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Carrier in Cable *

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In order to meet future demands for high-grade and economical circuits in cables, considerable carrier development work has been done which has included an extensive experimental installation on a 25-mile loop of underground cable. Sufficient pairs were provided in the cable and repeaters were installed to set up nine carrier telephone circuits 850 miles long. Tests on these circuits showed the quality of transmission to be satisfactory, while the methods and devices adopted to prevent interference between them were found to be adequate. The trial has, therefore, demonstrated that the obtaining of large numbers of carrier telephone circuits from cable is a practicable proposition.

This paper is largely devoted to a description of the trial installation and an account of the experimental work which has been done in this connection. Due to present business conditions, it is expected that this method will not have immediate commercial application.

This work is part of a general investigation of transmission systems which are characterized by the fact that each electrical path transmits a broad band of frequencies. Such systems offer important possibilities of economy particularly for routes carrying heavy traffic. The conducting circuit is non-loaded so that the velocity of transmission is much higher than present voice-frequency loaded cable circuits. This is particularly important for very long circuits where transmission delays tend to introduce serious difficulties.

A TRIAL installation was recently made in which, for the first time, carrier methods were applied to wires contained wholly in overland cable for the purpose of deriving a number of telephone circuits from each pair of wires. The trial centered at Morristown, New Jersey. A 25-mile length of underground cable was installed in the regular ducts on the New York-Chicago route in such a manner that both ends terminated in the Long Lines repeater station at Morristown. The cable contained 68 No. 16 A.W.G. (1.3 millimeter diameter) non-loaded pairs on which the carrier was applied. Sufficient repeaters and auxiliary equipment were provided at Morristown so that these 68 pairs could be connected together with repeaters at 25-mile intervals to form the equivalent of an 850-mile four-wire circuit.

From this 850-mile four-wire circuit nine carrier telephone circuits were derived, using frequencies between 4 and 40 kilocycles. The diagram of Fig. 1 shows the system simulated by the experimental setup.

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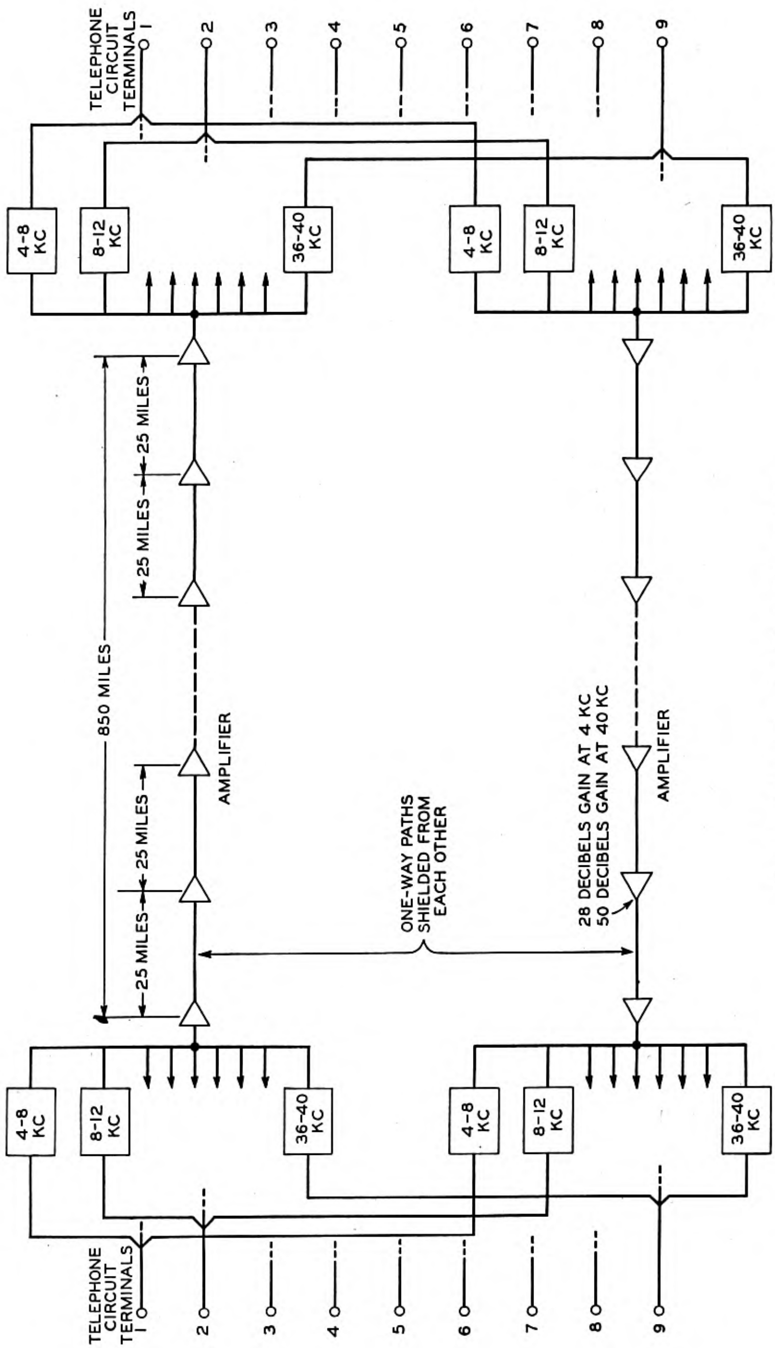

Fig. 1—Schematic of cable carrier system.

Note: In a practical installation the one-way paths would be shielded from each other either by placing them in separate cables or by placing them in a single cable divided into two electrical compartments by means of a specially arranged shield. In the setup at Morristown the circuit was necessarily arranged somewhat differently since only one cable was available. Transmission over all loops in this cable went in the same direction, half the loops then being connected in tandem to simulate one direction of transmission through a long circuit and the other half in tandem to simulate the other direction of transmission.

It will be noted that in this cable system the practical equivalent of two electrical paths was provided, one for transmission in each direction, the same range of frequencies being used in each direction. This differed from common open-wire practice in which the frequency range is split in two and used, one half for transmission in one direction, the other half for transmission in the other. Fig. 2 compares the

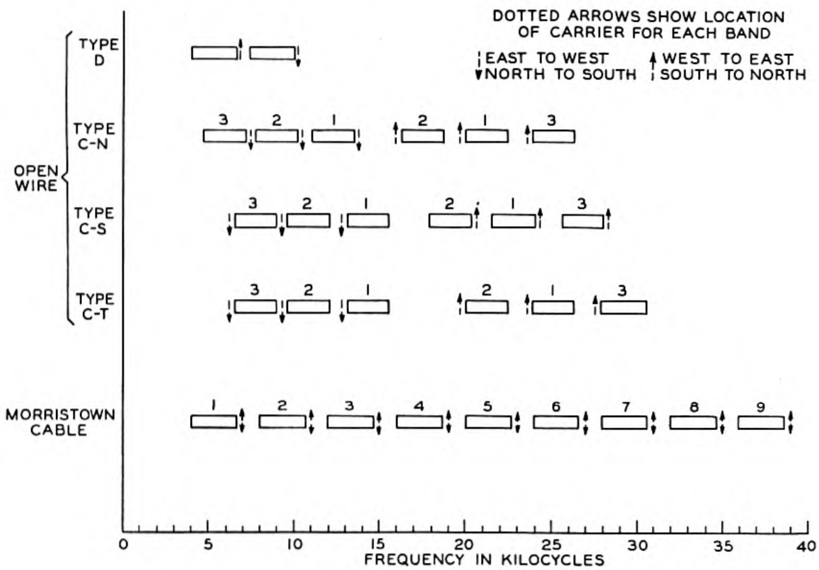

Fig. 2—Frequency allocations of carrier telephone systems.

frequency allocation of the Morristown cable carrier system with existing open-wire systems in this country. Except for this matter of difference in frequency allocation, the fundamental carrier methods used in this cable system did not differ in principle from those already used on open wires. As will be noted in Fig. 2 all of these carrier telephone systems use the single sideband method of transmission with the carrier suppressed.

Fig. 3 is a schematic diagram of the terminal apparatus used in deriving one of the telephone circuits. Its general resemblance to

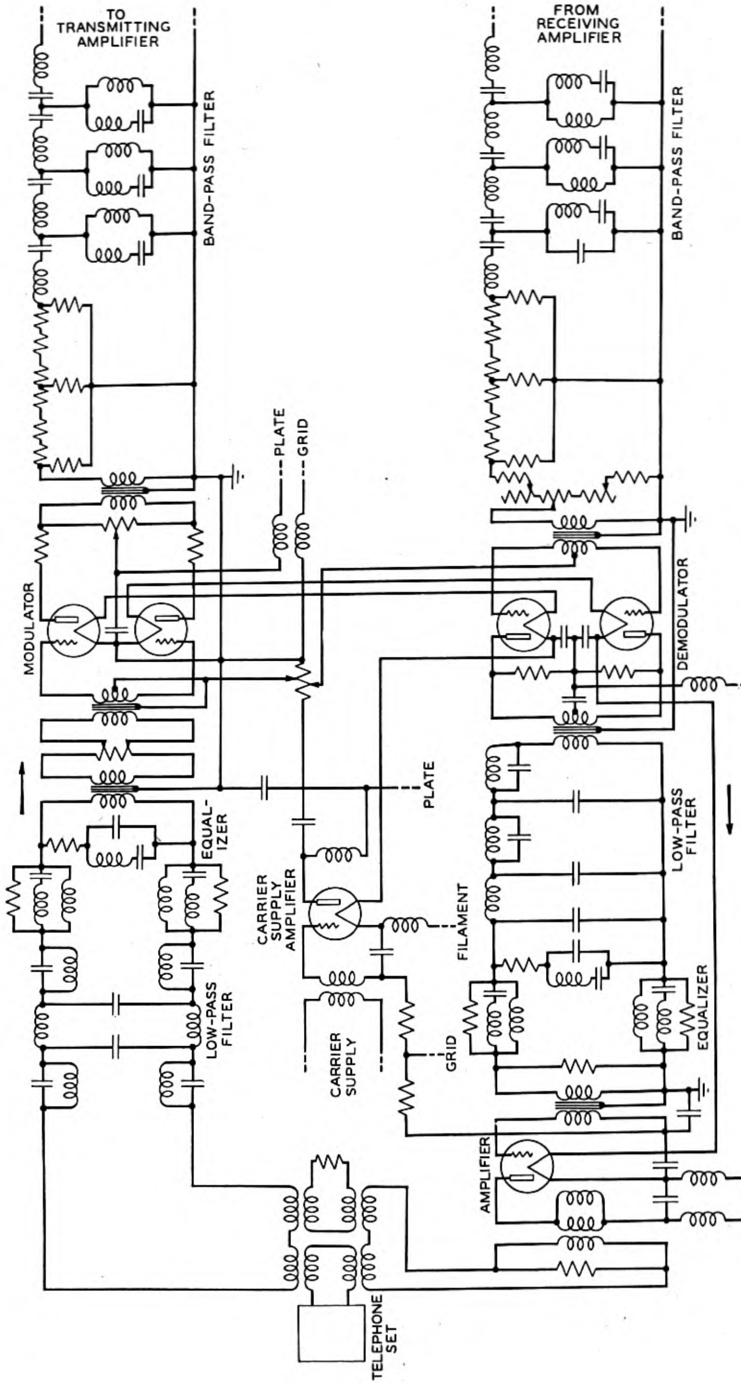

Fig. 3—Terminal of one telephone circuit.

the terminal apparatus used in present open-wire systems is evident so no further discussion of this seems required. Fig. 4 shows five relay rack bays carrying terminal equipment (exclusive of line amplifiers) for one system terminal yielding nine telephone circuits.

Important problems in cable carrier transmission are:

1. Keeping circuits electrically separated from each other, i.e., preventing troublesome crosstalk.
2. Maintaining stability of transmission.

Crosstalk

With respect to crosstalk, the first and most important requirement is to secure a very high degree of electrical separation between paths transmitting in opposite directions. Careful crosstalk tests demonstrated that by placing east-going circuits in one cable and west-going circuits in another, the necessary degree of separation could be obtained even though the two cables were carried in adjacent ducts. Tests on short cable lengths indicate that adequate separation can probably be secured by means of a properly designed shield; one practical form of such a shield consists of alternate layers of copper and iron tapes. With such a shield a cable may be divided into two compartments and thus carry both directions of transmission.

Having thus separated opposite bound transmissions there is left the problem of keeping the crosstalk between same direction transmissions within proper bounds. In the cable used for the Morristown trial the 16 A.W.G. pairs used for the carrier were separated from each other by sandwiching them in between No. 19 A.W.G. (.9 millimeter diameter) quads of the usual construction. These quads served as partial shields between the carrier circuits and would in a commercial installation have been suitable for regular voice-frequency use. Thus a considerable reduction in the crosstalk between the carrier pairs was effected.

When the problem of keeping crosstalk between circuits transmitting in the same direction within proper bounds is examined it becomes evident that no matter how high the line amplifier gains may be, these gains do not augment this crosstalk since if all of the circuits are alike transmission remains at the same level on all circuits. Not so evident perhaps is another fact that crosstalk currents due to unbalances at different points tend to arrive at the distant end of the disturbed circuit at the same time. This makes it possible to neutralize a good part of the crosstalk over a wide range of frequency by introducing compensating unbalances at only a comparatively few points.

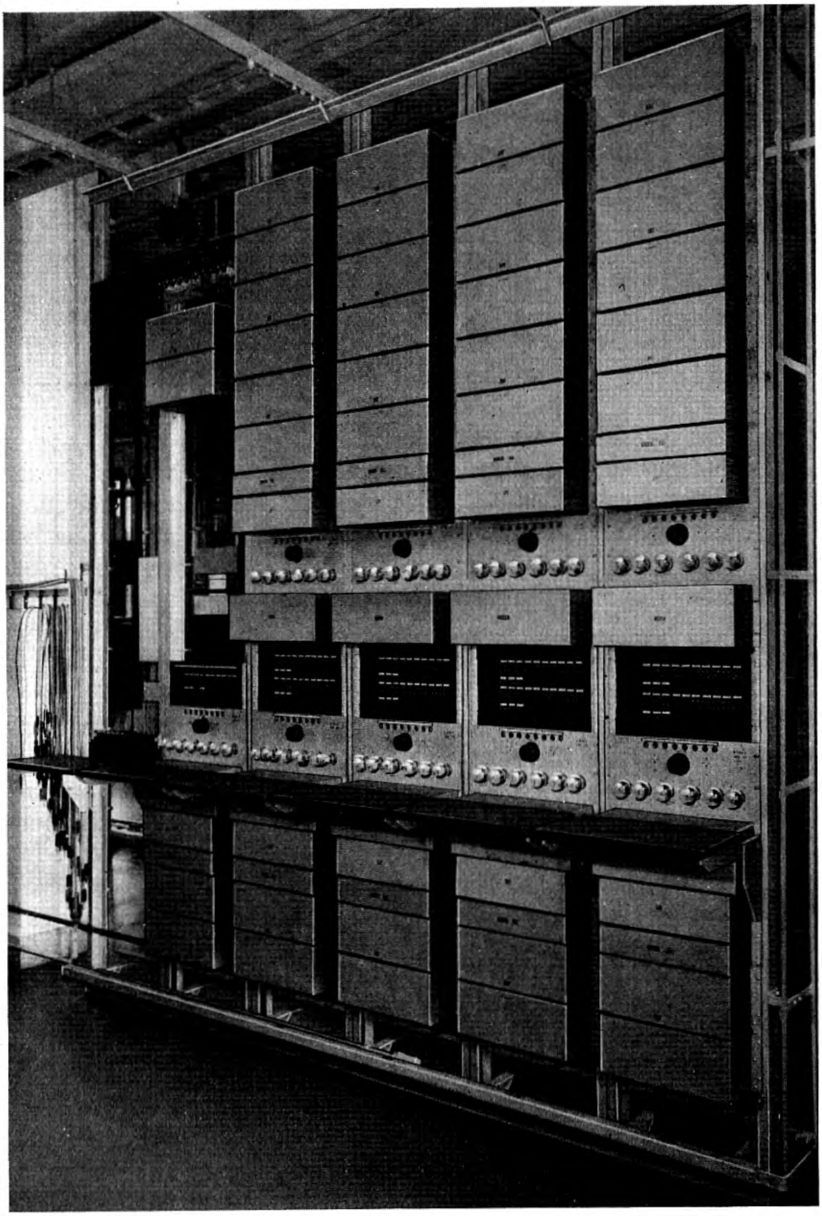

Fig. 4—Terminal equipment for 9 telephone circuits.

In practice, balancing at only one point in a repeater section (which may be an intermediate point or either extremity) serves to make possible considerable reduction of the crosstalk. In the Morristown setup balancing arrangements were applied at an intermediate point in the cable and found to be entirely adequate for the frequency range involved; in fact, transmission of considerably higher frequencies would have been possible without undue crosstalk. Other tests have indicated that, thanks to these balancing means, the 19-gauge quads used in the Morristown cable for separating the 16-gauge pairs from each other can probably be dispensed with, even for frequencies considerably above those used in the trial.

The photograph of Fig. 5 shows the experimental panel on which the circuits were brought together for balancing. This panel was

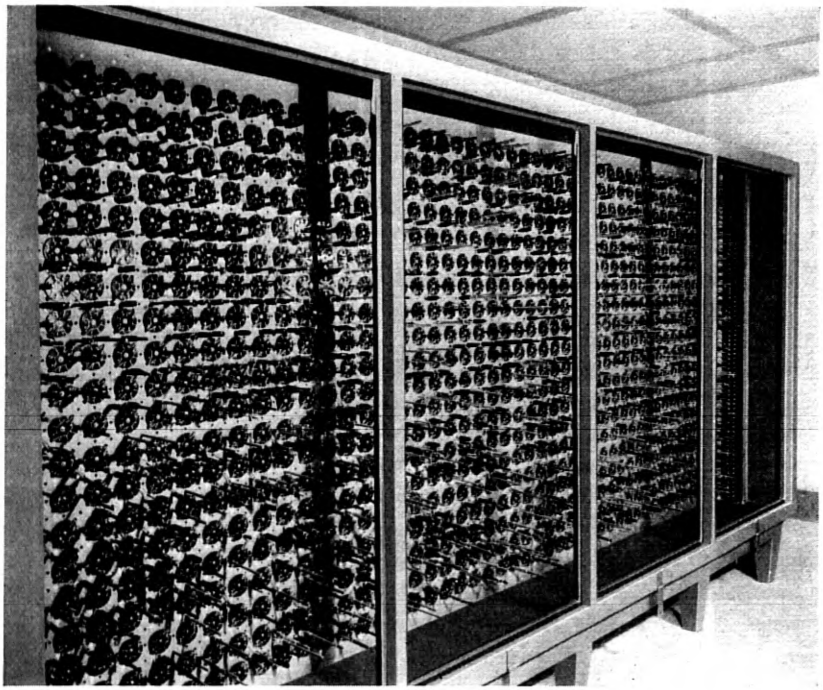

Fig. 5—Special crosstalk balancing panel.

installed in a weather-proof hut near the center of the 25-mile repeater section. By this means all pair to pair combinations in the group to be balanced were brought into proximity so that the leads to the balancing devices could be kept short. The actual balancing was accomplished by connecting small condensers made up of twisted pairs, between wires of different cable circuits and/or by coupling wires of different circuits together through small air-core transformers. Each unit was individually adjusted after measurement of the cross-talk between the various combinations.

Maintaining Stability of Transmission

Referring to the problem of stability, the importance of this will be appreciated from the fact that the average attenuation at the carrier frequencies employed in the 850-mile circuit as set up at Morristown was about 1300 db. A circuit was actually set up and tested consisting of nine of the carrier links in tandem, giving 7650 miles of two-way telephone circuit whose total attenuation without amplifiers was about 12,000 db. This attenuation, on an energy basis, amounts to 10^{1200}. This ratio, representing the amplification necessary, quite transcends ratios such as the size of the total universe to the size of the smallest known particle of matter.

Balancing this huge amplification against the correspondingly huge loss, to the required precision, one or two db, is a difficult problem. Fortunately, a new form of amplifier employing the principle of negative feedback has been invented by Mr. H. S. Black of the Bell Telephone Laboratories and may be described later in an Institute paper. By making use of this negative feedback principle, amplifiers were produced for this job giving an amplification of 50–60 db and this amplification did not change more than .01 db with normal battery and tube variations. This is ample stability even when it is considered that, with amplifiers spaced 25 miles apart, there would be 160 of these in tandem on a circuit 4000 miles long.

As is well known, the losses introduced by cable circuits do not remain constant even though the circuits are kept dry by means of the airtight lead cable sheaths. Variation in temperature is principally responsible for the variation in efficiency of the circuits. The change in temperature, of course, alters the resistance of the wires and to a lesser extent changes the other primary constants, particularly the dielectric conductance. Fig. 6 shows the transmission loss plotted against frequency of a 25-mile length of 16-gauge cable pair at average temperature (taken as 55° F.) and also the effect of changing this temperature $\pm$ 18° F. which is about the variation experienced in underground cable in this section of the country. For a circuit 1000 miles long the yearly variation amounts to about 100 db.

The transmission loss at any frequency is a simple function of the d-c. resistance. Consequently, measurement of the d-c. resistance of a pilot wire circuit exposed to the same temperature variations can be used to control gains and equalizer adjustments to overcome the effect of this temperature variation. Fig. 7 shows a schematic diagram of the pilot wire transmission regulation system used in the Morristown experiments, while the photograph of Fig. 8 indicates the appearance of the apparatus. This pilot wire regulation system takes care of a 25-mile length of cable. The arrangement of the regulating networks is such that variation of a single resistance causes the transmission loss to be varied a different amount at different frequencies

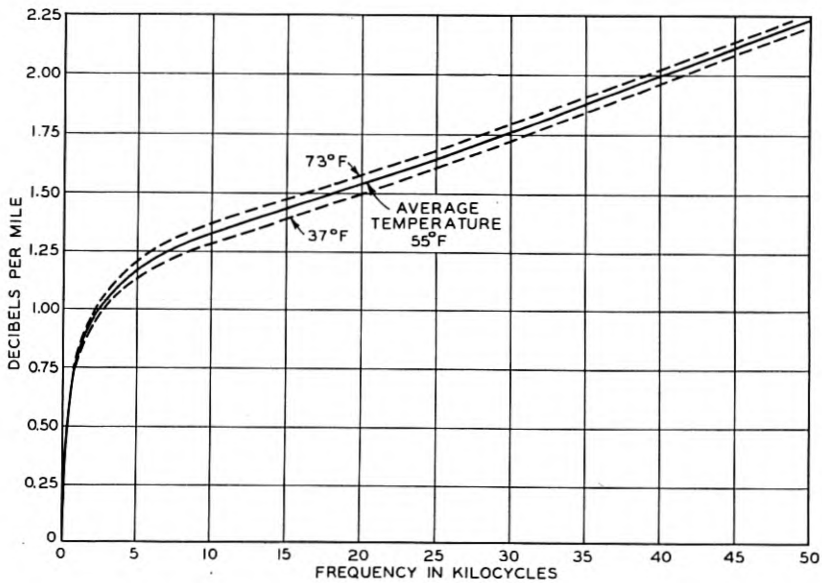

Fig. 6—Transmission loss of 16-gauge cable pair.

as required by the variation in the line loss shown in Fig. 6 above. In Fig. 7 the relay system is omitted for the sake of simplicity. The function of the relay system is, of course, to control the rotation of the shaft carrying the variable resistances so that it follows the rotation of the shaft associated with the master mechanism. The centering cam is provided to avoid "hunting."

The Morristown experiments have shown that this form of regulation is adequate when underground cables are employed. Similar regulation of aerial cables in which the transmission variation with time is three times as large and several hundred times as rapid presents greater but not insuperable difficulties.

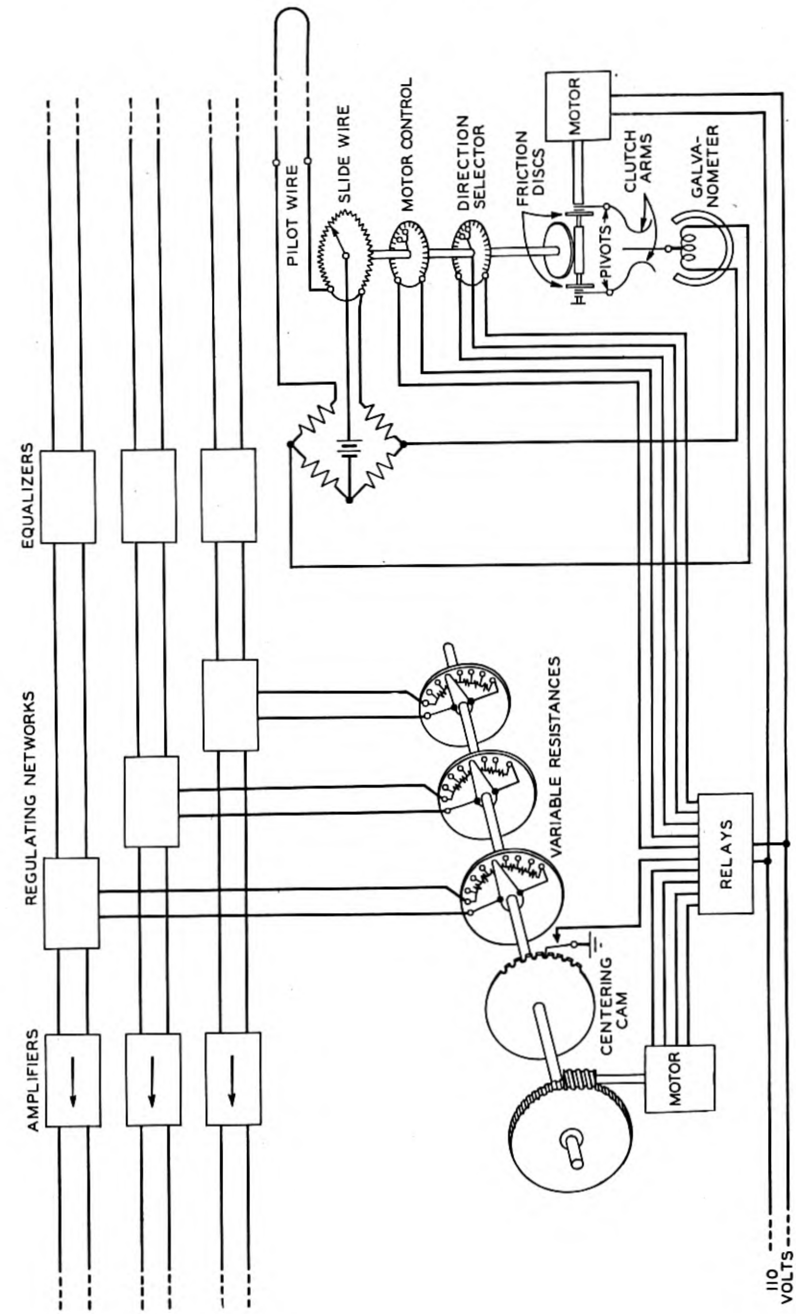

Fig. 7—Automatic transmission regulating system.

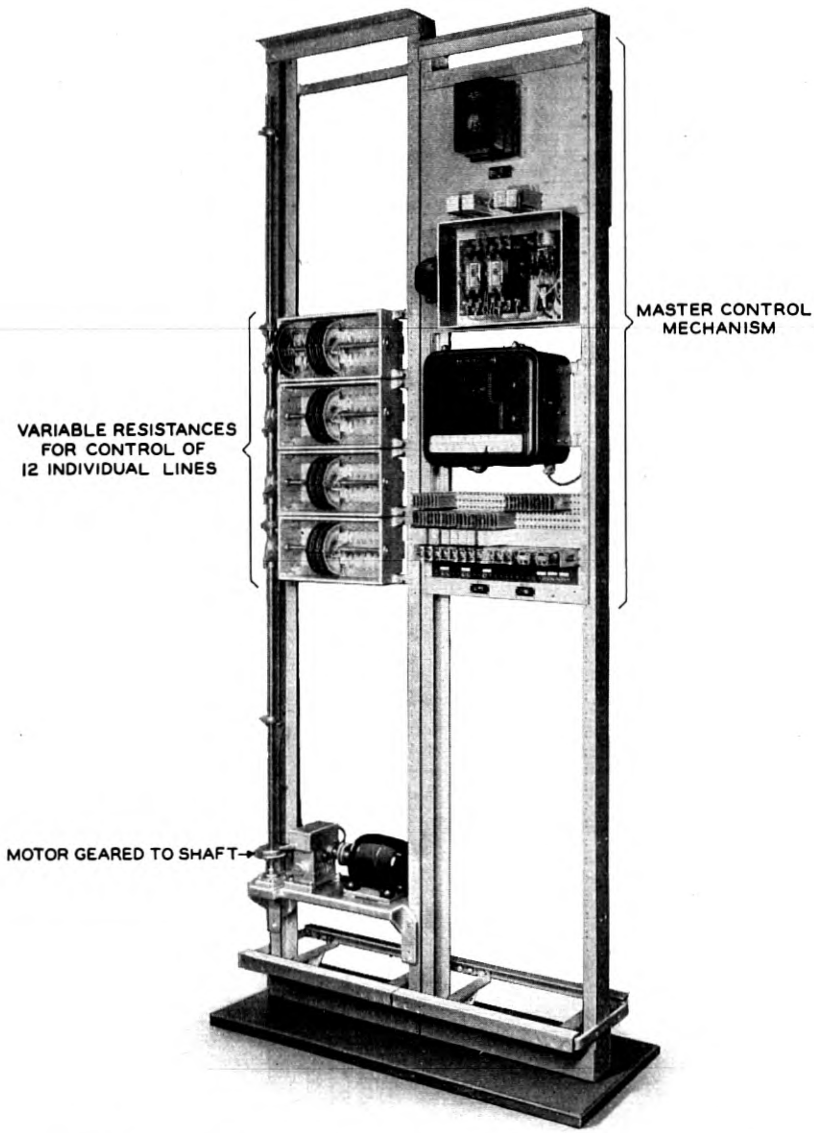

Fig. 8—Automatic transmission regulating equipment—covers removed.

Obtaining High Amplifications

The attenuation of cable pairs being inherently high at carrier frequencies, high amplifier gains are called for, otherwise the cost of the carrier circuits goes up very materially. Since as the power carrying capacity of the repeaters is increased a point is soon reached where it becomes very expensive to go further, high amplifications must be secured by letting the transmitted currents become very weak before amplifying them. A natural limit to this is found in the so-called thermal or resistance noise [1] generated by all conductors. Similar natural and largely insuperable noises are introduced by the vacuum tubes in the amplifiers. Other sources of noise are:

1. Telegraph and signaling circuits worked on other pairs in the same cable with the carrier circuits.
2. Radio stations.
3. Noise from power systems, particularly electric railways.

The latter two disturbances originate outside the cable so that they are subject to the shielding effect of the lead sheath which increases rapidly with increasing frequency. Generally speaking, in a new cable both of these and also the noises from other circuits in the same cable may be relegated by location and design to comparatively minor importance. On existing cables, however, they may require special treatment. In all cases, however, the lower levels at the upper frequencies, which largely determine the repeater spacings, are established primarily by the thermal noise in the conductors and by the corresponding noises in the vacuum tubes. In the Morristown installation the amplifications were kept small enough and the levels high enough so that noise was not an important factor.

Experimental Results

A large number and wide variety of tests have been made using the setup at Morristown. These were generally of too technical a character to be of interest in a general paper such as this one. It will be of chief interest to note that no serious difficulty was experienced in setting up the 850-mile four-wire 4 to 40-kc. circuit with the necessary constancy of transmission loss at different frequencies, although the equalizer arrangements which made this possible presented intricate and difficult problems of design. Nine separate carrier telephone con-

[1] "Thermal Agitation of Electricity in Conductors," by J. B. Johnson, *Phys. Rev.*, Vol. 32, p. 97, 1928, and "Thermal Agitation of Electric Charge in Conductors," by H. Nyquist, *Phys. Rev.*, Vol. 32, p. 110, 1928.

versations were transmitted over this broad band circuit without difficulty due to cross-modulation.

Each carrier telephone circuit was designed to yield a frequency band at least 2500 cycles wide, extending from about 250 cycles to somewhat above 2750 cycles when five such carrier links are connected in tandem. This liberal frequency band and the very satisfactory linearity of transmission over the entire system, gave a very excellent quality of transmission. In order to exaggerate any quality impairment which might have been present the nine carrier circuits were, as noted previously, connected for test in tandem giving a total length of about 7650 miles of two-way telephone circuit. The quality of transmission over this circuit was also found very satisfactory. In fact, the quality was not greatly impaired even when twice this length of one-way circuit was established by connecting all the lengths in tandem, giving a 15,300-mile circuit whose overall loss without amplifiers was about 24,000 db.

As noted previously, the fact that the cable pairs are left non-loaded gives the cable carrier circuits the advantage of very high transmission velocity. Including the effect of the apparatus this velocity is approximately 100,000 miles per second—five or six times as great as the highest velocity loaded voice-frequency toll cable circuits now employed in the U.S.A. This velocity is ample for telephoning satisfactorily over any distances possible on this earth.

Conclusion

Under the present economic conditions there is no immediate demand for the installation of systems of this type. Consequently development work is being pursued further before preparing a system for commercial use. The final embodiment or embodiments of the cable carrier system will probably differ widely, therefore, from the system described in this paper. Since the transmission performance of the experimental system was so completely satisfactory, emphasis is now being directed toward producing more economical systems which will be applicable to shorter circuits. Preliminary indications from this work are that some form of cable carrier system will ultimately find important application on circuits measured in tens rather than hundreds of miles.

Mutual Impedance of Grounded Wires Lying On or Above the Surface of the Earth *

By RONALD M. FOSTER

This paper presents a formula for the mutual impedance of any insulated wires of negligible diameter lying in horizontal planes above the surface of the earth and grounded by vertical wires at their four end-points. The formula holds for frequencies which are not too high to allow all displacement currents to be neglected. Tables and curves are given to facilitate numerical computation by means of the formula.

In the expansion of this formula for low frequencies and for any heights the first two terms give the direct-current mutual impedance; the third term is independent of the heights, thus being identically the same as that previously found for wires on the surface. The mutual impedance for wires at any heights H and h, with separations large in comparison with these heights, is found to be approximately equal to the mutual impedance for wires on the surface multiplied by the complex factor $[1 + \Gamma(H + h)]$, where Γ is the propagation constant in the earth.

THE formula established in a previous paper [1] for the mutual impedance of any grounded thin wires lying on the surface of the earth has now been extended to include wires lying in horizontal planes above the surface of the earth and grounded by vertical wires at their four end-points. As before, we assume the earth to be flat, semi-infinite in extent, of negligible capacitivity, of uniform resistivity ρ, and of inductivity ν equal to that of free space. The air is also assumed to be of negligible capacitivity and of inductivity ν equal to that of free space. All displacement currents are thus neglected both in the earth and in the air; this is the assumption which is ordinarily made as a first approximation at power frequencies for the shorter transmission lines.

By the same general method of derivation as before, the extended formula is found to be:

$$Z_{12} = \int\int \left[\frac{d^2P(r,H,h)}{dSds} + \cos\epsilon\, M(r,H,h) \right] dSds, \qquad (A)$$

where

$$P(r,H,h) = P_0(r) + P_1(r,H + h) - P_2(r,|H - h|),$$

* A brief report of the principal theoretical result obtained in this paper was recently published in the *Physical Review* (2), *41*, 536–537 (August 15, 1932). An error in the formula for $N_0(r)$, as printed there, should be corrected: in the denominator of the fraction, r^2 should be r^3.

[1] R. M. Foster, "Mutual Impedance of Grounded Wires Lying on the Surface of the Earth," *Bell System Technical Journal, 10*, 408–419 (July, 1931); see also *Bulletin of the American Mathematical Society, 36*, 367–368 (May, 1930).

$$M(r,H,h) = M_0(r) + M_1(r,H+h) - M_2(r,|H-h|),$$

$$P_0(r) = \frac{\rho}{2\pi r},$$

$$P_1(r,s) = \frac{i\omega\nu}{4\pi}\int_0^\infty \left\{ \frac{s}{\mu} - \frac{1-e^{-s\mu}}{\mu^2}\left[\frac{(\mu^2+\Gamma^2)^{1/2}-\mu}{(\mu^2+\Gamma^2)^{1/2}+\mu}\right]\right\} J_0(r\mu)d\mu,$$

$$P_2(r,d) = \frac{i\omega\nu}{4\pi}\left[d\log\frac{(r^2+d^2)^{1/2}+d}{r} - (r^2+d^2)^{1/2} + r\right],$$

$$M_0(r) = \frac{\rho}{2\pi r^3}[1-(1+\Gamma r)e^{-\Gamma r}],$$

$$M_1(r,s) = \frac{i\omega\nu}{4\pi}\int_0^\infty (1-e^{-s\mu})\left[\frac{(\mu^2+\Gamma^2)^{1/2}-\mu}{(\mu^2+\Gamma^2)^{1/2}+\mu}\right] J_0(r\mu)d\mu,$$

$$M_2(r,d) = \frac{i\omega\nu}{4\pi}\left[\frac{1}{r} - \frac{1}{(r^2+d^2)^{1/2}}\right].$$

The integrations in the iterated integral are extended over the two wires S and s, lying in planes at heights H and h, respectively. The elements dS and ds are separated by the horizontal distance r and include the angle ϵ between their directions. The propagation constant of plane electromagnetic waves in the earth, varying with the time as $e^{i\omega t}$, is Γ, which equals $(i\omega\nu/\rho)^{1/2}$. All distances are measured in meters, Z_{12} in ohms and ρ in meter-ohms; ν has the value 1.256 $\times 10^{-6}$ henries per meter; ω is equal to 2π times the frequency; J_0 is the Bessel function of order zero. The derivation of the formula is outlined in the latter part of this paper.

The functions P and M are divided into three parts: first, P_0 and M_0, which are functions only of the horizontal distance r; secondly, P_1 and M_1, which are functions of r and of the sum of the two heights H and h; and thirdly, P_2 and M_2, which are functions of r and of the numerical difference of the two heights. These three parts are arranged in the order of relative importance when the heights are reasonably small. For zero heights, the functions P and M reduce to P_0 and M_0, which are the values previously obtained for wires on the surface. For small values of the heights, P_1 and M_1 are of the order of magnitude of the sum of the heights, whereas P_2 and M_2 are of the order of magnitude of the square of the difference.

For some purposes it is convenient to transform formula (A) into the alternative expression:

$$Z_{12} = \int\int\left[\frac{d^2P(r,H,h)}{dSds} + \cos\epsilon\, M(r,H,h)\right]dSds, \qquad (B)$$

where

$$P(r,H,h) = P_0(r) + P^0(r,H,h) + P_3(r,H+h),$$

$$M(r,H,h) = M^0(r,H,h) + M_3(r,H+h),$$

$$P_0(r) = \frac{\rho}{2\pi r},$$

$$P^0(r,H,h) = \frac{i\omega\nu}{4\pi}\left\{ H \log \frac{[r^2+(H+h)^2]^{1/2}+H+h}{[r^2+(H-h)^2]^{1/2}+H-h} + h \log \frac{[r^2+(H+h)^2]^{1/2}+H+h}{[r^2+(H-h)^2]^{1/2}-H+h} + [r^2+(H-h)^2]^{1/2} - [r^2+(H+h)^2]^{1/2} \right\}$$

$$P_3(r,s) = \frac{i\omega\nu}{2\pi}\int_0^\infty \frac{1-e^{-s\mu}}{\mu[(\mu^2+\Gamma^2)^{1/2}+\mu]} J_0(r\mu)d\mu,$$

$$M^0(r,H,h) = \frac{i\omega\nu}{4\pi}\left\{ \frac{1}{[r^2+(H-h)^2]^{1/2}} - \frac{1}{[r^2+(H+h)^2]^{1/2}} \right\},$$

$$M_3(r,s) = \frac{i\omega\nu}{2\pi}\int_0^\infty \frac{\mu e^{-s\mu}}{(\mu^2+\Gamma^2)^{1/2}+\mu} J_0(r\mu)d\mu.$$

The functions P and M are again divided into three parts: first, P_0, the term giving the direct-current mutual resistance; secondly, P^0 and M^0, terms giving the mutual impedance on the assumption of a perfectly conducting earth; and thirdly, P_3 and M_3, the correction terms for the finite conductivity of the earth. The P^0 and M^0 terms thus give $i\omega\nu/8\pi$ times the mutual Neumann integral of the two complete circuits formed from the actual wire circuits by adding to them their reflections in the surface of the earth.

For small values of Γ, the P_3 and M_3 terms can be expanded as follows:

$$\left.\begin{aligned} P_3(r,s) &= \frac{i\omega\nu}{4\pi}\{- s\log\Gamma - s\log[(r^2+s^2)^{1/2}+s] - r \\ &\qquad + (r^2+s^2)^{1/2} + [2\log 2 + \psi(1) + \tfrac{1}{2}]s \\ &\qquad + \tfrac{1}{3}s^2\Gamma - \tfrac{1}{48}s(3r^2-2s^2)\Gamma^2\log\Gamma + \cdots\}, \\ M_3(r,s) &= \frac{i\omega\nu}{4\pi}\left\{ \frac{1}{(r^2+s^2)^{1/2}} - \tfrac{2}{3}\Gamma - \tfrac{1}{4}s\Gamma^2\log\Gamma + \cdots \right\}. \end{aligned}\right\} \quad (1)$$

By means of these expansions, the complete P and M functions, as given by formula (B), can be put into the form:

$$\left.\begin{aligned} P(r,H,h) &= \frac{\rho}{2\pi r} + \frac{i\omega\nu}{4\pi}\{-H\log\{[r^2+(H-h)^2]^{1/2}+H-h\} \\ &\qquad -h\log\{[r^2+(H-h)^2]^{1/2}-H+h\} \\ &\qquad +[r^2+(H-h)^2]^{1/2}-r \\ &\qquad +F(H,h,\Gamma)+O(\Gamma^2\log\Gamma)\}, \\ M(r,H,h) &= \frac{i\omega\nu}{4\pi}\left\{\frac{1}{[r^2+(H-h)^2]^{1/2}}-\tfrac{2}{3}\Gamma+O(\Gamma^2\log\Gamma)\right\}. \end{aligned}\right\} \quad (2)$$

The function $F(H,h,\Gamma)$ is of no consequence, since it does not involve r; it contributes nothing to the value of the impedance. The remaining terms are infinitesimals of order $(\Gamma^2 \log \Gamma)$ for infinitesimal values of Γ; they are thus of higher order than Γ itself.

By means of equation (2) we can now show that the first three terms in the expansion of Z_{12} for low frequencies and for any heights are given by

$$Z_{12} = \frac{\rho}{2\pi}\left(\frac{1}{Aa}-\frac{1}{Ab}-\frac{1}{Ba}+\frac{1}{Bb}\right)+\frac{i\omega\nu}{4\pi}N_{(S-E)(s-e)} + \frac{1-i}{6\pi}\left(\frac{\omega^3\nu^3}{2\rho}\right)^{1/2} AB\,ab\cos\theta+\cdots, \quad (3)$$

where $N_{(S-E)(s-e)}$ is the mutual Neumann integral between the two circuits formed by the wires S and s, lying in planes at heights H and h above the earth, grounded by vertical wires at their four end-points, and with earth returns,—the four grounding points on the surface of the earth being A,B and a,b, respectively. The angle between the straight lines AB and ab is designated by θ. $N_{(S-E)(s-e)}$ is equal to N_{Ss}, the mutual Neumann integral between the two wires S and s, augmented by terms which depend only on the arithmetical distances between eight points,—the four end-points and the four grounding points.

The first two terms in the expansion (3) are precisely the direct-current mutual impedance as given ten years ago by G. A. Campbell.[2] The third term is independent of the heights of the wires; it is thus identically the same as the third term previously found for wires on the surface.

The leading term in the expansion of Z_{12} for a long straight wire S and any wire s located near the midpoint of S, for any heights, is

$$\int\left\{\frac{i\omega\nu}{2\pi}\log\frac{[x^2+(H+h)^2]^{1/2}}{[x^2+(H-h)^2]^{1/2}} + \frac{i\omega\nu}{\pi}\int_0^\infty \frac{e^{-(H+h)\mu}}{(\mu^2+\Gamma^2)^{1/2}+\mu}\cos x\mu\,d\mu\right\}\cos\epsilon\,ds, \quad (4)$$

[2] G. A. Campbell, "Mutual Impedances of Grounded Circuits," *Bell System Technical Journal*, 2, (no. 4), 1–30 (October, 1923).

x being the positive horizontal distance from ds to S, and ϵ the angle between ds and S.

This result is derived immediately from formula (B) upon assuming S to be doubly infinite and then integrating over its entire length. The first part of the expression comes from the M^0 function, the second part from the M_3 function. The other functions contribute nothing to the leading term in the expansion.

The expression enclosed in braces in (4) is the mutual impedance gradient parallel to an infinite wire at a positive horizontal distance x from the wire. It agrees with the results published independently by F. Pollaczek,[3] J. R. Carson,[4] and G. Haberland.[5] Pollaczek has also investigated the case of two grounded circuits of finite length, with certain modifications.[6]

For purposes of computation, however, formula (A) is better, in general, than formula (B). A distinct improvement is effected by multiplying all distances which occur in (A) by the attenuation constant, that is, by $(\omega\nu/2\rho)^{1/2}$. The numerical value thus obtained for any one distance is indicated by a prime accent on the corresponding letter. We then find the mutual impedance expressed in the following form:

$$Z_{12} = \frac{(\omega\nu\rho/2)^{1/2}}{2\pi}\int\int\left[\frac{d^2Q(r',H',h')}{dS'ds'} + \cos\epsilon\, N(r',H',h')\right]dS'ds', \quad (C)$$

where

$$Q(r',H',h') = Q_0(r') + Q_1(r',H'+h') - Q_2(r',|H'-h'|),$$

$$N(r',H',h') = N_0(r') + N_1(r',H'+h') - N_2(r',|H'-h'|),$$

$$Q_0(r') = \frac{1}{r'},$$

$$Q_1(r',s') = i\int_0^\infty\left\{\frac{s'}{\mu} - \frac{1-e^{-s'\mu}}{\mu^2}\left[\frac{(\mu^2+2i)^{1/2}-\mu}{(\mu^2+2i)^{1/2}+\mu}\right]\right\}J_0(r'\mu)d\mu,$$

$$Q_2(r',d') = i\left[d'\log\frac{(r'^2+d'^2)^{1/2}+d'}{r'} - (r'^2+d'^2)^{1/2} + r'\right],$$

[3] F. Pollaczek, "Über das Feld einer unendlich langen wechselstromdurchflossenen Einfachleitung," *Elektrische Nachrichten-technik*, *3*, 339–359 (September, 1926).

[4] J. R. Carson, "Wave Propagation in Overhead Wires with Ground Return," *Bell System Technical Journal*, *5*, 539–554 (October, 1926).

[5] G. Haberland, "Theorie der Leitung von Wechselstrom durch die Erde," *Zeitschrift für angewandte Mathematik und Mechanik*, *6*, 366–379 (October, 1926).

[6] F. Pollaczek, "Gegenseitige Induktion zwischen Wechselstromfreileitungen von endlicher Länge," *Annalen der Physik* (4), *87*, 965–999 (December, 1928). His assumptions regarding conditions at the ground connections seem to depart considerably from the conditions assumed in the present paper, and moreover his results are not expressed in convenient form for direct comparison with the formula given above for Z_{12}.

$$N_0(r') = \frac{1}{r'^3}\{1 - [1 + (1 + i)r']e^{-(1+i)r'}\},$$

$$N_1(r',s') = i\int_0^\infty (1 - e^{-s'\mu})\left[\frac{(\mu^2 + 2i)^{1/2} - \mu}{(\mu^2 + 2i)^{1/2} + \mu}\right] J_0(r'\mu)d\mu,$$

$$N_2(r',d') = i\left[\frac{1}{r'} - \frac{1}{(r'^2 + d'^2)^{1/2}}\right];$$

the prime accent applied to any length L indicating the corresponding modified length $L' = (\omega\nu/2\rho)^{1/2}L$.

As in formula (A), the six constituent functions involved in formula (C) are arranged in order of importance: first, Q_0 and N_0, functions only of the modified horizontal distance r'; secondly, Q_1 and N_1, functions of r' and of the sum of the two modified heights H' and h'; and thirdly, Q_2 and N_2, functions of r' and of the numerical difference of the two modified heights.

To assist in the numerical application of this formula, a table of values of the real and imaginary parts of N_0 has been computed, for all values of r' from 0 to 10, in steps of 0.1. Beyond this range, the function is practically equal to the leading term in its asymptotic expansion, namely, $1/r'^3$. These computed values are also shown graphically in Fig. 1. The imaginary part changes sign at approximately $r' = 3.8$, and again at 7.0, oscillating for increasing values of r', although approaching zero very rapidly indeed.

The real and imaginary parts of the functions $Q_1(r',s')$ and $N_1(r',s')$ are shown in Figs. 2, 3, 4, and 5, for the range of r' from 0 to 10, and for the set of values of s' from 0 to 0.2 in steps of 0.02. It is believed that this will cover the range of heights likely to be encountered in ordinary problems. To cover this range adequately it was necessary to show portions of the N_1 curves with the horizontal scale enlarged two and a half times, and with a greatly reduced vertical scale, in Figs. 4-A and 5-A.

For actual computation, the Q_2 and N_2 functions are already expressed in (A) in convenient, closed form, but for purposes of comparison with Q_1 and N_1, the corresponding values of Q_2 and N_2 are shown in Figs. 6 and 7, which are drawn to the same scales as Figs. 3 and 5.

Tables II and III give the corresponding numerical values of Q_1 and N_1 for the range of r' from 0 to 1, in steps of 0.1, as well as the values for 1.5 and 2.

TABLE I

Real and Imaginary Parts of $N_0(r')$

r'	Real	Imag.	r'	Real	Imag.
0	0.66667	∞	5.0	0.0081667	−0.00038659
0.1	0.61800	9.33461	5.1	0.0076496	−0.00034816
0.2	0.57199	4.33824	5.2	0.0071782	−0.00031048
0.3	0.52860	2.67724	5.3	0.0067477	−0.00027431
0.4	0.48778	1.85135	5.4	0.0063538	−0.00024017
0.5	0.44949	1.36031	5.5	0.0059927	−0.00020839
0.6	0.41363	1.03722	5.6	0.0056609	−0.00017918
0.7	0.38014	0.81043	5.7	0.0053554	−0.00015262
0.8	0.34892	0.64405	5.8	0.0050736	−0.00012872
0.9	0.31987	0.51804	5.9	0.0048131	−0.00010740
1.0	0.29291	0.42035	6.0	0.0045717	−0.000088557
1.1	0.26792	0.34327	6.1	0.0043477	−0.000072047
1.2	0.24480	0.28161	6.2	0.0041392	−0.000057706
1.3	0.22346	0.23177	6.3	0.0039449	−0.000045358
1.4	0.20379	0.19116	6.4	0.0037634	−0.000034822
1.5	0.18568	0.15785	6.5	0.0035936	−0.000025919
1.6	0.16905	0.13041	6.6	0.0034344	−0.000018472
1.7	0.15379	0.10770	6.7	0.0032850	−0.000012315
1.8	0.13981	0.088878	6.8	0.0031444	−0.0000072892
1.9	0.12703	0.073237	6.9	0.0030120	−0.0000032482
2.0	0.11535	0.060227	7.0	0.0028872	−0.0000000570
2.1	0.10470	0.049402	7.1	0.0027693	0.0000024076
2.2	0.095002	0.040395	7.2	0.0026578	0.0000042564
2.3	0.086174	0.032905	7.3	0.0025522	0.0000055886
2.4	0.078152	0.026685	7.4	0.0024522	0.0000064921
2.5	0.070871	0.021526	7.5	0.0023573	0.0000070444
2.6	0.064268	0.017257	7.6	0.0022672	0.0000073128
2.7	0.058287	0.013734	7.7	0.0021816	0.0000073559
2.8	0.052874	0.010835	7.8	0.0021001	0.0000072236
2.9	0.047980	0.0084577	7.9	0.0020226	0.0000069586
3.0	0.043558	0.0065174	8.0	0.0019488	0.0000065967
3.1	0.039567	0.0049415	8.1	0.0018785	0.0000061678
3.2	0.035966	0.0036689	8.2	0.0018114	0.0000056965
3.3	0.032719	0.0026483	8.3	0.0017474	0.0000052028
3.4	0.029792	0.0018364	8.4	0.0016863	0.0000047027
3.5	0.027156	0.0011967	8.5	0.0016280	0.0000042086
3.6	0.024782	0.00069852	8.6	0.0015722	0.0000037302
3.7	0.022645	0.00031616	8.7	0.0015190	0.0000032745
3.8	0.020720	0.00002803	8.8	0.0014680	0.0000028466
3.9	0.018987	−0.00018390	8.9	0.0014193	0.0000024498
4.0	0.017427	−0.00033467	9.0	0.0013727	0.0000020858
4.1	0.016021	−0.00043678	9.1	0.0013280	0.0000017555
4.2	0.014754	−0.00050056	9.2	0.0012852	0.0000014587
4.3	0.013612	−0.00053454	9.3	0.0012443	0.0000011945
4·4	0.012582	−0.00054572	9.4	0.0012050	0.00000096159
4.5	0.011652	−0.00053979	9.5	0.0011673	0.00000075815
4.6	0.010811	−0.00052138	9.6	0.0011312	0.00000058219
4.7	0.010050	−0.00049420	9.7	0.0010966	0.00000043156
4.8	0.0093603	−0.00046121	9.8	0.0010633	0.00000030402
4.9	0.0087349	−0.00042473	9.9	0.0010313	0.00000019732
5.0	0.0081667	−0.00038659	10.0	0.0010007	0.00000010925

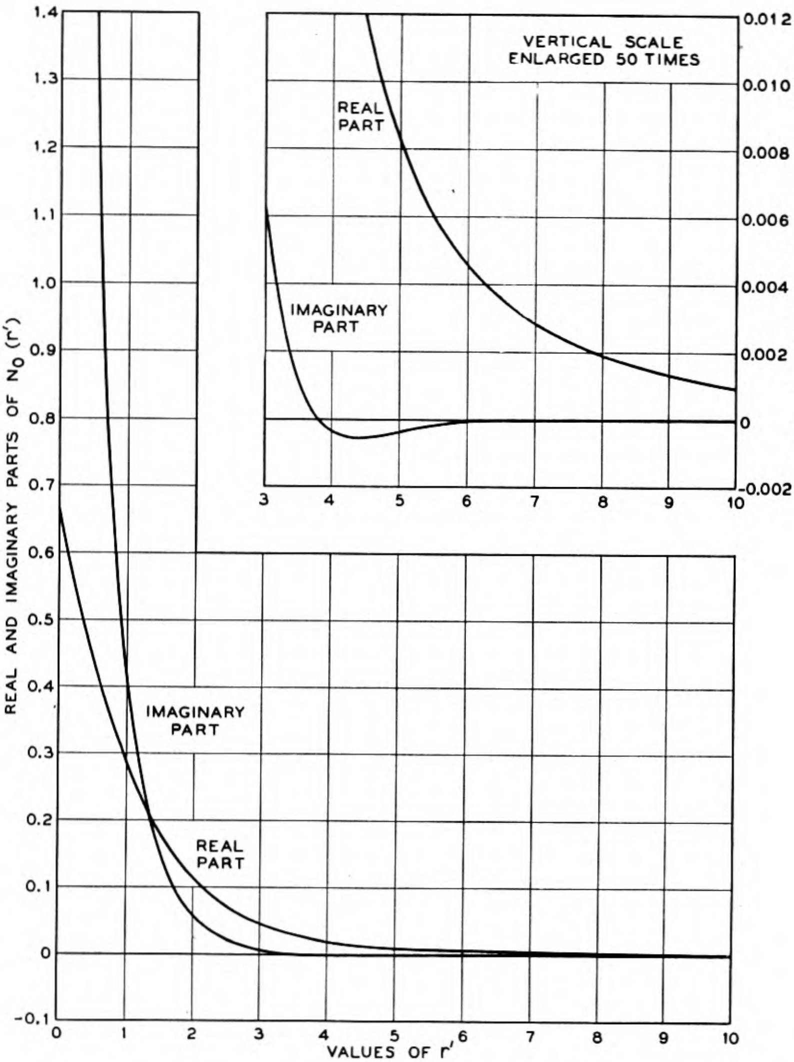

Fig. 1—Real and imaginary parts of $N_0(r')$.

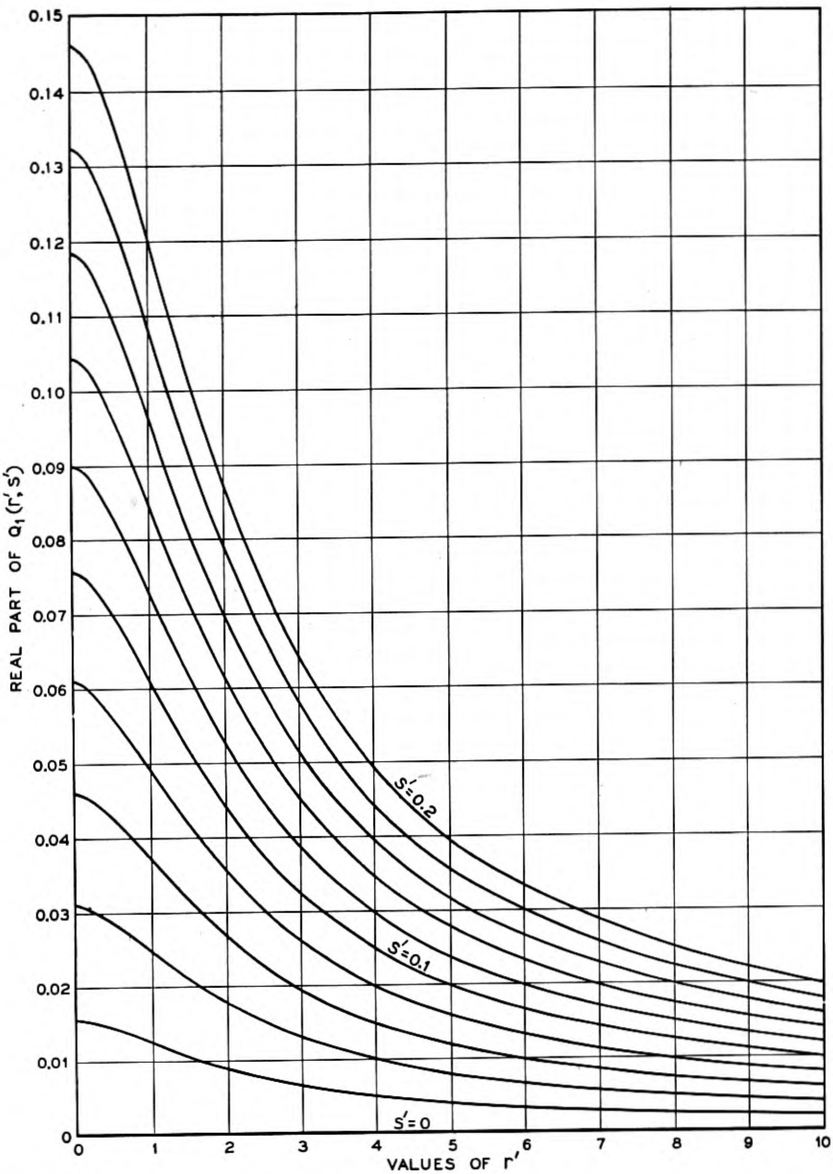

Fig. 2—Real part of $Q_1(r', s')$.

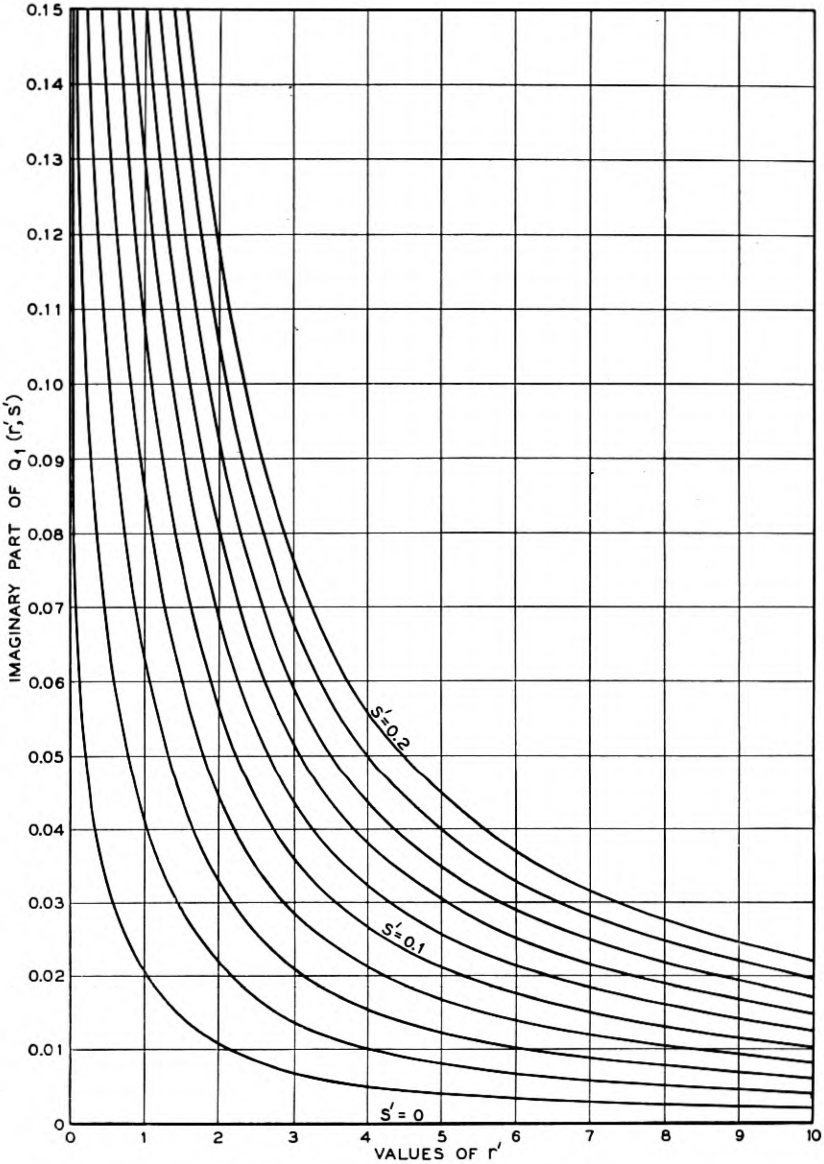

Fig. 3—Imaginary part of $Q_1(r', s')$.

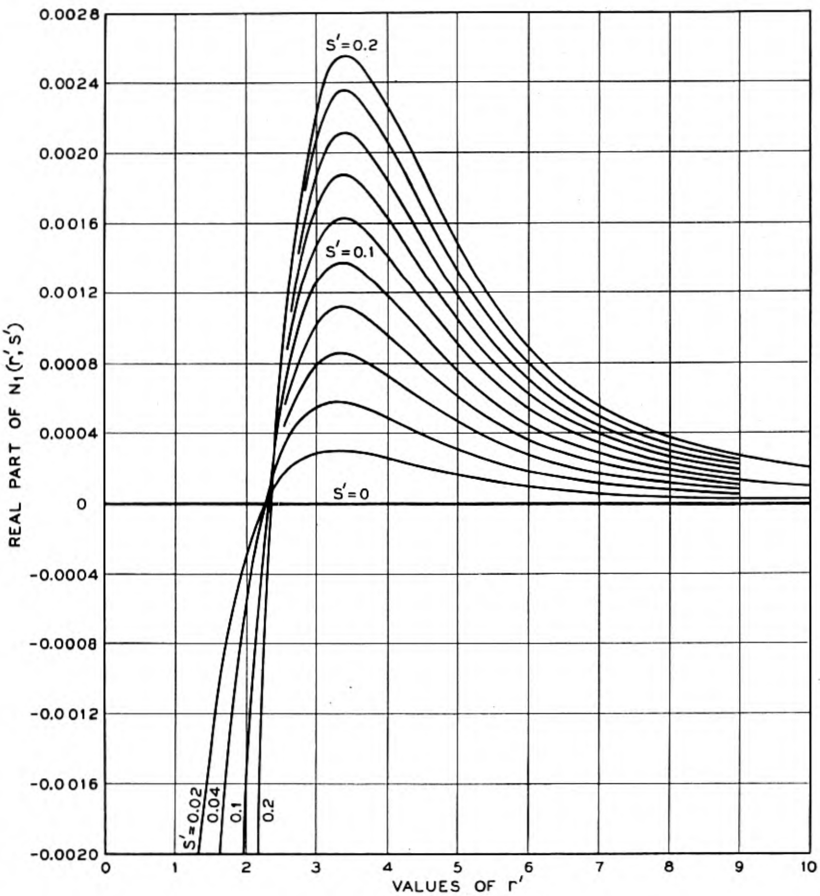

Fig. 4—Real part of $N_1(r', s')$.

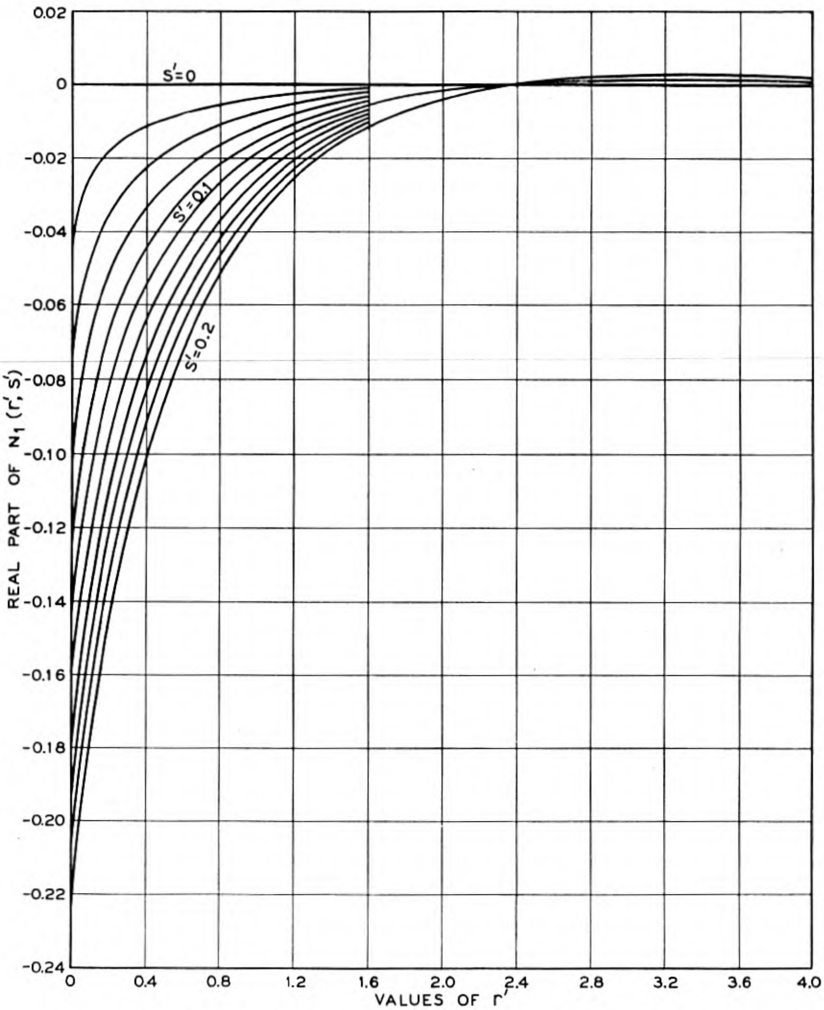

Fig. 4-A—Real part of $N_1(r', s')$, enlarged horizontal scale.

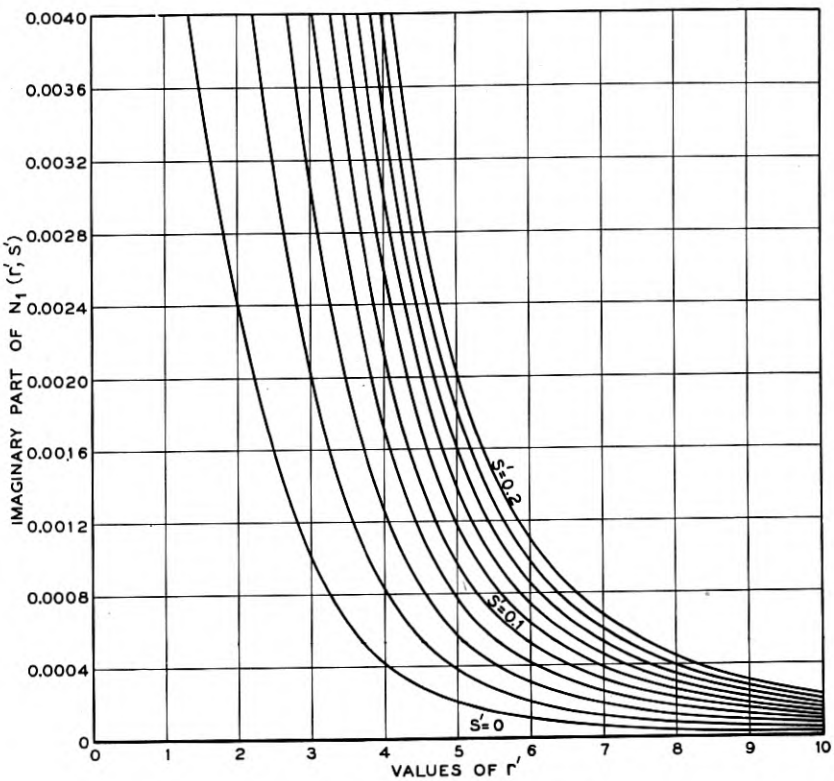

Fig. 5—Imaginary part of $N_1(r', s')$.

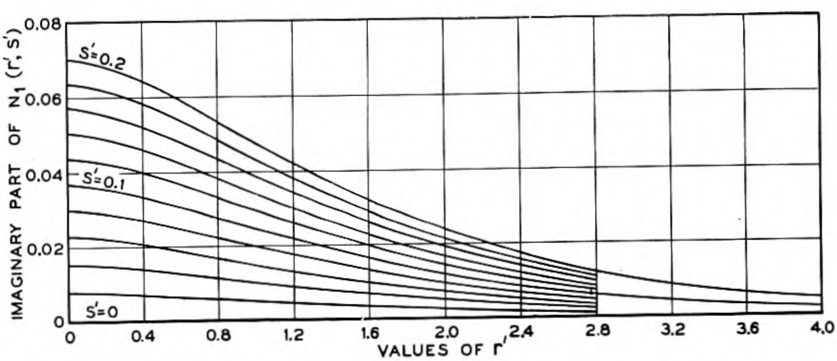

Fig. 5-A—Imaginary part of $N_1(r', s')$, enlarged horizontal scale.

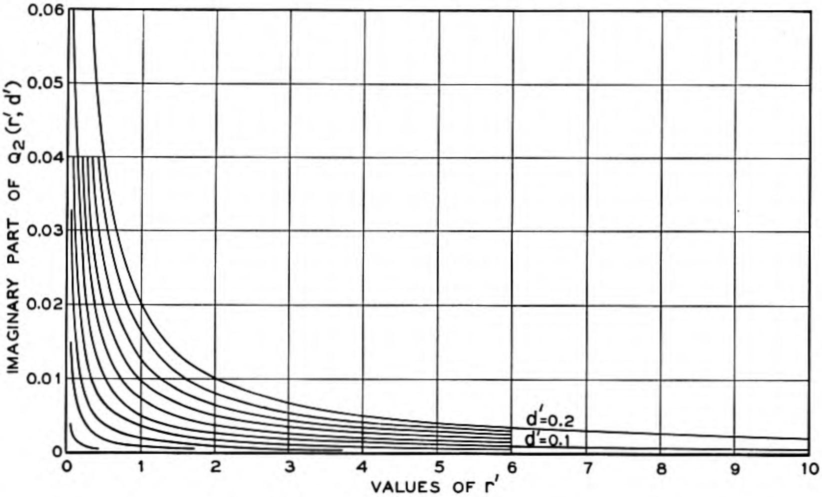

Fig. 6—Imaginary part of $Q_2(r', d')$.

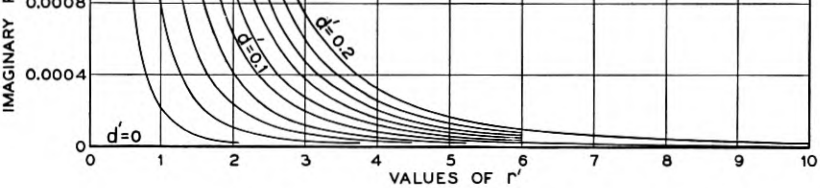

Fig. 7—Imaginary part of $N_2(r', d')$.

TABLE II

REAL PART OF $Q_1(r',s')$

r'	$s' = 0.02$	$s' = 0.04$	$s' = 0.06$	$s' = 0.08$	$s' = 0.10$	$s' = 0.12$	$s' = 0.14$	$s' = 0.16$	$s' = 0.18$	$s' = 0.20$
0	0.0156	0.0309	0.0460	0.0609	0.0755	0.0900	0.1042	0.1182	0.1321	0.1458
0.1	0.0155	0.0308	0.0458	0.0606	0.0752	0.0896	0.1038	0.1178	0.1316	0.1453
0.2	0.0153	0.0304	0.0453	0.0599	0.0744	0.0887	0.1028	0.1167	0.1304	0.1439
0.3	0.0151	0.0299	0.0446	0.0590	0.0733	0.0874	0.1013	0.1150	0.1286	0.1419
0.4	0.0148	0.0293	0.0437	0.0579	0.0719	0.0858	0.0994	0.1130	0.1263	0.1395
0.5	0.0144	0.0287	0.0427	0.0566	0.0704	0.0840	0.0974	0.1106	0.1237	0.1367
0.6	0.0141	0.0280	0.0417	0.0553	0.0687	0.0820	0.0951	0.1081	0.1209	0.1336
0.7	0.0137	0.0272	0.0406	0.0538	0.0669	0.0799	0.0927	0.1054	0.1180	0.1304
0.8	0.0133	0.0265	0.0395	0.0524	0.0651	0.0777	0.0902	0.1026	0.1149	0.1270
0.9	0.0129	0.0257	0.0383	0.0509	0.0633	0.0755	0.0877	0.0998	0.1117	0.1235
1.0	0.0125	0.0249	0.0372	0.0493	0.0614	0.0733	0.0852	0.0969	0.1085	0.1200
1.5	0.0106	0.0211	0.0316	0.0419	0.0523	0.0625	0.0727	0.0828	0.0928	0.1027
2.0	0.0089	0.0178	0.0267	0.0355	0.0442	0.0529	0.0616	0.0702	0.0788	0.0873

IMAGINARY PART OF $Q_1(r',s')$

r'	$s' = 0.02$	$s' = 0.04$	$s' = 0.06$	$s' = 0.08$	$s' = 0.10$	$s' = 0.12$	$s' = 0.14$	$s' = 0.16$	$s' = 0.18$	$s' = 0.20$
0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.1	0.0655	0.1312	0.1971	0.2634	0.3299	0.3966	0.4636	0.5308	0.5983	0.6660
0.2	0.0516	0.1036	0.1557	0.2082	0.2608	0.3138	0.3669	0.4204	0.4740	0.5279
0.3	0.0436	0.0875	0.1317	0.1761	0.2207	0.2656	0.3108	0.3562	0.4018	0.4477
0.4	0.0380	0.0763	0.1148	0.1536	0.1926	0.2319	0.2714	0.3111	0.3511	0.3913
0.5	0.0337	0.0677	0.1019	0.1363	0.1711	0.2060	0.2412	0.2766	0.3123	0.3482
0.6	0.0303	0.0608	0.0915	0.1225	0.1538	0.1852	0.2169	0.2489	0.2811	0.3135
0.7	0.0274	0.0550	0.0829	0.1110	0.1394	0.1680	0.1968	0.2259	0.2552	0.2847
0.8	0.0250	0.0502	0.0756	0.1013	0.1273	0.1534	0.1798	0.2064	0.2332	0.2603
0.9	0.0229	0.0460	0.0694	0.0930	0.1168	0.1409	0.1651	0.1896	0.2143	0.2393
1.0	0.0211	0.0424	0.0639	0.0857	0.1077	0.1299	0.1523	0.1750	0.1979	0.2209
1.5	0.0147	0.0296	0.0447	0.0601	0.0756	0.0913	0.1062	0.1233	0.1396	0.1561
2.0	0.0110	0.0221	0.0334	0.0449	0.0566	0.0684	0.0804	0.0926	0.1049	0.1174

TABLE III

REAL PART OF $N_1(r',s')$

r'	$s'=0.02$	$s'=0.04$	$s'=0.06$	$s'=0.08$	$s'=0.10$	$s'=0.12$	$s'=0.14$	$s'=0.16$	$s'=0.18$	$s'=0.20$
0	−0.0444	−0.0752	−0.1009	−0.1235	−0.1437	−0.1621	−0.1790	−0.1947	−0.2094	−0.2231
0.1	−0.0243	−0.0468	−0.0677	−0.0871	−0.1051	−0.1219	−0.1376	−0.1523	−0.1662	−0.1793
0.2	−0.0179	−0.0350	−0.0514	−0.0669	−0.0818	−0.0960	−0.1095	−0.1223	−0.1346	−0.1463
0.3	−0.0141	−0.0278	−0.0409	−0.0537	−0.0660	−0.0778	−0.0893	−0.1003	−0.1109	−0.1211
0.4	−0.0114	−0.0226	−0.0334	−0.0440	−0.0542	−0.0642	−0.0739	−0.0833	−0.0924	−0.1012
0.5	−0.0094	−0.0186	−0.0277	−0.0365	−0.0451	−0.0535	−0.0617	−0.0697	−0.0775	−0.0851
0.6	−0.0078	−0.0155	−0.0230	−0.0304	−0.0377	−0.0448	−0.0518	−0.0586	−0.0653	−0.0718
0.7	−0.0065	−0.0129	−0.0193	−0.0255	−0.0316	−0.0376	−0.0436	−0.0494	−0.0551	−0.0607
0.8	−0.0054	−0.0108	−0.0161	−0.0214	−0.0265	−0.0316	−0.0367	−0.0416	−0.0465	−0.0513
0.9	−0.0045	−0.0090	−0.0135	−0.0179	−0.0223	−0.0266	−0.0308	−0.0350	−0.0392	−0.0433
1.0	−0.0038	−0.0075	−0.0113	−0.0150	−0.0186	−0.0223	−0.0259	−0.0294	−0.0330	−0.0365
1.5	−0.0014	−0.0028	−0.0042	−0.0056	−0.0070	−0.0084	−0.0099	−0.0113	−0.0127	−0.0142
2.0	−0.0003	−0.0006	−0.0010	−0.0013	−0.0017	−0.0021	−0.0025	−0.0029	−0.0033	−0.0038

IMAGINARY PART OF $N_1(r',s')$

r'	$s'=0.02$	$s'=0.04$	$s'=0.06$	$s'=0.08$	$s'=0.10$	$s'=0.12$	$s'=0.14$	$s'=0.16$	$s'=0.18$	$s'=0.20$
0	0.0078	0.0153	0.0227	0.0299	0.0369	0.0438	0.0505	0.0571	0.0635	0.0698
0.1	0.0077	0.0152	0.0225	0.0296	0.0366	0.0434	0.0501	0.0566	0.0630	0.0693
0.2	0.0075	0.0148	0.0220	0.0290	0.0359	0.0426	0.0492	0.0556	0.0619	0.0681
0.3	0.0073	0.0144	0.0213	0.0282	0.0348	0.0414	0.0478	0.0541	0.0603	0.0663
0.4	0.0070	0.0139	0.0206	0.0272	0.0336	0.0400	0.0462	0.0523	0.0583	0.0641
0.5	0.0067	0.0133	0.0197	0.0260	0.0322	0.0383	0.0443	0.0502	0.0560	0.0617
0.6	0.0064	0.0126	0.0188	0.0248	0.0308	0.0366	0.0424	0.0480	0.0536	0.0590
0.7	0.0061	0.0120	0.0179	0.0236	0.0293	0.0348	0.0403	0.0457	0.0510	0.0562
0.8	0.0057	0.0114	0.0169	0.0224	0.0277	0.0330	0.0382	0.0434	0.0484	0.0534
0.9	0.0054	0.0107	0.0159	0.0211	0.0262	0.0312	0.0361	0.0410	0.0458	0.0505
1.0	0.0051	0.0101	0.0150	0.0198	0.0246	0.0294	0.0340	0.0386	0.0432	0.0476
1.5	0.0036	0.0071	0.0106	0.0141	0.0176	0.0210	0.0243	0.0277	0.0310	0.0343
2.0	0.0024	0.0048	0.0072	0.0096	0.0119	0.0143	0.0166	0.0190	0.0213	0.0235

These tabulated values were computed from the corresponding convergent series, the first few terms of which are:

$$\left.\begin{aligned} Q_1(r',s') &= \tfrac{1}{4}\pi s' - \tfrac{1}{3}s'^2 + \cdots \\ &\quad + i\{- s' \log r' + [\tfrac{3}{2}\log 2 + \psi(1) + \tfrac{1}{2}]s' \\ &\qquad + \tfrac{1}{3}s'^2 + \cdots\}, \\ N_1(r',s') &= \tfrac{1}{2}s' \log [(r'^2 + s'^2)^{1/2} + s'] \\ &\qquad - \tfrac{1}{2}[\tfrac{3}{2}\log 2 + \psi(1) - \tfrac{1}{4}]s' \\ &\qquad + \tfrac{1}{2}r' - \tfrac{1}{2}(r'^2 + s'^2)^{1/2} - \tfrac{4}{15}s'^2 + \cdots \\ &\quad + i\{\tfrac{1}{8}\pi s' - \tfrac{4}{15}s'^2 + \cdots\}. \end{aligned}\right\} \tag{5}$$

For values of r' greater than 2, sufficient accuracy for ordinary purposes is obtained by using the first two terms in the expansions in terms of s':

$$\left.\begin{aligned} Q_1(r',s') &= s'Q_1^{(1)}(r') + s'^2Q_1^{(2)}(r') + \cdots, \\ N_1(r',s') &= s'N_1^{(1)}(r') + s'^2N_1^{(2)}(r') + \cdots, \end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} Q_1^{(1)}(r') &= i[I_0(u)K_0(u) + I_1(u)K_1(u)], \\ Q_1^{(2)}(r') &= \frac{1-i}{8u^3}[(1 - 2u^2) - (1 + 2u)e^{-2u}], \\ N_1^{(1)}(r') &= \frac{1}{u^2}[1 - 2uI_1(u)K_0(u) - 2I_1(u)K_1(u)], \\ N_1^{(2)}(r') &= \frac{1+i}{16u^5}[(9 - 2u^2) - (9 + 18u + 16u^2 + 8u^3)e^{-2u}], \\ u &= \tfrac{1}{2}(1 + i)r'. \end{aligned}\right\} \tag{7}$$

For actual computation we note that

$$Q_1^{(2)}(r') = \tfrac{1}{2}\left[\frac{i}{r'} - N_0(r')\right].$$

The real and imaginary parts of these four functions are shown in Figs. 8, 9, 10, and 11. The dominating terms in the asymptotic expansions of Q_1 and N_1 are thus given by those of $Q_1^{(1)}$ and $N_1^{(1)}$. For large values of r', $Q_1^{(1)}$ approaches zero as $(1 + i)/r'$, and $N_1^{(1)}$ as $(1 + i)/r'^3$.

For very large values of r' it is convenient to express the functions as follows:

$$\left.\begin{aligned} Q_0(r') + Q_1(r',s') &= Q_0(r')\left[1 + \frac{Q_1{}^{(1)}(r')}{Q_0(r')}s' + \cdots\right], \\ N_0(r') + N_1(r',s') &= N_0(r')\left[1 + \frac{N_1{}^{(1)}(r')}{N_0(r')}s' + \cdots\right]. \end{aligned}\right\} \tag{8}$$

The real and imaginary parts of these ratios of functions—the coefficients of s' in the above expansions—are shown in Figs. 12 and 13. We note that each of these ratios approaches the value $(1 + i)$ as r' increases without limit. Hence, as a rough approximation, we may say that the mutual impedance for wires at heights H and h, with separations large in comparison with these heights, is equal to the impedance for wires at zero heights multiplied by the factor:

$$1 + (1 + i)(H' + h') = 1 + \Gamma(H + h). \tag{9}$$

The mutual impedance formula (A) was originally derived from first principles, following the method used in the previous paper for

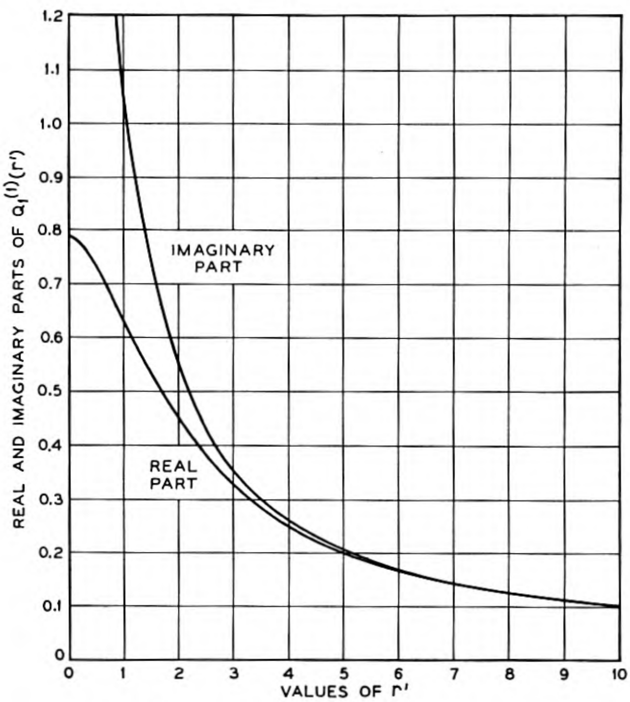

Fig. 8—Real and imaginary parts of $Q_1{}^{(1)}(r')$.

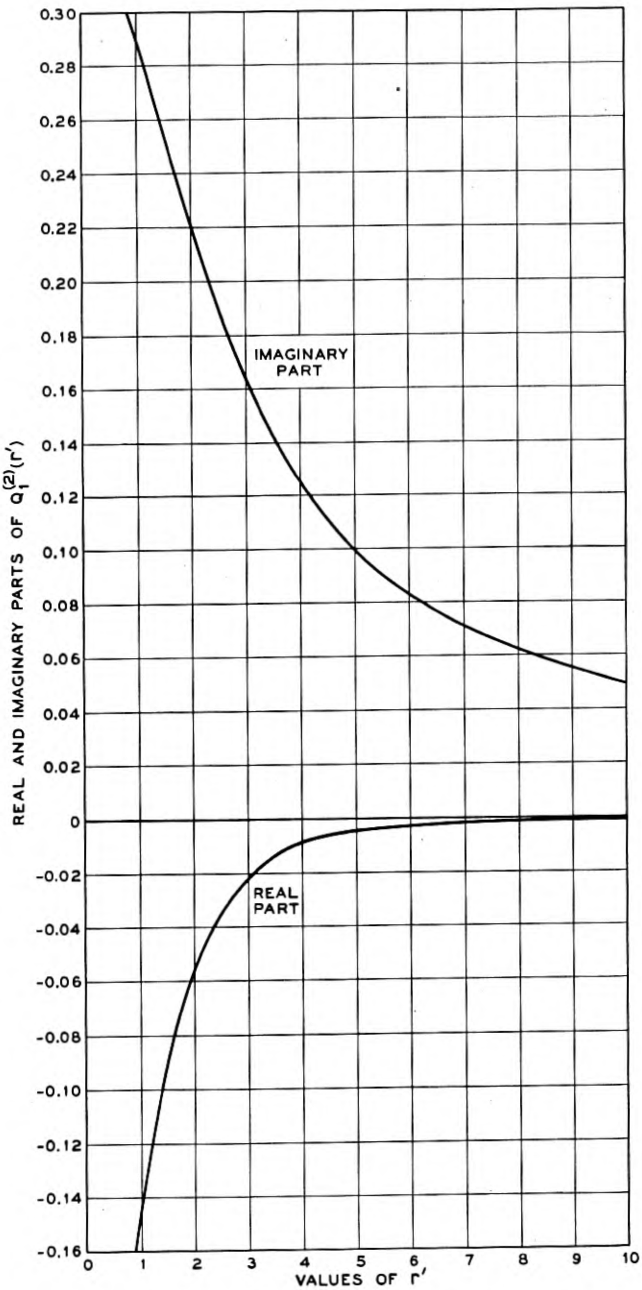

Fig. 9—Real and imaginary parts of $Q_1^{(2)}(r')$.

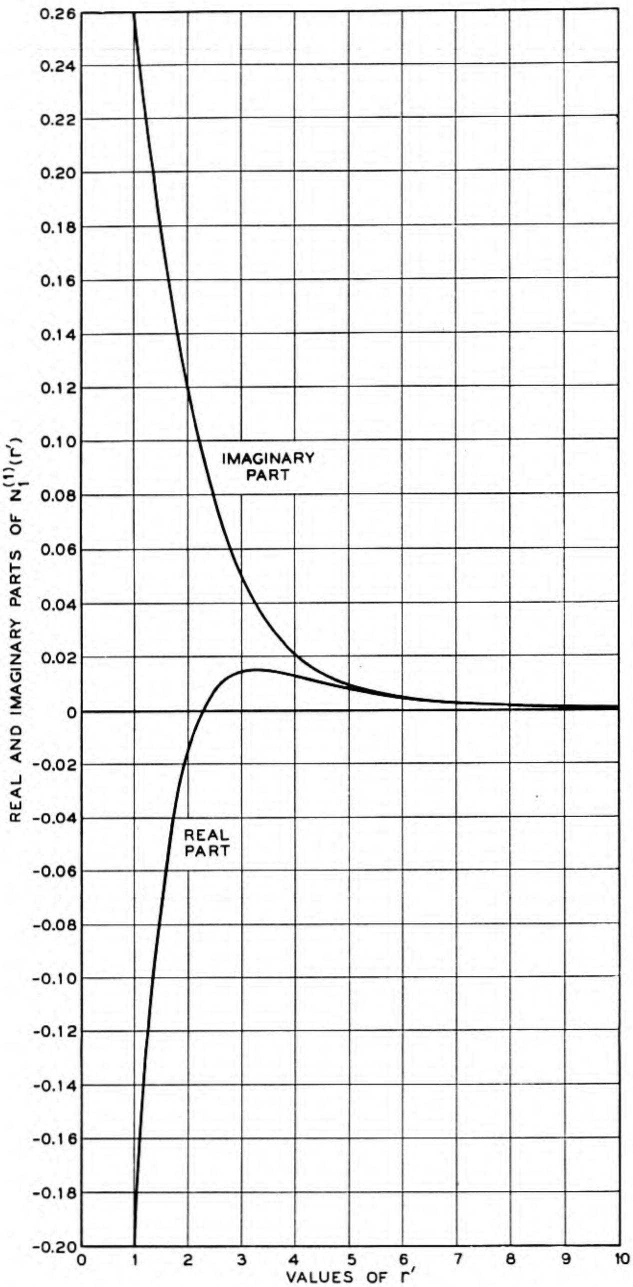

Fig. 10—Real and imaginary parts of $N_1^{(1)}(r')$.

wires on the surface. A brief outline of this derivation is given here. We first find the formulæ for the components of the electric field due to a current flowing in a straight wire of length $2a$ parallel to the surface of the earth and at the height H above it, assuming the air to be replaced by a medium of finite resistivity ρ_1. This part of the

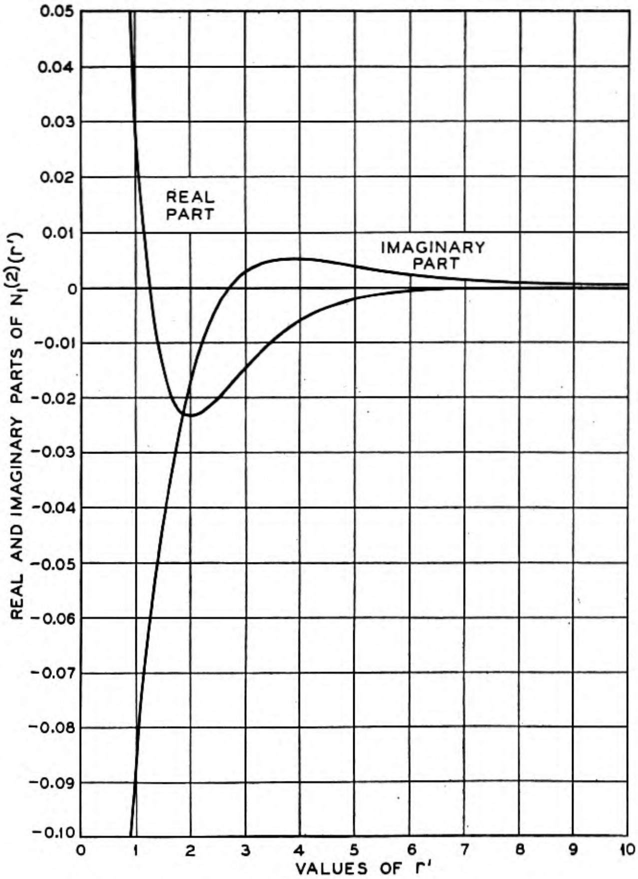

Fig. 11—Real and imaginary parts of $N_1^{(2)}(r')$.

derivation follows closely the work involved in the previous case of wires on the surface. We next find the electric field due to a current in a vertical wire extending from the surface of the earth up to the height H in the assumed medium. This part of the derivation is simpler since there is circular symmetry. Upon combining these two results, we obtain the field due to a current flowing through three sides of a rectangular circuit beginning and ending at the surface of the earth, extending up to the height H, and of width $2a$. We can now allow ρ_1 to become infinite, corresponding to the assumptions of our problem, since this circuit is completed through the earth. Upon allowing a to approach zero, such that $2a = dS$, we find the field corresponding to a rectangle of infinitesimal width. We then take the

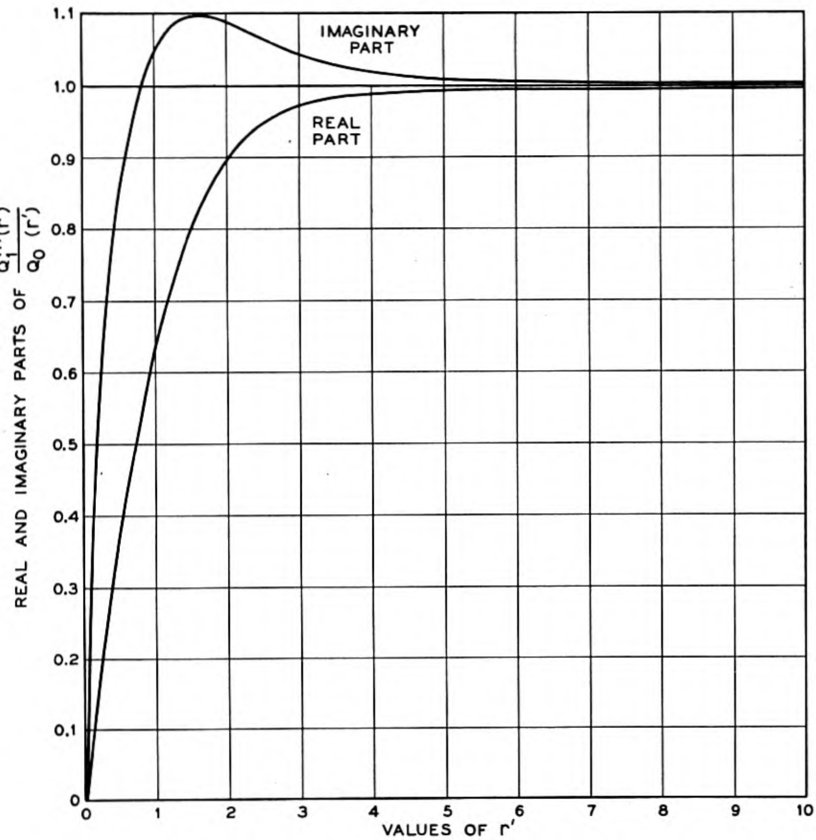

Fig. 12—Real and imaginary parts of $\dfrac{Q_1^{(1)}(r')}{Q_0(r')}$.

integral of this expression around a similar circuit consisting of a horizontal element of wire of length ds at the height h, grounded by wires at its end-points. Upon making various algebraic simplifications, we finally obtain the mutual impedance as given by formula (A).

It is perhaps more convenient to derive this formula from results obtained by H. von Hoerschelmann,[7] again following the method

[7] H. von Hoerschelmann, "Über die Wirkungsweise des geknickten Marconischen Senders der drahtlosen Telegraphie," *Jahrbuch der drahtlosen Telegraphie und Telephonie*, *5*, 14–34, 188–211 (September, November, 1911).

employed in the previous paper in a similar derivation for wires on the surface. For our present problem we use his formulæ for the Hertzian vectors due to horizontal and vertical electric antennæ above the surface of the earth. It is important, at first, to retain a non-

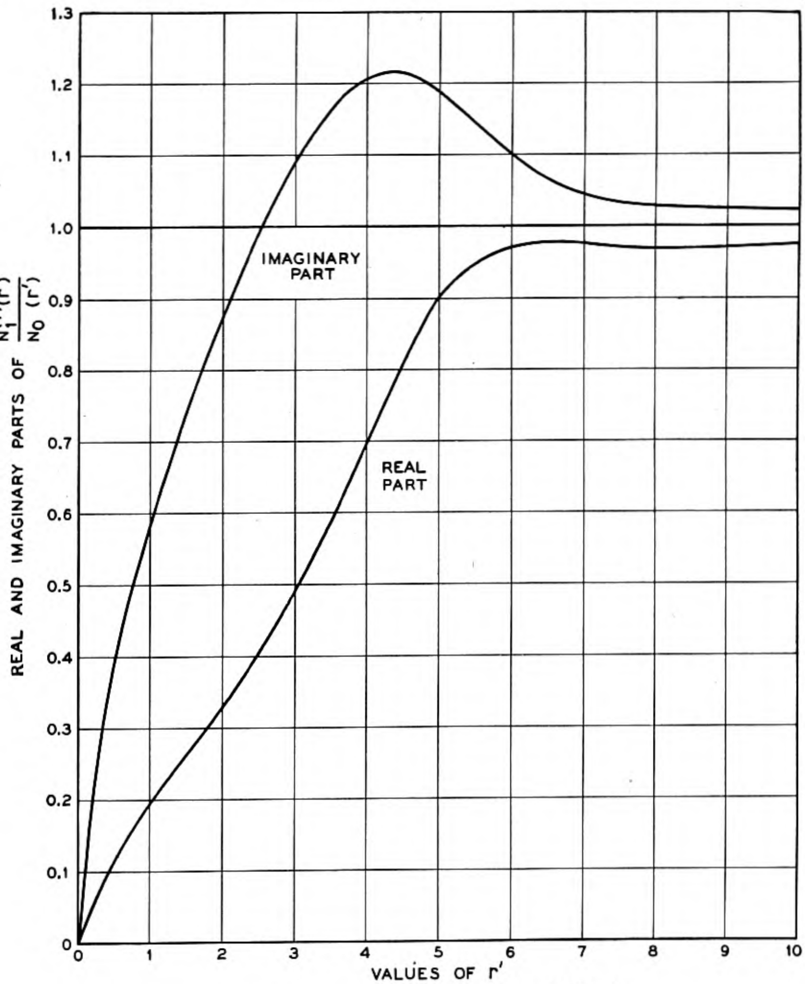

Fig. 13—Real and imaginary parts of $\dfrac{N_1^{(1)}(r')}{N_0(r')}$.

vanishing value of the capacitivity of the air. From these formulæ we obtain the vector Π, due to a current flowing through a horizontal element of wire of length dS at height H above the surface of the earth, grounded by vertical wires at its end-points. Next, we obtain the electric field $\mathbf{E}$ in the air by the relation:

$$\mathbf{E} = \text{grad div } \Pi - \Gamma_1^2 \Pi, \tag{10}$$

where Γ_1 is the propagation constant in the air. We can now allow Γ_1 to vanish, thus obtaining the expression for the field corresponding to the assumptions of our problem. We then proceed as before to find the expression for the mutual impedance.

I am greatly indebted to my colleagues, Dr. Marion C. Gray and Miss Helen M. Kammerer, for much valuable assistance in the preparation of this paper, particularly in the compilation of the tables and curves.

Contemporary Advances in Physics, XXVI
The Nucleus, First Part

By KARL K. DARROW

This article, like its forerunners on radioactivity and transmutation, is devoted to the beginnings of the oncoming stage of atomic physics: the study of the nucleus. The nucleus or kernel of an atom is in ultimate control of all its properties and features, for such of these as do not depend directly on it depend upon the number and arrangement of the orbital electrons, both of which are decided by the nuclear charge; further, the atomic weight is decided almost exclusively by the nuclear mass. Though in dealing with most of these properties it is usual to imagine the nucleus as a geometrical point endowed with mass and charge, the truth is far less simple and more interesting. Nuclei are structures built of elementary particles—some and maybe all of which are independently known to us—bound tightly together. It is of great importance to ascertain these structures, not only for their own sake, but because through understanding them we may become able to control and extend the transformations of nuclei from one kind to another—the processes of transmutation, some of which are already feasible. Several fields of research are apt to contribute to such an understanding. Accurate measurement of the masses of atoms, and of the masses and charges and other properties of the elementary particles, are the first two of these, and form the subject of the present article.

SOME thirty years have now elapsed since the atom-nucleus was first imagined. Before it could be conceived men had to discover and measure negative electrons, and evolve the idea that these corpuscles normally reside in atoms, which in that case must comprise positive charges as well. Since an electron is less than one one-thousandth as massive as the lightest kind of atom, it is natural to suppose that the positive charges within an atom are linked with the main mass thereof. From this it is but a step to the notion of a heavy positive nucleus serving as central sun of the atom, with electrons revolving around it after the fashion of planets. This step was taken in 1904 (by Rutherford, and on the other side of the world by Nagaoka). A few years later, the picture was made more precise by assigning a definite number of circling electrons to every kind of atom—that is to say, to the atoms of all the elements; this at first was rather vaguely estimated at about half of the atomic weight of the element in question; then in 1915 it was chosen equal to the atomic number (customarily called Z) which marks the place of the element in the periodic table. Everything since discovered has justified this choice. It necessarily fixes the positive charge of the nucleus, which must exactly balance the total of the charges on the Z electrons, since the atom as a whole is neutral; to the atom-nuclei of the Zth element of the periodic table it therefore assigns the positive charge Ze.

In so far as the circling or "orbital" electrons are concerned, the details of this atom-model have suffered change after change in the lapse of thirty years. Classical mechanics has given way to one form after another of "quantum" mechanics; the electron-orbits at times have been defined with the utmost exactitude, at other times they have been merged into wide and hazily-bounded zones; the electrons themselves have appeared sometimes as simple corpuscles, sometimes as corpuscles with a magnet superadded, sometimes as particles implicated with a wave-motion and sometimes as a continuous haze of fluid charge. All the while, however, some of the features of the model have remained undisturbed. Among these are the total number of the electrons chosen equal to Z, and the conception of the nucleus seated at the heart of the electronic system with the positive charge Ze and most of the mass of the atom concentrated upon itself. To the problems of this nucleus we now address ourselves.

First a few words about its size, which incidentally will recall the best of the evidence for its existence. The nuclear atom-model was transformed from a pretty speculation into almost a reality, when in 1913 Rutherford, Geiger and Marsden observed the deviations of a shower of alpha-particles projected against a sheet of gold foil.[1] Alpha-particles are atom-nuclei of the second element of the periodic table, helium ($Z = 2$); gold is the seventy-ninth element ($Z = 79$). The observed law of the deviations—that is to say, the distribution-in-angle of the deflected alpha-particles—is superbly well accounted for by assuming that within every atom of gold there is a center of force, the origin of just such an inverse-square central field as would surround a charge $+ 79e$; and that the alpha-particles are themselves point-charges of amount $+ 2e$, which are deflected by the forces which they suffer in passing through these fields. The concordance between the observed distribution-in-angle, and that which was deduced from these assumptions, extends to angles of deflection as great as 150°. Now under these assumptions, a particle which has had its path bent by as much as 150° has passed within $3.1 \cdot 10^{-12}$ cm. of the center of the central field. Inward as far as this, then, comes the inverse-square field; and whatever meaning we may later attach to such a vague expression as "size of the nucleus"—for size is an indefinite concept, in regard to anything which is neither tangible nor visible—the radius of the nucleus of the gold atom must assuredly be put at a value

[1] See my "Introduction to Contemporary Physics," pp. 72–92; or the second article of this series (*Bell Sys. Tech. Jour.*, January, 1924).

smaller than this. I will later speak more fully of the corresponding data for the few other kinds of atoms for which such studies have been made. In the meantime the reader may think of 10^{-12} and 10^{-13} cm. as reasonable guesses for the radii of atom-nuclei. They agree in order-of-magnitude with the value usually assigned for the radius of the electron, and are ten or a hundred thousandfold smaller than the radii of the atoms; so that, as many a writer has remarked, the nucleus and electrons bulk about as large in the atom which they make up as flies in a very great cathedral.

Small as it is, an atom-nucleus cannot be regarded as an elementary and an ultimate particle. No sooner had the physicists of a generation ago divided the "indivisible atom" of the nineteenth century mentally into electrons and a nucleus, than they found themselves obliged to go on with the division. The electron so far has escaped this surgery, but the nucleus has been resolved—mentally, again—into as many parts as the rest of the atom itself. The arguments are two. In respect of their masses, the nuclei of the many kinds of atoms which are known are so related among one another as to suggest that all of them are aggregates of diverse numbers of particles of a very few fundamental kinds, all those of a kind having quite the same charge and almost the same mass wherever they appear. Moreover, particles sometimes spring out of atoms—from certain elements spontaneously, from others only under the bombardment of such missiles as alpha-particles—which are of such a nature that their source must be sought in or about the nuclei of the atoms whence they come. The two arguments coalesce when it is noticed that the particles which must be postulated for the one are some of those which are observed in the phenomena on which the other is based. The masses of atom-nuclei imply that they are built out of certain kinds of bricks, and bricks of these very kinds are indeed observed at times, falling or plunging or being violently hurled out of disintegrating atoms.

The study of the nucleus therefore involves, to begin with, the measurement of its mass—the measurement of the masses of all the known kinds of nuclei, amounting by now to several hundreds. This seems to be the same as the basic task of chemistry, the task of measuring atomic weights. Yet in spite of the indescribable labor which numberless chemists have lavished upon atomic weights, their data are seldom of value in modern nuclear physics. This is because the atoms of most elements are of two or more different kinds (isotopes) with different masses. Chemical methods yield an average of their weights, but the student of the nucleus wants the mass of each kind separately; and this nearly always requires a physical method of measurement, which only of late years has been brought to the requisite grade of accuracy. Even by this method the datum is not the mass of a nucleus, but of an atom; from it one must subtract the masses of the orbital electrons.

Next comes the measurement of the masses and charges of the fragments of nuclei which have fallen apart of themselves or been broken apart by missiles; these being, as I said, the bricks out of which it is tentatively assumed that nuclei are built up. Three of them have been identified as the electron, the proton, and the alpha-particle. The two last-named are the nuclei of the two lightest elements, hydrogen and helium respectively; their masses have been determined as accurately as that of the electron itself, while their (positive) charges have been found equal to $+ e$ and to $+ 2e$ respectively. Further, there is the strange new uncharged particle called "neutron," discovered less than a year and a half ago among the rays proceeding from atoms of beryllium exposed to alpha-particle bombardment; and there is the yet newer "positive electron," springing out from what seem to be explosions provoked in nuclei by cosmic rays. Such a variety of bricks is not entirely welcome; it would be more elegant to design nucleus-models out of two fundamental particles only, say the proton and the negative electron, as once seemed possible; but we must take our building-materials as we find them. Perhaps, though, it will prove permissible to argue that some of these particles are not pre-existent in the nucleus, but are created when something crashes into it.

When fragments of charge and mass come out of a disintegrating nucleus, energy comes along with them; their kinetic energy in the first place, and in addition (in many cases) parcels of energy in the form of photons or corpuscles of light. A typical instance is that of the element radium C, of which a nucleus may disintegrate of its own volition, ejecting an alpha-particle and one or more corpuscles of light, and becoming—that is to say, the residue *is*—a nucleus of another element, radium D. The latter of these nuclei differs from the former in respect of the lost charge ($+ 2e$), the lost mass, and the lost energy. The third of these differences must be measured, along with the other two; to do this one must measure the velocity and mass of the emitted particle (or particles) of electricity and matter, and the wave-lengths of the emitted light.

It is not the custom to assume that when corpuscles of light are emitted from an atom, they must previously have existed as such within the atom. Protons and electrons are supposed to be durable, whether or not they are bound with one another into a nucleus; alpha-particles are supposed either to endure, or else to be resolved into durable protons and electrons; but photons are regarded as mere transitory vehicles of energy, which gathers itself up into them when they are emitted, and disperses itself into other forms when they are absorbed. The energy, however, is supposed to share in the mass of whatever atom or nucleus it inhabits. In relativistic mechanics, energy E is always endowed with mass E/c^2, and mass m with energy mc^2; so that when a quantity of energy ΔE departs from a nucleus in the form of a photon (or, for that matter, in any other form) the mass of what is left behind is automatically reduced by the amount $\Delta E/c^2$. Thus to compute the mass of a RaC nucleus from that of a RaD nucleus, we should have to subtract from the latter not only the mass of the alpha-particle, but also that which departed with the emitted light.

Of course these statements about energy and mass are not to be taken as *necessarily* true, albeit they are based directly on the restricted theory of relativity, for the validity of which there is excellent evidence. On the contrary, one of the most alluring promises of the study of nuclei—for the speculative physicist—is that of testing the interconnection of energy and mass which relativity suggests. In the meantime, it is quite generally taken for granted. Notice an interesting corollary: the mass of an aggregation of electrified particles (such as a nucleus is) will not in general be the sum of the masses which its individuals have when far away from one another, for as these particles come together they may radiate energy, whereof the mass must be deducted from the sum of their masses. We shall see that this is commonly accepted to explain the fact that the mass of a nucleus is not quite equal to the sum of the masses of the protons, electrons, and other "bricks" out of which there is reason for assuming it to be built.

Thus from stable nuclei, we may learn their masses; from unstable or self-disintegrating nuclei, something about their constituents, and the energy-difference and mass-difference between the nucleus before and its fragments after its collapse; from nuclei disrupted by impact of projectiles, something about their constituents and something about their energy-content. There is much more to be measured. Some kinds of nuclei endure for æons, others break up in a time measured in millionths of a second; some have alternative ways of breaking up, a certain fraction following one and the remainder the other; some may be disrupted by impact of alpha-particles, some by protons, some by both and some apparently by neither. It is certain that all of these things are indications of the structure of the nucleus, but most are still too difficult to read.

A great part of contemporary physics consists of the analysis and interpretation of spectra; one wonders whether in this vast and tangled array of data there is information about nuclei? The answer must be phrased with care. The spectrum of an atom is due to its orbital electrons, and of these the number and the arrangement are controlled by the nuclear charge, which therefore dominates the spectrum; spectroscopy is full of evidence for the theorem which I set down at the start, that $+ Ze$ is the nuclear charge of the element of atomic number Z. The mass of the nucleus is much less influential, owing to the enormous disparity between it and the masses of the electrons. Were it and they of the same order of magnitude, the nucleus would move like the electrons, revolving around the center of mass of the atom with a kinetic energy comparable with theirs. The emission of light would then entail a contribution from the kinetic energy of the nucleus as well as from those of the electrons, and the frequencies of the spectrum-lines would be affected by the nuclear mass. But the nucleus is so massive, its motion so slight and its kinetic energy so insignificant, that in nearly all atoms that contribution is too small to be appreciable, and the spectrum-lines are sensibly the same as if the electrons revolved around a perfectly motionless centre. The only exceptions are the three lightest kinds of atoms; I will later explain how the discovery of one of these was brought about, two years ago, by the influence of the mass of its nucleus upon the frequencies of its spectrum-lines.

The spectra of molecules are more dependent on nuclear masses than are those of atoms; for, when two (or more) nuclei and their attendant orbital electrons are combined into a single system, the balance of forces is such as to provide for each nucleus a position of equilibrium, from which it may be displaced and about which it will oscillate more or less like a pendulum. There are (for instance) two kinds of chlorine atoms, of nuclear masses standing to one another approximately as 35 to 37; consequently there are three kinds of diatomic molecules in ordinary chlorine gas, built as indicated by the symbols $Cl^{35}Cl^{35}$, $Cl^{35}Cl^{37}$, $Cl^{37}Cl^{37}$. In all of these three kinds of molecules the internal forces are very nearly the same, being determined by the charges of the nuclei and electrons which are identical for all three, and by their arrangement which is nearly identical; but the masses of the nuclei are different, and therefore so are their frequencies of oscillation, which appear in the spectra. The differences of nuclear masses also entail differences in the moments of inertia of these three kinds of molecules, which likewise are reflected in their spectra. The lines of molecular spectra are often doubled or tripled by virtue of the presence of two or three kinds of molecules differing only in nuclear masses.

More recondite is another influence of nuclei on spectra, which is due neither to their charge nor to their mass. It often happens that what appears with an ordinary spectroscope to be a single line is resolved by an excellent instrument into several, although the earlier theory affirmed quite decisively that it should be single and simple. By "the earlier theory" I mean one which was substantially like the atomic theory of today, except that it involved the assumption that the field whereby the nucleus acts upon its attendant electrons is purely an inverse-square electrostatic field. If we suppose that in addition to this there is a magnetic field—that the nucleus is not only a charged body, but also a minute magnet acting upon or (to use a commoner term) "coupled with" the orbital electrons by the magnetic as well as by the electric field—then the subdivision of these apparently simple lines into clusters begins to become intelligible. It is well known that spectrum lines are split into clusters by the action of an external magnetic field—the Zeeman effect; it is natural to expect a magnetic field applied to the orbital electrons from the center of the atom to have somewhat the same effect as one applied from without, and to produce these permanent splittings, which are known as "hyperfine structure." Magnetic moment is attended with angular momentum, inasmuch as magnetism is due to whirling of electric charge; and some physicists prefer to regard the latter as primary, and to say that the subdivision of the lines is due to some unspecified kind of an interaction between the angular momenta or the "spins" of the nucleus and the orbital electrons. To the ones, the hyperfine structure yields the spin of the nucleus; to the others, its magnetic moment. These are intricate questions, to which it will be necessary to devote much space.

The nucleus is a magnet; the incessant circlings of each electron in its orbit constitute another magnet, a charge revolving in a closed path being equivalent to a current flowing in a closed circuit; and finally, it has proved essential for spectrum analysis to assume that each electron is in itself, quite apart from its motion, a magnet. The magnetic moment of the atom as a whole is the resultant of these three component moments, or rather groups of moments, since there may be many electrons and many orbits to a single atom. Now, this resultant may be measured, for instance by the method of Gerlach and Stern, in which a stream of atoms is deflected by a non-uniform magnetic field; and if there is ground for believing that one knows what part of the resultant is due to the electronic moments, then one can deduce the magnetic moment of the nucleus itself. This has already been done in several cases. Perhaps it will be possible in time to attribute the magnetic properties of solid bodies, even of ferromagnetics, in part to their nuclei; but probably that is looking a long way ahead.

One more participation of the nucleus in phenomena remains to be recorded. The passage of X-rays and gamma-rays—that is to say, high-frequency light—through strata of matter has been abundantly studied. For the most part it is admirably well accounted for by supposing that the corpuscles of these rays possess the power, and only the power, of expelling orbital electrons from atoms through which they pass; any particular corpuscle either makes such an expulsion and vanishes or loses energy in doing so, or else it goes through the substance unaffected. There are two alternative modes of expulsion, but that is a detail into which we need not enter now. The relevant point now is, that with certain kinds of atoms and with particularly high frequencies of light it appears that these processes are not the whole of what is happening. The absorption and the scattering of X-rays are greater than they should be, if the photons interacted only with orbital electrons; and it is supposed that the excess is due to interactions with nuclei. Presumably it would be greater with the rays of immeasurably high frequency which probably form a part of the cosmic radiation.

Nuclei, then, contain almost the whole of the mass of ponderable matter. They are the seat of radioactivity. They may be disrupted by impacts of other and lighter nuclei, possibly by electrons and photons. They influence spectra through their charges and their masses, and through the closely-connected qualities of magnetic moment and angular momentum. Through their magnetic moments they are responsible in part for the magnetic properties of atoms and of larger pieces of matter. They interact with high-frequency X-rays. Such is the range of phenomena in which the nucleus takes a significant part, and out of which, therefore, the properties of the nucleus are to be derived.

In the present article I will describe and discuss these phenomena in succession. Some have been treated already in earlier articles in this journal, a fact of which I will avail myself to shorten this one, which nevertheless must extend into following issues.

The Elementary Particles

There are now six different kinds of material corpuscles known by direct experiment, of which there is more or less reason to believe that they enter into the structure of some at least among the nuclei. These are:

The *proton*, or nucleus of the most usual kind of hydrogen atom;

The *alpha-particle*, or nucleus of the helium atom;

The *electron* (that is to say, the negatively-charged corpuscle customarily known by that name);

The *neutron*;

The *positive electron*;

The H^2 *nucleus* or *deuton*, the nucleus of an unusual kind of hydrogen atom of double the mass of the usual kind.

Of these six the first three have been known for years. They have actually been observed to spring out of nuclei, spontaneously in some cases, in others elicited by bombardment; and this is one of the two major reasons for imagining them as parts of nuclear structures. It is true that this reason does not apply directly to all kernels. Those which are known to emit alpha-particles spontaneously are a small fraction, a tenth or thereabouts, of the total number; and all but possibly two belong to the uppermost end of the periodic table, to massive atoms of atomic weight superior to 200. Those which are known to emit electrons are yet fewer, and again all but two belong to the most massive group. (The two exceptions are potassium and rubidium.) No kernel is known to emit protons spontaneously; but a great many elements both light and heavy will yield charged particles out of their nuclei, when suitably bombarded; and these have been proved in some cases to be alpha-particles, in others to be protons. Moreover the bombarding particles which achieve these results are themselves alpha-particles and protons, and there is reason to believe that sometimes these are actually absorbed into nuclei which they strike.

The other major reason for inserting protons, alpha-particles and electrons into our tentative models of nuclei is deduced from the masses and the charges of these bodies. There is a certain well-known standard of mass, one sixteenth of the mass of an oxygen atom; and the masses of all nuclei come fairly close to being integer multiples of this standard. Of course this can also be said about any other mass lying within a certain (narrow) range of the standard just defined, and perhaps it would seem better to say that the nuclear masses come fairly close to having a greatest common divisor of that order of magnitude, and then to determine by the method of least squares what number had best be chosen for this greatest common divisor. This procedure, however, would not be wise, unless the departures of the various masses from the integer-multiple rule were casual, whereas it is extremely probable (to say the least) that they are systematic, and are indices of the structures of the nuclei. The choice of a definite standard must therefore be based on expediency or on theory, and none better than the present one has been proposed.

It would be pleasant to say that this standard is exactly the same as the mass of the proton, and thence to deduce that every nucleus consists of protons entirely. As a matter of fact, there is a difference of about three quarters of one per cent, the standard being lighter than the free proton; but this by itself is no bar to the hypothesis that all nuclei are made up of protons, since it is compatible with the general theory of electricity that charged particles when crowded close together should individually have smaller masses than when they are far apart. It is not, however, admissible to assume that these protons of reduced mass are all that the nucleus comprises. Were this so, the positive charge of a kernel of mass NM_s (M_s standing for the standard mass, N for any integer) would be $+ Ne$; but it is always (except in the case of hydrogen) observed to be less than this amount—it is equal to Ze, where Z stands for some integer less than N; and one must assume that there are $(N - Z)$ electrons present to cancel the difference between Ne and the actual charge. As for the alpha-particle, its mass and charge suggest that it consists of four protons and two electrons, and the masses and charges of certain heavier nuclei—carbon and oxygen supply the most vivid examples—suggest that within them the protons and electrons are united in groups of four and two to constitute alpha-particles, a substructure within the main structure.

Until a year or two ago, models of nuclei were constructed exclusively out of protons and electrons, sometimes grouped into alpha-particles and sometimes not. The discovery of the three new particles put an end to this era. The interlopers were not entirely welcome; deficient as the prevailing models had proved to be in many ways, people had become accustomed to them, and various eminent physicists were quoted as deploring—in informal and jocular words—the necessity of tearing them down and rebuilding with the new bricks among the old. Nevertheless, neutrons have been observed to spring out of nuclei, and positive electrons have been observed wandering about in space, sometimes among what seem to be the fragments of a kernel ruined by an impact so violent as to provoke an internal explosion. The new kind of hydrogen nucleus is sufficiently low in mass to suggest that it may be a building-stone in the construction of kernels heavier than itself.

The histories of the discoveries of these three particles have not yet been related in the pages of this journal, and as they are extremely interesting portions of the most strictly contemporary physics, they well deserve some pages of description.

The Neutron [2]

It had been known since 1919 that certain light elements emit protons when they are bombarded by alpha-particles; these, however, are not "penetrating" rays, in the sense in which that term is commonly used, inasmuch as they are completely stopped by a layer of metal a fraction of a millimetre thick. The discovery of the neutron was the outcome of an attempt to detect penetrating rays emitted by the bombarded atoms. Bothe and H. Becker made this attempt, surrounding the source of alpha-particles and the substance on which they impinged by two millimetres of zinc and brass, and detecting what got through this barrier by means of a Geiger point-counter. Four elements—lithium, boron, fluorine and especially beryllium—produced an unmistakable effect. Bothe and Becker ascribed this to high-frequency gamma-rays or photons. It was indeed largely due to such photons; but mingled with these there were particles of another nature, as the further experiments of Irène Curie, Joliot and Chadwick were to prove.

To appreciate the proof it is necessary to realize that what is observed is an indirect rather than the direct effect of the corpuscles coming from the atoms bombarded by the alpha-rays. It is ionization of gas which is observed—ionization coming in spurts, which may be separately observed and counted by use of a Geiger counter or a quick-acting electroscope with proper amplifiers or an expansion-chamber, or may be summed up by the accumulation of charge in a slow-acting electrometer. The spurts of ionization are due to the transits of corpuscles across the gas, corpuscles which sometimes at least are recognizably electrons or atom-nuclei. But it is not to be taken for granted that these directly-ionizing corpuscles spring from the source of the phenomena, the element bombarded by the alpha-particles. They start their flights in the matter environing the source, being launched on their courses by invisible agents which are presumably the true primary rays coming from the source. What is observed, therefore, depends on the matter surrounding the source; and the last step leading up to the identification of the neutron was taken when Curie and Joliot interposed thin screens of various substances in the path of the primary rays from the source to the ionization-chamber.

[2] For a fuller account cf. an article of mine in *Review of Scientific Instruments*, 4, 58–63 (February, 1933).

When the screens were of metal, nothing sensational happened; but *if they were of paraffin, water or cellophane*—materials containing *hydrogen—the ionization-current went up instead of down.* This was not the first time that a screen had been observed to enhance the effect of what supposedly were gamma-rays, but in the previous cases it was permissible to infer that the rays were expelling electrons from the substance of the screen. Here the substances were distinguished not by abundance of electrons, but by abundance of hydrogen atoms in their structure; and Curie and Joliot conceived the idea that the primary rays were ejecting protons from the screen, which entered the chamber and in it ionized abundantly. This theory they fortified at once by applying magnetic fields, and finding that the ionization persisted (electrons issuing from the paraffin would have been twisted back, unless extremely fast); by interposing 0.2 mm. of aluminium, and finding that the extra ionization ceased (electrons, if extremely fast, might have got through); and by taking cloud-chamber photographs, and observing tracks of the aspect of proton-tracks springing out of the paraffin and traversing the ionization-chamber partly or altogether.

At once it was guessed by Curie and Joliot that these protons were recoiling from elastic impacts of the high-energy photons which the primary rays were still supposed to be—that they had suffered, in fact, the very same sort of blow as electrons suffer in the well-known "Compton effect." So great, however, was the energy of the protons (as evinced by their range) that photons of energy almost incredibly great had to be postulated; such would probably have an even greater penetrating power than that of the primary rays, and there were other objections more or less solidly founded on theory, which now it would be scarcely worth while to discuss. The French physicists were aware of these difficulties, and published them; but it was reserved for one of the Cavendish group to reject the idea altogether, and supplant it with the one which at present is accepted. Chadwick seized upon the revelations from the Institut du Radium with such alacrity that within six weeks he was reporting data obtained by counters and by cloud-chambers—data which confirmed that the rays emitted from beryllium when bombarded by alpha-particles are able to confer great speeds not only upon protons, but on nuclei of other elements of low mass (a later list comprises Li, He, Be, B, C, N, O, A; and Kirsch has very recently detected emission of neutrons from many more). Out of these data emerges the fact which speaks most clearly for his theory that the corpuscles which impel the protons and other nuclei are material particles of nearly the mass of a proton, instead of being corpuscles of light.

The argument is as follows: For simplicity let us consider solely the nuclei which are projected in directions pointing straight away from the source of the primary rays, and therefore must have suffered central impacts. Specially, let us take the cases of hydrogen and nitrogen nuclei thus projected. The ranges of these have been measured (of N by Feather, of H by various physicists) and their maximum speeds deduced by means of knowledge earlier acquired of the range-vs.-speed relations of charged particles. The values of speed accepted by Chadwick are $3.3 \cdot 10^9$ and $4.72 \cdot 10^8$, respectively. Now if the corpuscles which in central impacts gave to these nuclei these speeds were photons, it is easy to compute by the Compton-effect equations the energy U of the photons; if the impinging corpuscles were material particles of mass M and speed v, it is easy to compute both v and M. It turns out that by the first procedure, one gets different values of U from the two cases (55 and 90 million electron-volts, respectively); by the second, one gets compatible values of M and v. With the first theory, then, one would have to say that nuclei of different kinds were struck by different photons. This is not quite inconceivable, as there *might* be a mixture of gamma-rays of different energies, and a greater likelihood of the higher-energy photons interacting with the more massive nuclei. But it seems less acceptable than the other theory, which permits one to postulate a single kind of corpuscle to explain the impacts against both kinds of nucleus. This corpuscle must be neutral, as a particle of charge e and the computed mass and speed could never penetrate nearly as thick a layer of matter as it can traverse; it is therefore called the "neutron."

The value of M deduced from the foregoing data is given as 1.15 times that of the hydrogen nucleus; the possible error in the estimate of the speed of the recoiling nitrogen nuclei is such that Chadwick says "it is legitimate to conclude that the mass of the neutron is very nearly the same as the mass of the proton." An estimate ostensibly much closer ($1.007 \pm .005$) has been made by a train of reasoning which I will later quote.

The Positive Electron

Whereas the discovery of the neutron came about through the study of transmutation, the positive electron came to light in the course of cosmic-ray research. The ionization of the atmosphere, whereby the cosmic rays are manifested, is due directly to fast-flying corpuscles which leave behind them trails of ionized molecules fairly close together (on the average, about a hundred ion-pairs per cm. in air at sea-level atmospheric pressure). The trails may be made visible by the classical method of the expansion-chamber (Figs. 1 and 2). The particles may be tested for their charge by having a magnetic field pervading the chamber. Some of the paths are then found to be smoothly curved, proving beyond a doubt that the corpuscles are charged.[3]

The sign of the curvature of a path in a magnetic field should disclose the sign of the charge of the responsible corpuscle; but here

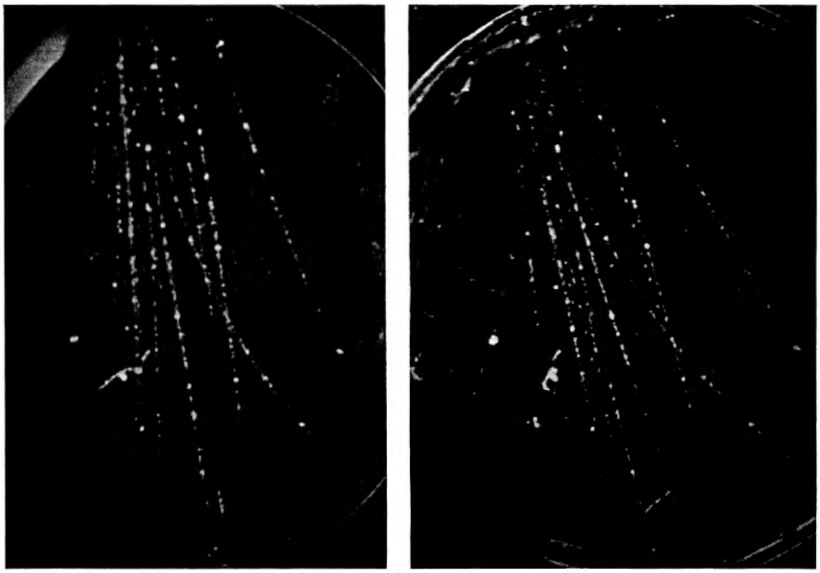

Fig. 1—Two photographs (taken from different viewpoints) of a nuclear explosion, probably that of a copper nucleus struck by a cosmic ray. The tracks on the right, and concave to the right, are those of positive electrons; others are due to negative electrons. (P. M. S. Blackett; *Proceedings of the Royal Society.*)

appears a difficulty: the sign cannot be inferred unless the sense in which the corpuscle described the path be known, and there is nothing whatever about the aspect of an ordinary trail to indicate that sense. It might be guessed that the particle is necessarily moving downward rather than upward, since the cosmic rays come from above. This, however, would be a bad guess, for some at least among the trail-making corpuscles are secondaries set into motion by the primary rays, as protons are known to be impelled by neutrons, and electrons by photons; and some of these secondaries may be, and indeed certainly

[3] Other paths seem quite straight, but there is strong reason to believe that a neutral particle would not produce anywhere nearly so great a density of ion-pairs as is observed along them, and it is inferred that they are due to charged particles which are moving with too much momentum to be sensibly deflected.

are, moving upward. One therefore has to await, or to produce, some unusual event to reveal the sense of the traversal of a path.

One such event is portrayed in Fig. 1. It is certainly one of the most deep-seated of human convictions that when tracks are seen to radiate from a common point, the objects which made them must have

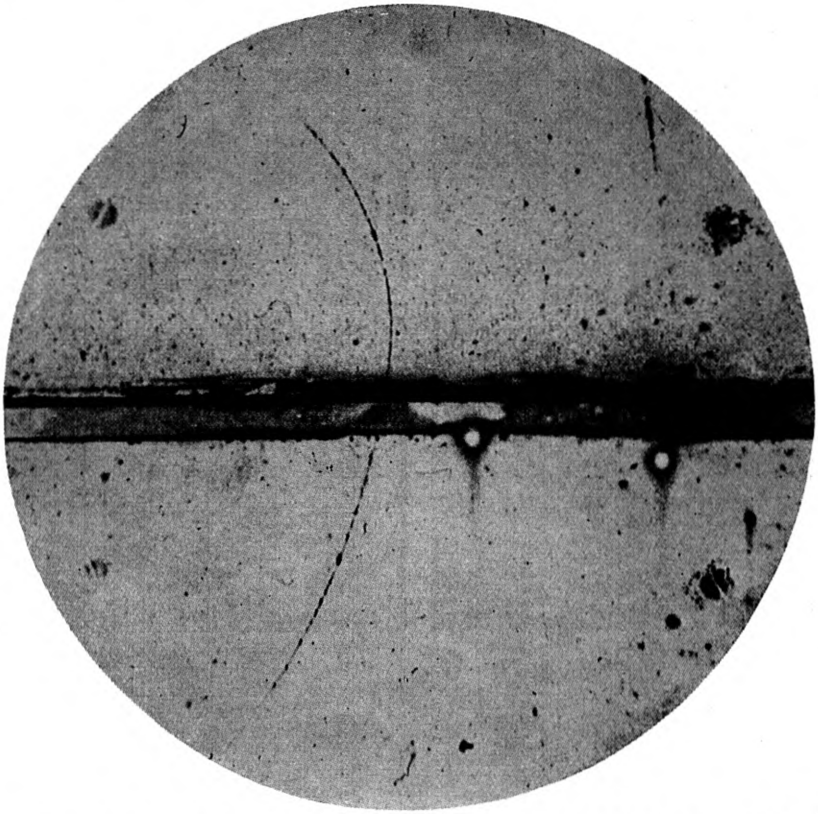

Fig. 2—Track of a positive electron which traverses a lead plate 6 mm thick, and has energy amounting to 63 million electron-volts before it enters the lead. (C. D. Anderson; *Physical Review.*)

travelled outward and not inward, except possibly for one which may have provoked the flying-asunder of the rest. Here is such a situation. The radiant point was in the midst of a mass of copper wire surrounding the expansion-chamber, and it is probable though not certain that the event was the explosion of a copper nucleus provoked by a cosmic ray. Among the radiating paths, curvatures of opposite senses occur; and this practically proves that charged particles of both signs are present.

Several other such photographs were taken by Blackett and Occhialini in Cambridge (England) and by Anderson in Pasadena.

Events of another type are observed, when the mixture of neutrons and photons emitted from beryllium bombarded by alpha-particles is allowed to fall upon a metal plate: the tracks of many ionizing corpuscles are noticed springing from the plate, and when there is a magnetic field applied, some are seen to be curved one way and some the other. Yet another is exemplified in Fig. 2. This is a historic photograph, the one from which the positive electron was first inferred (by C. D. Anderson); it is rarely that one can fix with such precision the moment of a major discovery, and perpetuate the very observation out of which it was made. Here obviously is the path of a single particle coming from below, which has cloven entirely through the lead plate of 6 mm. thickness, and has emerged from the upper side with diminished speed revealed by the augmented curvature of the trail. It is this change of curvature which fixes the sense of the traversal of the path, and the sign of the curvature thereupon fixes the sign of the particle's charge as positive.

But granted that many of the ionizing corpuscles which interlace the air are positively charged: are they not simply alpha-particles or protons, or of some other well-known type of positive ion? Here enters the second item of the evidence. Assuming (e.g.) the agent of the trail of Fig. 2 to be a proton, one may calculate the speed which it would necessarily have, in order to suffer a curvature-of-path equal in magnitude to that which is observed. One may then evaluate, from prior knowledge, the number of ion-pairs per unit length of trail which it would produce; and this turns out to be many times as great as that which is observed. A proton would produce a trail much denser, and also much shorter, than the actual one; its energy would be used up in a progress of 5 mm. away from the plate, whereas the visible course of this corpuscle extends for more than 5 cm. and shows no sign (in thickening or in increase of curvature) of being near its end.

The particle of Fig. 2 was therefore not a proton, nor, *a fortiori*, an alpha-particle or more massive ion; and the only way to reconcile the observed curvature with the observed density of path seems to be, to assume a particle about the same as the electron both in mass and in magnitude of charge, though not in sign of charge. This is not the same as saying that either the charge or the mass is accurately determined. Apparently it is certain that the charge must be less than $+2e$, which makes it equal either to $1e$ or to some non-integer multiple of e, and the latter alternative is too painful to be borne. As for the mass, it must be many times smaller than that of the proton (if the charge is e), but to say more would be premature. The basis for supposing it equal to the mass of the electron is the feeling that there ought not to be any other fundamental masses in Nature than we knew already, together with certain suggestions from the quantum-mechanical theories of Dirac. The estimation of the mass may be bettered, if it is possible to observe collisions between positive and negative electrons with the expansion-chamber and to trace the paths of the colliding particles; there are reports that this has already been done with some success.

The action of cosmic rays being something which we cannot intensify nor control, it is doubly fortunate that another agent has already been discovered which is capable of generating positive electrons; for these particles have been observed, by several people in several different schools, leaping out of sheets of metal bombarded by "hard" or high-frequency gamma-rays. At the first observations, the bombarding radiation was a mixture of gamma-rays with neutrons, and it was not unnatural to suppose that so novel a result must be due to the action of the novel kind of corpuscle. Perhaps in those experiments the neutron did participate in the effect; but it has now been found—by Anderson and Neddermeyer in Pasadena, by Meitner and Philipp in Berlin—that gamma-rays suffice. Those employed so far are chiefly, if not altogether, the radiation from thorium C″ consisting of photons of energy 2.6 millions of electron-volts.

A theory quite extraordinary, indeed by all prior concepts revolutionary, has been propounded by Blackett and Occhialini: it is the idea that the photon converts itself into a pair of electrons, positive and negative respectively. The net charge of the universe is not altered by such a process, since the two created charges balance one another; neither is the total mass of the universe, for the masses of the two electrons (including the kinetic energy wherewith they are endowed) are equal altogether to that of the vanished photon. For this theory it may be said, in the first place, that positive electrons frequently appear jointly with negatives, one particle of each kind springing forth from a single point: Anderson and Neddermeyer have observed no fewer than 22 of such cases. Moreover if the theory is true, the total kinetic energy of the two particles of such a pair—and *a fortiori* the kinetic energy of the positive electron by itself—must lie below a certain upper limit, which is computed by deducting a million electron-volts from the energy of the responsible photon; for this is the amount of energy which by Einstein's relation (which will figure prominently in the latter part of this article) must be used in building the electrons by themselves. Thus if in these experiments with gamma-rays, either positive electrons or electron-pairs were to be observed with energy greater than 1.6 million electron-volts, the theory would be contradicted; but it turns out that the energies seem to lie just below this figure, never certainly above it. Positive electrons should not be produced at all by gamma-rays of which the photons have less than a million electron-volts of energy; and in fact none was found when Meitner and Philipp applied such rays to a metal. A much greater number of cases should be observed before the idea is affirmed; but if it should be confirmed the consequences would be highly important, not only for its own sake but because it is an offshoot of basic quantum-mechanical theory, which would thus be greatly strengthened. Incidentally it would then not be necessary to provide for positive electrons in our models of nuclei.

The H^2 Nucleus

This particle, for which physicists are having difficulty in finding the perfect name (*deuton*, *diproton*, *hemialpha particle*, and *demihelion* are among those which have been suggested), is the nucleus of the newly-discovered isotope of hydrogen, "deuterium." I will defer the history of the discovery of this isotope to the end of the article, as there are several things which should be told before it. There is no definite reason as yet for assuming that the deuton enters as such into the composition of yet more massive nuclei, but it may well prove a convenient stone for the building of nuclear models.

The Masses of the Elementary Particles

The remaining "elementary" particles—proton, alpha-particle, negative electron—have been known too long to require a special description. I will therefore give only a table of their masses and their charges, along with those of the other three; prefacing it with the statement that I have not been using "elementary" in the sense

Corpuscle	Mass in Terms of Grammes [4]	Mass in Terms of One Sixteenth the Mass of the Oxygen Atom [5]	Charge
Proton	$1.66 \cdot 10^{-24}$	1.0078	$+e$
Alpha-particle	$6.60 \cdot 10^{-24}$	4.002	$+2e$
Electron	$9.03 \cdot 10^{-28}$	.00054	$-e$
Neutron	$1.66 \cdot 10^{-24}$ ca.	1.007 ca.	0
Positive electron		(see page 341)	
H^2 nucleus	$3.31 \cdot 10^{-24}$	2.0129	$+e$

[4] From Birge's critical tabulation; the probable errors amount mostly to less than one digit in the last place quoted.

[5] See the following pages for probable errors.

of "ultimate"! It is possible, nay probable, that some of these corpuscles are built up from others. Neutron may be proton plus electron; proton may be neutron plus positive electron; alpha-particle may be two protons plus two neutrons, or four protons plus two electrons.

Masses of Atoms and Their Nuclei

If all the atoms of an element were perfectly alike, we could take the relative values of their masses—relative to those of other elements, and in particular to that old familiar standard, one sixteenth the mass of an oxygen atom—straight from the chemists' tables of atomic weights. It happens, however, that there are two, three, or several different kinds of atom to almost every element, and they are nearly always so thoroughly intermingled in even the smallest analyzable samples as to suggest that the mixing was done while the earth was still a gas. Whatever chemical method of measuring "atomic weight" be applied to an element (and this includes the strictly physical scheme of measuring its density when it is gaseous) leads forthright and inevitably to a mean value of the masses of its "isotopes" or divers kinds of atoms. Not a simple average, of course! but rather a weighted mean, to which every isotope makes contribution in proportion to its relative abundance in the mixture.

The tables of the "chemical atomic weights" are just collections of these weighted means. They nearly all involve two or more varieties of atoms, and in most of the cases the weighted average is markedly different from the mass of any isotope. Sometimes one of the isotopes predominates so greatly that the others contribute very little to the mean, and the chemical atomic weight is not a bad approximation to the mass of this single kind of atom. This is not typical of the system of the elements as a whole, but it happens to be the case of no fewer than eight among the first eleven: a coincidence which has had some influence on the trend of scientific thought, for if it had not happened the chemical atomic weights of seven among these eight elements would not have been so nearly integer multiples of the standard as they actually are (*viz.* H 1.01, He 4.00, Be 9.02, C 12.00, N 14.01, F 19.00, Na 23.00) and then it would have been difficult to advance the idea that all atoms are built up from common particles. If oxygen itself were not of the group of these eight—if the rarer isotopes of oxygen were, say, a tenth or a third as abundant as the predominant one, instead of being less than 1/500 as abundant—we should either be suffering from a table of atomic weights in which there would be no integers unless by accident, or else we should be using some other

PERIODIC TABLE OF THE ELEMENTS

(Values of atomic weights taken from the Third Report of the Committee on Atomic Weights; G. P. Baxter, *J. Am. Chem. Soc.*, 55, p. 451)

I	II	III	IV	V	VI	VII	VIII	O
1 H 1.0078								2 He 4.002
3 Li 6.940	4 Be 9.02	5 B 10.82	6 C 12.00	7 N 14.008	8 O 16.000	9 F 19.00		10 Ne 20.183
11 Na 22.997	12 Mg 24.32	13 Al 26.97	14 Si 28.06	15 P 31.02	16 S 32.06	17 Cl 35.457		18 A 39.944
19 K 39.10 29 Cu 63.57	20 Ca 40.08 30 Zn 65.38	21 Sc 45.10 31 Ga 69.72	22 Ti 47.90 32 Ge 72.60	23 V 50.95 33 As 74.93	24 Cr 52.01 34 Se 79.2	25 Mn 54.93 35 Br 79.916	26 Fe 55.84 27 Co 58.94 28 Ni 58.69	36 Kr 83.7
37 Rb 85.44 47 Ag 107.880	38 Sr 87.63 48 Cd 112.41	39 Yt 88.92 49 In 114.8	40 Zr 91.22 50 Sn 118.70	41 Nb 93.3 51 Sb 121.76	42 Mo 96.0 52 Te 127.5	43 Ma 53 I 126.92	44 Ru 101.7 45 Rh 102.91 46 Pd 106.7	54 Xe 131.3
55 Cs 132.81 79 Au 197.2	56 Ba 137.36 80 Hg 200.61	RARE EARTHS 81 Tl 204.39	72 Hf 178.6 82 Pb 207.22	73 Ta 181.4 83 Bi 209.00	74 W 184.0 84 Po	75 Re 186.31 85—	76 Os 190.8 77 Ir 193.1 78 Pt 195.23	86 Rn 222
87—	88 Ra 225.97	89 Ac	90 Th 232.12	91 Pa	92 U 238.14			

RARE EARTHS

57 La 138.92	58 Ce 140.13	59 Pr 140.92	60 Nd 144.27	61 Il	62 Sm 150.43	63 Eu 152.0	64 Gd 157.3
65 Tb 159.2	66 Dy 162.46	67 Ho 163.5	68 Er 167.64	69 Tm 169.4	70 Yb 173.5	71 Lu 175.0	

standard; I must leave it to some chemist to say which is the likelier alternative.

Despite these particular cases, it is a general rule that the masses of the atoms of an element cannot be ascertained, unless its isotopes are separated from each other and separately measured. Indeed, the exceptions to the rule are more apparent than real. One cannot be quite sure that any element is an exception, without performing upon it such an experiment as would separate its isotopes if there were more than one existing in a sensible amount. It is true that there are different radioactive isotopes of one and the same element, which come into being from different sources and therefore are not mixed with one another; but these are generally so scanty in amount that their atomic weights have not been measured at all. Thus every valid measurement of what can properly be called the mass or the weight of an atom requires an "isotope analysis" of the element in question.

The way of separating isotopes and the way of measuring the masses of their atoms are happily the same, although of course the latter aim demands a great refinement of the method over what is needed for the former. One sends a stream of ions of the element through a sequence of electric and magnetic fields, the first of which accelerates them to a considerable speed, while in the remaining field or fields they are deflected. The deflection depends upon the mass, so that ions of equal charges and different masses—and thus, ionized atoms of the different isotopes of a single element—arrive at different points of the photographic plate which receives them and registers their presence. When the scheme was introduced by J. J. Thomson, he considered it a method of chemical analysis: it was applied to the ions found in electric discharges in ordinary gases and mixtures of gases, and he expected to observe—and did observe—ionized molecules of compounds too unstable to be durable. Unexpectedly it turned out to be a method of ultra-chemical analysis, for when applied to the ions of a discharge in neon, it disclosed two kinds instead of one. Efforts were made to identify one of the two as something else than neon, but when they all failed, neon was registered as the first of the elements to be separated into isotopes.

This discovery was made in 1912, and then occurred the great hiatus of the war. The later story will be an easy matter for historians to trace, at least as far as 1933; for despite its obvious importance, this subject of research invited incredibly few workers. I cannot guess why, in times when many physicists were looking for experimental problems, it was so seldom chosen. There are just three names to be mentioned (omitting the work of a few students on a special question, the relative abundance of the isotopes of lithium, and that of Bleakney on the isotopes of hydrogen and neon). Outstanding, and for years unique, is that of Aston of the Cavendish Laboratory, who took over the problem from Thomson and has bound up his name with isotopes by fourteen undeviating years of concentration. There are two stages of the post-war history: the period when isotopes were merely counted and their masses roughly estimated, and the period (in the midst of which we now stand) when their masses are measured with precision rivalling the vaunted accuracy of the chemical atomic weights, and also their relative proportions or "abundances" in the mixtures which we usually call elements. Aston initiated both these periods. In the earlier of them Dempster, who also had been trained before the war in the analysis of ions, separated several of the elements into their isotopes. Costa made a couple of very accurate measurements of mass, but then abandoned the field. Bainbridge entered it after the second period commenced, and is now measuring atomic masses with an exactitude equalling Aston's.

In Aston's apparatus the deflecting fields are disposed in an intricate and ingenious way, so that ions of equal mass shall be brought to the same point on the photographic plate even though their speeds be far from equal. This is because he usually derives his ions from a self-sustaining glow-discharge, of which the electric field serves as his accelerating field and imprints different speeds upon different ions of equal mass because they start from different places in the discharge. Much simpler are the schemes of Dempster and of Bainbridge, in which the sole deflecting agency is a uniform magnetic field, which swings the ions around in semicircles from the slit where they enter the deflection-chamber to the plate on which they impinge (Fig. 3). This, however, does not work properly unless all the ions of a particular mass have very nearly the same velocity, so that either they must leave the source with very low speeds and be subjected to the same and relatively large accelerating voltage (such was the case in Dempster's work) or else there must be some device for preventing all ions but those of a very narrow velocity-range from reaching the deflection-chamber. Bainbridge's device for this latter purpose is shown in Fig. 3; between the plates of the "velocity-filter" a transverse electric field is superposed on the magnetic field which is at right angles to the plane of the paper, and no charged particle gets through to the slit unless its speed is very nearly equal to the ratio of the field-strengths.

If a beam of ions all of identical mass M and charge e and speed v

were to enter the chamber through the slit they all would follow the same semicircle and assemble on the very same spot on the plate, the distance of which spot from the slit would tell the observer their mass. But when a beam of ions of a single element is projected through the slit, it is not usually a single spot which appears upon the plate. All students of physics have seen reproductions of such plates, chiefly from Aston's magnificently ample store. I reproduce

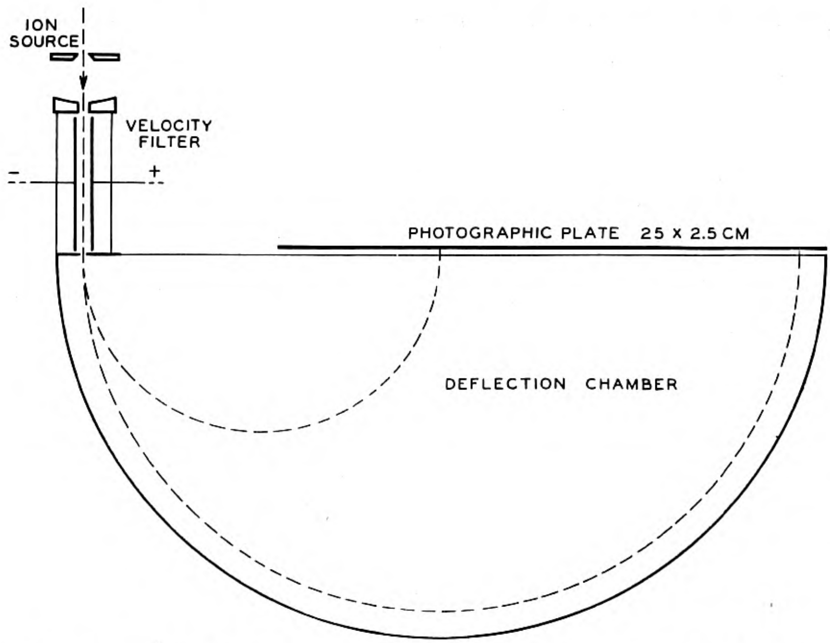

Fig. 3—Scheme of Bainbridge's apparatus for accurate measurement of the masses of isotopes.

here two from Bainbridge's, Fig. 4 for zinc and Fig. 5 for germanium. These are "mass-spectra" every spot or "line" of which is the evidence of a separate isotope of the element in question. Germanium and zinc are neither the least nor the most profuse in isotopes among the elements; there are still a few (fluorine and sodium, for instance) for which only one has been discovered, and at the other extreme there is tin with no fewer than eleven.

It is, of course, the charge-to-mass ratio of the ion rather than its mass which is deduced from the position of the spot and the strengths of the accelerating and deflecting fields. (There is no need of giving the formula here, as it is to be found in every textbook and is readily derived.) The charge is usually $+ e$ (singly-ionized atom), sometimes $+ 2e$ (doubly-ionized atom), rarely $+ 3e$ or greater; there is no difficulty in telling which. No one goes to the trouble of determining mass or charge-to-mass ratio absolutely, with the full precision of which the method might be capable; what is actually evaluated is

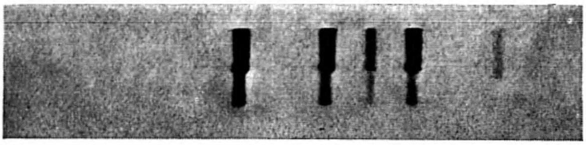

Fig. 4—Mass-spectrum of zinc (K. T. Bainbridge).

the ratio of the mass of each unknown to that of some familiar kind of atom, eventually always the atomic mass of the principal isotope of oxygen. There are various schemes and tricks for facilitating the comparisons, of interest chiefly to those who have some intention of imitating the experiments. Of more general interest is the problem of producing the ions.

The elements which are gaseous at ordinary temperatures, and those which have compounds that are gaseous at ordinary temperatures (such as carbon in CO and CO_2), and the metals which have high vapor-pressures such as mercury—these were analyzed early in the game.

Fig. 5—Mass-spectrum of germanium (K. T. Bainbridge).

They are introduced into the discharge-tube, alone or mixed with other gases, and the processes of the discharge ionize their atoms (or the molecules of their compounds, which serve the purpose just as well). Certain others, the alkali metals and the alkaline earths in particular, were conquered through the fact that their ionized atoms stream out of their solid salts when these are heated or bombarded by electrons. The easier cases thus disposed of, it became necessary to lay siege by special artifice to most of those which remained. Constant readers of *Nature* are acquainted with the letters, generally two to four in a year, in which Aston announces the capture of one fortress after another. Sometimes it is the gift of a sample of some rare element which makes possible the new result, but oftener the contribution of some unusual compound of a common element which, when introduced into the discharge-tube, vaporizes fast enough to supply the desired atoms to the discharge but not fast enough to inhibit the current or clog the tube. Curious observations have been made upon the behavior of some of these strange compounds in a current-carrying gas; of osmium tetroxide, for instance, Aston relates that it had upon the discharge an effect to be compared with the injection of a powerful drug into a living organism.

So much success has attended these efforts that the conquests yet remaining to be made are few, and it is a much quicker affair to list the as-yet-unanalyzed elements than the analyzed. In order of increasing atomic number (which I place in front of each symbol) they are: 43 Ma (a lately-discovered element); 45 Rh, 46 Pd (two members of the second of the "triads"); 61 to 71 inclusive, excepting 68 Er (ten rare-earth elements); 72 Hf (likewise lately discovered); 77 Ir, 78 Pt (two members of the third triad); 79 Au; and the elements beyond 84, of which all but three (88 Ra, 90 Th, 92 U), being unstable, are very scarce.[6] Some of these must owe their absence from the list of the conquered to their rarity, but many are common enough, and what is lacking is a way of driving their atoms into the open and ionizing them.

The other list, that of the analyzed elements, now comprises sixty-six. Among these are distributed nearly two hundred kinds of atoms of different masses. I count 198 in one of the tabulations, but of these some twelve or fifteen are marked as somewhat doubtful, because their ostensible lines on the plates are either very dim or else might be ascribed to some other kind of substance. (Thus if two kinds of ions are observed which differ in mass by one unit, it is often possible that the lighter may be an ionized atom and the heavier an ionized molecule of the hydride of that atom, instead of both of them being ionized atoms of unequal masses.) Among the 198 there are several of which the existence was first deduced from band-spectra; some of these have since been detected in mass-spectra, notably the minor isotopes of oxygen, O^{17} and O^{18} (I adopt the practice of writing atomic mass as a superscript to the chemical symbol); others, Be^{8} and C^{13} for example, have not yet been confirmed in this manner, but the evidence from the bands is strong.

These nearly two hundred isotopes do not exhaust the list. There are in addition the radioactive atoms, of which there are known at present thirty-six varieties, distributed over the last twelve places of

[6] At the recent Chicago meeting of the A. A. A. S., Aston announced that he had analyzed uranium, finding a single isotope of mass about 238. This does not speak against the extra isotope of mass 234 appearing in Fig. 7, which is inferred from the study of radioactivity and is known to be too scanty to appear on Aston's plate.

the table of the elements (Z 81 to Z 92); and there are the seventeen elements of atomic number inferior to 81 which have not yet been analyzed, to each of which we must assign at least one isotope. This makes the round figure *two hundred and fifty* a suitable choice for the number of different masses of atoms, *the number of different kinds of nuclei, already known.* It may be a little excessive, but is not likely to remain so for long.[7]

A graphical presentation of these atomic masses is more effective by far than a table. One naturally thinks first of plotting A, the atomic mass, against Z, the atomic number; but then it turns out that the diagram is inconveniently high. The inconvenience is lessened in Figs. 6 and 7 by plotting $(A - Z)$ against Z, a scheme which has also some value for theory. All the isotopes of an element are marked by dots along its vertical line, and their mutual differences of mass are properly given; but in comparing the isotopes of any element with those of any other, one must think of their dots as vertically displaced by an amount equal to the difference between the abscissæ of the elements. The two figures refer, one to the elements below and the other to those in and above the great gap which in a single figure occurs at the as-yet-unanalyzed group of the rare earth elements. The slanting lines in Fig. 7 connect the consecutive members of radioactive families; they are too crowded to be clear, but I have shown a much clearer diagram in an earlier article of this series.[8]

Such a diagram implies that the masses of the isotopes are integer multiples of a common unit, that unit which is one sixteenth the mass of an oxygen atom; we must now examine into this question. Before mass-spectra were observed, the non-integer "atomic weights" of the chemical tables—such as the 24.32 of magnesium and the 35.46 of chlorine—were regarded as the masses of individual atoms. The discovery of isotope-analysis must have created, in some minds at any rate, the transitory hope that all true atomic masses would be proved to be exactly integers,—if not in terms of one sixteenth the oxygen mass, then in terms of some other. I do not know whether this hope was ever widely formed; in any case, it was doomed to be dashed. The ratios of the masses of the isotopes to one sixteenth the

[7] Absence of an isotope from the list of those discovered means, of course, not that it is absolutely non-existent, but that the ratio of its abundance to those of the major isotopes of the element in question must be below some critical least-observable amount. This critical amount varies so much with the element, the method, and the experimenter that no generally-valid figure can be given. In the very best cases (e.g. helium, with which a vigorous search for He^5 has been made) it is as low as one part in 40,000; in others, apparently as high as one in a few hundred.

[8] Number 12 ("Radioactivity"), this *Journal, 6*, 55–99, January, 1927.

mass of the O^{16} isotope are much more nearly integers than many of the chemical atomic weights, but they are not exactly integers. The most famous of all the chemical misfits—the ratio 1.008 to 16.000 of the combining weights of hydrogen and oxygen—is almost exactly

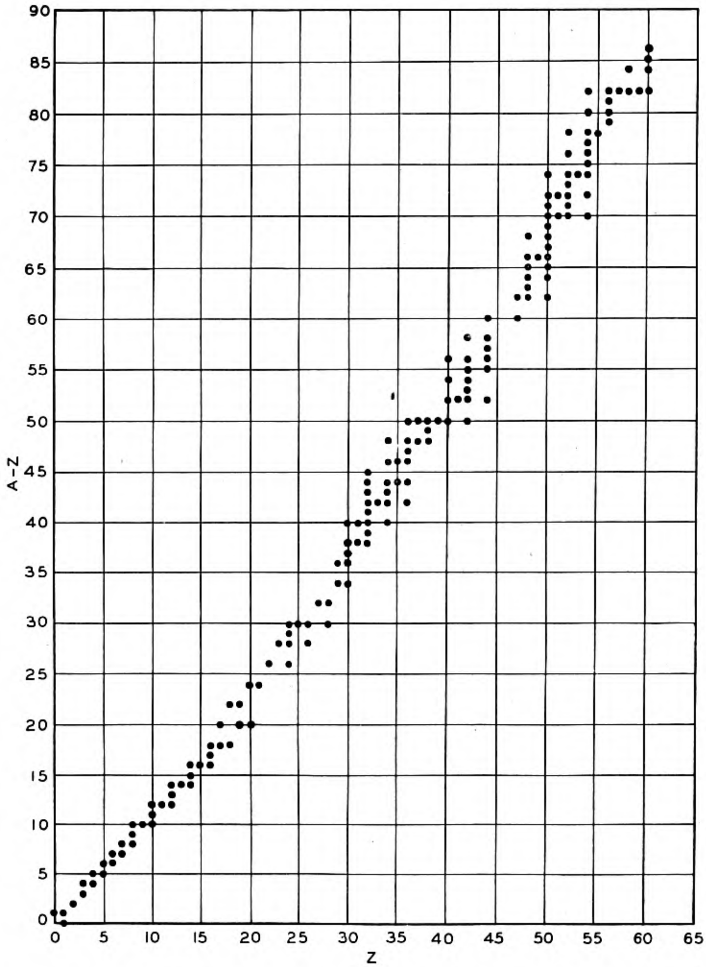

Fig. 6—Diagram of the isotopes of the elements of atomic numbers up to 60, the difference between mass-number A and atomic number Z being plotted against Z.

repeated between the isotopes; for both these elements are of the class in which one kind of atom predominates immensely over the rest. The ratio of the masses of the principal isotopes, H^1 and O^{16}, is one of those on which the highest resources of the technique of mass-spectroscopy have been lavished; and it turns out (according to Bainbridge) to be 1.007775 ± .000035 to 16.00000. From another part of the table of the elements, take caesium. This is one of the few

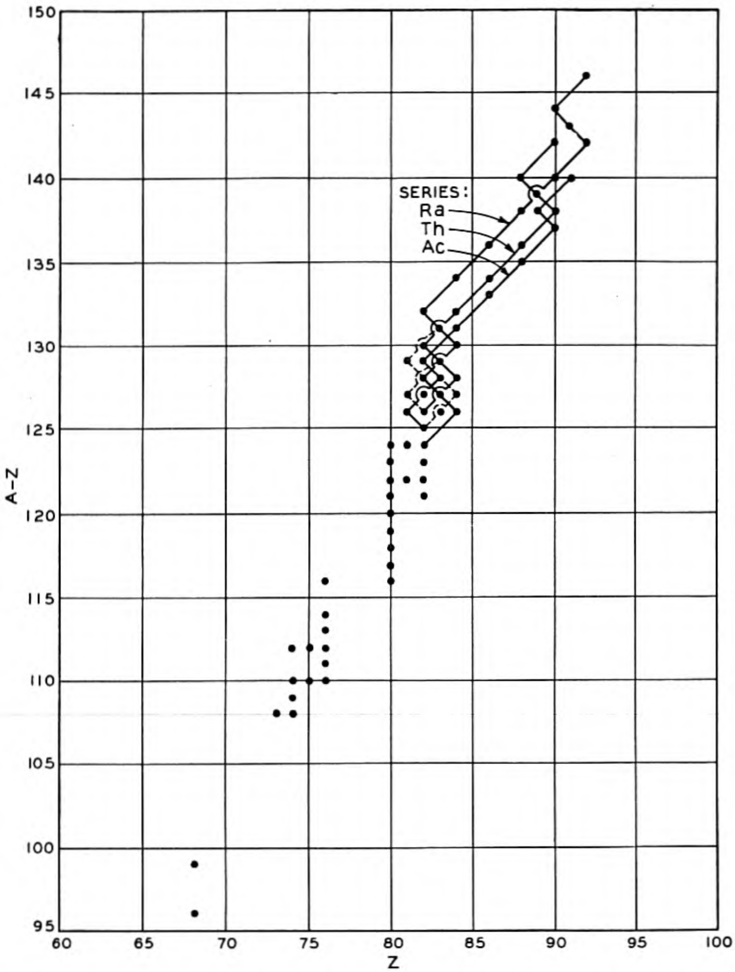

Fig. 7—Diagram of the isotopes of the elements of atomic numbers over 60. Lines connect isotopes belonging to one and the same radioactive series, most of which are known by their radioactivity alone. The mass of the end-product of the actinium series (AcD) is taken as 207 in accordance with Rutherford's opinion.

analyzed elements which as yet has disclosed no trace of more than one isotope, and the mass of this one amounts, in terms of "one sixteenth of O^{16}" to 132.93 ± 0.02.

Yet strange as it may seem, this failure of atomic masses to be integer multiples of either the mass of H^1 or one-sixteenth-the-mass-of-O^{16} is no detriment to theory, but rather the reverse. There is a very general hypothesis which may be phrased as follows: if a number of elementary particles cling together in a stable cluster, the mass of the cluster M is less than the sum Σm of the masses which the particles would have if they were free, and the difference $(\Sigma m - M)$ is the energy "of binding," the energy which would have to be given back to the particles of the cluster to disperse them again into freedom. I say "the difference of masses *is* energy," thus invoking Einstein's principle of the equivalence of energy and mass. By this principle a mass amounting to m grammes is an energy amounting to mc^2 ergs (c standing as usual for the speed of light in vacuo, $3 \cdot 10^{10}$) and an energy amounting to E ergs is a mass of E/c^2 grammes, whether it be kinetic energy or light or whatsoever other form.[9] If a nucleus be a cluster of, say, electrons and protons, then its mass must be less than the sum of their separated masses, for otherwise it would have no cohesion and would fall apart of itself; and its deficiency of mass is a measure of its stability.

At this point I ought to give some idea of the orders of magnitude involved. Nothing has been said thus far about the mass in grammes of any kind of atom, but we now require some such value in order to make the translations between energy expressed in ergs or in electron-volts, and mass expressed in terms of our standard one-sixteenth-of-O^{16}. The masses of atoms in grammes are not known nearly so well as their ratios to each other, but the three significant figures assured for oxygen are sufficient for our purpose. The mass of the oxygen atom is $2.64 \cdot 10^{-23}$ g, and it follows that one million electron-volts of energy amount to .00107 of one of our units of mass. Now the mass of the electron is .00054; the mass of the proton is that of the H^1 atom less that of its orbital electron, or say 1.0072; the mass of the O^{16} nucleus is that of the O^{16} atom less that of its eight orbital electrons, or say 15.9957. If we make the hypothesis that the O^{16} nucleus is a cluster of sixteen protons and eight electrons, the separate masses of these twenty-four particles add up to 16.1195, and there is a discrepancy of 0.1238 units; but this is perhaps no real discrepancy, but simply the energy which the twenty-four particles yielded up when they gathered into the cluster, and which must be restored to them if they are ever

[9] In the special case of a system of electrified particles acting on one another strictly according to the laws of classical electrodynamics, the equivalence of mass m and energy mc^2 can be derived from these laws; i.e. it can be deduced that two configurations of the system differing in energy by E differ in mass by E/c^2. However, such particles could not form a stable cluster; so that one is compelled to postulate Einstein's general principle, after all.

to disperse again. It amounts to about 115 millions of electron-volts, and this is not an unwelcome figure, for had the value been much smaller we might expect oxygen nuclei to be easily disrupted, which is not the case.

This evidently makes an extra reason for measuring atomic masses with the utmost care: not only are these masses important in themselves as constants of Nature, they may also be used as indices of the stability or the fragility of the various kinds of nuclei. Aston's first apparatus enabled him to measure them to one part in a thousand, an accuracy which may be valuable among the lightest elements but not among the heavier, where the uncertainty rises to one fifth of a unit of mass. His second apparatus proved itself competent to one part in ten thousand, and with its completion in 1925 the second period of isotope-analysis began. Bainbridge in measuring the ratio of He^4 to H^1 pushed onward to a precision severalfold greater, claiming a probable error of only one part in a hundred thousand. With such data as these, it is necessary at times to take account of the fact that what is measured is the ratio of masses of two ions, the unknown and the O^{16} ion; what is tabulated is usually the ratio of the corresponding atoms; but what is required for nuclear theory is the ratio of the masses of the nuclei. Even with contemporary accuracy, though, the correction is still trivial unless the very lightest atoms are involved.[10] It should be mentioned here that band-spectra occasionally permit the ratio of the nuclear masses of two or more isotopes of the same element to be evaluated, with an accuracy which may attain (in the case of the ratio C^{13}/C^{12}) one part in ten thousand.

Not nearly all of the known kinds of atoms have had their masses so precisely measured. Suitable data exist for nearly all of the isotopes of the first ten elements; beyond these there are but twenty-four elements of which even a single kind of atom has been measured, and deplorable gaps between them.

How best to plot these data? This is a difficult problem. Considering the inchoate state of nuclear theory, it would probably be best to plot the measured masses directly, as in Fig. 6—were it not that then the graph would have to be as large as a wall-map. It is

[10] This is due not entirely to the smallness of the electronic mass, but partly to the fact that the ratio of nuclear mass (in standard units) to number-of-orbital-electrons is always between 2 and 3 for all kinds of atoms excepting H^1 for which it is about one.

Aston until 1930 published his estimates of atomic masses coupled not with their probable errors, as the custom is, but with the extreme limits outside of which (in his opinion) the value of the mass in question cannot possibly lie—an unusually conservative policy, because of which some people who have used his values have underestimated their probable accuracy. The ratio of these "uncertainties" to the probable errors is commonly taken, with Aston's concurrence, as three.

much more convenient to plot the differences between each measured mass M and the nearest integer, which latter is the "mass-number" A of the kind of atom in question. Aston prefers to use the quantities $10^4(M - A)/A$, the differences aforesaid divided by the corresponding mass-numbers and "expressed in parts per ten thousand"; these he calls the "packing fractions" in allusion to the principle that elementary particles suffer changes in mass when they are clustered or packed closely together.

If one plots either $(M - A)$ or the packing fraction against A, it is immediately obvious that the values of either do not jump about at random as one progresses along the procession of the atoms in order of atomic mass. The packing fractions lie pretty closely along the sweeping curve in the lower part of Fig. 8, with its odd bifurcation (ordinates on the left!). Not all the available data are represented

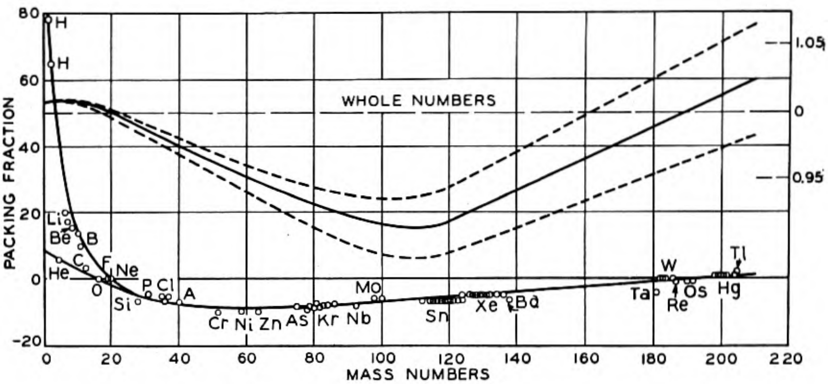

Fig. 8—Deviations of atomic masses from mass-numbers (upper curve) and packing-fractions (lower curve, with points of observation). Curves from the report of a Royal Society discussion of 1929, with subsequent observations filled in from Aston and from Bainbridge.

by dots, as some would fall too close to others to be distinguishable on this scale. The curve of $(M - A)$ is the full curve sketched above (ordinates on the right!).[11]

As the trend of either curve makes clear, the masses of the atoms near either end of the procession, the "light" end and the "heavy" end, exceed their nearest integers; while all through the middle (and

[11] The packing fractions from one end of the curve to the other are mostly uncertain by from one to three units, excepting those of H^1, H^2 and a few other very light atoms. The uncertainty of $(M - A)$ increases steadily with A, as the reader will easily understand; it is indicated by the space between the dashed curves. This is a reason for preferring packing-fraction to $(M - A)$ as a quantity for plotting.

The data omitted in Fig. 8 are: -5 for Cs^{133}, $+2$ for Tl^{203}, -7 for Se^{80}.

by much the largest) part of the procession they fall below their nearest integers. There is a minimum or greatest-negative-value of the difference $(M - A)$ near $A = 110$, and a minimum of the packing-fraction near $A = 60$. It may seem paradoxical that the two minima do not coincide, but the apparent paradox is easily understood.

If all the packing-fractions were negative, and all the atomic masses lay just below their nearest integers, we should infer that all the nuclei consist of particles having one sixteenth the mass of O^{16} when free, and that all the differences $(M - A)$ are losses of mass due to clustering or packing. The policy of plotting packing-fractions is open to criticism because it leads, or rather misleads, to that untenable idea—untenable, because so many of the nuclei show positive values of $(M - A)$. One is obliged to argue that the protons and neutrons which are presumably packed into nuclei undergo an *average* shrinkage in mass from 1.008 or 1.007 to 1.000, and in addition an *extra* change either positive or negative of which $(M - A)/A$ is a sort of a measure. This viewpoint has certain merits, but I think that the best thing to do with a packing-fraction is to retrace the steps whereby it was originally calculated, and thus obtain the mass of the atom in question, which then may be compared with the masses of adjacent atoms, or those of the elementary particles of which one supposes it built, or indeed with anything else whatever.[12]

The sort of reasoning that then is possible can best be shown by illustrations.

We start with H^1, nuclear mass 1.0072, and go ahead to H^2, nuclear mass (by Bainbridge's latest measurement) 2.0131. As $Z = 1$ for this latter nucleus, it might conceivably be either a cluster of two protons and an electron, or a proton and a neutron. Here the principle of the interrelation of mass and energy may prove important: if for either of these models the sum of the masses of the separated particles should be smaller than 2.0131, it would be necessary to discard either that model or the principle. There is no difficulty with the former model, the sum being 2.0149. As for the latter, not even the indirect estimates of the mass of the neutron are sufficiently close to permit the test. One may turn the argument around and deduce that if it is ever shown by other evidence that the H^2 nucleus is a proton plus a neutron, the mass of the latter when free must be more than 1.0058.

Many a search has been made for nuclei of mass-number 3, but all in vain; the non-existence of such kernels may be as significant to the

[12] The same remark goes for the so-called "mass-defect," which for a nucleus of mass-number $4n + b$ (n = any integer, b = any integer less than 4) is computed by adding the masses of n alpha-particles and b protons, and taking the difference between their sum and the actual mass of the nucleus.

theorists of the future as the existence of other kinds. We go on then to the kernel He^4, our indispensable friend the alpha-particle.

The ratios of the masses of the atoms H^1, He^4 and O^{16} are among the most important constants of physics. All are known by now with admirable precision: the three, mutually compatible values 1.0078 : 16 for H^1/O^{16}, 4.0022 : 16 for He^4/O^{16}, and 3.9713 : 1 for He^4/H^1—the two first from Aston, the last from Bainbridge—appear to be uncertain by not more than one place in the last significant figure, if so much as that.[13]

Forming a model for the He^4 nucleus out of four protons and two electrons, we find that not only is it stable by the principle aforesaid, but it is abundantly stable. The difference between Σm the sum of the separate masses and M the mass of the alpha-particle is positive and equal to .029 mass-units, or about twenty-seven million electron-volts! There is consequently no cause for worry over the fact, or rather the appearance, that when alpha-particles with as much as eight million electron-volts of kinetic energy crash into other nuclei, either nothing breaks or else the other nucleus gives way.[14] With the H^2 nucleus the difference $\Sigma m - M$ amounts to less than two million electron-volts, so that we should rather expect it to be broken under similar circumstances.

Mass-number 5 again is missing from the procession, in spite of an ardent and lately-stimulated search.

Mass-numbers 6 and 7 are isotopes of lithium, of which Bainbridge has determined the masses as 6.0145 and 7.0146, with uncertainties of 3 and 6 places in the last significant figure. One can picture the nucleus of Li^6 as a cluster of six protons and three electrons, which have lost altogether 0.033 of a unit of mass or something over thirty million electron-volts in combining. It is more usual, however, to apply a certain very general hypothesis, of which the validity is still quite uncertain: the hypothesis that in every nucleus there are nearly or quite as many completed alpha-particles as the mass will admit. In the case of Li^6 this suggests one alpha-particle, two "loose" protons and one "loose" electron; and about four million electron-volts would have been lost by the three last in attaching themselves to the alpha-

[13] The values and uncertainties as given are: (1.00778 ± .00015) : 16; (4.00216 ± .0004) : 16; and 3.971283 ± .000042; the uncertainties being the extreme ones in the first two cases, the probable error in the last (cf. footnote 10).

[14] When protons emerge from a substance bombarded by alpha-particles, why should we assume that they come from the bombarded nuclei and not from the projectiles? Chiefly, I suppose, because in the contrary case they would be expected to appear whatever the substance, whereas actually they vary exceedingly in amount and energy-distribution from one element to another. But there is some reason for thinking that the alpha-particle coalesces with the struck nucleus when the proton comes off, which makes the question rather meaningless.

particle. In the case of Li^7 it suggests one alpha-particle, three loose protons and two loose electrons. This would mean the addition to the Li^6 nucleus of a proton and an electron, originally of total mass 1.0078, which would shrink to 1.000 in process of being added. It seems as though our standard of mass had an objective existence in Li^7, but this is probably misleading.

Lithium can be transmuted by impact either of alpha-particles or of protons. In the former case, neutrons are emitted, together with gamma-rays; in the latter, alpha-particles come off in pairs. What can be inferred about the nuclei?

Here we meet with the great difficulty common to experiments on transmutation: with an element of two or more isotopes, one does not know which is or are being disintegrated. This is sometimes welcome to the theorist, who can ascribe the transmutation to whichever isotope happens best to fit his theory. Thus to explain what happens when protons strike lithium, it is very satisfactory to write:

$$Li^7 + H^1 = 2He^4, \qquad (1)$$

a quasi-chemical equation—an equation of nuclear chemistry—in which both masses and charges are balanced, and which implies that the proton and the constituents of the lithium nucleus fuse themselves into a pair of alpha-particles, which kick one another violently apart. Now consider what happens when alpha-particles strike lithium; using n as the symbol for the neutron, we may write either of two of these equations:

$$Li^6 + He^4 = x + n, \qquad Li^7 + He^4 = y + n, \qquad (2)$$

in which x would have to stand for a nucleus of atomic number 5—that is to say, a boron nucleus—and mass-number 9, while y would have to stand for a boron nucleus of mass-number 10. Now boron kernels B^{10} are familiar, but kernels B^9 are as yet among the missing; it is therefore much pleasanter to infer that it is the Li^7 isotope which is disintegrated by alpha-particles; and such inferences are often drawn.

Equation (1), as I intimated, should be a balancing of masses as well as of charges; but on putting the measured masses of the nuclei Li^7 and H^1 and He^4 into the equation, one gets 8.020 on one side and 8.004 on the other, and the discrepancy is far beyond the uncertainty of either. This is a very interesting case, because it affords evidence for the principle of the equivalence of energy and mass. According to this principle, we ought to introduce into the equation T_0 the kinetic energy of the particles before, and T_1 the kinetic energy of the particles after the impact:

$$Li^7 + H^1 + T_0 = 2He^4 + T_1. \quad (3)$$

It chances that T_1 is considerable, about seventeen million electron-volts (equally divided between the two He^4 nuclei) while T_0 is relatively negligible, since this transmutation can be effected by protons having even less than 10^5 electron-volts of vis viva. Translating T_1 into mass-units we find the right-hand member elevated to 8.018, which agrees within the uncertainty of experiment with the 8.020 on the left. Here is a reaction in which mass has truly been conserved, and there would appear to have been an actual loss thereof, if kinetic energy itself were not possessed of mass.[15]

The emergence of neutrons from proton-bombarded lithium—and beryllium, and boron, not to speak of other elements—is of course the strongest reason for supposing that they exist in these and other nuclei. We can as easily say that the Li^7 nucleus consists of an alpha-particle and a proton and two neutrons, and the alpha-particle of two protons and two neutrons, as we can say that they consist respectively of an alpha-particle and three loose protons and two loose electrons, and of four protons and two electrons respectively. But is there any real difference between the two models? any difference, that is to say, which might be tested by experiment? Or in other words: is there anything to be gained (or lost) by substituting, in a nucleus-model comprising both protons and electrons, a neutron for a proton-and-electron pair? If the mass of a neutron differed considerably (i.e. by a large fraction of a mass-unit) from the sum of those of an electron and a proton, there might be a definite gain (or loss); but this does not appear to be the case. There may however be a really important distinction resulting from the "spins" of these particles, which will be treated in a later instalment.

To return to the procession of the atoms: mass-number 8 is represented by an isotope of beryllium so rare that it has been detected only (possibly not with certainty) in band-spectra. Its nuclear charge and mass-number are such that one may suppose its kernel to be a pair of alpha-particles. It seems obvious to infer that Be^8 is rare, if not non-existent, because two alpha-particles repel one another too violently to hang together. This, however, is a dangerous line of thought, inasmuch as the kernels C^{12} and O^{16} would also be expected

[15] One should strictly define kinetic energy in the relativistic rather than the classical fashion, but the difference is much too small to be observable in these experiments. The test of equation (3) may be regarded by some as merely a new verification of the relativistic dependence of mass on speed so often verified by experiments on electrons, but it seems to me to contain something more.

to consist of nothing but alpha-particles—three and four respectively—and they are among the most stable and abundant varieties which there are. One begins already to guess that nuclear theory is not easy.

Mass-number 9 is represented by the principal isotope of beryllium, mass 9.0155 ± .0006 (Bainbridge). Beryllium is one of the elements which pour out neutrons most lavishly when assailed by alpha-particles, and one would like to infer that the Be^9 nucleus is a cluster of two alpha-particles and a neutron. Formerly the accepted model consisted of two alpha-particles, a loose proton and a loose electron, though this picture made it difficult to understand why beryllium is one of the few light elements which yield up few or no protons when alpha-particles bombard them. On forming the difference of the nuclear masses Be^9 and $2He^4$, we find 1.011, which is a very disconcerting figure, as it is greater than either the accepted value of the mass of the neutron or the sum of those of proton and electron. The excess is very small in each case, so small that without the present-day technique of measurement it would remain undetected; perhaps it is uncertain even yet; but unless and until someone proves that actually there is a deficiency instead of an excess with at least one of the two models, it will be questionable whether the Be^9 nucleus comprises two perfected alpha-particles.

Mass-numbers 10 and 11 are represented by isotopes of boron, masses given by Aston as 10.0135 and 11.0110 with maximum uncertainties of ± .0015. I will use these in explaining how the mass of the neutron is estimated. When bombarded by alpha-particles, boron emits neutrons. Again it is uncertain which isotope emits them; but if we write equations similar to (2), with allowance for kinetic energies:

$$B^{10} + He^4 + T_0 = x + n + T_1; \quad B^{11} + He^4 + T_0 = y + n + T_1, \quad (4)$$

we see that x and y would have to be isotopes of nitrogen, of mass-numbers 13 and 14 respectively. No atom N^{13} is known, but N^{14} is the principal isotope of nitrogen. These facts speak strongly in favor of the second of equations (4), and so does the fact that when nitrogen gas is bombarded by neutrons there are transmutations in which alpha-particles appear—evidently the converse of the process which that equation was first written down to describe.

If now in the second of equations (4) we put the nuclear mass of N^{14} for y, and then insert Aston's values for N^{14} and B^{11} and He^4, we get:

$$\text{mass of neutron} = (1.0051 \pm .005) + (T_0 - T_1). \quad (5)$$

Now in trying to evaluate $(T_0 - T_1)$ one encounters two difficulties which are of no great importance in this case, but may be serious in others. First, the incident alpha-particles do not all have the same speed and the expelled neutrons do not all have the same speed; it may be that $(T_0 - T_1)$ is the same for each individual event, but so long as we can only observe these events in multitudes we have to tolerate a wide distribution of T_0 and a wide distribution of T_1. Second, the kinetic energy of the neutrons is not measured; what is measured is the range of the particles (atom-nuclei) which they strike, and from this the speed of these struck particles is deduced, and from this the kinetic energy of the neutrons themselves, which thus is two steps away from the data! Luckily it is the difference between T_0 and T_1 which enters into the equation, and this is not nearly so large as either; Chadwick estimates it as .0016 mass-unit, so obtaining:

$$\text{mass of neutron} = (1.0067 \pm .005). \tag{6}$$

The alteration seems so much smaller than the uncertainty as to be not worth the making; but the latter again is Aston's extremely generous estimate of the uncertainty, which may be three or four times the probable error; so that perhaps the allowance for $(T_0 - T_1)$ is worth while. Similar computations can be made for the neutrons expelled from Be and Li, but perhaps had better be left for those who have personal acquaintance with the problem of estimating their kinetic energies.[16]

Mass-number 12 is the principal isotope of carbon; it would be the only one known, were it not for observations made on band-spectra of carbon compounds by King and Birge, who detected lines due to C^{13}. This latter nucleus is presumably the residue of the transmutation of B^{10} by the impact of an alpha-particle, which frees a proton and merges with what is left. The process permits another test of the mass-to-energy relation (not so good as the one described above) which I have treated elsewhere.[17]

Mass-numbers 14 and 15 are isotopes of nitrogen, the former being vastly the more abundant.

Mass-numbers 16, 17 and 18 are isotopes of oxygen, the first being much the most abundant. The other two were discovered (by Giauque and Johnston) through observation of faint lines in absorp-

[16] Neutrons are reported to have been expelled from many of the more massive elements by alpha-particle impact. It is interesting to notice that owing to the trend of the packing-fraction curve (Fig. 8) the application of the foregoing reasoning to these neutrons would lead to values of neutron-mass very much closer to 1.000, unless $(T_0 - T_1)$ were to amount to several millions of electron-volts.

[17] *Review of Scientific Instruments*, June, 1933.

tion-bands of oxygen, photographed with the brightest light and the thickest layer of oxygen on earth—the rays of the declining sun, shining obliquely through the air. Aston has since observed them in mass-spectra. They are probably the most unwelcome of all isotopes, since they necessitate an extra precaution in comparing chemical atomic weights with physical measurements of the masses of isotopes. The chemists' unit of atomic weight is one sixteenth the weighted mean of the masses of the oxygen isotopes, while the physicists' unit, as I have said so often, is one-sixteenth-of-O^{16}. The difference between the two, according to the latest estimates of the relative abundances of the three isotopes, is about 125 parts in a million. O^{17} is the presumable residue of the transmutation of N^{14} by impact of an alpha-particle, which frees a proton and fuses with what is left. This is the most completely analyzed of all transmutations, and Blackett, who first observed it in detail by the expansion-chamber method, might be regarded as the discoverer of O^{17}.

Mass-number 19 belongs to fluorine. The mass is given as 19.0000 $\pm$ .002, another remarkable example which might convince one of the objective existence of the unit of atomic mass.

As one goes onward along the list (which space forbids our scrutinizing henceforward in such fullness), one meets a novelty at atomic number 18. Here begin *overlappings* of the atomic masses of different elements: the isotope A^{40} of argon ($Z = 18$) is heavier than K^{39} of potassium ($Z = 19$), and K^{41} is heavier than Ca^{40} ($Z = 20$). The former of these overlappings is responsible for the formerly very surprising "inversion" whereby the chemical atomic weight of argon (39.44) is greater than that of its immediate follower potassium (39.10). Another inversion (involving tellurium and iodine) occurs farther along in the list and is similarly caused, and there are many other overlappings which do not produce so drastic a result.

Mass number 40 is shared by two atoms of different atomic number, different elements therefore, argon and calcium. The reader can pick out other examples from Figs. 6 and 7. There is even an instance of three "isobares," as atoms differing in Z but not in A are called: this is at $A = 124$, the three elements being tellurium, tin and xenon. (There is probably another at $A = 96$, but it is questionable as yet, as of the three in question (Mo^{96}, Zr^{96}, Ru^{96}) the two last are not positively affirmed by Aston.) It will be interesting to find whether measurements of mass can be pushed to such a degree of accuracy as to disclose small differences between isobares. Aston gives 79.926 and 79.941 for Se^{80} and Kr^{80}, but adds "the difference is too near the possible experimental error [one part in 10^4] to be of much significance." Groups of three or four isobares occur among the radioactive atoms beyond $A = 206$.

Also, as one goes onward along the list, one meets with elements having quite remarkable numbers of isotopes: lead with eight, xenon and mercury with nine, tin with no fewer than eleven put down as certain! At the same time one notices elements of apparently a single isotope only, up almost to the end of the procession; and there is a striking rule, perhaps the most definite yet found in this field: *there is no element of odd atomic number for which more than two stable isotopes are known.* The word *stable* must be inserted, as there are more than two radioactive isotopes for each of the elements 81 and 83. Moreover, for every such element past nitrogen the mass-numbers of the two isotopes (if more than one is known) differ by two units. It was also considered a rule that (past boron) the lighter isotope is the more abundant of the two; but Aston has lately discovered that the contrary is the case with rhenium ($Z = 75$) and thallium ($Z = 81$), so that this rule must be confined to the middle part of the list. This brings us to the question of abundances.

The relative plenty or scarcity of the various elements has been for many years a topic of inquiry among chemists, and also—or even more—among geologists and astrophysicists. It now becomes a subdivision of a larger topic, the relative plenty or scarcity of the various kinds of atoms. Better said, there are now two subjects of research—the relative abundances of the various isotopes within each element, the relative abundances of the elements with respect to one another—and by combining the data of the two one might hope to get the relative amounts of the many kinds of atoms in the whole of Nature.

The latter and older problem, however, is in much the more unsatisfactory state, and seems likely to remain so. We have only the earth's crust, the air, a few meteorites, some nebulæ, and the outermost layers of the stars available for the study; the nebulæ and the stars only by spectroscopic methods, of which the results are not always easy to interpret. The interior of the earth and the interiors of the stars remain impenetrable to us. The relative abundances of the elements in the five more or less accessible regions are by no means the same, and give us no sure basis for guessing what they may be in the inaccessible regions.

Nevertheless, there are rules for the relative abundances of the elements in the earth's crust, which are so strong that one is very much tempted to extend them to the whole of Nature. There is a great predominance of elements of even atomic number over elements of odd (Harkins' rule). There is a predominance of atoms of mass-numbers divisible by four. There is evident, in Fig. 6, a relative scantiness of atoms for which $(A - Z)$ is, or would be, odd; this would be even more obvious if the dots, instead of being all alike, were proportioned in size to the relative abundances of the isotopes within the elements. It seems unlikely that in the inaccessible parts of the earth and the stars these atoms should be so over-abundant as to restore the balance. Except for this unlikely possibility, we must infer that nuclei for which $(A - Z)$ is odd are not easily formed or else that they break up easily. Such nuclei, if imagined as clusters of protons and electrons, would have odd numbers of electrons; if imagined as clusters of protons and neutrons, they would have odd numbers of neutrons.

In comparing the relative abundances of the different isotopes of a single element, one feels on surer ground. It is a general rule (violated only by the radio-active elements, their end-products, possibly a few others) that these quantities are the same for every sample of a given element, wherever out of the earth's crust *or even out of meteorites* it may have been taken. It looks then as though the mixing of the isotopes within each element had been pretty thoroughly accomplished in the beginning of time, and as though the ratios of their relative amounts might have universal value.

Mostly the ratios are deduced from the darkness of the spots which the isotopes imprint upon mass-spectrum plates. The difficulties of inferring from the aspect of a spot the number of the particles which made it are like those which occur in photography, and are overcome in much the same way. The charges being exactly and the masses nearly the same for the isotopes of a heavy element, one may pretty safely suppose that equal numbers of atoms of such isotopes produce equal effects; but with very light elements this is not so sure. In occasional experiments the total charge which the ionized atoms bring with them is measured, and this is in principle the neater method. It may be carried out acceptably with apparatus not designed for making exceptionally accurate measurements of mass. Bleakney has employed it with hydrogen and neon.

About a couple of hundred abundance-ratios of isotopes in individual elements have now been measured, mostly by Aston. No rule has so far emerged from all these data, excepting the partial one about elements of odd atomic numbers which I cited earlier. There has, however, been a useful and entertaining set of by-products, in the form of revisions of the standard values of the chemical atomic weights. Obviously "physical" values for these can be obtained, if one can measure the masses and the relative abundances of all the isotopes. The highest attainable accuracy of this scheme in the most favorable cases now somewhat surpasses one part in ten thousand, which is about as good as the chemical methods can offer.

Many of the physical evaluations have been in beautiful agreement with the best-approved of the chemical, reflecting honor on both; but there have been striking temporary exceptions, with ultimate results very surprising to anyone brought up in the tradition that chemical atomic weights stand for the *ne plus ultra* in accuracy. The weights of krypton and xenon were formerly given as 130.2 and 82.9; Aston evaluated them as 83.77 ± 0.02 and 131.27 ± 0.04; within a year (1931) redeterminations of the densities of these gases (perhaps it would be justice to call this a physical rather than a chemical method) resulted in 83.7 and 131.3. Among the elements for which the analysis of isotopes has lately given a value markedly different from the accepted chemical atomic weight, are osmium, selenium, scandium and caesium. It will be interesting to see what happens to these values in the tables of atomic weights.

I left the story of the discovery of H^2 to the end, so as to make earlier mention of several things on which it depended. Apparently it was the joint result of two independent predictions. First, the ratio of the masses of the atoms H^1 and O^{16} agrees remarkably with the ratio of the chemical atomic weights of hydrogen and oxygen: both are certainly between 1.0077 and 1.0078. This agreement seems wonderful testimony to the accuracy of the measurements of physicists and chemists; but it turns out to be a mere coincidence. Such testimony it indeed would be, if H^1 and O^{16} were the sole isotopes of their respective elements; but from the moment when O^{17} and O^{18} were discovered, it could be taken as meaning one thing only (short of actual errors in the work): it could be taken only as meaning that there is an extra isotope (or more than one) of hydrogen, more massive than H^1. This idea came first to Birge and Menzel, who proceeded to compute in what ratio of abundances H^2 and H^1 must stand in order to produce the agreement in question, if H^2 be the only extra isotope. The result must depend, of course, on the ratios of the abundances of O^{17} and O^{18} to that of O^{16}. For these ratios the estimates (made from band-spectra, excepting for a preliminary one by Aston) are not in very good accord. At the time of the prediction of Birge and Menzel, they indicated a ratio of 4500 to 1 for the abundances of H^1 and H^2 in ordinary hydrogen. Second, the diagram of Fig. 6 shows a recurring uniformity, a stepwise pattern, in the broken line connecting the successive dots from $Z = 3$ to $Z = 8$. If isotopes H^2, H^3 and He^5 exist, then this pattern extends uninterrupted down to $Z = 1$.

These were the ideas which brought about the discovery of H^2 by

Urey, Brickwedde and Murphy. At first they did not expect to distinguish such an isotope in ordinary hydrogen; but they inferred from thermodynamical theory that a greater than the normal proportion (of H^1H^2 molecules among the ordinary H^1H^1 molecules) should be obtained by liquefying large quantities of gas and letting it re-evaporate at a low pressure, taking for their investigation the last two or three cubic centimetres of liquid out of several thousand. It turned out later that they could detect H^2, or "deuterium" as they have named it, in ordinary hydrogen; but in these special samples the evidence of it was far more patent.

This evidence is the advent of "shifted lines" in the ordinary line-spectrum of atomic hydrogen. The frequencies of atomic spectrum-lines depend on the ratio of the masses of electron and nucleus, in a manner which in times past has been of the utmost value in establishing the present-day model of the atom [18]; the difference between the values of this ratio for H^1 and H^2 is only that between 1/1850 and 1/3700, and results in a frequency-difference of less than three parts in ten thousand, and yet the corresponding wave-length-difference is easy to detect with spectroscopes. The existence of H^2 entails that each of the familiar spectrum-lines of H^1 should be attended by a faint companion, displaced by this percentage toward lesser wave-lengths. Urey, Brickwedde and Murphy observed the faint companions of the four most prominent of the Balmer lines; others have since observed them, and just before these pages started for the press, there appeared the photographs [19] taken by Ballard and White of four lines of the Lyman series with their companions, which I reproduce as Fig. 9.

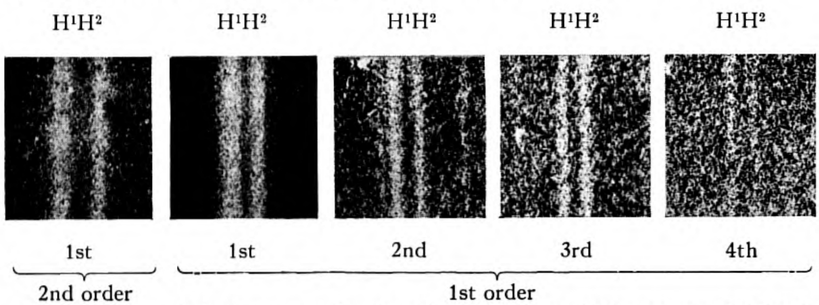

Fig. 9—Lines of the Lyman series of ordinary hydrogen (H^1) accompanied by the corresponding lines of H^2. The first picture on the left shows the first line of the series, photographed in second order; the others show the first four lines of the series from left to right, photographed in first order. (S. S. Ballard & H. E. White; *Physical Review.*)

[18] Cf. the ninth of this series of articles (October, 1925), or my *Introduction to Contemporary Physics*, pp. 308–312.

[19] I am indebted to Messrs. Ballard and White for sending me the original of this picture.

It will be noticed that in these photographs the lines of each pair are of approximately equal brightness; whereas in the original work of Urey, Brickwedde and Murphy, the one due to H^2 was always by far the fainter. The gain is due to the fact that G. N. Lewis has discovered an amazingly potent method for separating H^2 from H^1, of which the efficiency outruns by far anything that was formerly hoped for or dreamed of; it is said that samples of hydrogen or of hydrogen compounds may be obtained, in which the heavier isotope exceeds the lighter by more than one hundred to one! This appears to be a godsend to the chemists, as there is reason to suspect that the properties of "heavy" hydrogen and of its compounds may be markedly and even fantastically different from those of "light" hydrogen and *its* compounds; a whole new province of chemistry seems to be opened to explorers. Here, however, we are concerned only with the mass of the nucleus; and in the original samples there was sufficiently much of H^2, to permit of its mass being measured by Bainbridge with the result and with the accuracy which I have quoted already. As for the question of the relative abundance of H^2 and H^1 in "ordinary" hydrogen, it is now in a quite unsatisfactory state; for various experiments give various results, mostly disagreeing with the prediction which had a share in the discovery.

A System of Effective Transmission Data for Rating Telephone Circuits

By F. W. McKOWN and J. W. EMLING

A previous paper[1] introduced the idea of rating the transmission performance of telephone circuits on the basis of repetition observations and outlined briefly a method for expressing such ratings. The present paper describes in some detail a system for presenting these data in a form suitable for engineering use and the steps required for obtaining these data from the repetition observations.

Introduction

A TELEPHONE circuit may be described in terms of its physical characteristics, but these characteristics do not in themselves indicate the transmission results which will be obtained by its users in service. Laboratory talking tests, such as articulation tests,[2] indicate the ability of the circuit to transmit speech sounds under the conditions of the tests. In service, however, a wide and complex range of conditions is encountered and in the case of a new kind of instrument or circuit the service conditions may be modified in an unpredictable way due to the users' reactions. The complexity and a priori uncertainty of these conditions point to the advantage of ratings obtained during actual service.

The real criterion for rating a circuit is its transmission performance when in actual use as a link in the extremely complicated and variable communication channel between the brain of one telephone user and the brain of another telephone user. The paper by Mr. Martin[1] fully developed this idea and described a quantitative method for providing ratings on this basis of the transmission performance of a circuit, which method includes the effects of such circuit characteristics as volume loss, noise, distortion and sidetone. This method uses as a measure of circuit performance the number of repetitions requested by normal telephone users per unit time while using the circuits in actual service, on the basis that this number is a direct quantitative measure of the success with which telephone users carry on conversations. The previous paper also discussed other methods of rating the transmission performance of telephone circuits such as articulation

[1] "Rating the Transmission Performance of Telephone Circuits," W. H. Martin, *B. S. T. J.*, Vol. X, p. 116.
[2] "Articulation Testing Methods," H. Fletcher and J. C. Steinberg, *B. S. T. J.*, Vol. VIII, p. 806. "Developments in the Application of Articulation Testing," T. G. Castner and C. W. Carter, Jr., this issue of the *B. S. T. J.*

tests, volume tests and judgment tests and their relation to the repetition method.

The present paper describes the development of this rating method into a system of data for exchange area circuits which gives a convenient means of determining from the physical makeup of a complete telephone circuit a rating of the effectiveness of the transmission between normal subscribers using the circuit. Such data are called *effective* transmission data to distinguish them from previous transmission data which were based largely on volume losses. The effective data are expressed in terms of the db of effective loss relative to a reference circuit. An effective loss of 1 db is introduced into the reference circuit when the loss of the trunk is increased by 1 db at all frequencies without other change. Any other change in the circuit which has the same effect on its transmission performance as this distortionless change in volume loss also causes an effective loss of 1 db. The equality of performance is judged by the equality of repetition rates.

The problem of converting repetition data obtained from a relatively small number of circuits into usable transmission ratings for the very large number of practical circuit combinations resolves itself into two major parts: first, a choice of the form in which the data should be presented for use in laying out the telephone plant, and second, the actual preparation of the numerical data.

The preferable form for presenting the data is fixed by the nature of telephone exchange service. The general transmission problem is not to design a complete telephone circuit from one particular station to another particular station, but rather to design each circuit element separately in such a way that any complete circuit made up of these elements will give satisfactory transmission. Thus, each element, such as a subscriber loop or an interoffice trunk, must be designed to work as a part of any one of a large number of different connections. The technique for solving this problem on the basis of volume losses was worked out long ago in a satisfactory way and has been in use for many years. Volume loss data were prepared in convenient form for all available types of circuit elements, with the losses of the elements defined in such a way that when all the component losses were added the loss of the complete connection was obtained. These component volume losses were based, in general, on voice-ear tests or computations showing the effect on the volume of the received sound, of inserting the element to be rated in a reference system. With data set up in this way, it was possible to apportion the permissible overall rating between the different types of circuit elements and then to choose the facilities for each individual element separately so that its loss would not exceed the permissible loss. In a particular area, for example, the transmitting loss of each loop might be limited to 8 db or less, the receiving loss to 3 db or less, the total office losses on a connection to 1 db or less and the losses of an interoffice trunk to 6 db or less. In this case, no interoffice connection would have a loss of over 18 db regardless of which stations in the area were involved. This method makes it possible to design the circuit elements separately and also simplifies the presentation of data for the very large number of combinations of facilities available for the telephone plant.

A method has been developed for assigning effective loss ratings to parts of a circuit and presenting them in a form very similar to that used for the volume loss data. The advantages of this form are retained, therefore, even though the data include the effects of distortion, sidetone and noise in addition to volume losses.

The second problem, the preparation of numerical transmission rating data for the various types of facilities available for use in the telephone plant, requires a somewhat indirect attack because of the large amount of data required. Theoretically it would be possible to obtain relative effective ratings directly by means of repetition counts for all the circuit and instrument combinations which might be of interest in plant design. These ratings of complete circuits could then be broken down into ratings for the individual circuit elements. This method of attack, however, is entirely impractical because there is an almost infinite number of combinations of circuit elements and it would take some weeks of observation time on each combination. It is necessary, therefore, to make observations on a relatively small number of circuits chosen to cover the whole range of conditions and to obtain ratings for other circuits by interpolation between the ratings which have been obtained directly.

The method of interpolating which has been found practicable is based on the fact that the performance of a complete circuit can be described, with sufficient accuracy for most engineering work, as a function of the characteristics, volume loss, sidetone, distortion and noise. The magnitude of all of these characteristics, for any circuit using conventional types of instruments, can be derived from physical measurements. Since this can be done for each of the circuits used in the repetition tests, relations can be obtained for converting changes in noise, sidetone, or distortion into equivalent distortionless changes in volume loss. For any other complete circuit, it is necessary only to determine the magnitude of these characteristics by measurements and computations, and to convert them to effective ratings by means of the relations. These ratings of complete circuits can then be divided up into ratings of individual circuit elements.

Form of Effective Data

As stated before, the purpose of the effective data is to give a means of computing a transmission performance rating of a complete telephone circuit from the physical makeup of the circuit. These ratings, which are called effective transmission equivalents, are based on the definition that two complete circuits have the same effective transmission equivalent when under the same conditions of use they give the same grade of service as indicated by the repetition rate. The term *complete circuit* as used here includes the transmitter and receiver as well as the other elements of the electrical circuit. Circuit noise is, of course, one of the characteristics of a circuit and room noise may be treated as if it were a circuit characteristic, since it affects the transmission results obtained by the users of the circuit.

Reference System

In addition to adopting a criterion for the equality of two circuits, it is necessary to adopt a scale for the rating of circuits which differ in performance over a wide range. This has been done by a method analogous to that used for expressing volume equivalents. A reference circuit has been selected, to which a rating has been assigned as discussed below. This reference system may be varied from its normal adjustment by distortionless changes in the loss of the trunk, which forms a part of the system, until the reference system is equal in performance to the circuit being rated. Each change of 1 db in this trunk by definition changes the effective equivalent of the reference system by 1 db. Thus, the effective equivalent of circuits may be determined by comparison with the reference system.

The requirements of a reference system for effective transmission equivalents are: (1) that it be reproducible from simple physical measurements, and (2) that its performance can be compared with the performance of the circuits to be rated. Theoretically these are the two requirements for the reference system if it is to be used simply for rating complete circuits. As discussed later, since the system is to be used for determining effective losses of circuit elements as well as effective equivalents of complete circuits, there is the additional requirement (3) that it have characteristics similar to the commercial circuits to be rated. No system is available at present which fully meets all three requirements or even the first two, since present systems meeting requirement (1) are essentially laboratory devices and cannot be compared with other circuits under typical operating conditions. In order to meet the last two requirements, it has been necessary to adopt for the present a working reference system which is described in terms of particular instrumentalities instead of in terms of physical measurements. As plant conditions change other working reference systems may be required. If any other reference system is adopted it may be rated in terms of the existing working reference system, in which case the overall rating of any complete circuit which can be rated in terms of both systems will be approximately the same in terms of either reference system.

The working reference system which has been adopted is shown schematically in Fig. 1. It consists of two representative subscriber

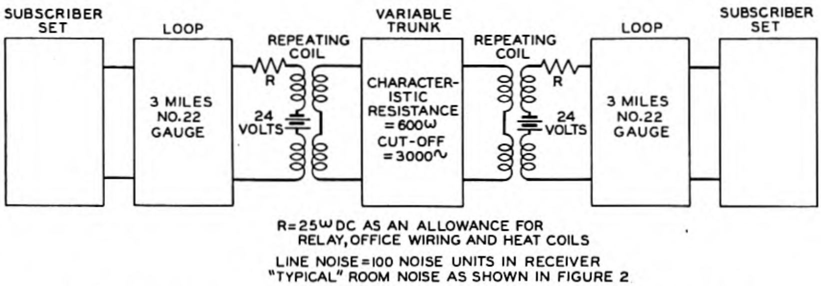

Fig. 1—Working reference system for the specification of effective losses.

sets on three-mile 22-gauge cable loops connected, through repeating coils supplying 24-volt talking battery, to a trunk having a pure resistance characteristic impedance and an adjustable attenuation. The present working reference trunk has a very high attenuation above 3000 cycles to simulate loaded lines. Below this frequency the attenuation is independent of frequency and can be varied distortionlessly so that differences in effective ratings can be expressed in terms of differences in trunk attenuation. It has a 600-ohm characteristic impedance. The circuit noise in the receiver of the working reference system is 100 noise units. The room noise associated with this reference system is that distribution of noise which normally will be found in relatively quiet offices and in relatively noisy residences. The average magnitude of noise in such locations relative to other familiar conditions is shown by Fig. 2.

Any convenient rating may be assigned to the working reference system. The Standard Cable Reference System with a trunk of zero length, which was used for specifying volume losses, was the best circuit commercially available at the time it was adopted and was given a rating of zero. This reference point was continued essentially unchanged long after the time when this significance of the zero had been lost by the introduction of improved facilities, and after the Standard Reference System was replaced by the Master Reference

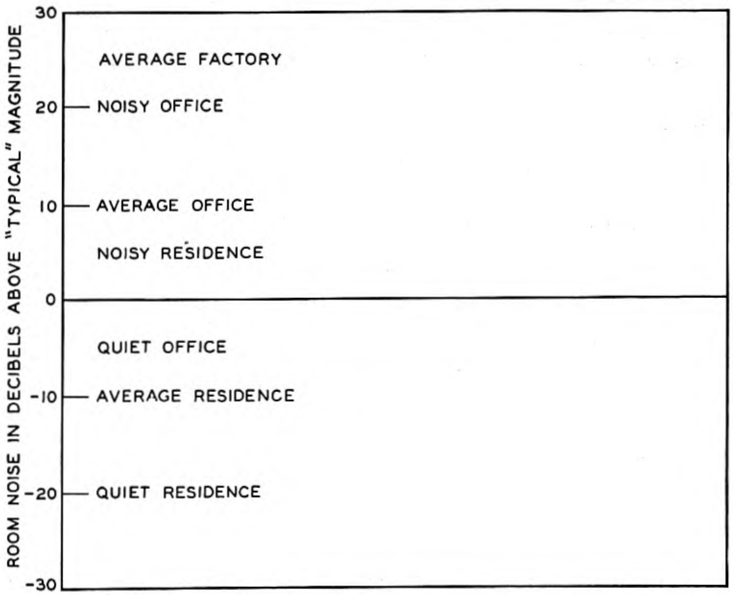

Fig. 2—Room noise in familiar locations relative to typical magnitude.

System.[3] Because of the long use of this reference point both for rating circuits and for specifying standards of transmission, the numerical values of the volume equivalents of the circuit and the volume losses of the component parts, expressed by this system, have become associated with transmission performance and it is considered desirable, at least for the present, to retain this significance of the numbers as far as possible with the new method of describing circuit characteristics. This has been accomplished, first, by selecting typical limiting conditions for the working reference system and, second, by making the effective equivalent of this system numerically equal to the volume equivalent obtained from the volume loss data. This numerical equality holds for the working reference system with any adjustment of the line provided that the trunk contains enough attenuation to prevent material effects due to the interaction between the

[3] "The Transmission Unit and Telephone Transmission Reference System," W. H. Martin, *B. S. T. J.*, Vol. III, p. 400. "Master Reference System for Telephone Transmission," W. H. Martin and C. H. G. Gray, *B. S. T. J.*, Vol. VIII, p. 536.

terminals. The normal adjustment is that for which the working reference system has an 18 db volume equivalent. The effective equivalent of any other complete telephone circuit is also equal to 18 db if it provides the same grade of service as the normal adjustment of the working reference system.

Ratings for Circuit Elements

The division of the effective equivalent of a complete circuit into its various parts, which are called effective losses, could be done in any one of several ways. The procedure described below appears to be the most suitable considering convenience, significance of the losses assigned to each element, and consistency with the form of previous transmission data.

The individual effective losses which in general make up the effective equivalent of a complete circuit include the following:

1. Transmitting loop loss.
2. Receiving loop loss.
3. Trunk loss.
4. Terminal junction loss.
5. Central office loss.
6. Intermediate junction loss.
7. Circuit noise loss.
8. Room noise loss.

The apportionment of the total normal rating of 18 db among the parts of the reference system can be done in any way which is convenient. The performance significance of the numerical values assigned by the volume loss method of rating has been retained by making the effective ratings of the working reference trunk and transmitting and receiving loops equal to those obtained from the previous volume loss data.

The loop losses are ratings of a subscriber station, subscriber loop, and a basic central office circuit. They are determined by comparison with the corresponding element of the working reference system, in each case using the remaining elements of the working reference system to complete the circuit and using the same electrical circuit noise and the same room noise as specified for the reference system. For example, any transmitting loop, which is substituted for the reference transmitting loop and which gives the same grade of service, also has an effective loss equal to the loss of the reference loop. Any loop which gives service effectively X db better has an effective loss equal to the assigned loss of the reference loop minus X db, and any loop which gives service Y db poorer has a loss of Y db more than the loss of the reference loop. Receiving loops are rated relative to the reference receiving loop in the same way, the difference in effective loss of the two conditions being subtracted from or added to the assigned effective loss of the reference receiving loop. These losses are given in curve form, similar to Fig. 3. They include the effects of variation

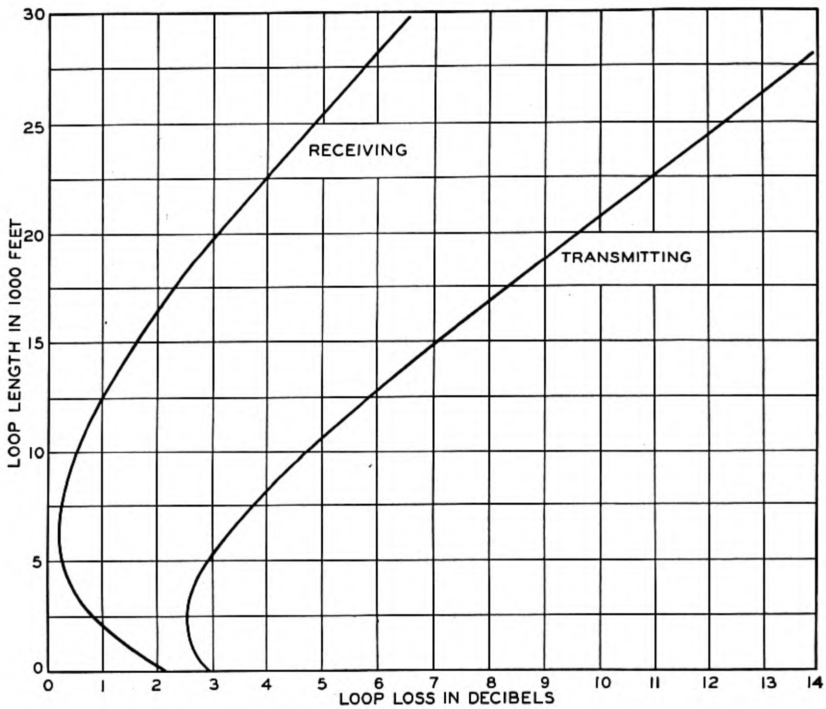

Fig. 3—Effective loop losses.

in sidetone and distortion with loop length as well as the variation in volume loss. In some cases the loss on extremely short loops is greater than on loops of intermediate length because of the rapid increase in sidetone with decrease in loop length.

The effective loss due to substituting any type of trunk for the reference trunk in the reference system could be determined and the loss data for various lengths presented in curve form in the same way that effective loop losses are presented, but the same curve would not apply for the loss of this type of trunk between other than the reference loops. If two or more such curves, as shown in Fig. 4, are determined for a particular type of trunk when used with different loops, two important facts are evident: (1) the curves are practically straight over the more important range (solid lines in Fig. 4), and (2) the straight parts of the curves are practically parallel. For trunks which

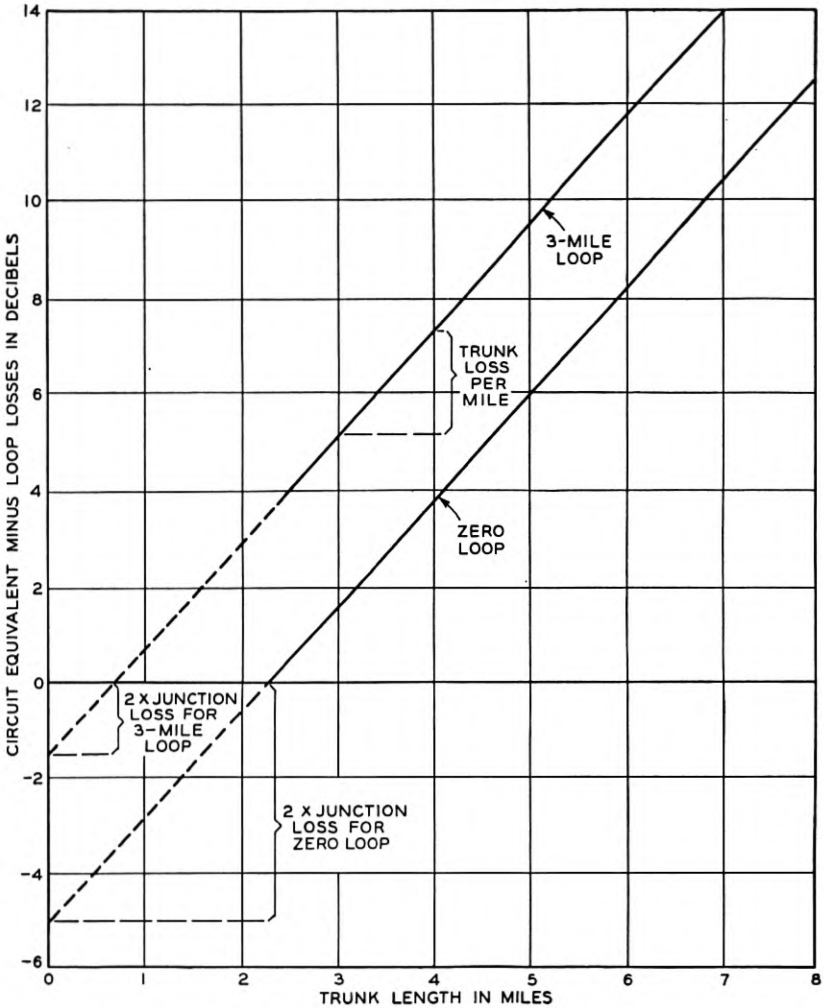

Fig. 4—Effective trunk and terminal junction losses.

are electrically short the straight line relation does not hold, but in most cases the exact loss of such short trunks is relatively unimportant. It is possible, therefore, to describe the loss of a particular type of trunk over a wide range of conditions in terms of a series of linear equations all having the same slope but with intercepts which depend on the type and length of the subscriber loop. This is exactly the method used in the volume data where its use followed logically from the mathematics which give the loss of a line at a single frequency. The use of this method with effective data is permissible only because the effects of trunk distortion as well as volume loss can be treated, with a satisfactory degree of approximation, as a linear function.

The trunk loss per unit length equals the slope of the curves and can be defined, therefore, as the increase in effective loss per mile increase in length of a trunk, which is initially electrically long, when used between the reference loops. In the case of loaded trunks, this increase in length must be accomplished without change in end section. The trunk loss per unit length, although measured between the reference loops, can be treated as independent of loop. It includes two component losses, the volume loss per unit length, and the effect of the increase in distortion per unit length.

Effective terminal junction losses are corrections associated with the junction of a loop and trunk which are added in computing the effective equivalent of a circuit employing a trunk other than the reference trunk. For a circuit with two equal loops each loss equals one-half the Y intercept in Fig. 4. It is dependent on the type of set, the type and length of loop and on the type of trunk but is independent of trunk length. It contains a volume reflection correction, the effects of that part of the trunk distortion which is independent of length, and the effects of trunk impedance on sidetone. Fig. 5 is a sample of the form in which these losses are presented.

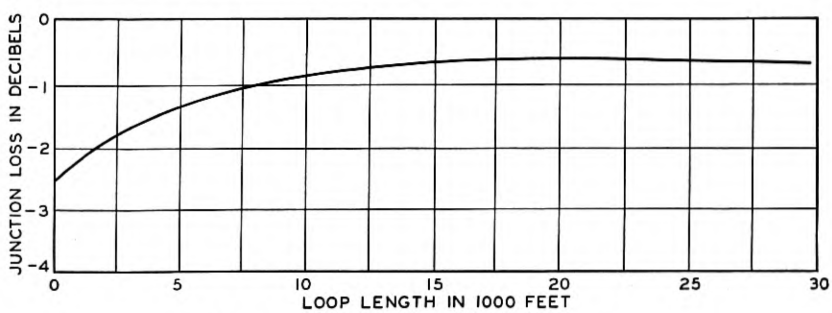

Fig. 5—Effective terminal junction loss.

The central office losses present data relative to the loss of central office apparatus and cabling other than that included in the loop losses. They are determined by substituting the apparatus to be rated for the corresponding parts of the working reference system, and equal the effective loss of this condition relative to the working reference condition. Somewhat different losses would be obtained with other loops and trunks, but the values obtained under the reference conditions give sufficiently good approximations for practical purposes.

The intermediate junction losses are corrections to be added to the other elementary losses when the trunk is made up of more than one type of facility. They include the volume reflection loss at the junction of the two facilities and a distortion correction which together with the other distortion losses of the elements will give a total equal to the distortion rating of the complete circuit relative to the reference.

The data covering the six types of losses discussed above are set up on the basis of the same electrical line noise and the same room noise as those specified for the working reference circuit.

Effective losses due to circuit noise may be presented in the form of curves showing the loss due to any amount of circuit noise. These losses are added to the other effective losses when the noise at the receiving loop terminal differs from the noise on the reference circuit. The amount of circuit noise on a particular circuit cannot usually be predicted accurately from the design constants of the circuit, since it depends largely on the characteristics of the disturbing circuits, the coupling between disturbing and disturbed circuits, and characteristics of the disturbed circuit which include random unbalances. Effective losses due to circuit noise, therefore, must, in general, be based on noise measurements rather than on the design constants of the telephone circuits.

Effective losses due to room noise may be added to the other effective losses when the room noise at the receiving end differs from the room noise associated with the reference system. Since the magnitude of the room noise at a particular station is in no way a function of the design of the telephone circuit this type of loss is in a somewhat different class from the others. More than one curve is required for presenting room noise loss since the loss depends to some extent on the sidetone of the telephone set.

The determination of a circuit rating from effective loss data is simpler than may appear from the description of the data. Exchange area circuits involve at most eight types of losses and most of these circuits involve a smaller number. Three of these losses, the transmitting and receiving loop losses and the terminal junction losses, are determined from curves similar to Figs. 3 and 5. The remaining losses, that is, the trunk, office, intermediate junction and noise losses, are obtained from simple tables or curves.

The definitions of losses have been set up so that the rating of an element is obtained when the element is substituted for the corresponding element in the working reference system, that is, when it is part of a typical telephone system. This method of determining losses was adopted because the effects of distortion, noise and sidetone in any one element depend to a greater or less extent on all the characteristics of the remainder of the circuit. It is therefore essential that each element of the reference system be fairly representative of the corresponding component of the telephone plant, if the ratings are to be approximately additive. This has been taken into account in the choice of the working reference system. Certain approximations are involved in the system of data outlined, but they are minor and are justified in the interest of simplification of the method.

Preparation of Effective Data

Data for preparing effective loss ratings have been obtained primarily from repetition counts made during a series of special transmission observations on calls between telephone employees in the regular course of their business. These calls were made over special facilities which permitted the variation of the circuit constants over a wide range and the rating of the various conditions relative to each other in terms of repetitions. During these tests, different types of instruments were used and changes were made in the sidetone characteristics of the sets, and the attenuation and type of trunk. Each type of change covered rather completely the whole range found in the present telephone plant and to some extent that expected in the future plant, but it has been practicable to cover only a small portion of the combinations of instruments and circuits which might be used together in commercial service.

In the preparation of the necessarily large quantities of effective transmission data from the transmission observations, the principal problem is to determine the rating of any complete circuit from the limited number of complete circuits which have been rated directly. The determination of these additional ratings is relatively easy if the circuits can be described in terms of a few simple characteristics which will serve as a basis for interpolating between the ratings obtained directly from observations. The physical measurements which can readily be made in the required quantity describe a circuit in a complex manner, namely, in terms of the efficiency, at each frequency in the voice range, of the several speech and noise transmission paths of the complete circuit. These data must, therefore, be combined in some way to give a relatively small number of parameters for describing the circuit.

The definitions of these parameters may be more or less arbitrary, provided that the circuit performance is the same for circuits having numerically equal parameters regardless of differences in physical characteristics. The number of parameters required to describe a circuit is largely a matter of convenience. A small number tends to make the interpolation simple, but the derivation from the measurements complex. The converse holds for a large number. For preparing effective transmission data for the local plant, the circuit description has been expressed in terms of five parameters as follows: the volume loss from the transmitter of one set to the receiver of the other, the sidetone volume loss at the talking end, the sidetone volume loss at the listening end, the circuit noise efficiency of the station at the listening end, and the distortion. An important advantage of using these particular parameters is that each represents a circuit characteristic generally recognized as affecting transmission performance. The electrical line noise at the station end of each subscriber loop and the average room noise at the station are, of course, two other parameters which are maintained at the reference value except when computing line noise and room noise losses.

The computation of effective ratings from repetition observations and circuit measurements may be summarized as follows: The transmission observations are preferably made on various series of circuits, in each of which one parameter is varied while the others are kept constant. First, a series of observations is made on circuits which are identical except that the volume loss is varied by distortionless changes in the trunk attenuation; preferably these should be various adjustments of the working reference system. A second series of observations is made with a constant volume loss but with variations in some one of the other parameters; for example, the sidetone at one end of the circuit might be varied. From this series of tests in conjunction with the first series, the distortionless change in volume loss, which is equivalent to each change in sidetone, is determined both for transmitting and receiving and curves of effective loss versus sidetone may be established. Such curves are shown in Fig. 6. These effective losses apply only for this particular volume loss, but tests with other constant volume losses give essentially the same relations. The change in effective transmitting efficiency is due to the fact that telephone users raise their talking volume when the sidetone is reduced. The change in effective receiving efficiency is due to the fact that a reduction in sidetone reduces the interfering effect of room noise.

In the same way, distortion and noise are varied separately and the effect of each of these changes in terms of equivalent change in volume loss is determined.

Room noise losses are difficult to determine with any degree of accuracy by observing on working circuits since it is impractical to introduce artificial room noise, and natural variations in room noise

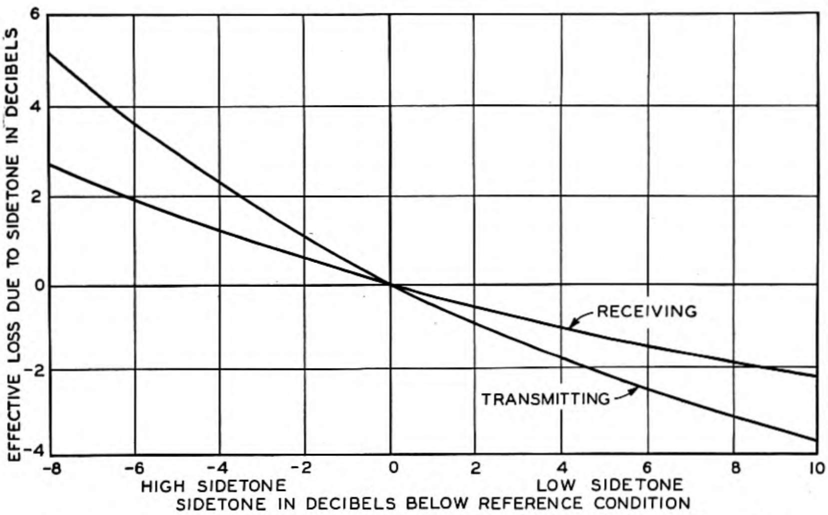

Fig. 6—Effective loss due to sidetone relative to working reference condition.

are, in general, accompanied by other variations affecting repetitions. Laboratory or theoretical methods of determining these losses are not satisfactory since, as room noise changes, the subscriber consciously or unconsciously tries to counteract the effect and such reactions cannot be studied satisfactorily under controlled laboratory conditions. Each method, however, helps to determine the general magnitude of these effects.

In practice it is seldom possible to vary one parameter over a wide range without causing some variation in the other parameters, but this does not add any serious complication to the derivation of the relations. It is merely necessary to make successive, approximate corrections when deriving each relation.

After these relations have been established, it is possible to predict the performance of any ordinary circuit by computing the five parameters from the physical measurements. The magnitude of each parameter may then be compared with the magnitude of the same parameter in the reference system, and the difference between the two magnitudes may be converted into equivalent distortionless changes in volume loss by means of the curves. These ratings may then be combined to give a single effective loss rating of the circuit.

For the normal range of conditions existing in the telephone plant simple algebraic addition of the component ratings gives the combined rating with sufficient accuracy provided the individual ratings are obtained under typical conditions. Ratings can be obtained only for complete circuits, since some of these parameters, such as distortion, have no meaning except when applied to a complete circuit, and others, such as sidetone, depend on two or more circuit elements. However, by choosing for computation complete circuits in which the elements to be rated are substituted for the corresponding element of the reference circuit, it is possible to determine the effective losses of individual circuit elements in accordance with the definitions given previously.

The parameters used for describing circuits have been used previously in a qualitative sense and several have quantitative definitions based on listening tests. A problem is presented, however, by the necessity for computing them from physical measurements. Volume loss, for example, has been defined in terms of voice-ear comparisons with an adjustable reference condition. Such a definition is satisfactory for describing this parameter but the testing method is cumbersome for obtaining the large amount of data required for engineering purposes. If, however, a series of such volume loss measurements is made on a large number of circuit conditions for which the physical characteristics are varied systematically, an empirical formula can be derived for weighting and combining the efficiencies measured over the voice-frequency range. Using this formula, the volume loss of other conditions can be readily computed. Similar methods may be used for deriving empirical formulas for computing noise and sidetone volume losses from the basic physical characteristics.

The use of distortion in a quantitative sense requires the adoption of a scale for this parameter. The only requirement for such a scale is that any two circuits having the same amount of distortion, all other parameters being equal, will give the same repetition rate. The term distortion factor is applied to the particular scale used in this work and its definition is derived from laboratory articulation tests. These tests are made on a large number of circuits which have equal volume losses but which differ from each other in frequency characteristics. The empirical formula for computing the distortion factor from the basic circuit measurements is set up so that all of the circuits which give the same articulation rate will have the same distortion factor. From service observations made on a number of different types of circuits it has been shown that, with all other parameters constant, circuits having the same distortion factor will give essentially the same repetition rate. Presumably, other scales for expressing distortion could be used, with other formulas, and possibly these can be derived directly from repetition observations if a great enough variety of circuits is covered. In any case, it appears that some such distortion factor is needed since distortion in any complete telephone connection is too complicated to be classified by any simple means, such as specifying a cutoff frequency.

It should be pointed out again that the computations and measurements described provide merely a means of interpolating between transmission observation results and have both the limitations and advantages of an interpolation method. They cannot be used to predict performance of any circuit radically different from those covered by repetition counts, but on the other hand, any inaccuracy in the formulas used in computing the parameters is of only secondary importance since they are used only for interpolating between observed points. For limited applications to simpler circuits more direct methods might be satisfactory. For more complicated circuits, such as present-day long toll circuits, other parameters are needed to describe such characteristics as delay distortion and echoes.

Conclusion

The effective transmission data can be applied in practically the same manner as the volume loss data which they replace. Consequently, the effects on transmission service of distortion, noise and sidetone, as well as of volume loss, can all be taken into account in the design of the plant in a simple, systematic way. Such comprehensive transmission ratings are required to utilize properly the various types of facilities now employed in the telephone plant, to direct future developments, such as further reductions in distortion, and to incorporate into the plant the new types of facilities resulting from these developments.

Developments in the Application of Articulation Testing

By T. G. CASTNER and C. W. CARTER, JR.

The first part of this paper discusses the control and measurement of variable factors involved in testing telephone circuits by the articulation method and the simulation of the testing conditions in the laboratory to those of actual use; the second part describes the auxiliary apparatus by which these controls and measurements are effected. This apparatus includes a caller's control circuit, by which the caller's speech intensity may be measured and regulated independently of the circuit tested; a switching system which automatically reverses the direction of transmission between test sentences; devices for automatic and uniform agitation of carbon button transmitters; equipment for automatic measurement of the magnitude of the speech and noise waves on the circuits tested; phonographic sources of line and room noise; and a control board at which circuit elements and conditions can be changed and measured quickly.

In addition, the time required to carry out a program of tests has been materially reduced by the use of equipment which analyzes the articulation data automatically and provides the test results in typewritten form immediately after each list is called.

Introduction

ARTICULATION tests have been used for many years as one of a number of laboratory methods of measuring the performance of telephone circuits. The continued application of this method to comprehensive programs of laboratory tests has emphasized the importance of reducing the time required to obtain results of the desired precision, and has led to the development of methods for accomplishing this. By carrying to further refinement the control and measurement of certain factors which have caused variations in the results, and by the systematic use of certain modifications in the testing routine, the precision of the tests has been increased with no increase in testing time. In addition, the development of automatic equipment for recording and analyzing the data, so that the results of the test are immediately available in the form of a typewritten record, has reduced by half the time required for carrying out a program of tests. At the same time a number of features have been introduced to improve the simulation of the testing conditions to those of actual service.

In the present paper it is proposed first to discuss the objectives which it has seemed desirable to reach and the methods which have been adopted for doing so. The auxiliary equipment which is used in articulation testing will be described in some detail in the second part.

Methods of Control and Analysis

The Articulation Testing Method

The articulation testing method itself remains essentially as it has been described previously in this journal.[1] Briefly a test is carried out as follows: Each of a number of persons, referred to as callers, speaks a list of meaningless monosyllables of the consonant-vowel-consonant type over the circuit tested. The test syllables are spoken as part of a sentence by inserting them at the time of calling in blank spaces left for that purpose in the middle of a number of short sentences called "carrier sentences"; as an example we have the sentence "When will *nud* be done," the test syllable being *nud*. A group of observers at the receiving end of the circuit record their understanding of the test syllable; their records are compared with the syllable called and any errors are noted. The results of the test may be expressed in various ways: Sound articulation, meaning the percentage of sounds correctly understood, (or its complement, sound error) is generally preferred.

The obtaining of a useful numerical index expressing the articulation performance of a telephone system is made difficult by the combined effect of a large number of variable factors: The sounds of speech are numerous and subtle, covering a wide range both in frequency and volume. Voices differ from each other and the same voice may vary considerably in its characteristics from time to time. Hearing abilities differ also, both among individuals and for the same person at different times. Finally, attentiveness and skill in perception vary greatly.

Because of these variable factors the attainment of precision, in the sense of the closeness with which a numerical result can be reproduced, depends upon the careful formulation of the technique and incessant watchfulness in supervision. Precision, however, is not enough. Two persons, one acting as caller, the other as observer, might, with training, learn to reproduce their experimental data closely but with very little practical value. It is essential that the results be representative of a large number of persons, as well as precise. Both men and women should be represented, for example, because distortion at high frequencies affects the reproduction of their voices quite differently. In the present testing group four men and four women serve as callers, each calling a list of 66 syllables to four observers; this constitutes a single test. In the paper referred to above a detailed discussion is given of the precautions which it is necessary to take in the selection of voices, the testing of the observers' hearing, the training of the crew, and the formulation of the lists of syllables used in testing.

[1] "Articulation Testing Methods," H. Fletcher and J. C. Steinberg, *B. S. T. J.*, VIII, p. 806, October, 1929.

The results obtained by a single caller-observer pair, even when well-trained, are far from constant in successive tests on the same circuit. These fluctuations may be ascribed in part to the smallness of the sample of speech contained in one list, in part to the inability of a caller to repeat speech sounds uniformly and in part to variations in the concentration exercised by the observer. When a large crew is used the effects of such fluctuations among the individual pairs tend to neutralize each other more than with a small crew, so that the additional time consumed by using eight callers is offset by the fact that with fewer callers more tests must be made to reach the same precision. At the same time the use of eight callers provides a more representative result. However, even with eight callers and four observers the effects of the individual fluctuations are still of such importance that it is the regular practice to repeat each test in a program at least once.

Interleaved Tests on Pairs of Circuits

In addition to repetition of the tests on each circuit, a further control over the fluctuations in crew skill is provided by the practice of testing circuits not singly, but in pairs. That is, instead of completing a test on one circuit before proceeding to another, two circuits are tested almost simultaneously by interleaving the two tests. The first caller reads a list of testing syllables on one circuit and follows it immediately by reading a list on the comparison circuit. The second caller then reads a list on the comparison circuit and follows it immediately by a list on the first circuit. This procedure is followed by the other six callers. Thus when the eighth caller has ended his second list, two single tests have been made. Such a pair of interleaved tests on two circuits is called, for convenience, a double test. By suitably arranging the schedule for the callers and observers the effects of erratic fluctuations can be neutralized to a large degree.

The testing equipment and the circuit elements are arranged so as to facilitate this method of carrying out the tests. A single master key, controlling a switching system, is provided so that the change from one circuit to the comparison circuit may be made almost instantaneously. The two circuits may be different throughout, even to the magnitudes of noise under which they are tested, and the intensity used by the caller; they may differ in two or three elements, such as transmitters, types of set, and length of trunk; or the difference may be in a single element, such as trunk length, or in one of the operating conditions, such as the magnitude of line noise. The data recording apparatus types out with the analyzed data whether the list has been called on "Condition A" or "Condition B."

When a single circuit is to be investigated it is usual to make the comparison circuit a standard reference circuit, the characteristics of which are well known, and to which the data would naturally be referred. In a series of tests it is customary to choose one of the test conditions of the series as the circuit with which the other test conditions are compared. In a large program, which may include many series of tests, a circuit condition from each series can be chosen as temporary reference conditions to which the data from the various series are referred. These circuits may then be related to each other and also to a standard reference circuit by direct comparisons, providing base points for whatever residual corrections are needed to reduce the results of the whole program to a common basis of crew skill; just as in a triangulation survey of land certain base points are especially well determined for use in the final adjustment of the whole survey.

The substantial advantages of interleaving the tests on pairs of circuits are indicated by an analysis of the results from more than 100 tests, representing from 2 to 5 comparisons on 42 different sets of conditions. This analysis showed a reduction from 1.3 to 0.8 per cent sound articulation in the average deviation of the differences between two conditions when they were tested simultaneously as compared with the average deviation of the differences obtained in an equal amount of time from non-simultaneous comparisons. Since the average deviation is inversely proportional to the square root of the amount of data obtained, it would have required from two to three times as much testing to attain the same precision with non-simultaneous testing.

Caller's Control Circuit

One of the principal variables in articulation testing is the intensity with which the caller speaks. With practice a caller can learn to preserve a moderately steady intensity throughout a list, but it is unlikely to be the same intensity from day to day, and it is fairly certain to differ from the intensities of the other seven callers. Some sort of control is necessary.

Control and measurement of the caller's intensity by means of measuring instruments located in the circuit under test have obvious disadvantages. A primary difficulty is that of specifying and comparing such measurements on circuits having characteristics varying with frequency in different ways. When, under such circumstances, are two intensities to be called equal? There is the secondary difficulty that with carbon button transmitters variations in the reproduction efficiency of the button occur, and it is desirable to be able to follow these independently of variations in the speaker's intensity.

When the measuring device is located in the circuit tested a sudden increase in the reading may indicate that the caller has increased his intensity or, on the other hand, he may have spoken in the desired way but the transmitter may have had a momentary change in efficiency. In the first case the caller should be instructed (except in special types of tests) to lower his intensity; in the second case the variation is simply one of the factors affecting the result which should be known but not compensated for.

To direct the caller as to the intensity of his speech, independently of variations in the circuit tested, an arrangement known as the caller's control circuit has been developed. This is essentially a high quality circuit which is inserted between the caller's lips and the transmitter of the circuit to be tested.

Normally the caller's control circuit is so operated that the output of the artificial mouth [2] which terminates it is a faithful copy both in intensity and frequency (between 100 and 7000 cycles per second) of the output of the caller's voice. It contains a gain control, however, so that if desired the output of the artificial mouth may be adjusted independently of the caller's intensity. Such a control is desirable, for example, in testing the load characteristics of certain circuit elements, since if a marked change in intensity is made by the voice itself, it is accompanied by distinct changes in the characteristics of the voice.

An essential part of the caller's control circuit is an automatic device for measuring the magnitude of the caller's speech wave and for indicating to the caller whether or not he is maintaining the desired value. There are two objectives for the caller to meet: The average intensity for the list should be the desired value, and the deviations from the average during the list should be small. An average obtained by calling the first half 10 db high and the second half 10 db low would evidently be undesirable. The caller is instructed to avoid abrupt changes and, when trained, is very successful in doing so. Some deviations from the desired value are inevitable, however, and the caller is informed of these by a system of signal lamps in the calling booth, which may be seen in Fig. 1.

The automatic volume indicator which controls the signal lamps is of a special type. Instead of indicating a separate measurement for each test sentence the volume indicator of the control circuit is arranged to show the algebraic sum of the deviations measured from the desired value. As long as the center lamp alone is illuminated the caller knows that his intensity has been maintained correctly up to that point

[2] "A Voice and Ear for Telephone Measurements," A. H. Inglis, C. H. G. Gray and R. T. Jenkins, *B. S. T. J.*, XI, p. 293, April, 1932.

in the list. If now he should call a sentence 2 db low, the lamp to the left will light. If he persists in calling 2 db low the illumination will move farther to the left, but if he raises his voice to the correct value

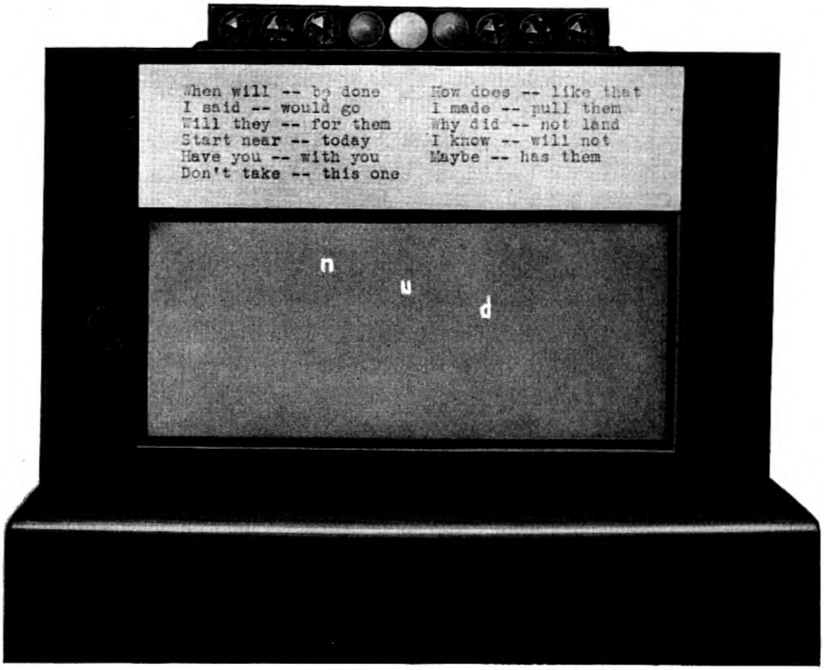

Fig. 1—Visual syllable indicator and signal lamps of automatic volume indicator—in calling booth.

the lights will remain unchanged. By raising his voice 2 db higher the light can be brought back to the center. Thus changes in the position of the lighted lamp show the departure of the sentence called from the desired value and the position itself shows the algebraic sum of these departures. With very little training the caller learns, under the guidance of the signal lamps, to keep the individual deviations small and at the same time to approach the desired average closely. No difficulty is found in attaining an average value for the 66 syllables of the list differing by less than 0.1 db from the desired value.

Experience shows that this method of assisting a caller to maintain a given intensity is capable of much greater precision than methods which indicate the intensity for each sentence and rely on the caller to make each sentence reach the desired value. It is particularly difficult for the caller, when using calling intensities which are higher or lower than normal, to maintain the desired average unless he is continuously informed as to the amount by which he has failed to reach his objective.

In addition to providing a satisfactory means of control of the speech intensity applied to the carbon transmitters in tests on a commercial telephone system, the caller's control circuit can also be readily adapted to the use of phonograph records of articulation lists instead of callers.

Control of Carbon Button Transmitters

Because the working of a carbon button transmitter depends to some degree on the physical treatment it receives, steps have been taken to subject the transmitters of the circuits tested to a cyclic routine simulating that given them in an actual telephone conversation. The insertion of the testing syllable in a carrier sentence is one step in this direction since this subjects the transmitter to a conversational flow of speech, rather than an abrupt monosyllabic impulse. The length of the list, 66 syllables, is another, since this takes 3.9 minutes to call, which is of the order of the duration of many telephone connections.

A third step is prompted by the fact that in conversation the transmitter at one end of the circuit is being agitated by speech while that at the other end is being agitated by room noise, and as the conversation proceeds these agitations alternate: speech, room noise, speech, room noise at one end; room noise, speech, room noise, speech at the other. This is simulated in the present articulation testing routine by having alternate test sentences called from opposite ends of the circuit, while room noise is applied to both transmitters.

The reversal of the direction of transmission is carried out automatically every 3.4 seconds, which is about the average length of utterance in a telephone conversation.

In order to minimize the effects of differences which are inherent in any group of shop-product transmitters, a number of them, rather than a single specimen, are used in each articulation test. These are selected as typical from a group of 25 or 50, and are changed frequently in testing, either between lists or between callers.

When changing them it is desirable to agitate them in such a way as to avoid using them under conditions of extreme variations in sensitivity. In actual service this agitation is provided by the jar of the switchhook when a deskstand is used, or by the movement of lifting if the instrument is a handset, as well as by the introductory remarks which usually start the conversation. To simulate this agitation and to change the transmitters an automatic device has been developed.

Automatic Measurement of Magnitudes of Received Speech and Noise

Even when the intensity of the wave reaching the transmitter of the system tested is controlled, variations may occur momentarily in the operation of certain types of carbon button transmitters. Such variations affect an articulation test in several ways. The reproduced speech may be momentarily greater or less than the average as to magnitude, and possibly may even suffer momentary changes in frequency composition. The receiving end transmitter may vary in the amount of room noise which it picks up and conveys to the listener's ear by the sidetone path. Both transmitters may occasionally contribute burning noise. The combined effect of such variations is sufficient to make it desirable to measure them in order to state accurately the conditions prevailing in the tests. Also, when the variations are large, it is sometimes desired to correct the test results to an average level of speech and noise.

Because of the rapidity with which the testing is carried out automatic equipment is used to make the measurements. This has been arranged to measure two quantities, the average magnitude of the speech wave on the circuit and the average magnitude of the noise reaching the receiver. The time at which the measurements take place is under the control of a timing commutator which regulates all of the automatic equipment. While the test sentence is called the apparatus measures and records the speech magnitude. This ordinarily is measured at the receiver terminals, but if the noise magnitude is comparable with that of the speech wave the measurement may be made at the input to the trunk. After the test sentence the apparatus measures and records the magnitude of the noise at the receiver terminals. At the conclusion of a list the average readings, through interconnection with the data recording equipment, are automatically typed with the analyzed data.

In addition to recording the average magnitude of the noise throughout a list of sentences it is frequently desirable to obtain data on the distribution of noise magnitudes about the average. Such information is valuable for explaining unusual variations in the recorded average values. This distribution is obtained by a series of electromagnetically operated counters which count and record the number of noise magnitudes occurring in each 2 db interval over a 30 db range. No automatic means of making a typewritten record of these values have been provided at the present time since they have been used only as a check on the operation of the testing equipment.

Room noise leakage under the caps of the observers' receivers is also an important factor affecting articulation results. Since it may vary considerably depending on the manner in which the receivers are held, care should be taken that this factor is controlled.

Simulation of Typical Noise Conditions

It is essential to have apparatus available to supply noise of various kinds, in order to simulate typical conditions under which telephone circuits are used. A dependable and convenient source is provided by phonograph records.

As a source of room noise a single record is used, except in special investigations. The material on the record is a combination of street noise as heard through a window, speech from a number of persons talking at once and other common noises. Violent changes in magnitude, such as slamming doors, are eliminated from the record in order to avoid making the test results dependent upon the purely fortuitous coincidence of such peaks with the test syllables. If tests are desired with particular types of room noise, special records of such noise can be used.

A number of records of line noise have been prepared in connection with work on specific projects and are now available for general testing. They include records of noise due to inductive interference from power systems, radio static, resistance noise and several forms of crosstalk.

Automatic Analysis of Data

The advantages of mechanical apparatus which analyzes and records the data of an articulation test become evident when it is considered that in a single test eight callers each call a list of 66 syllables, in each case to four observers. There are, accordingly, $8 \times 66 \times 4 = 2112$ syllables observed, comprising 6336 sounds, to be analyzed. Since in each case the test is ordinarily repeated at least once and in critical cases several times, the time required to correct and analyze the data when written records are used becomes an important consideration. This is particularly true when extensive programs of tests on commercial and experimental telephone circuits are contemplated, since many factors may require variation. Automatic equipment to perform the analysis makes it possible to deal with such situations economically.

The time saved by such automatic equipment is important in itself, but there are also other reasons which make it very desirable. The articulation testing method, if precision is desired, requires careful supervision and strict adherence to the details of the testing routine. This is greatly facilitated when the engineer in charge can be provided with the test result within a few minutes after the last syllable is called. Inconsistent data permit early discovery of circuit trouble, which may not have been shown by electrical test, and provide a close check on the testing personnel. Quick access to the final index permits intelligent control during the testing program. Some of the tests planned may be dropped and others added according to the nature of the data.

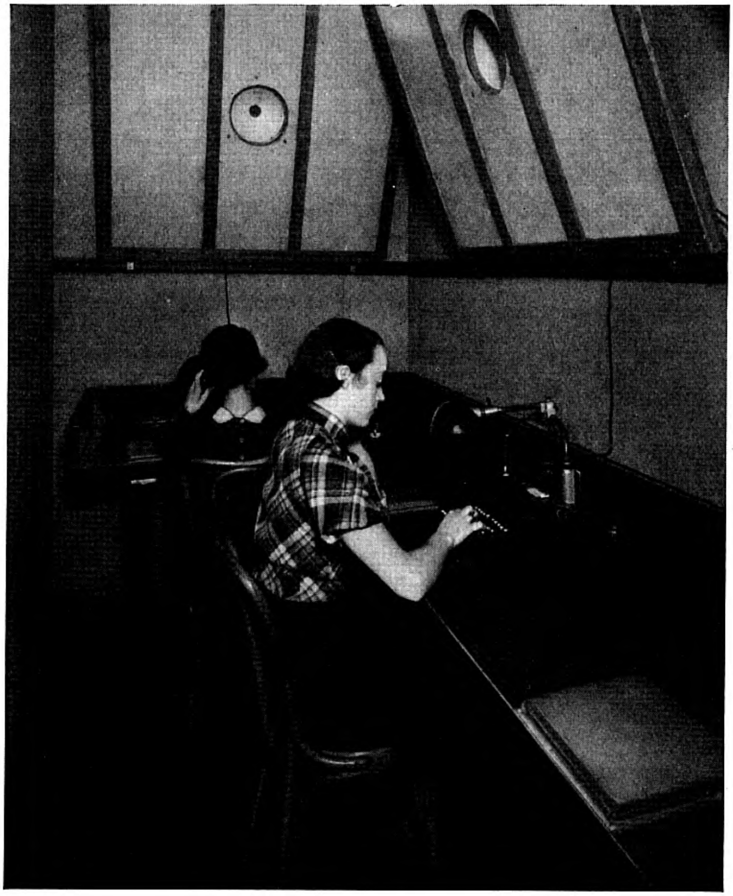

Fig. 2—Interior of observing booth.

Another important benefit is the control provided over the inevitable changes in the skill of the testing crew. Even the most experienced crew is likely to show a quick growth in proficiency in the early stages of testing a strange circuit, or may display a temporary loss of skill when returned after some time to a circuit previously familiar. During these stages the data ought usually to be discarded. The number of practice runs naturally depends, however, on the circuits being tested. Unless the engineer in charge receives the data promptly, the number of tests made may be insufficient or more than necessary. In either case the successful completion of the program is delayed.

The equipment used to analyze the data will be described in some detail later in this paper, but its operating principles briefly are as follows: A perforated tape, which is used with a standard printing telegraph tape sender, performs, under the control of the master timing commutator, two functions. It causes the syllable to be called to appear visually before the caller (Fig. 1) at regular intervals and it controls a relay system associated with four keyboards provided for the observers (Fig. 2). The observers, on hearing the syllable, press successively the keys labelled with the sounds which they believe were called. If the correct keys are pressed a certain set of relays operates; if the wrong keys are pressed another set operates. The operation of the relays in turn controls a standard page printer which types in succession the number of errors made on each sound.

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    100-310-321-111-300-401-042-111-201-112-410-010-300-000-001-110-401-000-
    303-311-000-010-200-100-003-313-200-103-001-101-320-001-102-303-013-402-
    302-003-001-102-102-400-001-010-304-110-112-101-200-130-000-310-202-103-
    301-301-411-000-002-001-201-211-104-001-402-011-
MCD 450 B18-1-50-2-50-3-60-4-46 T3 N46.0 S70.1
    403-400-204-310-404-111-000-401-101-321-101-203-323-203-004-200-204-112-
    001-101-310-002-332-221-402-001-312-111-403-301-203-213-401-001-410-443-
    303-403-422-001-202-002-102-400-304-422-200-104-313-433-301-311-410-104-
    201-324-440-321-300-100-304-301-001-413-004-320-
MCD 451 A19-1-83-2-67-3-93-4-74 T3 N44.1 S58.8
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Fig. 3—Record of articulation test as made by page printer.

A typical typed record may be seen in Fig. 3. Each set of three figures refers to a syllable. The first digit of the first number, 100, shows that on the first syllable called in this list one observer mistook the initial consonant, while the other three observers recorded it correctly. The second and third digits indicate that no errors were made on the two succeeding sounds. The third number, 321, shows that three observers missed the initial consonant, two missed the vowel and one missed the final consonant.

Under the control of the timing commutator this procedure of flashing a syllable which is spoken by the caller, heard and acted upon by the observers, whose opinion is analyzed and recorded upon the page printer, goes on until the list of 66 syllables has been called. Immediately afterward the mechanical apparatus, through interconnection of the tape sender and other relay circuits, proceeds to type out on the page printer a summary of the test results, which may also be seen in Fig. 3, in the fifth line.

This line of the record may be translated as follows: The list number is 450. The circuit condition is B. The next number, 18, indicates the number of lists which have been called since the beginning of the group of tests. Observer No. 1 made 50 errors, observer No. 2 also made 50 errors, No. 3 made 60 and No. 4 made 46. The transmitters used at each end are identified by the entry T3. The average magnitude of the noise at the receiver terminals was 46.0 db; that of the speech was 70.1 db, in each case above an arbitrary reference value. The initials of the caller are added afterwards in ink. The printed data in the next 5 lines refer to the comparison circuit over which the same caller immediately called the next list, which for convenience is on the same strip of perforated tape.

Some specific figures concerning the advantages of mechanical analysis may be of interest. When the observers make written records which the crew itself analyzes, a usual rate of testing (using eight callers and four observers) is about one comparison of two circuit conditions per day. This rate can be greatly exceeded in a short series covering a few conditions, but for long programs of tests involving many variable factors this is a representative figure. Using mechanical analysis, the rate of testing can easily be made to cover two such comparisons a day. This includes a liberal allowance for time used in circuit maintenance and clerical work. Mechanical analysis, then, permits, in a given number of weeks, a program with at least twice the number of test conditions; or, a more usual disposition of this time, the entire program can be repeated at least once in the same number of weeks, and the tests on the basic circuit conditions several times.

It is also interesting to note that the use of the mechanical recording system has had several beneficial effects on the members of the testing crew. Not only do they find the work less fatiguing than when written records were used but their ability to see the results of their observations immediately after the calling of a list has noticeably increased their interest in doing a good job.

Apparatus for Control and Analysis

The Control Board

A control board, which is shown at the left in Fig. 4, permits the engineer in charge easily and quickly to supervise the testing. By a master switch on the control board the circuit conditions may be switched rapidly from one to another to be compared with it. The conditions changed by the master switch depend on the previous set-

Fig. 4—The control board and page printer.

ting of other keys which select the individual circuit elements. By these keys telephone sets of various types, which are installed on racks, may be selected. Likewise, trunks with different losses and cutoff frequencies are mounted on racks and are accessible through keys. The magnitude of the room noise is determined by two attenuators, one for each of the two circuits compared, which are adjusted before the test begins and are then controlled by the master switch. Quick change from one magnitude of line noise to another is managed in the same way. Likewise, the setting of the automatic volume indicator in the caller's control circuit is under the control of the master switch so that different calling intensities can be used on successive conditions.

Before each list is called the operation of the switching apparatus and the circuit elements are checked rapidly at the control board by the application of a test tone to the transmitter terminals at one end of the circuit. A volume indicator, connected across the receiver terminals at the other end of the circuit, shows by the deflection on its meter, which may be seen in Fig. 4, whether or not the power received has a specified value. The same volume indicator is used to check the magnitudes of the room noise and line noise.

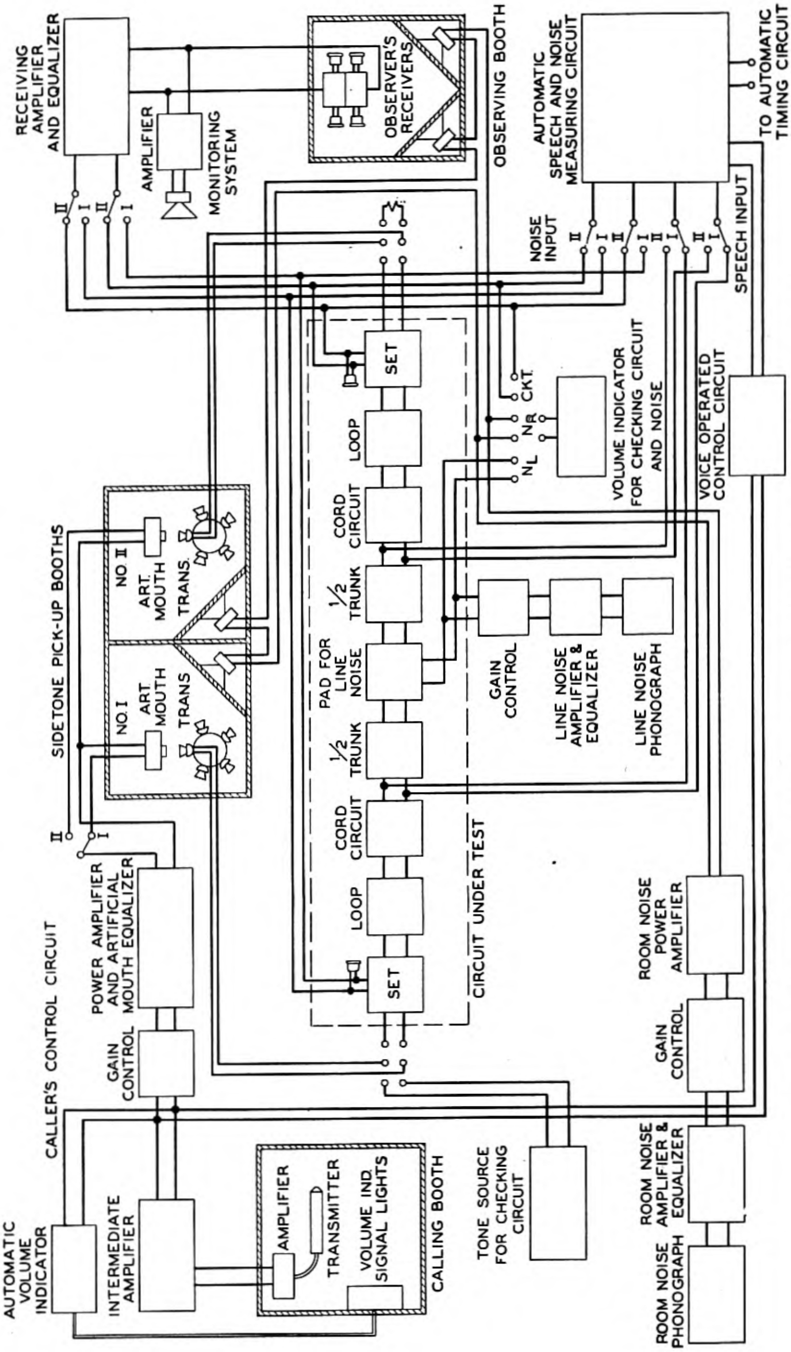

Fig. 5—Schematic diagram of the transmission circuits used in articulation testing.

As shown at the right in Fig. 4, the page printer of the data analyzing equipment is located near the control board, so that the recording may be followed as the list is called. As a further check on the operation of the circuit a loudspeaker is mounted nearby. This is bridged across the amplifier of the observer's circuit and permits an aural check on the received speech and noise. The dial controlling the changing and agitation of the transmitters is also located on the control board. The push buttons shown are used for summoning the callers and observers.

Caller's Control Circuit

The caller's control circuit is shown schematically in Fig. 5 and in greater detail in Fig. 6. A small condenser transmitter [3] is used for

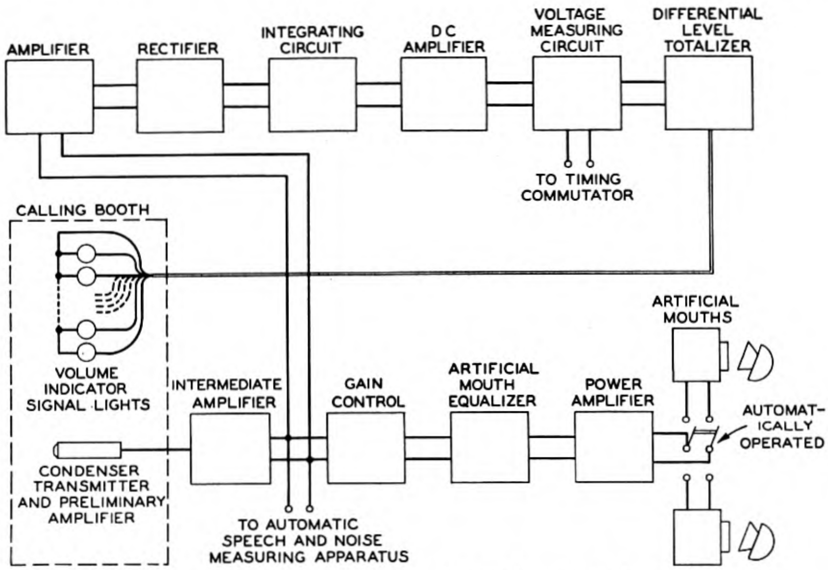

Fig. 6—Schematic diagram of caller's control circuit.

picking up the speech of the caller. This, with its directly associated preliminary amplifier, is located in the calling booth. The electrical output passes through an intermediate amplifier, a gain control, a power amplifier and an equalizer into one of a pair of artificial mouths. One of the artificial mouths is shown mounted in front of a transmitter under test in Fig. 7. The characteristics of the artificial mouth and associated equipment are fully described in the paper referred to previously.

[3] "An Efficient Miniature Condenser Microphone System," H. C. Harrison and P. B. Flanders, *B. S. T. J.*, XI, p. 451, July, 1932.

The automatic volume indicator is connected across the output of the intermediate amplifier of the caller's control circuit. This is a high level point in the circuit at which the form of the electrical wave is essentially a duplicate of that of the acoustic speech wave which produces it.

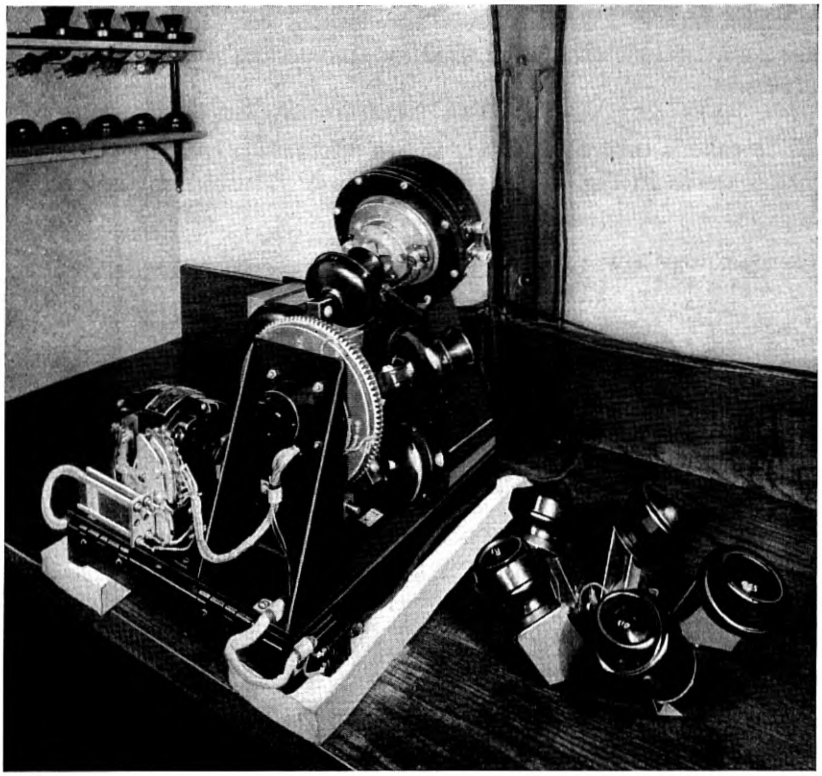

Fig. 7—Interior of sidetone booth—showing artificial mouth and transmitter changer and agitator.

The device works as follows: The speech voltage applied across the input is amplified, rectified, and applied to an integrating circuit, the output of which, after further amplification, is applied to a voltage measuring circuit. The maximum value of this voltage for each sentence is measured in terms of the number of 2 db steps by which it exceeds an arbitrary minimum, a relay corresponding to each step being operated through contacts made by the master timing commutator on the automatic system for recording and analyzing the data. The relay system is arranged so that if more than a predetermined number are operated an electromagnetically stepped selector switch is actuated. If less than this number of relays are operated another selector switch is actuated. If exactly the right number of relays are operated neither selector switch is actuated. These selector switches control the moving parts of a differential switch, which consists essentially of a rotating set of contact points and a rotating contact arm which may move over these contact points, the direction of rotation being the same for both. Each contact point is connected to one of the signal lamps in the calling booth, the lamp circuit being completed through the contact arm to a battery. At the beginning of a list the contact arm rests on the contact which illuminates the center lamp. If a test sentence is called at too high an intensity the contact arm moves over a number of contact points governed by the number of relays operated in excess of the desired number, illuminating the coordinated lamp. If the following sentence is called at the proper intensity neither selector moves and, therefore, the same lamp remains illuminated. If, on the next sentence, the calling intensity is low, less than the specified number of relays are operated; consequently, the other selector switch moves the contact points the required number of steps while the contact arm remains stationary. This, in effect, moves the contact arm backward to the contact point corresponding to the algebraic sum of the deviations from the desired value up to this point in the list and this is shown in the calling booth by a movement of the light to the left. In a similar way the succeeding deviations from the average cause the two parts of the differential switch to move in such relation to each other that the signal light indicates the cumulative departure from the desired average value.

Reversing the Direction of Transmission

The use of the caller's control circuit and an automatic switching system permits reversal of the direction of transmission over the circuits tested without loss of time. In addition to soundproof booths for the caller and the observers, two others are used. These two booths, which are in all respects identical, are referred to as sidetone pickup booths. Each contains an artificial mouth, a number of transmitters which may be connected to the circuit tested, and a loudspeaker which reproduces room noise. These booths are shown schematically in Fig. 5 and a photograph of the interior of one is shown in Fig. 7.

The timing commutator, which governs the other automatic devices used, controls switches which make the following connections (see Fig. 5): The artificial mouth in Booth I is connected to the caller's control circuit, the other artificial mouth is disconnected, and the observer's amplifier is connected to the receiver at the same end of the circuit as the transmitter in Booth II. Thus, when a sentence is called it is picked up by the transmitter in Booth I while the transmitter in Booth II is picking up room noise. At the end of the time allotted to the sentence and the various measuring and recording operations the automatic switches reverse the circuit simply by transferring the caller's control circuit to the artificial mouth in Booth II and the observer's amplifier to the receiver at the other end of the circuit. This reversal is repeated after each of the 66 sentences.

Automatic Change and Agitation of Transmitters

Two motor-driven devices serve to change the transmitters used in testing, agitate them in a uniform way and then center them properly in front of the artificial mouths. This apparatus is under the control of a dial at the control board so that it is unnecessary to enter the sidetone booths to make the changes. The apparatus in the sidetone booth is shown in Fig. 7.

The transmitters to be used at each end of the circuit are mounted on circular plates, which may be removed as units from the rotating devices. An unmounted disk holding a set of transmitters is shown beside the rotating device in the photograph. When direct comparisons between two different types of transmitters are to be made, two transmitters of one type and two of the other are mounted on each disk.

Just before the calling of a list is started the engineer at the control board manipulates the dial, which is merely a modified telephone set dial, by which the transmitters to be used are selected. Following the dial pulses a series of relays causes the transmitters in both sidetone pickup booths to rotate through an angle of not less than 360 degrees, which has been found to supply adequate agitation. After this the rotation continues until the desired transmitters are exactly in front of the artificial mouths. In directly comparing two circuits which make use of the same type of transmitters, the same transmitters are used for two successive lists, but the rotating agitation is applied between lists.

Automatic Measurement of Received Speech and Noise

The apparatus used for the automatic measurement of speech and noise on the circuit under test is similar, except for the input circuit and recording circuit, to the automatic volume indicator just described, but is designed to handle a larger range in volume. A peak voltage measuring circuit is used alternately to measure the rectified voltage resulting from the speech energy and that from the noise. The voltages to be measured are arranged to be negative. The measurement, in principle, is a determination of the amount of positive voltage which must be added to the negative voltage so that the net voltage applied to the grid of a "trigger tube" reaches the operating point of the tube. The positive voltage is obtained from a rotary potentiometer which is driven by a synchronous motor through a magnetic clutch (which is required to start the apparatus in synchronism with the data recording and analyzing equipment). It makes one complete revolution for a measurement of speech and another for the measurement of noise.

The fraction of a complete revolution which the potentiometer makes between the start of a cycle and the time at which the "trigger circuit" operates indicates the magnitude of the rectified voltage being measured and is arranged to be directly proportional to the number of db that the magnitude of the speech or noise is above some previously chosen reference value. The sum of these fractional rotations for the 66 sentences of a list gives a measure of the average speech magnitude during the calling of a list. This sum is obtained mechanically by a totalizing device which is essentially a revolution counter coupled to the shaft of the rotary potentiometer through a magnetic clutch. As each of the 66 sentences is called the totalizer moves ahead. Its final reading shows the average volume for the whole list. Exactly the same operations take place to give the average noise volume throughout the calling of a list.

The switching of the apparatus from the condition in which it is set up to measure the speech magnitudes to that for the measurement of noise magnitudes is under the control of timing contacts on the shaft of the motor-driven potentiometer. However, since callers may occasionally fail to finish calling a sentence in the allotted time and as a result deliver some speech energy during the time when noise only is supposed to be measured, an auxiliary voice operated control is required to keep the equipment from starting a noise measurement until the speech has stopped. The auxiliary control is operated by the voltage produced by the speech wave in the caller's control circuit at the point where the automatic volume indicator is connected.

Observers' Circuit

A special circuit, shown in Fig. 5, is needed at the receiving end to enable four observers to work at the same time. An amplifier is necessary to preserve the proper relationships between the magnitudes of speech, sidetone noise, and room noise leaking under the receiver cap. This amplifier, which gives a gain of 6 db (offsetting the loss of the receivers in series-parallel), has a high input impedance, so that it may be bridged across the receivers used in the telephone sets.

Phonographic Sources of Noise

Both the room noise and line noise phonographs are controlled by the automatic data recording system, so that the reproducers are automatically set down on the records at the beginning of each list and lifted and returned to the starting position at the end. As may be seen from Fig. 5, the output of the room noise phonograph, after passing through various controls, is reproduced by loudspeakers, of which four are located in the observing booth, and one in each of the sidetone booths. In this way the receiving end transmitters and the observers are both exposed to the same noise. Because of the highly absorptive character of the walls of the testing booth it is necessary to use equalizing networks in the reproducing amplifier in order to insure that the frequency distribution of the reproduced noise is that desired. Throughout the test the electrical volume supplied to the loudspeakers may be checked by a volume indicator located at the control board.

The output of the line noise phonograph is applied to the circuit tested through a high impedance bridging coil or through a resistance network ordinarily at the middle of the trunk. Line noise magnitudes are adjusted to the desired value with the help of a circuit noise meter and are then continually checked throughout a test by means of the control board volume indicator.

Automatic Data Analyzer

Two different systems of automatic analyzing equipment have been designed and used. The first, on which active development work was initiated in 1930, was used in the routine laboratory testing of commercial telephone circuits from 1931 to the early part of 1933. The present system, simpler in design and embodying a large number of improvements, was then installed and has been used since that time.[4]

As pointed out before, the present data handling machine embodies a number of the parts and operating principles of standard printing telegraph systems. The testing lists are previously prepared in the form of perforated tapes,[5] of which a large number are available.

[4] Another type of analyzing equipment for articulation testing has been described by J. Collard, *Electrical Communication*, X, p. 140, January, 1932.

[5] In making up such tapes it is necessary to apply a code to the various articulation sounds since the keyboard of the tape perforator contains only the standard English alphabet. Each syllable appears on the tape as three consecutive sets of perforations, one for each sound. Additional perforations are used in some portions of the tape to control various functions of the automatic recording apparatus. Two lists of 66 syllables are recorded on each separate tape to make possible comparison tests on two different systems with a minimum of delay.

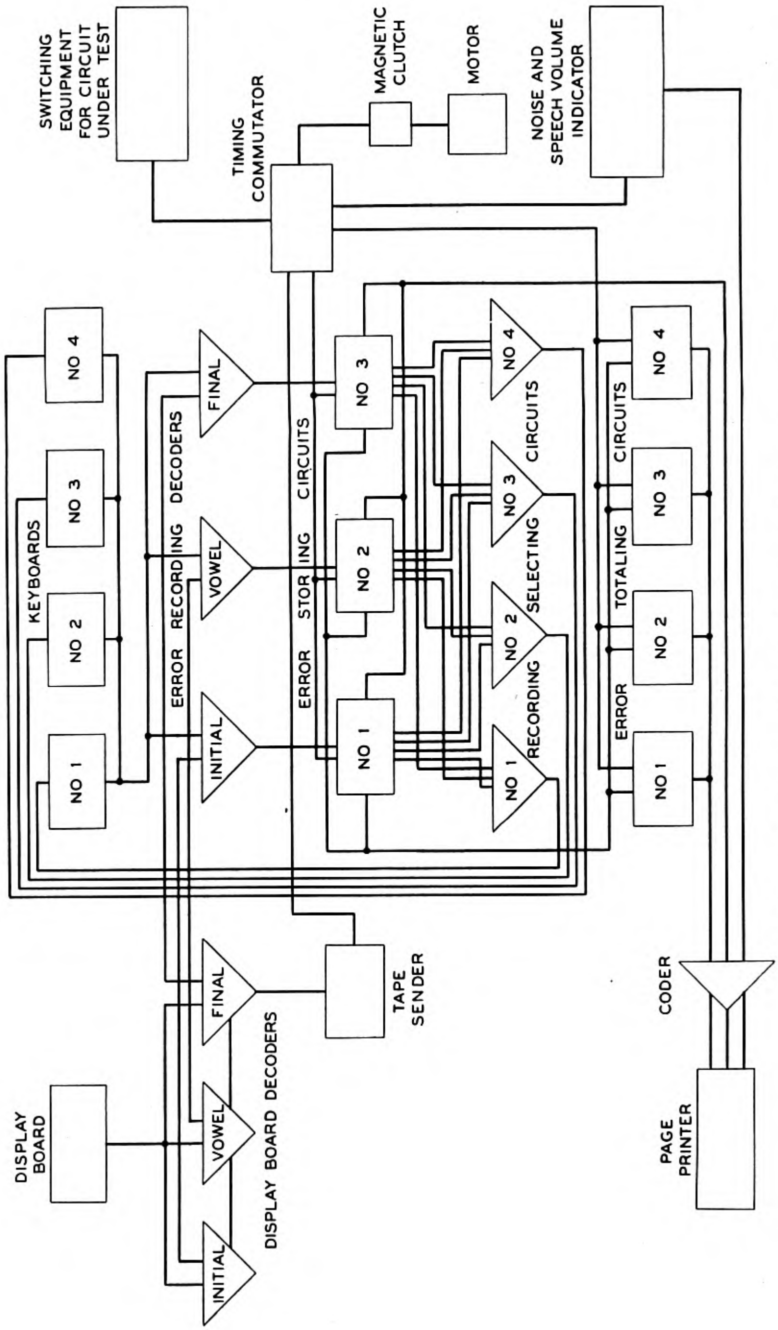

Fig. 8—Schematic diagram of automatic data analyzing apparatus.

These are used with a standard tape sender to convert the tape record to suitable electrical impulses. The use of tape gives flexibility to the testing method since it is equally adaptable to syllable lists of any desired type and length. It can also be easily arranged for synchronous operation with a phonograph calling system.

The operation of the automatic data handling apparatus may be followed on the schematic diagram of Fig. 8. After a tape has been threaded in the tape sender the caller starts the machine by pressing a button. This causes a magnetic clutch to engage, which couples a synchronous motor to the timing commutator, starts the speech and noise volume measuring apparatus, resets the automatic volume indicator and the other counting circuits, puts the room noise and line noise phonographs into operation and signals the observers that a list is about to be called.

The timing commutator then causes the tape sender to advance three steps and set up the first group of three duplex sets of decoding relays, one set for the initial consonants, one set for the vowels and one set for the final consonants. This group of decoding relays connects the proper lamps to illuminate the syllable to be called on the screen in front of the caller, the bank of lamps used for this purpose being covered by a mask on which the various letters are printed. The length of the flash is controlled in order to guide the caller in his rate of calling. The translucent screen, with a syllable set up, may be seen in Fig. 1.

The caller now calls the first of the carrier sentences with the test syllable inserted in the middle, starting to call as the syllable is flashed on the screen and finishing when it goes off. The automatic volume indicator simultaneously measures the volume of the sentences called and soon after signal lamps indicate to the caller by how much he has deviated from the prescribed volume. At the same time the automatic speech volume recorder measures the speech volume on the circuit under test. In the meantime the second group of decoding relays has been set up and the observers are signaled by a light in front of each that it is time to record what they think they have heard.

The equipment immediately in front of the observers is a set of keyboards, one for each observer, which may be seen in Fig. 2. Each keyboard is provided with a key for each of the speech sounds used. On hearing the test syllable and receiving the signal the observers operate in succession three keys corresponding to the three sounds of the syllable as they understand them. The keyboards are so arranged by interconnection with the decoding relays that if the proper keys are pressed short circuits are applied to prevent the operation of a system of error counting relays, but if the wrong keys are pressed the related relays act to indicate the errors. There are twelve of these relays, giving a separate count of each observer's errors on each of the three sounds of the test syllable.

The relay circuits are arranged so that the same keys are used for both initial and final consonants and also so that the observers are prevented from getting more than one opportunity to record on each sound.

While the observers are recording, various other automatic operations are going on under the control of the timing commutator. The automatic noise measuring apparatus measures the amount of noise delivered to the observers' receivers. Shortly afterwards the telephone system under test is switched so that the next sentence to be called will go over it in the opposite direction from the first. Also during the recording period, the first group of decoding relays is knocked down and set up again as the tape moves forward to flash the next syllable to the caller.

Immediately after the recording period, a set of error totaling circuits counts the total number of errors made by each observer. These circuits operate cumulatively, that is, errors on the next syllable will be added to these, and so on until the end of the list. Another set of error totaling circuits, however, acts at once to cause the page printer to type the total number of errors made on each sound of the syllable by all of the observers. After this the second group of decoding relays is knocked down. By this time the caller has finished calling the second sentence and the cycle is repeated.

These operations continue for 66 syllables. At the conclusion of the cycle covering the last syllable the page printer under the joint control of the paper tape and the timing commutator, as was pointed out before, records the number of the list, the serial number of the calling, a code letter A or B to denote to which of two circuits being compared the data pertain, the total errors made by each observer on all sounds, and the average values of speech and noise as measured at the observers' receivers by the automatic measuring apparatus.

Mechanical Formulation of Testing Lists

The first automatic analyzer, although no longer in use, was distinguished by a special feature which has aroused some interest and will therefore be described.

The machine was arranged to make up the lists automatically as they were needed and also to censor the list automatically before using it.

This was provided for by a system of selectors (of types used in dial telephone apparatus) and relays. Sets of contacts corresponding to the appropriate lamps on the translucent screen in front of the caller, that is, to the 22 initial consonants, 22 final consonants, and the 11 vowels (in duplicate), were arranged so that the order in which they were swept over was varied mechanically in a random way. The mechanical rearrangement of the contacts, which prepared a group of 22 syllables, required about two seconds. Of this time, only 0.3 second was needed to set up a new group. The remaining 1.7 seconds were used by the machine in checking over the group to see that the syllables were satisfactory. The need for this is plain. Certain combinations of sounds, being impossible to pronounce, must be rejected, and certain others must be eliminated, as having undesirable meanings in English. Additional relay systems were provided so that during a rapid preliminary run the presence of such undesired combinations or syllables would cause the group to be rejected automatically, that is, the machine would be returned to its normal position and a new group would be set up. The checking process was repeated until a suitable list was obtained.

The apparatus used by the callers and observers with the first machine is the same as that used with the present equipment. The final record differed in being a photograph of a bank of 60 message registers (electromechanical counters), which classified the errors by individual sounds in addition to showing the total number of sound errors made by each observer. This bank of message registers was photographed automatically at the beginning and at the end of the calling of a list of 66 syllables. At the end of a test the photographs were developed, washed and dried by other mechanical apparatus, the final record appearing in about two minutes.

While this equipment demonstrated effectively the value of rapid mechanical analysis of the data, it was felt desirable to simplify it and extend its usefulness by adding certain other features. This seems to be satisfactorily attained by the present machine, which not only is simpler to operate and maintain, but offers as well greater flexibility in the lists which may be used and in its adaptability to phonographic calling.

Abstracts of Technical Articles from Bell System Sources

Theory of the Detection of Two Modulated Waves by a Linear Rectifier.[1] CHARLES B. AIKEN. In this paper there is developed a mathematical analysis of the detection, by a linear rectifier, of two modulated waves. Solutions are obtained which are manageable over wide ranges of values of carrier ratio and degrees of modulation. These solutions are of greater applicability and are more convenient than those previously obtained, and give a full treatment of the action of an ideal linear rectifier under the action of two modulated waves.

The development is first made in terms of the derivatives of zonal harmonics of an angle which is directly related to the phase difference between the carriers. As these derivatives are tabulated functions the solution is convenient.

The solutions are limited by the condition that $K < (1 - M)/(1 + m)$, K being the carrier ratio, M the degree of modulation of the stronger carrier, and m that of the weaker. Two methods of attack are developed, one of which is applicable when K is small and M and m large, and the other when M and m are small and K large.

The cases of identical and of different programs are both considered and a number of curves are given showing the magnitudes of various output frequency components under typical operating conditions.

In the latter part of the paper the phase angle between the carriers is set equal to μt so that a beat note exists. There is then considered the effect of a noise background on the reception of signals on shared channels, and it is shown that much less "flutter" effect and much less distortion of the desired signal will result from the use of a linear rectifier than from the use of a square-law rectifier under the same conditions.

Finally, brief consideration is given to heterodyne detection and to "masking" effects.

Thermionic and Adsorption Characteristics of Thorium on Tungsten.[2] WALTER H. BRATTAIN and JOSEPH A. BECKER. Variation of thermionic emission of tungsten with surface density of adsorbed thorium.—Thorium was deposited on a tungsten ribbon by evaporation from a thorium wire. A study was made of the dependence of the thermionic

[1] *Proc. I. R. E.*, April, 1933.
[2] *Phys. Rev.*, March 15, 1933.

emission on the two parameters: T, the temperature, and f, a quantity which is proportional to the amount of thorium on the tungsten surface. At a fixed temperature 1274° K it was found that as the amount of thorium on the tungsten surface was increased, the thermionic emission increased to a maximum, then decreased, and asymptotically approached a constant value. For the maximum, f is defined to be 1.0. The maximum value and the final constant value of the emission current were respectively 5.7×10^5 and 5.7×10^4 times the value of emission current characteristic of clean tungsten. Moreover the final constant value of the emission agreed to within a factor of 2 with the value characteristic of clean thorium. From $f = 0.0$ to $f = 0.8$ the relation between the emission current and f satisfied the following empirical equation

$$\log_{10} i = -3.14 - 6.54\epsilon^{-2.38f},$$

where i is the emission current in amperes per cm². For $0.8 < f < 2.0$, the values of emission currents are tabulated. For any fixed f, the emission obeys Richardson's equation. All the Richardson lines for $0 < f < 1$ intersect in a common point at an extrapolated temperature of 12,500° K, and for $f \geq 1$ the lines intersect in a common point for which the temperature is 3250° K. These results obtained by depositing thorium on a tungsten ribbon have been compared with results obtained from thoriated tungsten wire. Thoriated tungsten wire can be activated by diffusion of thorium from the interior to the surface. For a while every atom that diffuses to the surface sticks to it so that f increases linearly with the time; later when evaporation is no longer negligible the rate of accumulation, df/dt, gets less and less; a steady state is reached when the diffusion rate equals the evaporation rate. It is unnecessary to assume "induced evaporation" to explain these results.

Variation of emission from thoriated tungsten with applied field.—It was found that for both the ribbon and the thoriated tungsten wire the dependence of emission on applied field changed as f was varied. For the thoriated tungsten wire the dependence of the thermionic constants A and b on applied field was most pronounced for $0.3 < f < 0.6$.

Evaporation and migration of thorium on tungsten surface.—Evaporation and migration of thorium on the tungsten surface were studied. The evaporation rate depends on the temperature and the fraction of the surface covered (f). For $0.2 < f < 1.0$ the rate of evaporation is approximately an exponential function of f. At 2200° K and $f = 0.2$ the rate of evaporation was 10^{-4} layers/sec. and at $f = 0.8$ was 31×10^{-4} layers/sec. It was found that thorium could be deposited on one side of the tungsten ribbon and then made to migrate to the other side of the ribbon. This migration occurred at an appreciable rate above 1500° K and was not complicated by evaporation up to 1655° K. It was found that the migration coefficient depended on f as well as on T. For a given set of conditions an approximate value of the heat of migration was calculated to be 110,000 calories per mol.

Diffraction of Electrons by Metal Surfaces.[3] L. H. GERMER. Fast electrons scattered from polished metal surfaces do not form diffraction patterns. A strong Debye-Scherrer pattern is produced, however, by electrons scattered from a surface which has been mechanically roughened in such a manner that electrons are able to pass directly through projecting irregularities. Small ridges extending from wires, which have been drawn through an imperfect die, also give rise to a diffraction pattern. These experiments indicate: (1) that there is no considerable layer of amorphous material (Beilby layer) on a polished metal surface, and (2) that Debye-Scherrer diffraction patterns are formed only by transmitted electrons. Fast electrons scattered at a small glancing angle from an etched polycrystalline surface form a diffraction pattern if the surface appears mat or roughened, but no pattern is formed if the surface shows metallic luster. Here again diffraction patterns appear to be produced only by transmission. A probable explanation is given for the fact that diffraction rings are not formed by electrons scattered from smooth polycrystalline surfaces.

Perfect Transmission and Reproduction of Symphonic Music in Auditory Perspective.[4] F. B. JEWETT, W. B. SNOW and H. S. HAMILTON. The demonstration in Constitution Hall, Washington, on April 27th, of the perfect transmission and reproduction in full auditory perspective of a symphony concert produced in Philadelphia by the Philadelphia Orchestra and transmitted to Washington over underground telephone wires, marked the completion of several years' work by the research and engineering forces of the American Telephone and Telegraph Company and Bell Telephone Laboratories.

In this paper is a foreword by Dr. Jewett. The features of the demonstration and some description of the equipment are presented by Mr. Snow. Mr. Hamilton discusses some of the details of the complex line circuits used in the electrical transmission of the music.

[3] *Phys. Rev.*, May 1, 1933.
[4] *Bell Telephone Quarterly*, July, 1933.

A New Reverberation Time Formula.[5] W. J. Sette. The earliest work relating decay of sound in an auditorium and the acoustic absorption of the surfaces was done by W. C. Sabine who developed the formula which has recently been shown to be applicable to only "live" rooms. More recently Fokker in Holland, Schuster and Waetzmann in Germany and Eyring in this country derived an expression to hold for "dead" rooms also. The assumption of continuous absorption at the auditorium boundaries made in the Sabine formula was replaced by the conception of intermittent absorption, which is more in accord with actual conditions of decay.

Both of these formulae presuppose in their derivation uniform distribution of energy at each incidence, although Eyring observed that ordered states would necessitate assigning proper weights in computing the average surface absorption. The new formula is based on a similar assumption, but shifts the point of view to another kind of uniform distribution. Instead of each surface receiving a proportional share of the total energy in the room at each reflection, it is assumed that any ray of sound, after repeated reflection will have struck any one surface in proportion to the ratio of the area of that surface to the total room surface. This formulation of the process of decay leads to an alternative reverberation equation and some further extension of reverberation theory. The new equation is, of course, necessarily specialized and limited to those instances where the fundamental assumptions are fulfilled, as is brought out in the body of the paper.

[5] *Jour. Acous. Soc. Am.*, January, 1933.

Contributors to this Issue

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