.
[7] *Jour. Acous. Soc. Amer.*, April 1941.
[8] *Phys. Rev.*, June 15, 1941.

Bozorth and H. Christensen. This paper describes measurements of the changes in certain physical properties of 68 Permalloy that result from different thermal and mechanical treatments and considers them in relation to the domain theory. The magnetostriction varied with heat treatment from 2.5×10^{-6} to 22×10^{-6}. The change in Young's modulus with magnetization to saturation varied from 0.09 to 10.5 per cent. The damping of mechanical vibrations was also measured as dependent on magnetization and heat treatment. Young's modulus and the damping constant were determined by measuring the natural frequency of vibration and the width of the resonance curve of a hollow rectangle magnetized parallel to its sides so that the magnetic circuit was complete without air gaps or end effects.

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